Deposit Insurance without Commitment:
Wall St. vs. Main St. *

Russell Cooper† and Hubert Kempf‡

December 19, 2009

Abstract

This paper studies the provision of deposit insurance without commitment. We ask whether a government has an ex post incentive to provide deposit insurance in the face of a bank-run. We find that deposit insurance will not be provided if it requires a (socially) undesirable redistribution of consumption or its financing through taxes is too costly. Else, the insurance gains to deposit insurance will be realized even without a government commitment to its provision.

1 Introduction

Within the framework of Diamond and Dybvig (1983), the implications of deposit insurance are well understood. If agents believe that deposit insurance will be provided, then bank runs, driven by beliefs, will not occur. In equilibrium, the government need not act: deposit insurance is never provided. Instead, deposit insurance works through its effects on beliefs, supported by the commitment of a government to its provision.

Yet, recent events during the financial crisis leads one to question this commitment of the government. In many countries, such as the US, the parameters of deposit insurance were adjusted during the crisis period. In other countries, such as UK, ambiguities about the deposit insurance program contributed to banking instability. In other countries, such as China, the exact nature of deposit insurance is not explicit. And, in Europe, the combination of a common currency, the commitment of the ECB not to bailout member governments and fiscal restrictions, casts some doubt upon the ability of individual countries to provide deposit insurance if needed.

Finally, in all of these instances, there is also the question of how broadly to define a bank and thus the financial arrangements deposit insurance (in some cases interpretable as an ex post bailout) might cover.

*We are grateful to the NSF for financial support. Comments from seminar participants at the Banque de France are appreciated.
†Economics Department, European University Institute, Florence, Italy and Department of Economics, University of Texas, Austin, russellcoop@gmail.com
‡Banque de France and Paris School of Economics, kempf@univ-paris1.fr
The bailout of AIG, for example, along with the choice not to bailout Lehman Brothers, makes clear that some form of deposit insurance is possible \textit{ex post} for some, but not all, financial intermediaries.

The argument here is not that these events show deposit insurance requires commitment. Rather, we think these circumstances open the question of whether deposit insurance will be provided \textit{ex post}.

There are two central building blocks for our analysis. First there is the standard argument about gains to deposit insurance, as in Diamond and Dybvig (1983). These are present in the \textit{ex post} choice of providing deposit insurance since agents face the risk of obtaining a zero return on deposits in the event of a run.

Second, there are potential costs of redistribution across heterogeneous households that may not be desired. This depends on the social objective function. These costs of redistribution also played a key role in the Cooper, Kempf, and Peled (2008) study of bailout of one region by others in a fiscal federation. That analysis highlights two motives for a bailout, the smoothing of consumption and the smoothing of distortionary taxes across regions.

Here, instead of regions, we have heterogeneous households. The central tradeoff we study is between the insurance gains of deposit insurance and the costs of the redistribution that may be entailed in the conduct of this policy. The redistribution arises both from the distribution of deposits across heterogeneous households and the tax obligations needed to finance deposit insurance. As long as the insurance gains dominate, deposit insurance will be provided \textit{ex post} and there is no commitment problem. But, if the deposit insurance entails a redistribution from relatively poor households to richer households and the social welfare function places sufficient weight on poor households, then deposit insurance will not be provided.

In the bank runs literature following Diamond and Dybvig (1983), Keister (2009) studies the tradeoff between the incentive effects to take risky actions by banks relative to the stabilizing influence of a bailout for current depositors. Ennis and Keister (2009) also look at the \textit{ex post} incentive for a bailout. Neither of those papers focus on the heterogeneity across households and thus the redistributive aspects of deposit insurance that is highlighted here.

\section{Planning}

The model is a version of Diamond and Dybvig (1983) with heterogeneity across agents. We first study the optimal allocation as the solution of a planner’s problem and then turn to a decentralized version of the model.

\subsection{Environment}

There are three periods, with $t = 0, 1, 2$. In periods 0 and 1, each household receives an endowment of the single good denoted $\alpha = (\alpha^0, \bar{\alpha})$. We index households by their period 0 endowment and refer to them as type $\alpha^0$. Let $f(\alpha^0)$ be the pdf and $F(\alpha^0)$ the cdf of the endowment distribution in period 0.

Households consume in either period 1, an early consumer, or in period 2, a late consumer. The fraction
of early consumers for each type of household is $\pi$.\(^1\) The preferences of the household is determined at the start of period 1, after any saving decision. The utility from period 0 consumption is represented by $u(c^0)$. Utility in periods 1 and 2 is given by $v(c^E)$ if the household is an early consumer and by $v(c^L)$ if the household is a late consumer. Both $u(\cdot)$ and $v(\cdot)$ are assumed to be strictly increasing and strictly concave.

There are two storage technologies available in the economy. There is a one period technology which generates a unit of the good in period $t+1$ from each unit stored in period $t$. Late households can store their period 1 endowment using this technology.

There is a two period technology which yields a return of $R>1$ in period 2 for each unit stored in period 0. This technology is illiquid though and has a return of 0 if it is interrupted in period 1.\(^2\)

### 2.2 Optimal Allocation

For the planner’s problem, we assume that the household type is observable so that the contract is contingent on the household’s endowment $\alpha^0$. In contrast, the household’s preferences are not assumed to be observed by the planner. So, though the contract is dependent upon realized household preferences, the allocation must be incentive compatible.

The planner chooses the type dependent functions $(d(\alpha^0), x^E(\alpha^0), x^L(\alpha^0))$ and the fraction of deposits to invest in the one period technology, $\phi$, to maximize:

$$\int \omega(\alpha^0)[u(\alpha^0 - d(\alpha^0)) + \pi v(\bar{\alpha} + x^E(\alpha^0)) + (1 - \pi)v(\bar{\alpha} + x^L(\alpha^0))]f(\alpha^0)d\alpha^0. \quad (1)$$

Here the period 0 consumption of the household is its endowment less a deposit, $\alpha^0 - d(\alpha^0)$. The period 1 consumption for an early consumer is the household’s endowment plus its transfer under the contract, $\bar{\alpha} + x^E(\alpha^0)$. Likewise the period 2 consumption if the household is a late consumer is $\bar{\alpha} + x^L(\alpha^0)$.

The resource constraints for the planner are:

$$\phi D = \pi \int x^E(\alpha^0)f(\alpha^0)d\alpha^0 \quad (2)$$

and

$$(1 - \phi) DR = (1 - \pi) \int x^L(\alpha^0)f(\alpha^0)d\alpha^0. \quad (3)$$

Here $\phi$ is the fraction of the overall deposits put into the one-period technology and $d(\alpha^0)$ is the “deposit” of agent of type $\alpha^0$. Total deposits are denoted $D = \int d(\alpha^0)f(\alpha^0)d\alpha^0$. In (1) the welfare weight of a type $\alpha^0$ agent is $\omega(\alpha^0)$.

The first order condition with respect to $d(\alpha^0)$ for this problem is:

$$\omega(\alpha^0)u'(\alpha^0 - d(\alpha^0)) = \lambda \quad (4)$$

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\(^1\)Here there are two important assumptions. First, $\pi$ is independent of $\alpha^0$ and second there is no aggregate uncertainty in $\pi$.

\(^2\)As in Cooper and Ross (1998), there could be some period 1 liquidation value for this technology as well.
for all $\alpha^0$ where $\lambda$ is the multiplier on (2). This condition implies that the marginal utility of period 0 consumption, weighted by $\omega(\alpha^0)$, is equal across households. It is natural to think of this as reflecting redistribution across the heterogeneous agents.

The other first order conditions are:

$$v'(\bar{\alpha} + x^E(\alpha^0)) = Rv'(\bar{\alpha} + x^L(\alpha^0))$$

and

$$v'(\bar{\alpha} + x^E(\alpha^0)) = u'(\alpha^0 - d(\alpha^0)).$$

Condition (5) stipulates optimal insurance across being an early and a late consumer. The final condition ties down the intertemporal dimension of the consumption profile. Further, from (5), $x^E(\alpha^0) < x^L(\alpha^0)$ and thus $c^E(\alpha^0) < c^L(\alpha^0)$ as $R > 1$.

As a special case, suppose the weights are independent of the household endowment, i.e. $\omega(\alpha^0) = \bar{\omega}$. Then these conditions would imply that the consumption levels of all agents were independent of $\alpha^0$: there would be complete redistribution.

### 2.3 Runs and Deposit Insurance

Though this is the planner’s problem, there is still the possibility of “runs”. Since we do not assume that planner observes the tastes of each household, we implement this allocation through a direct mechanism in which households announce their taste types to the planner.

One equilibrium is truth-telling which implements the above allocations. Since $c^L(\alpha^0) > c^E(\alpha^0)$, late households have no incentive to claim to be early households as long as all others tell the truth.

But there is the possibility that each household would announce their taste to be “early” consumer, given that others are doing the same. If so, this is akin to a bank run. In the spirit of sequential service, households would line up to obtain their promised allocation of $x^E(\alpha^0)$. Those near the front of the line would be served, others would not.

In fact, with $\pi < 1$, there is always a bank run equilibrium. To see this, note that (2) implies $\phi D < \int x^E(\alpha^0)f(\alpha^0)d\alpha^0$. The left side is the total amount of resources available to the economy while the right side, which is larger, is the total demands for consumption in period 1 if all agents announce they are early consumers. Since there are not enough resources to meet the demands of the households, each would strictly prefer to announce they are early consumers rather than late consumers in order to have a positive probability of obtaining positive consumption.

In the event of a run, the planner redistributes the available resources, $\phi D$, to households. Let $\tilde{x}(\alpha^0)$ denote the resources transferred to household of type $\alpha^0$. The planner chooses the transfers to maximize:

$$\int \omega(\alpha^0)[v(\bar{\alpha} + \tilde{x}(\alpha^0))]f(\alpha^0)d\alpha^0.$$
The resource constraint is:

$$\int  \tilde{x}(\alpha^0) f(\alpha^0) d\alpha^0 = \phi D. $$

(8)

The first-order condition implies that

$$\omega(\alpha^0)v'(\tilde{\alpha} + \tilde{x}(\alpha^0)) = \kappa$$

(9)

and thus is independent of $\alpha^0$. The key here is not this allocation per se but rather that the planner will intervene to reallocate resources in the event of a run. That is, if the planner did not act, agents would, following sequential service face a probability of not being served. Some agents would receive their promised $x^E(\alpha^0)$ while others would receive nothing. Clearly this allocation does not satisfy (9).

Further, we can ask if the planner would have an incentive to provide deposit insurance so that each agent received the promised allocation of $x^E(\alpha^0)$. To do so, the planner would have to tax the period 1 endowment of the agents to generate enough resources to provide $x^E(\alpha^0)$ to all agents. Note that since the run is not anticipated at the time the allocation is chosen, the tax system used to finance deposit insurance is not part of the ex ante optimal arrangement.

Letting $T(\alpha^0)$ be the tax on a type $\alpha^0$ household, the constraint that taxes must cover the difference between the promised allocation and the available resources is

$$\int T(\alpha^0) f(\alpha^0) d\alpha^0 = \int x^E(\alpha^0) f(\alpha^0) d\alpha^0 - \phi D.$$ 

(10)

The difference between expected utility with deposit insurance, $\Delta$, and one with runs is given by

$$\Delta \equiv \int \omega(\alpha^0)\nu(c^E(\alpha^0) - T(\alpha^0)) f(\alpha^0) d\alpha^0 - \int \omega(\alpha^0)[\zeta\nu(c^E(\alpha^0)) + (1 - \zeta)\nu(\bar{\alpha})] f(\alpha^0) d\alpha^0. $$

(11)

In this expression, $\zeta$ is the probability that a household will obtain the promised allocation of $x^E(\alpha^0)$. Since the total resources in the event of a run are $\phi D$, then $\zeta = \int x^E(\alpha^0) f(\alpha^0) d\alpha^0$.

With this structure, there are conditions under which the planner will provide deposit insurance ex post. These propositions are useful in understanding why deposit insurance may not be provided in the decentralized banking case that follows.

**Proposition 1** If $\omega(\alpha^0) = \bar{\omega}$ and $T(\alpha^0) = \bar{T}$ for all $\alpha^0$, then deposit insurance will be provided ex post.

**Proof.** When welfare weights are equal across households, the first-order conditions for the planner imply that consumption allocations are equal as well: $(c^0(\alpha^0), c^E(\alpha^0), c^L(\alpha^0)) = (c^0, c^E, c^L)$ for all $\alpha^0$. Using this equalization of consumptions along with the lump-sum tax, $\Delta \geq 0$ if and only if

$$v(c^E - \bar{T}) \geq \zeta v(c^E) + (1 - \zeta)v(\bar{\alpha}).$$

(12)

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3Deposit insurance is less general than the redistribution in (7) as it entails full restoration of deposits, financed by some tax system.

4See Cooper and Ross (1998) for a model in which the prospect of bank runs are understood ex ante.
From (10), $\bar{T} = x^E - \phi D = x^E(1 - \zeta)$, using the definition of $\zeta$. If $c^E$ is independent of $\alpha^0$, so is $x^E$. The comparison of utility becomes

$$v(c^E - x^E(1 - \zeta)) = v(\zeta x^E + \bar{\alpha}) \geq \zeta v(c^E) + (1 - \zeta) v(\bar{\alpha}).$$

(13)

The inequality is implied by the strict concavity of $v(\cdot)$. Hence deposit insurance is welfare enhancing ex post.

**Proposition 2** If the deposit insurance can be designed for each household type separately, then deposit insurance will be provided ex post.

**Proof.** In this case, we show that $\Delta > 0$ by arguing that there are gains to deposit insurance for each type. There are, by assumption, no interactions across the groups so no costs to redistribution. Thus we show

$$v(c^E(\alpha^0) - T(\alpha^0)) > \zeta_{\alpha^0} v(c^E(\alpha^0)) + (1 - \zeta_{\alpha^0}) v(\bar{\alpha})$$

for each $\alpha^0$. If there are no interactions across groups, the deposit insurance for each type of household must be financed by a tax on those households alone, $T(\alpha^0) = x^E(\alpha^0)(1 - \zeta_{\alpha^0})$. Inserting this tax, we have

$$v(c^E(\alpha^0) - x^E(\alpha^0)(1 - \zeta_{\alpha^0})) = v(\zeta_{\alpha^0} x^E(\alpha^0) + \bar{\alpha}) > \zeta_{\alpha^0} v(c^E(\alpha^0)) + (1 - \zeta_{\alpha^0}) v(\bar{\alpha})$$

The inequality comes from the strict concavity of $v(\cdot)$. Since this holds for all $\alpha^0$ types, $\Delta > 0$.

An important theme highlighted here is that there are two factors to consider ex post: redistribution and insurance. For the planner, the redistribution is taken care of in terms of the allocation $(x^0(\alpha^0), x^E(\alpha^0), x^L(\alpha^0))$ and in terms of the optimal allocation in the event of a run, (9). In this case, a deposit insurance scheme will then provide the valued insurance, as in Propositions 1 and 2.

### 3 Decentralization

Instead of the optimal allocation from the planner’s perspective, we can also study the decentralized allocation through bank contracts. Suppose there are competitive banks offering contracts to households. Through this competition, the equilibrium outcome will maximize household utility subject to a zero expected profit constraint. Since household types are observable, the contracts will be dependent on $\alpha^0$.

For now, as in Diamond and Dybvig (1983), assume that neither the bank nor its customers places positive probability on a bank run. We study the possibility of runs given this optimal contract.

#### 3.1 Household Optimization

Given a contract stipulating a return on deposits in the two periods, $(r^1(\alpha^0), r^2(\alpha^0))$, the type $\alpha^0$ household solves:
The first-order condition for the household is
\[ u'(\alpha_0 - d) = \pi r^1(\alpha_0)v'(\bar{\alpha} + r^1(\alpha_0)d) + (1 - \pi)r^2(\alpha_0)v'(\bar{\alpha} + r^2(\alpha_0)d) \]  
(15)

Since the returns depend upon \( \alpha_0 \), denote the solution as \( d(\alpha_0) \) and the value of this problem as \( U_{\alpha_0}(r^1(\alpha_0), r^2(\alpha_0)) \).

### 3.2 Banks

The bank will choose a contract and an investment plan, \((r^1(\alpha_0), r^2(\alpha_0), \phi(\alpha_0))\) to maximize household utility, \( U_{\alpha_0}(\cdot) \), subject to a zero expected profit constraint for each type \( \alpha_0 \). The bank will place a fraction \( \phi(\alpha_0) \) of deposits, \( \phi(\alpha_0) \), into the liquid storage technology which yields a unit in either period 1 per unit deposited in period 0. The remainder is deposited into the illiquid technology.

The zero expected profit condition for a type \( \alpha_0 \) contract is:
\[ r^1(\alpha_0)\pi d(\alpha_0) + r^2(\alpha_0)(1 - \pi)d(\alpha_0) = \phi(\alpha_0)d(\alpha_0) + (1 - \phi(\alpha_0))d(\alpha_0)R. \]  
(16)

To be sure the bank can meet the needs of customers, the following constraints must hold as well:
\[ \phi(\alpha_0)d(\alpha_0) \geq r^1(\alpha_0)d(\alpha_0)\pi \quad \text{and} \quad (1 - \phi(\alpha_0))d(\alpha_0)R \geq r^2(\alpha_0)(1 - \pi)d(\alpha_0). \]  
(17)

Clearly if the two constraints in (17) hold with equality, then the zero expected profit condition is met. Note that these conditions hold for any level of deposits.

### 3.3 Decentralized Allocation

The decentralized allocation maximizes \( U_{\alpha_0}(r^1(\alpha_0), r^2(\alpha_0)) \) subject to (16) and (17). The first-order condition implies
\[ v'(\bar{\alpha} + r^1(\alpha_0)d(\alpha_0)) = Rv'(\bar{\alpha} + r^2(\alpha_0)d(\alpha_0)). \]  
(18)

In addition, the constraints in (17) are binding so that (16) holds.

In general, the contract is allowed to vary with the agents type. If preferences are CARA, \( v(c) = \frac{c^{1-\sigma}}{1-\sigma} \), then the marginal utility of consumption is proportional to \( d(\alpha_0)^{-\sigma} \). In this case, the solution to (18) has \( \phi(\alpha_0) \) independent of \( \alpha_0 \). From the feasibility conditions, this would imply that the returns are also independent of \( \alpha_0 \).

Condition (18) is similar to condition (5) from the planner’s problem. Both conditions characterize optimal insurance across the two preference states for a household of type \( \alpha_0 \). Of course, the levels of

\[ \text{To show this, replace } r^1 \text{ and } r^2 \text{ in (18) with their values from the feasibility constraints, leaving } \phi(\alpha_0) \text{ as the unknown function. In the case of CARA preferences, } d(\alpha_0) \text{ factors out of (18) so that } \phi(\alpha_0) \text{ is a constant.} \]
consumption need not be the same in the two solutions since the planner’s allocation allowed for redistribution through the choice of $d(\alpha^0)$. Importantly, the welfare weights, $\omega(\alpha^0)$ are not present in the decentralized allocation.

4 Systemic Runs and Deposit Insurance

Given the optimal contract written without any consideration of bank runs. We ask two questions. First, can there be a run without Deposit Insurance (DI)? Second, if so, will the government have an incentive to provide DI \textit{ex post}?

In this section, we assume there are runs on all banks in the system. Later we study the case where there are runs on only a subset of the banks.

4.1 Are there runs?

For the decentralization given above, the answer to the first question is simple: as long as $\phi(\alpha^0) < r^1(\alpha^0)$, the bank does not have enough resources to allow all agents to withdraw $r^1(\alpha^0)d(\alpha^0)$. Since (17) is binding, $\pi < 1$ implies $\phi(\alpha^0) < r^1(\alpha^0)$.

4.2 Deposit Insurance

The run can be avoided if the government will provide deposit insurance, but will it have an incentive to do so \textit{ex post}? For now, define deposit insurance as a scheme which provides to each household its promised return of $r^1(\alpha^0)d(\alpha^0)$ under its deposit contract. This insurance is funded by the levy of a tax, $T(\alpha^0)$, on households.

4.2.1 Household Period 1 Utility under Deposit Insurance

If, \textit{ex post} the government provides deposit insurance, then welfare is:

$$W^{DI} = \int \omega(\alpha^0)v(\tilde{\alpha} + \chi(\alpha^0) - T(\alpha^0))f(\alpha^0)d\alpha^0$$

where $\chi(\alpha^0) \equiv r^1(\alpha^0)d(\alpha^0)$ is the total promised by the bank to the household. If $\omega(\alpha^0)$ is a constant, then the objective of the government is just a population weighted average of household expected utility. In general, the structure of $\omega(\alpha^0)$ will be relevant for gauging the costs and benefits of the redistribution associated with DI.

Another key element in the redistribution is the tax system used to pay for DI. In (19), $T(\alpha^0)$ is the tax paid by an agent of type $\alpha^0$. Government budget balance requires $\int [T(\alpha^0) + \phi(\alpha^0)d(\alpha^0)]f(\alpha^0)d\alpha^0 = \ldots$

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\textsuperscript{6}As in Cooper and Ross (1998), we could also study the choice of a deposit contract given that runs are possible. This is of interest if the government does not have an incentive to provide deposit insurance.
\[ \int \chi(\alpha^0)f(\alpha^0)d\alpha^0. \] The left side of this expression is the total tax revenues collected by the government plus the liquidated invested and the right side is the total paid to each depositor \( \chi(\alpha^0) \).

If, \textit{ex post}, there is no deposit insurance, then welfare is given by:

\[ W^{NI} = \int \omega(\alpha^0)[\zeta v(\bar{\alpha} + \chi(\alpha^0)) + (1 - \zeta)v(\bar{\alpha})]f(\alpha^0)d\alpha^0. \tag{20} \]

Here \( \zeta \) is the probability a household obtains the full return on its deposit, \( \zeta \equiv \frac{\int \phi(\alpha^0)d(\alpha^0)f(\alpha^0)d\alpha^0}{\int \chi(\alpha^0)f(\alpha^0)d\alpha^0} \). \tag{21}

We assume the likelihood of getting paid by the bank is independent of \( \alpha^0 \).

The values of DI and not providing DI are both calculated at the start of period 1. This is because the government lacks the ability to commit to DI before agents make their deposit decisions. The government can only react to a bank run in period 1.

### 4.2.2 Will DI be provided?

The government has an incentive to provide deposit insurance iff \( \Delta \equiv W^{DI} - W^{NI} \geq 0 \). We study this condition under the assumption of a lump-sum tax, \( T(\alpha^0) = \bar{T} \) for all \( \alpha^0 \) and return to other forms of taxation below. If taxes are lump-sum, \( \bar{T} = \int [\chi(\alpha^0) - \phi(\alpha^0)d(\alpha^0)]f(\alpha^0)d\alpha^0 \).

Under this tax system, the difference in utilities, using (19) and (20), becomes:

\[ \Delta = \int \omega(\alpha^0)[v(\chi(\alpha^0) - T + \bar{T} + \bar{\alpha}) - v(\zeta \chi(\alpha^0) + \bar{\alpha})]f(\alpha^0)d(\alpha^0) + \]

\[ \int \omega(\alpha^0)[v(\zeta \chi(\alpha^0) + \bar{\alpha}) - \zeta v(\chi(\alpha^0) + \bar{\alpha}) + (1 - \zeta)v(\bar{\alpha})]f(\alpha^0)d\alpha^0. \tag{22} \]

This decomposition allows us to isolate the two influences on the gains to deposit insurance.

The first term captures the negative effects of \textbf{redistribution}. Letting \( \hat{c}(\alpha^0) = \zeta \chi(\alpha^0) + \bar{\alpha} \) and \( \hat{c} \equiv \int \hat{c}(\alpha^0)f(\alpha^0)d\alpha^0 \), it can be written as

\[ \int \omega(\alpha^0)[v(\frac{1}{\zeta}(\hat{c}(\alpha^0) - \bar{c}) + \bar{c}) - v(\hat{c}(\alpha^0))]f(\alpha^0)d\alpha^0. \tag{23} \]

The first consumption allocation, \( \frac{1}{\zeta}(\hat{c}(\alpha^0) - \bar{c}) + \bar{c} \), is a mean-preserving spread of the second, \( \hat{c}(\alpha^0) \). Both have the same mean of \( \bar{c} \) and since \( 1 > \zeta \) the variance of the first consumption allocation is larger. From the results on mean preserving spreads, if \( v(c) \) is strictly concave

\[ \int [v(\chi(\alpha^0) - T + \bar{T} + \bar{\alpha}) - v(\zeta \chi(\alpha^0) + \bar{\alpha})]f(\alpha^0)d(\alpha^0) < 0. \tag{24} \]

The expressions (23) and (24) are not the same, \( \omega(\alpha^0) \) is absent from (24). In particular, for some \( \omega(\alpha^0) \) functions, (23) could be positive. As we shall see, there may be a loss from the redistribution if \( \omega(\alpha^0) \) is either decreasing in \( \alpha^0 \) or is constant.
The second term captures the insurance gains from DI. This is clearly positive if $v(c)$ is strictly concave. These gains are independent of the shape of $\omega(\alpha^0)$.

Thus the key tradeoff to the provision of DI ex post is whether the insurance gains dominate the redistribution effects. Importantly, this tradeoff was not present in the discussion of the planner’s solution. In that case, the ability of the planner to redistribute across the heterogenous households implied that the insurance gains from DI where independent of the redistribution. But, in this decentralized economy they are coupled.

To better understand this condition, we turn to some special cases. First, assume there is no heterogeneity across households, so $F(\alpha^0)$ is degenerate. In this case, deposit insurance is valued as it provides risk sharing across households of the uncertainty coming from sequential service.

**Proposition 3** If $F(\alpha^0)$ is degenerate, $v(c)$ is strictly concave and taxes are lump sum, then the government will have an incentive to provide deposit insurance.

**Proof.** In this case, the first term of (22) is zero, leaving only the second part. This second part is strictly positive since $v(\cdot)$ is strictly concave. Hence $\Delta > 0$.

This is a standard result in the Diamond and Dybvig (1983) model. It highlights the insurance gain from DI when there are no costs of redistribution.

It is comparable to the result in the planner’s problem reported in Proposition 1. For that result, any heterogeneity across households was offset by taxes and transfers so that, as noted earlier, consumption allocations were independent of $\alpha^0$ in the optimal allocation. Hence DI was provided for insurance reasons alone, as in Proposition 3.

To explore the other extreme, suppose agents are risk neutral so that there are no insurance gains whatsoever. Nonetheless DI is not irrelevant if it has redistributive effects.

**Proposition 4** If $F(\alpha^0)$ is not degenerate, $v(c)$ is linear and taxes are lump sum, then a government has a strict incentive to provide deposit insurance, $\Delta > 0$, if $\omega(\alpha^0)$ is increasing. However, if $\omega(\alpha^0)$ is decreasing, deposit insurance will not be provided, $\Delta < 0$.

**Proof.**

Let $\Delta(\alpha^0) \equiv (1 - \zeta)(\chi(\alpha^0) - \bar{\chi})$ be the difference in expected consumption between insurance and no insurance for type $\alpha^0$. With $v(c)$ linear, $\Delta \equiv (W^{DI} - W^{NI}) = \int \omega(\alpha^0)\Delta(\alpha^0)f(\alpha^0)d\alpha^0$.

We now show $\chi(\alpha^0)$, and thus $\Delta(\alpha^0)$, is increasing. We can write the first-order condition of the household, (15), as

$$u'(\alpha^0 - d) = \pi r^1 v'(\chi(\alpha^0) + \bar{\chi}) + (1 - \pi)r^2 v'(\bar{\alpha} + r^2(\alpha^0)d)$$

(25)

The feasibility constraint for the bank, (17), along with the first-order condition for the optimal deposit contract, (18), implies
From this expression, an increase in $\alpha^0$ will lead to an increase in consumption in both period 0 and in period 1, for early consumers. For this to be the case, $\chi(\alpha^0)$ must increase with $\alpha^0$.

Since $\chi(\alpha^0)$ is increasing, $\Delta(\alpha^0)$ is increasing and the provision of deposit insurance redistributes from low to high $\alpha^0$ households. This redistribution increases social welfare if $\omega(\alpha^0)$ is increasing.

If $\omega(\alpha^0)$ is independent of $\alpha^0$, then $W^{DI} - W^{NI} = \int \omega(\alpha^0) \Delta(\alpha^0) f(\alpha^0) d\alpha^0 = 0$. In this case, the provision of deposit insurance has no effects on welfare. If $\omega(\alpha^0)$ is decreasing in $\alpha^0$, then the redistribution strictly reduces welfare.

This proposition highlights the redistributive aspect of DI. Since the total resources in the economy are predetermined and agents are risk neutral, the only role of DI is to redistribute consumption. The nature of that redistribution depends on the deposits of each type, $d(\alpha^0)$ and the tax system. The social value of the redistribution is determined by $\omega(\alpha^0)$.

If households are both risk averse and heterogeneous, there will be insurance gains and potentially losses from redistribution from DI. From Proposition 3 the risk aversion of households will determine the magnitude of the insurance gain to deposit insurance. But from an extension of Proposition 4, the redistribution of consumption from poor to rich through the provision of deposit insurance will be welfare reducing. Which effect will dominate?

**Proposition 5** If households are not too risk averse and $\omega(\alpha^0)$ is decreasing in $\alpha^0$, then a government will not have an incentive to provide deposit insurance.

**Proof.** Using (19), work from the case of $v(c)$ linear and $\omega(\alpha^0)$ flat. We know that if $\omega(\alpha^0)$ is decreasing, then $W^{DI} < W^{NI}$. Then allow a small insurance gain via $v(c)$ strictly concave. This will have two effects: (i) there is a small insurance gain from DI and (ii) a redistribution cost which is higher. Deposit insurance is not provided.

### 5 Bank Specific Runs and Deposit Insurance

Bank runs are not always systemic but instead may initially impact only a subset of banks. In this section we explore the issue of whether DI will be provided in the event of bank specific runs.

The fact that runs occur in a subset of banks implies that there is a second dimension for redistribution: across groups of agents depending on the state of their bank as well as across types of agents based on their endowments.

To isolate the redistribution from a subset of banks experiencing a run, we place some additional restrictions on our model. In particular, we assume $v(c) = \frac{c^{1-\sigma}}{1-\sigma}$ so that, as argued earlier, the banking contract...
and investment choice is not dependent upon $\alpha^0$. Further, we assume that all banks in the economy have the same distribution of household types among depositors. That is, banks are replicas of each other. We also assume that $\omega(\alpha^0)$ is constant.

Suppose there is a run at a set of banks with a fraction $n$ households. This creates two groups of agents, one group experiencing a bank run and the other with no run. Then we can write the payoff from DI and no DI as:

$$ W^{DI} = n \int [v(\bar{\alpha} + \chi(\alpha^0) - \bar{T}) - \bar{T}) f(\alpha^0) d\alpha^0 + (1 - n) \int v(\bar{\alpha} + \chi(\alpha^0) - \bar{T}) f(\alpha^0) d\alpha^0. $$

(27)

The two terms here highlight the two regions though with DI the consumption levels are the same for each type.

As before, we start with analysis of a lump-sum tax. If a fraction $n$ of households are involved in a bank run, the lump-sum tax per household would be given by $\bar{T} = \int [\chi(\alpha^0) - \phi(\alpha^0) d\alpha^0] f(\alpha) d\alpha^0$.

If, *ex post*, there is no deposit insurance, then social welfare is given by:

$$ W^{NI} = n \int [v(\bar{\alpha} + \chi(\alpha^0)) + (1 - \zeta)v(\alpha)] f(\alpha^0) d\alpha^0$$

(28)

$$ + (1 - n) \int v(\bar{\alpha} + \chi(\alpha^0)) f(\alpha^0) d\alpha^0. $$

The two terms here indicate the differential treatment across groups: in one there are runs and the uncertainty created by sequential service while in the other there is stability.

The key point of the multiple regions is that the tax paid by those in the failed bank is smaller due to the presence of the other banks because those in the other banks pay a share of the deposit insurance. Whether deposit insurance is then paid *ex post* depends, in part, on the relative size of these gains and costs.

Drawing upon the arguments in Cooper, Kempf, and Peled (2008) that consumption smoothing across regions will lead to bailouts, DI will in fact be provided if the only differences across households is due to the status of their bank. Here, instead of the regions in Cooper, Kempf, and Peled (2008), we have groups of households distinguished by whether their deposits are in a failed bank or not.

For this case, the gain to deposit insurance is:

$$ \Delta = \int n[v(e^E(\alpha^0) - \bar{T}) - \zeta v(\bar{\alpha} + \chi(\alpha^0)) - (1 - \zeta)v(\bar{\alpha})] +$$

$$ (1 - n) [v(e^E(\alpha^0) - \bar{T}) - v(E(\alpha^0))] f(\alpha^0) d\alpha^0.$$  

(29)

The following results use this definition of the utility differential.

**Proposition 6** If $F(\alpha^0)$ is degenerate, then the gains from deposit insurance are positive.

**Proof.** When all households are identical, from (29), the expected utility difference across regions is given by:

$$ \Delta = [v(e^E - \bar{T}) - n[\zeta v(\bar{\alpha} + \chi^E) + (1 - \zeta)v(\bar{\alpha})] - (1 - n)v(E)].$$

(30)
To see that \( \Delta > 0 \), combine the second group of terms, subtracted from the first term. Since \( v(\cdot) \) is strictly concave, this combining of terms decreases \( \Delta \). Hence we have

\[
\Delta > [v(c^E - \bar{T}) - v(c^E(1 - n(1 - \zeta) + n(1 - \zeta)\bar{\alpha}))].
\] (31)

Using the definitions of \( \zeta \) and \( \bar{T} \) and arranging terms,

\[
\Delta > [v(c^E - \bar{\chi}) - v(c^E - (1 - \zeta)n(c^E - \bar{\alpha}))].
\] (32)

As \( \chi^E = c^E - \bar{\alpha} \), the term on the right of (32) is zero implying \( \Delta > 0 \). ■

If the distribution of \( \alpha^0 \) is not degenerate, then the provision of DI entails redistribution in two dimensions: across regions and across household types. Proposition 6 makes clear that if there is only redistribution across regions, then DI will be provided. But we know from section 5 that in some cases, the redistribution across types created by DI may be welfare reducing so that this insurance is not provided.

Now we replace our assumption of lump-sum taxes and allow them to depend upon the household type. In particular, suppose

\[
T(\alpha^0) = (\chi^E(\alpha^0) - \phi(\alpha^0)d(\alpha^0))n.
\] (33)

With this tax scheme, the lump sum tax of household type \( \alpha^0 \) is proportional then it is also the case that deposit insurance will be provided.

**Proposition 7** If taxes satisfy (33), then the gains from deposit insurance are positive.

**Proof.** Consider a particular household type. From (29), the expected utility difference for that type is given by:

\[
\Delta(\alpha^0) = [v(c^E(\alpha^0) - T(\alpha^0)) - n(\zeta v(\bar{\alpha} + \chi(\alpha^0)) + (1 - \zeta)v(\bar{\alpha})) - (1 - n)v(c^E(\alpha^0))].
\] (34)

To see that \( \Delta(\alpha^0) > 0 \) for all \( \alpha^0 \), combine the second group of terms, subtracted from the first term. Since \( v(\cdot) \) is strictly concave, this combining of terms decreases \( \Delta(\alpha^0) \). Hence we have

\[
\Delta(\alpha^0) > [v(c^E(\alpha^0) - T(\alpha^0)) - v(c^E(\alpha^0)(1 - n(1 - \zeta) + n(1 - \zeta)\bar{\alpha}))].
\] (35)

Using the definition of \( \zeta \), \( T(\alpha^0) \) from (33) and arranging terms, we write

\[
\Delta(\alpha^0) > [v(c^E(\alpha^0) - \chi^E(\alpha^0)(1 - \zeta)n) - v(c^E - (1 - \zeta)n(c^E(\alpha^0) - \bar{\alpha}))].
\] (36)

As, \( \chi^E(\alpha^0) = c^E(\alpha^0) - \bar{\alpha} \), the term on the right of (36) is zero so that \( \Delta(\alpha^0) > 0 \).

This argument is true for each type \( (\alpha^0) \). Hence \( \Delta > 0 \). ■

6 Extending the results

[In Process]
6.1 Equity Holders
6.2 Too big to fail
6.3 More general taxation
6.4 Cap on Deposit Insurance
6.5 Distortionary taxes

7 Conclusion

Will there be deposit insurance if the government does not have the ability to commit?

• yes for insurance reasons
• no if redistribution is not desired
• no if taxes are too distortionary
References


