Gains from trade in a Hotelling model of differentiated duopoly

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Abstract

Constructing a two-country Hotelling model of spatial duopoly, this paper explores welfare effects of bilateral reductions in transport costs. We show that welfare in trade is necessarily less than welfare in autarky for any level of trade cost, which sharply contrasts Clarke and Collie (2003) prove Pareto superiority of *any* trade over autarky in a non-address model. We discuss that gainfulness of trade is crucially affected by whether demand is elastic.

Keywords: F10, F12.

JEL classification: gains/losses from trade, duopoly, Hotelling model.

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1 Introduction

Is trade liberalization beneficial? This is one of the central topics in international economics and an affirmative answer is provided by the gains-from-trade proposition.[1] While the gains-from-trade proposition commonly assumes that a country moves from autarky to free trade, 'trade liberalization is typically progressive $\cdots$, it would seem more interesting to consider freer trade from an equilibrium in which exchanges already occur' (Schmitt, 1990, p. 655). This motivation has generated a large literature on trade liberalization under oligopoly and market segmentation.

Brander (1981) and Brander and Krugman (1983) are the first to examine welfare effects of trade liberalization in a homogeneous product Cournot model.[2] Their noteworthy conclusion is that freer trade can harm welfare if the initial transport cost is too high. This is because the negative effects of expanding wasteful resources through transport cost reductions and of profit-shifting from the domestic firm to the foreign firm both dominate the positive effect of promoting competition. However, trade liberalization benefits welfare if the initial level of transport cost is low enough. To sum, welfare depends on the transport cost in a U-shaped fashion and free trade (zero transport cost) Pareto dominates autarky.[3]

To our knowledge, Clarke and Collie (2003) are the first to incorporate differentiated Bertrand duopoly to reconsider gains from trade liberalization. Assuming a representative consumer with a quadratic utility function, they prove that welfare in trade is larger than welfare in autarky for *any* transport cost, i.e., the initial level of transport cost no longer matters and any trade

[1]See Kemp and Wan (1972) and Grandmont and McFadden (1972) for the gains-from-trade proposition in an Arrow-Debreu model. The proposition is extended to accommodate Cournot-Nash oligopoly and increasing returns to scale by Kemp and Shimomura (2001).

[2]See also Helpman and Krugman (1985, pp. 104-110).

[3]While the papers cited above commonly assume segmented markets, Markusen (1981) and Lahiri and Ono (1995) prove gains from trade in integrated markets.

improves welfare as compared to autarky.[4] The main driving force in their positive evaluation of trade liberalization is product differentiation. In other words, the consumer enjoys variety expansion by freer trade, which, together with increasing competition, overweighs the negative welfare effects.

This paper reconsiders Clarke and Collie's (2003) result in Hotelling's (1929) spatial model of differentiated Bertrand duopoly, which is another benchmark model in industrial organization. Our main motivation is the same as Schmitt's (1990, p. 656) who states 'the address approach captures very realistic consumer's and producer's features.' Within this framework, we prove that welfare in trade is necessarily smaller than welfare under autarky for any transport cost, i.e., any trade is Pareto inferior to autarky. This sharp contrast between Clarke and Collie (2003) and this paper is mainly driven by whether demand is elastic or not. Thus, one should be careful in assessing trade liberalization even if differentiated Bertrand models are commonly adopted.

By comparing free trade and autarky, Eaton and Kierzkowski (1984) point out a possibility of trade losses. Schmitt (1990), whose motivation and interest are closest to ours, considers the effects of tariff reductions in a spatial model with free entry. One of the novel results in Schmitt (1990) is that welfare depends on tariffs in an inverted-U way. Therefore, tariff reductions can lower welfare if the initial level of tariff is lower than the welfare-maximizing one.[5] This paper is differentiated from these papers in the following respects. First, we allow for arbitrary levels of trade barriers while Eaton and Kierzkovski (1984) focuses on the comparison between free trade and no trade. Second, we assume that trade barriers take a form of transport costs rather than tariffs.[6] This assumption allows us to facilitate the comparison

[4]Figure 3 diagrammatically summarizes their result.

[5]A related paper, Schmitt (1995) pays attention to implications of location choices for the international product configuration whereas Schmitt (1990) focuses on the situation where 'firms cannot change the geographical location' (p. 658).

[6]The case of tariffs is addressed in a companion paper, Fujiwara (2009).

our result with that of Brander and Krugman (1983) and Clarke and Collie (2003). Third, we assume neither location choice nor free entry.

This paper is organized as follows. Section 2 presents a model and derives an autarkic equilibrium. Extending the model to two symmetric countries, Section 3 characterizes the Bertrand-Nash equilibrium. Based on the results in these sections, Section 4 considers how welfare depends on transport costs and compares it with welfare under autarky. Section 5 concludes the paper. Appendix 1 derives the reaction curves of each duopolistic firm and Appendix 2 proves our losses-from-trade proposition.

2 A model: an autarkic equilibrium

Consider a linear model of horizontal product differentiation. There is a unit mass of consumers which is uniformly distributed in $[0, 1]$. Without loss of generality, the Home monopolistic firm is located at point 0.[7] Thus, the consumer surplus of consumer $x \in [0, 1]$ is $a - p - \tau x$, where $a > 0$ is intrinsic utility, p the price of the monopolized good and $\tau > 0$ a per-unit transport cost. Since any consumer x with $a - p - \tau x \geq 0$ is willing to purchase the good, the resulting demand is determined when $a - p - \tau x = 0$ holds from which $x = (a - p)/\tau$.

Assuming a constant marginal cost $c \leq a$ of the monopolistic firm, its profit under autarky is defined by $(p - c)x = (p - c)(a - p)/\tau$. Hence, the profit-maximizing price is determined as $p^A = (a + c)/2$, where superscript A indicates an autarkic equilibrium. Substituting this into the demand function, the equilibrium supply under autarky is $x^A = (a - c)/2\tau$ and the autarkic equilibrium welfare W^A becomes

$$\begin{aligned} W^A &= CS^A + \pi^A \\ &= \int_0^{x^A} \left(a - p^A - \tau x\right) dx + \left(p^A - c\right) x^A \end{aligned}$$

[7] Each firm's optimal choice of location is assumed away.

$$= \frac{(a-c)^2}{8\tau} + \frac{(a-c)^2}{4\tau} = \frac{3(a-c)^2}{8\tau}, \quad (1)$$

where CS and π denote the consumer surplus and the monopolistic firm's profit, respectively.

3 Trade equilibria

This section extends the model to duopoly by the Home and Foreign firms.[8] After the opening of trade, the market is duopolized by the Home firm located at point 0 and the Foreign firm located at point 1 so that the model is of the Hotelling type. Denoting p^* be the Foreign firm's price, the consumer who is indifferent between the Home and Foreign products is determined by $a - p - \tau x = a - p^* - \tau(1-x)$. Solving this equation for x, the demand of each good is

$$x = \frac{p^* - p + \tau}{2\tau}, \quad x^* = 1 - x = \frac{p - p^* + \tau}{2\tau}.$$

Let us assume that exporting is costly and $t \geq 0$ denote a per-unit transport cost. Then, each firm's profit is defined by

$$\begin{aligned}\pi &= (p-c)x = \frac{(p-c)(p^*-p+\tau)}{2\tau} \\ \pi^* &= (p^*-c-t)x^* = \frac{(p^*-c-t)(p-p^*+\tau)}{2\tau}.\end{aligned}$$

If both firms are active in equilibrium, the Bertrand-Nash equilibrium prices are determined through $\partial\pi/\partial p = \partial\pi^*/\partial p^* = 0$ as follows.[9]

$$p^T = \frac{3(c+\tau)+t}{3}, \quad p^{*T} = \frac{3(c+\tau)+2t}{3}, \quad (2)$$

[8] As in Helpman and Krugman (1985) and Clarke and Collie (2003), we need not explicitly consider the Foreign market because of the assumption of constant marginal cost and symmetry between countries.

[9] The reaction curve in the present model has kinks as Clarke and Collie (2003) argue in details. See Appendix 1 for the formal derivation of reaction functions.

where superscript T refers to the trade equilibrium. Substituting (2) into the demand functions above yields the equilibrium supplies:

$$x^T = \frac{p^{*T} - p^T + \tau}{2\tau} = \frac{3\tau + t}{6\tau} \tag{3}$$

$$x^{*T} = \frac{p^T - p^{*T} + \tau}{2\tau} = \frac{3\tau - t}{6\tau}. \tag{4}$$

Making use of (2)-(4), each firm's equilibrium profit is obtained as

$$\pi^T = \left(p^T - c\right) x^T = \frac{(3\tau + t)^2}{18\tau} \tag{5}$$

$$\pi^{*T} = \left(p^{*T} - c - t\right) x^{*T} = \frac{(3\tau - t)^2}{18\tau}. \tag{6}$$

In addition, we can calculate the consumer surplus as follows.

$$\begin{aligned} CS^T &= \int_0^{x^T} \left(a - p^T - \tau x\right) dx + \int_{x^T}^1 \left[a - p^{*T} - \tau(1-x)\right] dx \\ &= a - c - \frac{3\tau}{2} - \frac{2t}{3} + \frac{(3\tau + t)^2}{36\tau}. \end{aligned} \tag{7}$$

Summing up (5)-(7), we see that welfare in the trade equilibrium is[10]

$$\begin{aligned} W^T &= CS^T + \pi^T + \pi^{*T} \\ &= a - c - \frac{3\tau}{2} - \frac{2t}{3} + \frac{(3\tau + t)^2}{36\tau} + \frac{(3\tau + t)^2}{18\tau} + \frac{(3\tau - t)^2}{18\tau} \\ &= \frac{5t^2 - 18\tau t + 9\tau[4(a-c) - \tau]}{36\tau} \equiv W(t). \end{aligned} \tag{8}$$

In the Cournot model, the autarkic equilibrium is rehabilitated by evaluating $W(t)$ at the prohibitive trade cost $\bar{t} \equiv 3\tau$.[11] However, the same no longer survives the Bertrand model. Instead, the autarkic equilibrium is reestablished by substituting $\tilde{t}$ into $W(t)$, where $\tilde{t} \equiv [3(a-c) - 6\tau]/2$.[12] If trade costs are between $\bar{t}$ and $\tilde{t}$, imports are zero but the Home firm charges

[10] Note that π^*, the Foreign firm's profit, is included in the Home welfare since the Home firm earns π^* in the Foreign market due to the symmetry assumption of countries.

[11] $\bar{t}$ is obtained by setting $x^{*T} = 0$ in (4).

[12] $\tilde{t}$ is derived by equating the Bertrand-Nash equilibrium price p^T in (2) with the autarkic price $p^A = (a+c)/2$.

prices below the monopoly price because the presence of the Foreign firm is a threat to the Home firm. That is, potential competition with the Foreign firm leads the Home firm to charge a price below the autarkic price. In what follows, in order to ensure $\widetilde{t} > \bar{t}$, we make:

Assumption. $a - c > 4\tau$.

Given the foregoing argument, the welfare level for $t \in \left[\bar{t}, \widetilde{t}\right]$ is derived as follows.

$$\begin{aligned} W^T &= CS^T + \pi^T \\ &= \int_0^{x^T} \left(a - p^T - \tau x\right) dx + \left(p^T - c\right) x^T \\ &= \frac{-t^2 + 6[2(a-c) - \tau]t + 9\tau[4(a-c) - \tau]}{72\tau} \equiv \widetilde{W}(t). \qquad (9) \end{aligned}$$

Summarizing the arguments so far, welfare depends on trade costs as follows.

$$W^T = \begin{cases} W(t) & \text{for} \quad t < \bar{t} \\ \widetilde{W}(t) & \text{for} \quad \bar{t} \leq t < \widetilde{t} \\ W^A & \text{for} \quad t \geq \widetilde{t} \end{cases}. \qquad (10)$$

In the subsequent section, we will make use of (10) to consider welfare effects of trade liberalization, i.e., reductions in t.

4 Gains/losses from trade

This section states the main result and discusses its implications by comparing it with Clarke and Collie's (2004) gains-from-trade proposition. We begin by stating the main result.

Proposition. *Welfare under trade depends on transport costs in a U-shaped way as Figure 2 depicts for* $t \in [0, \bar{t})$*. In contrast, trade liberalization monotonically harms welfare for* $t \in \left[\bar{t}, \widetilde{t}\right)$*. Moreover, welfare in trade is less than* *welfare in autarky for any trade costs.*

Proof. See Appendix 2.

This result is sharply contrasting to Clarke and Collie's (2003) gains-from-trade proposition which is diagrammatically given by Figure 3. As has been already mentioned in Introduction, their novel result is that *any* trade is Pareto superior to autarky. However, our result predicts completely the opposite. While they employ a representative consumer (non-address) model with a quadratic utility function, the rest of assumptions is the same between them and this paper, e.g., Bertrand competition, constant marginal cost, and the trade barrier is a transport cost.

In order to find out what leads to the above deviation of implications, let us note that welfare effects of trade liberalization are decomposed as follows. First, freer trade promotes competition and raises consumer surplus. Second, foreign entry associated with trade liberalization shifts a part of the Home firm's profit to Foreign, which negatively affects welfare. Third, trade liberalization inevitably increases wasteful resources since a trade barrier takes a form of a transport cost. All of these effects arise regardless of the mode of competition (Cournot or Bertrand).

In addition to these effects, differentiated Bertrand competition induces an important effect. In the non-address model, the consumer in each country can enjoy gains from variety expansion by the opening of trade. In other words, the consumer can consume not only the domestic product but also the foreign product under trade. This contributes to positive welfare gains. On the other hand, a distinct effect emerges in the Hotelling model. Trade liberalization enables those who can not buy the differentiated good under autarky to consume it. In this sense, consumers gain from trade in both the address and non-address approaches although the source is different.

However, profit-shifting is stronger in the Hotelling model than in the quadratic utility model because the price elasticity of demand is zero in the former model. That is, the decrease in the domestic firm's market share has a considerable and dominate effect on welfare in the Hotelling model.[13] Consequently, the total effect largely depends on the profit-shifting effect and any trade unnecessarily results in welfare losses as compared to autarky.

5 Concluding remarks

We have shown that a Hotelling (1929) model brings a quite different prediction on welfare effects of trade liberalization although we are common with Clarke and Collie (2003) in assuming differentiated Bertrand duopoly. Our attempt clearly reports that gainfulness of trade is quite sensitive to the underlying assumptions and settings.

However, our primary purpose of this paper is not to criticize Clarke and Collie's (2003) argument. What we emphasize is that a subtle difference in assumptions can make it possible to obtain dractically contrasting welfare implications of trade liberalization. In view of the fact that the non-address model has been exclusively adopted in the literature mainly for tractability, the predictions based on it should be carefully revisited.

Of course, our result admittedly rests on numerous simplifying assumptions some of which may be far from reality. As mentioned in Introduction, we assume away both location choices and free entry in the oligopolistic industry, which are addressed in Schmitt (1990, 1995). Besides, it is of some use to revisit our result by interpreting trade barriers as tariffs instead of transport costs. These extensions are left as future research agenda.

[13] Schmitt (1990) also makes clear this effect in a spatial model with free entry.

Appendix 1: reaction functions

This appendix formally derives reaction functions of the Home and Foreign firms.[14] To this end, let us first note that the first-order condition for profit maximization $\partial\pi/\partial p = 0$ yields

$$p = \frac{p^* + c + \tau}{2}.$$

However, we must note that this equation defines the Home firm's reaction function only *partially*. This is because the Home firm has an incentive to set a limit price such that $x^* = 0$:

$$p = p^* - \tau,$$

if the Foreign firm chooses a too high price. The two upward-sloping lines in Figure 1 depict $p = (p^* + c + \tau)/2$ and $p = p^* - \tau$, respectively. The Home firm chooses the price along locus $x^* = 0$ when the Foreign firm charges a price above $c+3\tau$. In other words, the Home firm is tempted to deter foreign entry by charging a lower price than the monopoly price even if the Home firm captures the whole market. As a result, the Home firm's reaction curve is grammatically given by the solid locus in Figure 1 and algebraically given by

$$p = \begin{cases} \frac{p^*+c+\tau}{2} & \text{if} \quad p^* < c + 3\tau \\ p^* - \tau & \text{if} \quad c + 3\tau \leq p^* < \frac{a+c+2\tau}{2} \\ \frac{a+c}{2} & \text{if} \quad p^* \geq \frac{a+c+2\tau}{2} \end{cases}.$$

In summarizing the argument, potential competition with the Foreign firm makes the Home firm charge a lower price than the monopoly price even if the market is monopolized by the Home firm.

Applying the same argument to the Foreign firm, its reaction function is formally obtained as

$$p^* = \begin{cases} \frac{p+c+\tau+t}{2} & \text{if} \quad p < c + 3\tau + t \\ p - \tau & \text{if} \quad c + 3\tau + t \leq p^* < \frac{a+c+2\tau+t}{2} \\ \frac{a+c+t}{2} & \text{if} \quad p^* \geq \frac{a+c+2\tau+t}{2} \end{cases}.$$

[14] See also Clarke and Collie (2003) and Long and Wong (2009).

Appendix 2: proof of Proposition

As (10) indicates, the dependence of welfare on t takes different forms depending on t. We begin with the case in which both firms bilaterally export , namely, $t < \bar{t}$. In this case, welfare depends on t as in (8). Evaluating $W(t)$ at $t = 0$ and $t = \bar{t}$, we have

$$W(0) = \frac{4(a-c) - \tau}{4}, \quad W(\bar{t}) = \frac{2(a-c) - \tau}{2}. \tag{11}$$

Comparing these, it is easy to know that $W(0) > W(\bar{t})$. In words, free trade (zero transport cost) involves higher welfare than autarky (prohibitive transport cost).

Let us next obtain the first and second derivatives of $W(t)$:

$$W'(t) = \frac{5t - 9\tau}{18\tau}, \quad W''(t) = \frac{5}{18\tau}. \tag{12}$$

Eq. (12), together with (8), allows us to know that $W(t)$ is strictly convex and U-shaped in $[0, \bar{t})$ because evaluating $W'(t)$ at $t = 0$ and $t = \bar{t}$ yields

$$W'(0) = \frac{-1}{2}, \quad W'(\bar{t}) = \frac{1}{3}.$$

Having characterized $W(t)$ for $t < \bar{t}$, let us move on to the characterization of $\widetilde{W}(t)$ for $\tilde{t} > t \geq \bar{t}$. Eq. (9) applies to this range of t. Evaluating $\widetilde{W}(t)$ at $t = \bar{t}$ and $\tilde{t}$, we have

$$\widetilde{W}(\bar{t}) = \frac{2(a-c) - \tau}{2} = W(\bar{t}), \quad \widetilde{W}\left(\tilde{t}\right) = \frac{7(a-c)^2}{32\tau}. \tag{13}$$

The first and second derivatives of $\widetilde{W}(t)$ are obtained as

$$\widetilde{W}'(t) = \frac{-2t + 6[2(a-c) - \tau]}{72\tau}, \quad \widetilde{W}''(t) = \frac{-1}{36\tau}, \tag{14}$$

from which we see that $\widetilde{W}(t)$ is a strictly concave function. Moreover, $\widetilde{W}(t)$ is monotonically increasing for any $t \in \left[\bar{t}, \tilde{t}\right]$ by noting that $\widetilde{W}(\bar{t}) < \widetilde{W}\left(\tilde{t}\right)$ and that

$$\widetilde{W}'\left(\bar{t}\right) = \frac{a - c - \tau}{6\tau} > 0, \quad \widetilde{W}'\left(\tilde{t}\right) = \frac{a-c}{8\tau} > 0.$$

Finally, we compare three welfare levels, $W(0), \widetilde{W}\left(\tilde{t}\right)$ and W^A. In view of Assumption, straightforward comparisons yield

$$\frac{W(0)}{W^A} = \frac{2\tau[4(a-c)-\tau]}{3(a-c)^2} < 1, \quad \frac{\widetilde{W}\left(\tilde{t}\right)}{W^A} = \frac{7}{12},$$

from which we find that welfare under autarky Pareto dominates both welfare under free trade (zero trade cost) and welfare evaluated at $\tilde{t}$. Summarizing these results, the diagrammatic characterization of welfare as a function of t is depicted by Figure 2.

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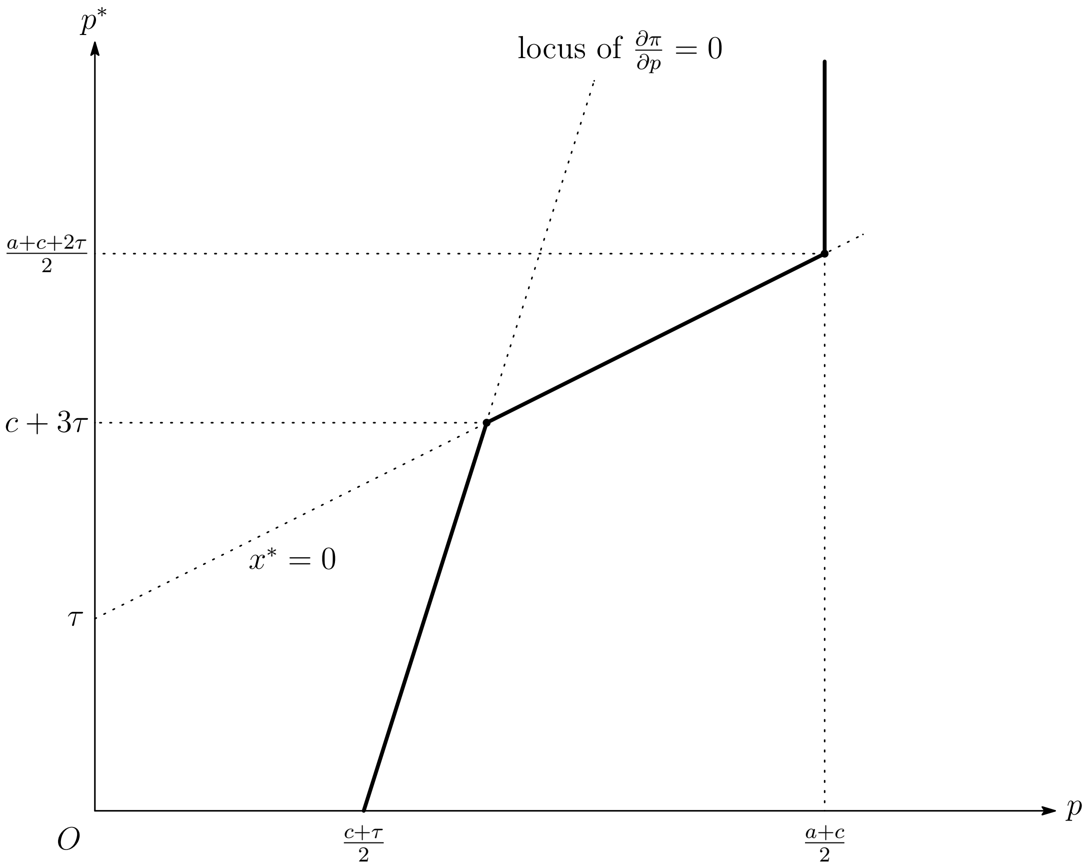

Figure 1: The Home firm's reaction curve

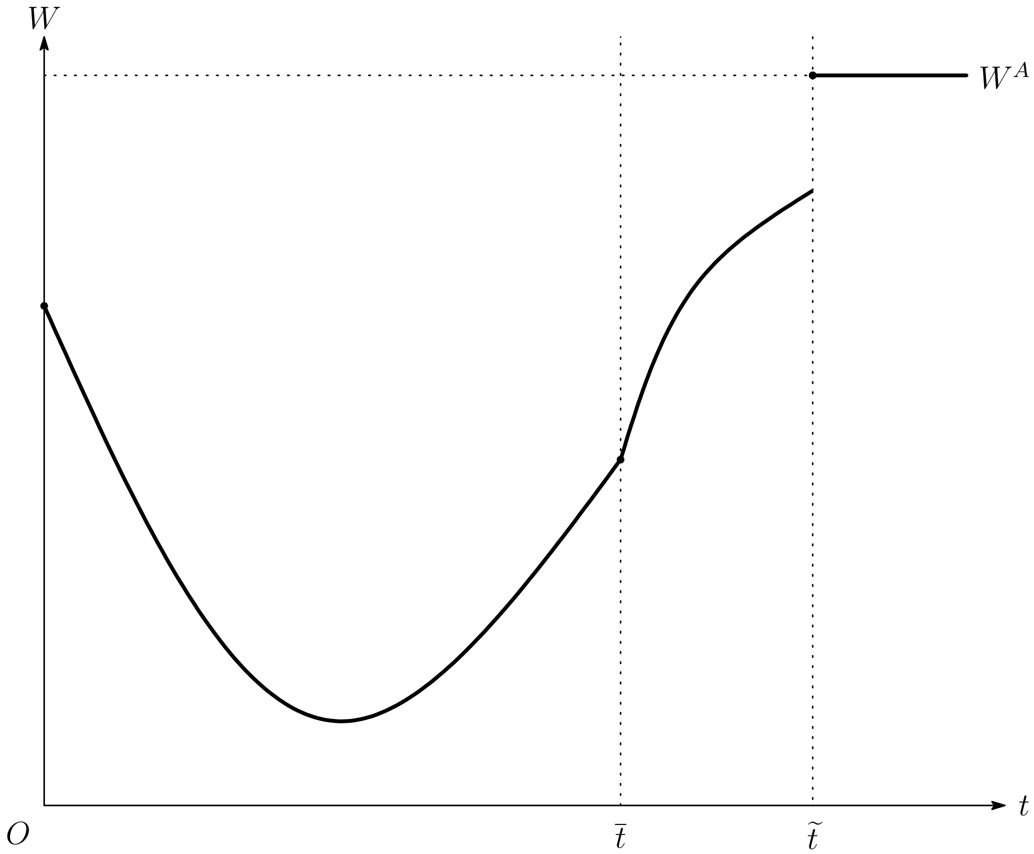

Figure 2: Welfare as a function of transport costs

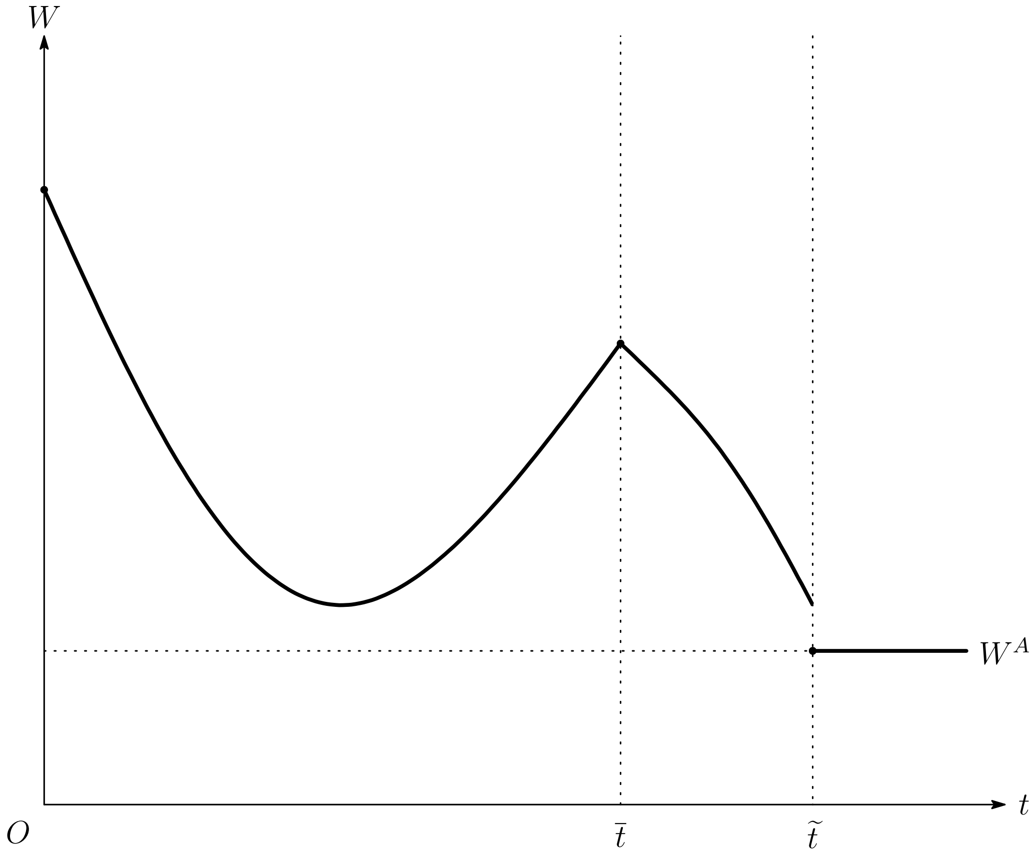

Figure 3: Clarke and Collie's (2003) gains-from-trade proposition