The Macroeconomics of Firms’ Savings*

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Abstract

The U.S. non-financial corporate sector became a net lender vis-a-vis the rest of the economy in the early 2000s. We document this fact in the aggregate and firm-level data. We then develop a structural dynamic model with investment to study the firms’ financing decisions. Debt is fiscally advantageous but subject to a no-default borrowing constraint. Equity payouts comove positively with the firm’s cash flow. We show firms accumulate financial assets for precautionary reasons, yet value equity as partial insurance against shocks. The calibrated model replicates the large fraction of publicly-traded firms with net savings observed in the period 2000-2007 as well as several other empirical features. Finally we exploit the higher fiscal cost of equity in the 1970s as a test of the theory: we find the model’s predictions to line up very well with the data.

Keywords: Corporate savings, debt, equity, dividend taxation.

1 Introduction

In the last 40 years a number of developed economies have experienced large changes in the level and composition of private savings. For the U.S., the private savings rate dropped from 10 percent in the 1970s and 1980s to less than 4 percent at the beginning of the 2000s. The composition of private savings has undergone even more dramatic changes. While U.S. households have set out on a path of lower and lower savings, the corporate sector has experienced a secular shift in its financial position, becoming a net lender to the rest of the economy in the 2000s.

In this paper we study the emergence of the U.S. corporate sector as a net lender to the rest of the economy. We show that the corporate net financial asset (NFA) position, defined as

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the difference between financial assets and debt liabilities, has exceeded the value of its tangible assets (capital henceforth) by over 5 percent for the period 2003-2007.\textsuperscript{1} In the 1970s and 1980s the corporate sector was a net debtor, borrowing as much as 20 percent of its capital.

In order to understand better why firms become net savers, we work with the Compustat database. It contains detailed historical information about balance sheets of publicly traded firms in the U.S. We show that the patterns in the aggregate data for the period 1970-2007 also emerge for publicly traded firms\textsuperscript{2} The average firm carries financial assets, in net terms, equal to about 7 percent of its capital in the 2000s. Net lending is quite widespread: about 44 percent of the firms in our sample have a positive NFA position during 2000-2007.

Why do we observe so many firms with net financial savings—even as they have outstanding equity? Internal funds appear to be preferable to external funds and, if the latter are needed, debt offers several advantages over equity. Interest payments are tax deductible, while dividends and capital gains are taxed. In addition, equity has significant floatation costs and can worsen agency problems by bringing external ownership into the company\textsuperscript{3} Thus from a cost perspective firms should adhere to a hierarchy of financing sources: first they should rely on internal funds; if external finance is needed, debt should be preferred to equity, which becomes a finance source of last resort. Surprisingly, the data suggest the opposite pattern, with firms relying on equity even though internal funds are available.

We argue that firms accumulate financial assets to avoid being financially constrained in the future—a precautionary motive—and simultaneously value equity as it provides partial insurance against negative cash flows. In fact, we show that firms find it optimal to fund additional financial asset holdings with equity revenues, despite their higher cost. There are two key premises to our theory. First, firms face a non-default constraint on its fixed-income liabilities, so on occasion they must resort to costly equity to finance their investment needs. As a result, the firms’ value function is strictly concave even if the underlying objective function is linear. Second, equity payouts are positively correlated with the firm’s cash flow. The concavity of the value function implies that firms are willing to pay an insurance premium for equity and, at the same time, strive to accumulate net worth—in the form of financial assets.

Using equity to fund acquisitions of financial assets increases the internal funds available to the firm in the event of negative cash flow shocks, safeguarding the firm from having to issue further equity. The intuition is as follows. A firm with low net worth has no choice but to issue

\textsuperscript{1}We define the corporate sector as non-farm, non-financial corporations. Data are from the U.S. Flow of Funds (FoF henceforth).
\textsuperscript{2}We restrict our sample to non-financial companies, excluding the technology and regulated utilities sectors. More details on the data are provided in Section 2 and in Appendix A.
\textsuperscript{3}This is a necessarily very short list of the main advantages and disadvantages of debt and equity. Frank and Goyal (2005) offer an overview of the corporate debt literature, noting that existing theories struggle to explain the low demand for debt observed in the data.
equity to satisfy its financing needs. Since a large fraction of the cash flow is then committed to shareholders, the firm’s net worth increases only very slowly, preventing the firm from reducing outstanding equity, and so on. Thus, one additional dollar of internal funds allows the firm to reduce equity reliance in the present and future periods after a negative cash flow shock. In other words, the firm values internal funds above the one-time cost of equity and is thus willing to raise equity revenues to build its financial asset holdings.

We develop a general-equilibrium model where risk-neutral entrepreneurs own firms operating a decreasing-returns-to-scale technology using labor and capital. Firms are heterogeneous regarding their net worth and productivity, which evolves stochastically. Households choose how much to consume, save and work. Labor can be contracted in spot markets, but capital is determined by the firm’s investment in the previous period.

There are two external sources of finance: debt and equity. The return on debt is risk-free: to ensure debt repayment is feasible in all states firms face a borrowing constraint—derived from the model primitives. Whenever the constraint is binding, firms must resort to equity to finance the desired level of investment. We model equity as an infinitely-lived claim on firm’s net revenues and capita, and assume an exogenous payout policy. Importantly, shareholder distributions depend positively on the firm’s cash flow—providing some financial relief to the firm if it experiences a negative cash flow—an event we label an “operational loss.”

We focus on fiscal considerations to calibrate the cost of equity relative to debt. Shareholder distributions, whether in the form of dividends or capital gains, are taxed while interest payments are deductible from corporate tax liabilities. We derive the price of equity such that the after-tax return of debt and equity are equalized for a risk-neutral household. From the firm’s point of view, this implies that shareholders demand a higher expected return than creditors.

We show that our model provides an excellent match to the distribution of NFA across firms in the period 2000-2007. The model predicts a large share of firms with positive NFA, very close to the data: 43 percent versus 44 percent in the data. It also matches the mean, standard deviation and various percentiles of NFA to capital distribution. Importantly, the model generates the fat right tail of the NFA distribution found in the data.

The quantitative success of our model rests on its ability to generate realistic levels of financing needs. Motivated by the data, we introduce two novel features to our calibration. First, firms suffer infrequent but costly operational losses that reduce their net worth. Second, firms occasionally learn of investment opportunities, which we model as movements up a produc-

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Our decision to focus on fiscal considerations is driven by the availability of independent estimates of tax rates across time. We recognize there are other important factors influencing the relative costs and benefits of equity, such as floatation costs, agency considerations, and deadweight losses associated with liquidation. Unfortunately, there are no independent, reliable measurements across time for these factors. For a review of empirical and theoretical work, see [Frank and Goyal (2008)] and [Tirole (2006)].
tivity ladder in the next period. These investment opportunities increase the desired capital levels by the firm without contemporaneously increasing its cash flow. We calibrate the parameters governing operational losses and investment opportunities to match, respectively, the observed transition probability into losses and the fraction of firms with investment expenditures exceeding their positive cash flow. Both investment opportunities and operational losses lead firms to accumulate financial assets while simultaneously keeping net worth growth in check. Despite our parsimonious calibration, the model reproduces various unconditional moments of investment, revenues, employment, cash flows and equity.

We then check the model’s predictions along two important dimensions. First, we show the model predicts the correct joint distribution of NFA with several variables. In particular, the model replicates the fact that firms with net saving have higher revenues, investment, and book equity, as well as higher revenue growth rates relative to firms with negative NFA positions. This suggests that the model captures well the key determinants of NFA in the data. We then go a step further and ask whether the adjustments in the firms balance sheet positions are consistent with the data. In particular, we investigate the co-movements between changes in equity, changes in NFA, and investment. We show that in the model changes in book equity are positively correlated with changes in NFA, as firms use some of the equity they raise towards accumulation of financial assets. At the same time, firms de-accumulate NFA when their investment is high. We find that the signs and the size of these correlations predicted by the model are very close to those we find in the Compustat sample.

Lastly, we exploit the pronounced changes in the fiscal cost of equity over the last forty years to provide a final test of the model. According to our calculations, reductions in dividend taxes in the 1980s and 1990s, up to the tax reform of 2003, reduced by half the cost of equity relative to debt. Once the higher relative cost of equity in the 1970s is accounted for, our model predicts lower firms’ reliance on equity to accumulate financial assets, and thus smaller positions in both equity and NFA. Quantitatively, we find the mean ratio of NFA to capital to be negative, at $-0.10$ — just slightly above the value observed in the data for the 1970s. The predicted share of firms with positive NFA in the model is also quite close to the data: 32 percent in the model compared to 27 percent in the data. We view this as further evidence that our model correctly captures the key determinants of the financial positions of the corporate sector.

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5 Standard specifications in the literature are calibrated to match revenue dynamics. These specifications do not generate enough finance demand because investment expansions are driven by positive productivity shocks, which also bring a cash flow windfall. It is thus too easy for the firms to self-finance. The role of negative cash flows is also emphasized in Gorbenko and Strebulaev (2010). In the data, the importance of such shocks for firms’ cash holdings has been documented by Opler et al. (1999) and Bates et al. (2009).

6 See also Poterba (2004) for further discussion on the taxation of corporate distributions. McGrattan and Prescott (2005) link changes in the U.S. tax and regulatory system regarding corporate distributions to large secular movements in corporate equity. Following the Jobs and Growth Tax Relief Reconciliation Act of 2003 there has been a renewed interest in how dividend and capital gains taxes affect capital structure and investment. See, for example, Chetty and Saez (2005, 2006), Gourio and Miao (2010), and Gourio and Miao (2011).
Our paper is related to a large literature on dynamic corporate finance, represented by Hennessy and Whited (2005, 2007), among others. A key insight from this literature is that dynamic structural models can explain many “puzzling” findings in empirical corporate finance. For example, Hennessy and Whited (2005) propose a model that generates a negative relationship between leverage and lagged measures of cash-flows, debt hysteresis, and path-dependence in financing policy. While our model also replicates these facts qualitatively, our main focus is on characterizing the aggregate distribution of firms’ financing choices. We therefore choose to dispense with several structural features typically used in this literature to match firm-level elasticities, such as adjustment costs or liquidation costs. As a result, our framework is substantially simpler and more parsimonious without compromising its quantitative performance.

Our work is also related to a growing literature studying the interaction of financing decisions with the real aggregate variables. Thus, Cooley and Quadrini (2001) use a model of industry dynamics to study the role of financial frictions and persistent productivity shocks for firm dynamics and their dependence on firms’ characteristics, such as initial size and age. Cooley and Quadrini (2001), however, do not allow for capital accumulation and abstract from the role of taxes. Jermann and Quadrini (2012) also formalize a model of debt and equity financing, but are interested in the cyclical properties of external finance and the effects of ‘financial shocks’. Karabarbounis and Neiman (2012) relate secular changes in labor income share to changes in corporate savings. Relative to these studies our contribution is to focus on firms’ financing positions and their cross-sectional distribution.

The paper is organized as follows. Section 2 documents the key facts regarding corporate NFA for the period 2000-2007. Section 3 describes the model setup and defines the industry equilibrium. We discuss how our model generates a simultaneous demand for equity and net savings in Section 4. We then turn to our quantitative analysis. Section 5 documents our calibration and Section 6 discusses the model fit and the key quantitative determinants of positive NFA. Section 7 documents and contrasts the model predictions for the high cost of equity environment of the 1970s. We conclude in Section 8. The Appendix contains a more detailed description of the data as well as several technical results regarding the model. We also consider a model extension that allows for firm heterogeneity based on age. This allows us to study the association between NFA and age.

Papers with similar focus include Gomes (2001), Whited (2006), Gamba and Triantis (2008), and DeAngelo et al. (2011).

There is an extensive empirical literature that focuses on cross-sectional determinants of corporate leverage and aims to distinguish among various capital structure theories (e.g., Titman and Wessels (1988), Rajan and Zingales (1995), Fama and French (2002), Shyam-Sunder and Myers (1999), and Welch (2004) among others).

Other papers that feature endogenous dynamic financing and investment policies include Brennan and Schwartz (1984), Titman and Tsyplakov (2007), and Riddick and Whited (2009).

Other studies that focus on the business cycle properties of external finance include Covas and Den Haan (2007), Choe et al. (1993) among others.
2 The US corporate sector as a net lender

In this section we document the key empirical developments in the capital structure of the U.S. corporate sector. We start with the aggregate data, drawn from the Flow of Funds (FoF) accounts of the United States. We focus on the non-farm, non-financial corporate business sector data on the levels of financial assets, tangible assets, liabilities and net worth during 1970-2007 period. We compute net financial assets as the difference between financial assets and liabilities. In all cases, we scale the variables by tangible assets, which provide a measure of the sector’s capital stock. All variables are measured at market value.\footnote{The Flow of Funds data set also contains the value of non-financial assets at historical cost. We find that using these variables does not change the trends in the ratios of NFA and equity to capital but raises their (absolute) levels.}

Figure\footnotemark presents the dynamics of the NFA to capital ratio during the 1970-2007 period. It shows that aggregate NFA to capital was relatively stable around -0.15 during the 1970s and 1980s, experienced a dramatic run-up during the 1990s, and stabilized again at 0.03 in the 2000s.\footnote{Interestingly, during the 1950s and 1960s, the NFA to capital ratio in the FoF was above its level in the 1970s and 1980s. However, it remained negative throughout the period, making the qualitative switch of the NFA position in the 2000s unprecedented.} These developments highlight the transition of the U.S. corporate sector from a net debtor into a net creditor at the turn of the century.\footnote{We provide a detailed account of these trends, their various decompositions and robustness checks using both aggregate and firm-level data in the online appendix available at \url{http://faculty.arts.ubc.ca/vhatkovska/research.htm}.} The increase in NFA was also accompanied by a rise in equity financing, where the net worth of the U.S. corporate sector as a share of its capital has increased from 0.85 in the 1970s and 1980s to 1.03 in the 2000s.\footnote{Both asset and liability positions of the corporate sector rose over the period, with assets rising faster than liabilities. Unfortunately, the Flow of Funds data provide only a few disaggregated components for both assets and liabilities, preventing us from an in-depth look into the factors behind the rise in aggregate NFA in the U.S. We include a discussion of the trends and conduct some decompositions based on the available Flow of Funds data in the online appendix.}

Which firms are net lenders? To answer this question we turn to disaggregated firm-level data from Compustat. We focus on U.S. firms only; we exclude technology and financial firms, as well as regulated utilities.\footnote{We exclude technology firms from our analysis due to a potentially serious mismeasurement of their capital stock, which is predominantly intangible.} We also drop the firms whose capital is below 50,000 USD, those with negative equity, and zero sales.\footnote{When computing statistics that are easily influenced by outliers we also eliminated the top and bottom 1 percent of observations in NFA and capital distributions.} This selection leaves us with a sample of 6535 firms in the 2000s. In line with the definitions used in the Flow of Funds data, we construct our measure of net financial assets in the Compustat database. Financial assets are obtained as the sum of cash and short-term investments, total other current assets, and account receivables. Liabilities are computed as the sum of current and long-term debt, accounts payable, and taxes payable. Our measure of tangible assets, or capital, includes firms’ gross property, plant and equipment,
investment and advances, intangible assets, and inventories.

In terms of the capital-output ratio, our Compustat sample comes very close to matching that ratio in the aggregate economy – the capital-output ratio in our sample is equal to 2 across all industries and is equal to 3 for the largest sector, manufacturing. In terms of overall size, non-financial Compustat firms employ about 36 percent of the aggregate U.S. labor force and hold 60 percent of the aggregate U.S. capital stock during the 2000s.  

We begin by reporting the mean and median of the NFA to capital ratio in our Compustat sample. Figure 2 presents our results.

It is easy to see that Compustat firms show a pronounced increase in NFA ratios, mirroring the trends we uncovered in the aggregate data. Both the mean and median NFA to capital are rising steadily over time. The mean ratio turns positive in the mid-1990s, reaching about 12 percent in 2006-2007 and averaging 7 percent from year 2000. The median NFA to capital ratio, has also risen sharply over the past 40 years, although it did not turn positive in the 2000s.

While publicly traded companies included in Compustat are not a representative sample of the

\[ \text{Source: U.S. Flow of Funds} \]

\[ \text{Corporate Net Financial Assets (NFA) / Capital} \]

\[ 1970 \quad 1980 \quad 1990 \quad 2000 \quad 2010 \]

\[ \text{year} \]

\[ \text{Figure 1: U.S. non-farm, non-financial corporate NFA to K} \]

\[ 17 \text{See the online appendix for details.} \]

\[ 18 \text{We have also looked at the ratio of mean net savings to mean capital, and the same ratio for medians. We found that the ratio of medians exhibits the same trends as presented here, while the ratio of means does not exhibit any pronounced trends, suggesting that small and medium-size firms, as opposed to large firms, are behind the rise of net savings in the Compustat data set. Those results can be found in Appendix A.} \]

\[ 19 \text{The Compustat data do not exhibit as dramatic a run-up in the NFA to capital ratio in the 1990s that we observed for the FoF series. In the FoF, the run-up is driven by the “Miscellaneous” component – the largest of all components of assets and liabilities reported in those data. We attempt to decompose the “Miscellaneous” NFA in the online appendix.} \]
U.S. firms, it is very reassuring that the central moments are roughly similar to the aggregate data and exhibit similar trends. At the same time, the analysis of the Compustat firms makes our findings of positive NFAs even more noteworthy as these firms also have equity outstanding. Table 1 takes a closer look at the distribution of the NFA to capital ratios across firms in the 2000s. During this period, the standard deviation is quite large, equal to 0.65. The distribution of NFA to capital is skewed to the right: the top ten percent of firms in our data set have NFA positions exceeding 138 percent of their tangible assets. However, positive NFA are not confined to a small set of firms, driving the central moments: about 44 percent of all firms in the 2000s have positive NFA positions. The distribution also features a small left-tail, with about ten percent of the firms borrowing more than half their tangible assets.

Are positive NFA positions concentrated within a particular segment of public firms or has the phenomenon been widespread? Thus, we look at NFA positions conditional on firm size, age, industry, and entry cohort. We find that firms in all sectors have experienced an increase in their NFA, with manufacturing firms seeing their net asset positions turn positive in the 2000s. We also find that small to medium size firms, younger firms, and entrants into Compustat contributed the most for the U.S. sector becoming a net lender during the 2000s. Detailed results and discussion of these findings are provided in Appendix A.  

We next develop

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20The corresponding number was only 27 percent in the 1970s.
21We also investigate whether firms with foreign operations are responsible for the large positive NFA positions in the 2000s, as these firms may choose not to repatriate their foreign profits for tax reasons and instead keep the funds in their savings accounts. We find no evidence for this in the Compustat sample. In fact, NFA to capital ratios of firms with foreign operations, as reported in the income statements, are lower than those for the firms with domestic operations only. Detailed statistics are presented in the online appendix.
Table 1: Moments of corporate NFA/capital distribution

<table>
<thead>
<tr>
<th>NFA/K</th>
<th>2000s</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.07</td>
</tr>
<tr>
<td>median</td>
<td>-0.07</td>
</tr>
<tr>
<td>Pr(NFA&gt;0)</td>
<td>43.55</td>
</tr>
<tr>
<td>skeweness</td>
<td>1.81</td>
</tr>
<tr>
<td>std dev</td>
<td>0.65</td>
</tr>
<tr>
<td>10pct</td>
<td>-0.51</td>
</tr>
<tr>
<td>25pct</td>
<td>-0.31</td>
</tr>
<tr>
<td>75pct</td>
<td>0.35</td>
</tr>
<tr>
<td>90pct</td>
<td>1.38</td>
</tr>
</tbody>
</table>

a theoretical framework through which we hope to understand the savings behavior of the U.S. publicly-traded firms.

3 The model

The economy is populated by a representative household, entrepreneurs, and the government. Time is discrete and denoted by $t = 0, 1, \ldots$. We abstract from aggregate shocks.

The entrepreneurs are subject to idiosyncratic shocks and make the core decisions in our model: how much to invest and how to finance themselves. Our description of the model accordingly starts with them. The representative household supplies labor and funds to the entrepreneurs, and is used to derive factor and asset prices. Finally the government balance budget constraint closes the model.

3.1 Entrepreneurs

There is a continuum of risk-neutral entrepreneurs, with mass normalized to one. Each period a fraction $\nu > 0$ die and an identical measure of new entrepreneurs are born.

3.1.1 Production

Each entrepreneur owns a firm that combines capital $k$ and labor $l$ into final output according to the production function

$$f(l, k; \sigma) = \frac{z(\sigma)^{\nu+\eta} l^{\nu} k^{1-\nu-\eta}}{\nu + \eta},$$
where \(z(\sigma) \in Z\) is an idiosyncratic productivity shock governed by the exogenous state \(\sigma \in \Sigma\), which follows a first-order Markov stochastic process. Parameters \(\nu, \eta > 0\) satisfy \(\nu + \eta < 1\) and determine the income shares of labor, capital, and the entrepreneur’s rents.

Labor is hired at a spot market at wage rate \(w_t\). The firm pays a corporate tax rate \(\tau^c\) on earnings minus capital depreciation expenses, \(\delta k_t\), where \(\delta > 0\) is the depreciation rate of capital. Investment is set one period in advance. In addition we introduce the possibility that a firm suffers a cash flow loss by allowing for additional after-tax expenses \(c^f(k_t; \sigma)\). Then, the firm’s after-tax net revenues and capital net of depreciation are given by

\[
\pi(k; \sigma) = \max_l (1 - \tau^c) \left( f(l, k; \sigma) - wl - \delta k \right) + k - c^f(k; \sigma). \tag{1}
\]

The additional expenses may be due to overhead costs, minimum scale requirements, product obsolescence, or, more exceptionally, liabilities or accidents. We must note that operational losses play an important role in our model. Entrepreneurs will periodically have to use finance to cover cash shortfalls, possibly in states of the world where their immediate revenue prospects are poor.

### 3.1.2 Financing

In order to obtain finance, an entrepreneur may rely on internal funds, debt, or equity issuance. Let \(a_t\) denote financial asset position at date \(t\), that is, \(a_t > 0\) denotes positive net savings (and thus internal funds), and \(a_t < 0\) denotes debt. The pre-tax gross return of savings/debt is \(1 + \tilde{r} > 1\). Since interest expenses are deductible from corporate taxes due, the after-tax gross return is \(1 + r = 1 + (1 - \tau^c)\tilde{r}\).

We consider only risk-free, fixed-return debt. Hence we must ensure it is feasible to repay outstanding debt with probability one. The no-default condition implies the following borrowing constraint:

\[
a_{t+1} \geq -\alpha, \tag{2}
\]

where \(\alpha\) is derived from the primitives of the model. In the Appendix we discuss the steps to derive the borrowing constraint, as well as conditions such that \(\alpha\) is strictly positive and constant across firms.

We model equity financing as follows. The entrepreneur can issue claims on the firm’s value to the households. In the tradition of the incomplete markets literature, the terms on these claims—the shareholder payout policy—are exogenously specified. We also assume the entrepreneur retains full control of the firm’s decision-making and is the residual claimant of the value of the firm at all times. In doing so we abstract from a host of corporate governance and agency issues. Both assumptions also ensure that the entrepreneur’s problem—akin to a
portfolio problem with investment—remains tractable for a quantitative analysis.

Let $s_{t+1}$ be the number of equity claims, or shares, issued at date $t$. At date $t+1$, after the realization of the firm’s state $\sigma_{t+1}$, the present value of all shareholder distributions, per claim, is exogenously given by function $q(k_{t+1}, \sigma_{t+1}) : \mathbb{R}_+ \times \Sigma \to \mathbb{R}_+$. Total equity payouts are thus $q(k_{t+1}, \sigma_{t+1})s_t$. Note we are subsuming all the various forms shareholder payout can take, e.g., dividends, shares buy-backs, capital gains... in the present value of distributions, $q$. While an exogenous payout policy is less than ideal, our approach is very flexible without compromising the tractability of the model—and it is thus very well suited to quantitative analysis. In Section 4 we will show that firms will value costly equity as long as payouts depend positively on the firm’s performance on the current period. Finally we assume that entrepreneurs cannot short themselves, $s_{t+1} \geq 0$, and total claims are bounded above, $s_{t+1} \leq 1$.

Investors price shares according to function $p(k_{t+1}, \sigma_t) : \mathbb{R}_+ \times \Sigma \to \mathbb{R}_+$. We will derive the price schedule later from the arbitrage condition of the representative household. In addition, we introduce a “markdown” parameter $\xi > 0$ that summarizes any wedge on the relative cost of equity and debt from the point of view of the firm. This cost may reflect issuance and floatation costs, fiscal considerations, or other frictions. Net revenues from equity issuance $e_t$ are then given by

$$e_t = \xi p(k_{t+1}, \sigma_t)s_{t+1}.\]

This specification gives us a lot of flexibility to capture the costs and benefits of equity over debt. If $\xi = 1$ the firm is indeed indifferent between debt and equity, and the Miller-Modigliani theorem holds. If $\xi < 1$, equity is relatively costly, so, absent any other friction, the firm prefers to use debt to finance itself.

### 3.1.3 The entrepreneur’s problem

We are now ready to set up the entrepreneur’s problem. We assume entrepreneurs have risk-neutral preferences and choose plans for asset holdings $a_t$, capital $k_t$, equity $s_t$, and consumption $c_t$ to maximize

$$E_t \left\{ \sum_{j=0}^{\infty} (\beta_e(1 - \kappa))^j c_{t+j} \right\},$$

subject to budget constraint

$$c_t + a_{t+1} + k_{t+1} + q(k_t, \sigma_t)s_t \leq \pi(k_t; \sigma_t) + (1 + r)a_t + \xi p(k_{t+1}, \sigma_t)s_{t+1} \quad (3)$$
as well as

\[\begin{align*}
    c_t & \geq 0 \\
    a_{t+1} & \geq -\alpha \\
    s_{t+1} & \in [0, 1]
\end{align*}\]

at all dates \(t \geq 0\), where \(\beta_e \in (0, 1)\) is the inter-temporal discount factor of the entrepreneurs.

The entrepreneur’s problem can be stated recursively by defining net worth,

\[\omega_{t+1} = \pi(k_{t+1}; \sigma_{t+1}) + (1 + r)a_{t+1} - q(k_{t+1}, \sigma_{t+1})s_{t+1},\]

as the endogenous state variable for the firm’s problem. Net worth summarizes all the cash inflows as well as payment obligations of the firm entering in period \(t+1\). It is thus a concise summary of the internal funds the entrepreneur can tap into. Since cash flow and net financial assets are bounded below, we can show that net worth is bounded below, \(\omega \geq \omega^b\). There is no upper bound for net worth, and thus the support for net worth is \(\Omega = \{\omega \geq \omega^b\}\).

We proceed by splitting the recursive problem into two stages. Given state \(\{\omega, \sigma\}\), the entrepreneur decides how much to invest:

\[V(\omega, \sigma) = \max_{k' \in \Gamma(\omega, \sigma)} J(k', \omega, \sigma),\]

where \(V : \Omega \times \Sigma \to \mathbb{R}_+\) is bounded and \(\Gamma(\omega, \sigma) : \Omega \times \Sigma \rightrightarrows \mathbb{R}_+\) is a correspondence with a non-empty compact image.\(^{22}\) With \(k'\) as given, the entrepreneur decides the best way to finance investment, and whether to consume

\[J(k', \omega, \sigma) = \max_{c,a',s'} c + \beta E_{\sigma} V(\omega'(\sigma'), \sigma')\]

subject to the following constraints

\[\begin{align*}
    c + a' + k' & \leq \omega + \xi p(k'; \sigma)s', \\
    c & \geq 0, \\
    a' & \geq -\alpha, \\
    s' & \in [0, 1],
\end{align*}\]

where

\[\omega'(\sigma') = \pi(k'; \sigma') + (1 + r)a' - q(k', \sigma')s'.\]

\(^{22}\)See the Appendix for a derivation of \(\Gamma(\omega, \sigma)\) as well as a detailed discussion of the recursive formulation.
for all \( \sigma' \in \Sigma \). We denote by \( \psi^x : \Omega \times \Sigma \to \mathbb{R} \) the resulting policy functions for \( x \in \{ c, k', a', s' \} \). We also obtain a law of motion for net worth, \( \psi^\omega (\omega, \sigma, \sigma') \).

### 3.1.4 Entry, exit, and firm distribution

Each period a fraction \( \kappa \) of entrepreneurs exit and an identical measure of entrants replace them. The net worth of exiting entrepreneurs is redistributed among the new entrepreneurs according to the joint distribution \( G(\omega, \sigma) \) over net worth and productivity. Entering entrepreneurs must incur a fixed entry cost, \( f_e \), that takes the form of an initial investment necessary to start up production. We set \( f_e \) such that all new entrepreneurs find it profitable to enter.\(^{23}\)

Let \( F_t (\omega, \sigma) \) be the cumulative distribution function of firms defined over net worth and productivity, with support \( \Omega \times \Sigma \). The borrowing constraint indeed ensures that a firm retains positive value at all dates, and thus liquidation is never optimal.

To obtain the law of motion for the firm distribution, we combine the exit and entry dynamics with the law of motion for net worth,

\[
F_{t+1}(\omega', \sigma') = \kappa G(\omega', \sigma') + (1 - \kappa) \sum_{\sigma \in \Sigma} \mu(\sigma' | \sigma) F_t (\phi(\omega', \sigma, \sigma'))
\]

for all \( \omega', \sigma' \), where \( \phi(\omega', \sigma, \sigma') = \sup \{ \omega \in \Omega : \psi^\omega (\omega, \sigma, \sigma') \leq \omega' \} \).

### 3.2 The representative household

The representative household is infinitely-lived and values non-negative consumption \( c^h_t \) and labor \( l^h_t \) sequences according to

\[
\sum_{t=0}^{\infty} \beta^t u \left( c^h_t, l^h_t \right)
\]

where \( u \) is a utility function with the standard properties and \( \beta \) is the intertemporal discount factor of the household, which is set equal to \( \beta_e (1 - \kappa) \), so both entrepreneurs and the representative household have the same discount factor.

The household budget constraint is

\[
c^h_t + a^h_t \leq w_t l^h_t + (1 + \bar{r}) a^h_{t-1} + \int_{\Omega \times \Sigma} \left( s^h_t (\omega, \sigma) q (k_t(\omega), \sigma) - s^h_{t+1} (\omega, \sigma) p (k_{t+1}(\omega), \sigma) \right) dF_t + T_t
\]

where \( a^h_t \) are the financial assets held by the household, \( s^h_{t+1} (\omega, \sigma) \) the shares held of firms with

\(^{23}\)For the sake of exposition, we do not explicitly write out the underlying bequest system across entrepreneurs. To be clear, there is no equilibrium condition associated with entry. The rationale for the fixed cost is to close the balance sheet of the firm, by accruing the entrepreneur’s rents to the initial investment.
net worth $\omega$ and state $\sigma$, and $T_t$ transfers from the government.

The optimality conditions from the household are used to derive the wage as well as the price of shares and bonds:

$$w_t = -\frac{u^l_t}{u^c_t},$$

$$1 + \tilde{r} = \left(\frac{u^c_{t+1}}{u^c_t}\right)^{-1},$$

$$p(k_{t+1}(\omega), \sigma) = (1 + \tilde{r})^{-1}E \{ q(k_{t+1}(\omega), \sigma') | \sigma \}.$$

Here $u^c$ and $u^l$ denote marginal utility of consumption and marginal disutility of work, respectively. There is no risk premium in the equity price since the representative household is perfectly diversified and there is no aggregate uncertainty.

### 3.3 Government and stationary equilibrium

Finally, the government collects tax revenues from corporate taxes and rebates them as transfers to the household

$$\tau^c \int_{\Omega \times \Sigma} ( f(l_t(\omega, \sigma), k_t(\omega, \sigma); \sigma) - w_t l_t(\omega, \sigma) - \delta k_t(\omega, \sigma) - r a_{t-1}(\omega, \sigma) ) dF_t (\omega, \sigma) \leq T_t.$$

Government policy $\tau^c, T_t$ is taken as given by all agents in the economy. The government budget constraint, together with market clearing, ensures aggregate resource constraints are satisfied.

Our focus in this paper is a stationary equilibrium with $F_t = F_{t+1}$ and constant aggregate consumption and output.

**Definition 1** A **stationary equilibrium** is a stationary distribution $F$, prices $\{ p, \tilde{r}, w_t \}$, policy functions $\{ \psi^a, \psi^c, \psi^s, \psi^k, \psi^\omega \}$, and household allocations $\{ c^h, l^h, a^h, s^h \}$ such that policy functions solve the entrepreneur’s problem given prices and taxes, $F$ satisfies the law of motion (4), markets clear, and the household optimality conditions and government budget constraint are satisfied.

### 4 Net Savings and Equity

As simple as our model is, it can generate strong demand for both net savings and equity. To understand how the model works, we first roll back the borrowing constraint and let the entrepreneur tap into as much debt or equity as needed. We then explore how the firm chooses
to finance itself as we vary the cost of equity relative to debt.

Consider first the case with $\xi = 1$. The Miller-Modigliani theorem applies and thus the capital structure of the firm is undetermined as the entrepreneur is indifferent between the two financing sources. If $\xi \neq 1$, then the risk-neutral entrepreneur will rely exclusively on the cheaper asset. For our case of interest, equity is relatively costly, $\xi < 1$, and thus the entrepreneur would finance investment exclusively with debt.\footnote{If $\xi > 1$, then the return on equity is lower than the return on debt (and thus savings). The entrepreneur would engage in arbitrage in this case: she would raise as much funds as possible from shareholders and simply save the proceeds.}

We now re-introduce the borrowing constraint for the case of costly equity, $\xi < 1$. At first pass this seems of little help to generate a demand for net savings and additional equity. Debt-holders require a lower return, and the entrepreneur prefers to finance fully with debt. Only if the firm is at debt capacity the entrepreneur would have to resort to equity for additional funding. Thus the firm would follow a “pecking order” among finance sources, where internal funds would be preferred to external funds and, among the latter, debt would be preferred to equity. We would observe most firms relying heavily on debt and resorting to equity issuance only if they are at debt capacity. No firm would carry financial assets without retiring as much equity as possible.

However, this argument misses a key observation: the entrepreneur’s problem becomes strictly concave, and thus risk considerations come into play, due to the interplay between the borrowing constraint and costly equity. Consider a firm following the pecking order described above to finance a given amount of investment. If the firm has a high net worth, investment can be financed by the firm’s own savings or debt. Thus the firm values an additional dollar of net worth at the risk-free return $1 + r$. A firm with low net worth, though, will hit debt capacity and will have to rely on equity. The higher finance cost not only reduces the value of the firm, but it also increases the value of an additional dollar of net worth: now one dollar allows the firm to save the expected return to equity, $(1 + r)/\xi$. Thus the firm values a dollar more when it has low net worth than when it has high net worth. Indeed, the differences in the value of an additional dollar get larger once the full dynamic program is considered. A firm with low net worth will find a large share of its cash flow committed to shareholders and will build its net worth only slowly, and thus may need to repeatedly tap into equity financing. Hence, one more dollar of net worth allows the firm not only to save equity issuance in the present period but also in future periods, and thus the firm values the additional dollar well above $(1 + r)/\xi$.

We are now in place to tackle the main mechanism in the model. Firms will strive to accumulate net financial assets for precautionary reasons, that is, to avoid finding themselves at debt capacity at future dates. Simultaneously firms will be willing to pay an insurance premium for equity if dividend distributions and net worth are positively correlated. In fact, firms will find...
it useful to fund additional financial asset holdings with equity revenues. This large deviation from the pecking order is indeed crucial for the model to match the high levels of net financial assets observed in the 2000s.

The precautionary motive resembles closely the one found in models of household finance. Firms want to build their net worth up rapidly in order to decrease the probability that they find themselves at debt capacity at future dates. Indeed, the entrepreneur delays any distributions to herself until the firm can self-finance at all future dates. Consider the first-order condition associated with the risk-free asset,

$$\lambda \geq \beta (1 + r) E \{ V' (\omega' (\sigma'), \sigma') | \sigma \}$$

with strict equality if the firm is not at debt capacity, \( a' > -\alpha \), where \( \lambda \) is the Lagrangian multiplier associated with the budget constraint and thus the marginal benefit of net savings. The first-order condition associated with consumption implies that \( \lambda \geq 1 \). Using the envelope theorem, we can rewrite the previous first-order condition as

$$\lambda \geq E \{ \lambda' | \sigma \}$$

where we have also used the condition \((1+r)\beta = 1\). Thus \( \lambda \) is a supermartingale, and \( \lambda \) converges almost surely to its lower bound. Whenever the firm is at debt capacity, one more dollar would allow it to relax the borrowing constraint, and thus it is more valuable, \( \lambda > 1 \). Thus the firm seeks to save as much net worth as possible in anticipation of states of the world where the debt capacity will bind. Only when there is zero probability that the borrowing constraint is ever binding, that is, when

$$\lambda = E \{ \lambda' | \sigma \} = 1$$

for all \( \sigma \in \Sigma \), there will be distributions to the entrepreneur.25 Financial assets allow firms to build up net worth over time without introducing further risk or incurring decreasing returns to capital.

We turn now our attention to the demand for equity. In particular, we are interested in isolating the conditions under which the demand for both net savings and equity can coexist. Consider the first-order condition associated with equity issuance,

$$\xi p(k', \sigma) \lambda = \beta E \{ V' (\omega' (\sigma'), \sigma') q(k', \sigma') | \sigma \},$$

where we have assumed positive issuance, \( s' > 0 \), and dropped the arguments where there is no

25There exists a level of financial assets, \( a^* \), such that the net return \( ra^* \) is sufficient to cover all finance needs in all states. Thus the entrepreneur can maintain the financial asset position \( a^* \) with probability one and consume the excess cash flow.
confusion possible. We can rewrite this expression in terms of the covariance,

$$\xi_p(k', \sigma) \lambda = \beta E \{ V'(\omega'(\sigma'), \sigma') \} E \{ q(k', \sigma') \} + \beta \text{Cov} \left( V'(\omega'(\sigma'), \sigma'), q(k', \sigma') \right).$$

Now assume that the firm is not at debt capacity, $a > -\alpha$, and thus the last dollar of equity revenues is effectively funding the financial assets of the firm. Combining the pricing equation for equity obtained from the household’s problem with the first-order condition for net savings $a$, we obtain

$$E \{ V'(\omega'(\sigma'), \sigma') \} E \left\{ \frac{q(k', \sigma')}{\xi_p(k', \sigma)} \right\} = \xi^{-1}(1 + \hat{r}) > 1 + r.$$

Clearly, both optimality conditions can be satisfied simultaneously only if

$$\text{Cov} \left( V'(\omega'(\sigma'), \sigma'), \frac{q(k', \sigma')}{\xi_p(k', \sigma)} \right) < 0.$$

This requires both that the value function $V$ is strictly concave, and shareholder payouts are positively correlated with net worth.

As discussed earlier, the concavity arises naturally in our model due to the borrowing constraint and the cost of equity. The positive correlation of equity payouts with net worth makes equity valuable to the firm due to its insurance properties. Namely, since shareholders payouts decrease when the firm has low cash flow or losses, equity delivers some financial relief to the entrepreneur exactly in the states where the firm will have lower net worth and thus is likely to face a higher finance cost. As a result, entrepreneurs are willing to pay an additional cost for equity—akin to an insurance premium. Therefore, in the calibration of the model we will assume that shareholder distributions and cash flows are positively correlated. This assumption has strong empirical support. Since the seminal work of Lintner (1956) who showed that earnings were the most important determinant of any change in dividends, a number of other studies have confirmed this result (see Fama and Babiak (1968), Fama and French (2001), Denis and Osobov (2008), among others, and Allen and Michaely (2003) for a comprehensive overview of this literature). Skinner (2008) generalized these findings by showing that corporate earnings now drive total firm payouts – dividends and repurchases – providing the basis for modeling approach we adopted in this paper.

It perhaps remains counterintuitive that firms find it useful to issue equity, at a cost, to insure themselves against the cost of equity financing in future periods. The key is that one additional dollar available for a firm with low net worth allows the firm to reduce equity reliance in the present and future periods. A firm with low net worth has no choice but to commit a large share of its cash flow to shareholder distributions in order to raise enough equity finance. Thus the firm finds itself crawling very slowly from debt capacity and resorting to equity repeatedly. One more dollar of net worth allows the firm to reduce equity issuance in the present period,
which in turn frees additional cash flow in the next period and again reduces equity outstanding in that period, and so on.

The logic of the model highlights the idea, emphasized by Hennessy and Whited (2005), that it is essential to view the capital structure decision in the context of a fully specified dynamic problem. Firms with a moderate level of net worth may have no chance of being at debt capacity next period or, more generally, in the short term. A model with a short horizon would need huge cash flow shocks in order to induce demand for equity among firms with some net savings. In a fully forward-looking model, even firms that can self-finance in the short term strive to accumulate further NFA and value the insurance properties of equity.

There remains the question, though, of whether our model can generate the observed positive net savings among firms that rely on equity. We answer this question with a quantitative evaluation of our model.

5 Calibration

We turn now to the core question of the paper: can our model generate positive NFA as observed for the period 2000-2007? As the model is taken to the task, we have to take a stand on two crucial aspects of the calibration. First, we have to quantify the relative cost of equity to debt. Second, we have to decide which moments to target with the productivity process. The remaining parameters regarding technology and entry are set to standard or straightforward values.

5.1 The fiscal cost of equity

We start with the relative cost of equity, ξ. We choose to base our calibration on fiscal considerations alone. There is no question those are significant and, more importantly, can be observed and quantified reliably from statutory rates and estimates from the public finance literature. We recognize that there are many factors affecting the costs and benefits of equity, e.g., floatation costs and agency problems, among others. However, it is not easy to quantify any of these factors. By focusing exclusively on fiscal considerations, we do not need to infer the equity markdown from the very same facts we seek to explain.

In the Appendix we derive the equity price households demand such that the after-tax return of debt and equity is equated. We account for dividend, capital-gains, and interest-income taxes, denoted τd, τg, and τi, respectively. We also need to take into consideration inflation as well as the split between dividends and capital gains for equity distributions. We then compute the
equity markdown by comparing the relative cost of equity and debt for the firm. The resulting markdown is
\[ \xi = \frac{(1 - \tau^d) \left( (1 - \tau^c) \tilde{R} - \gamma_a \right)}{(1 - \tau^i) \tilde{R} - (1 - \tau^g) \gamma_a}, \]
where \( \gamma_a \) is the growth rate of the equity price, and \( \tilde{R} \) is the interest rate on corporate debt, both in nominal terms. While the inflation rate does not enter the expression explicitly, both the nominal interest rate and the asset price growth rate vary with inflation.

We pick tax rates and interest rates representative of the period 2000-2007 for the U.S. Our choices are summarized in Table 2. Let us start with the corporate tax rate, \( \tau^c \). Due to investment not being expended for tax purposes, the corporate tax rate directly impacts the firm’s decision beyond its implications for the relative cost of equity. In the U.S. the corporate tax code specifies a flat tax rate of 34 percent from $335,000 to $10 million, and caps the marginal rate at 35 percent. The literature has an ample consensus on setting \( \tau^c = .34 \), and we follow suit.

Interest income is taxed at the federal income tax rate and thus varies across investors. Wealth, though, is heavily concentrated on the right tail, so we choose a tax rate close to the top rate, \( \tau^i = .34 \), which is slightly higher than estimates of the average marginal tax rate across households. The pre-tax nominal interest rate is set at 7 percent, while the inflation rate is at 2 percent. This results in an after-tax real rate of 2.5 percent.

Now we turn to the taxation of equity. The period 2000-2007 includes an important tax reform, the Jobs and Growth Tax Relief Reconciliation Act of 2003. The act equated dividend and capital gains tax rates at 15 percent, although there are several caveats. First, [Poterba (1987)] argues that the effective capital-gains tax rate is one fourth of the statutory rate, due to the gain referral and step-up basis at death. Second, some low-income households are subject to a lower dividend tax rate of 12 percent, while some other households may end up with a rate above 15 percent due to the alternative minimum tax. Third, some corporate investors do not pay dividend taxes, and the share of equity held by them has increased sharply over time. We note, though, that most estimates track closely the statutory rates in the decade of the 2000s. We thus decide to go with the statutory rates, \( \tau^d = .15 \) and \( \tau^g = .15 \). If anything, these rates

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26 The only tax rate explicitly included in the model is the corporate tax rate, as it affects investment decisions as well. We choose not to include the remaining taxes to keep our model as general possible and simultaneously save a substantial amount of notation. The Appendix provides the details for the full derivation.

27 Only small businesses and S corporations get a rate below 30 percent.

28 Poterba (2002) and NBER TAXSIM estimates tend to be just below 30 percent. Some bonds are tax-exempt, which reduces the average marginal tax rate. However, corporate bonds are always fully taxed.

29 For example, Poterba (2004) reports an average marginal tax rate on dividends of 18 percent. A similar situation arises regarding capital gains taxes.

30 For example, pension funds and other fiduciary institutions. See McGrattan and Prescott (2005) for a discussion.
Table 2: Taxes and interest rate — Baseline calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate tax $\tau^c$</td>
<td>0.34</td>
</tr>
<tr>
<td>Dividend tax $\tau^d$</td>
<td>0.15</td>
</tr>
<tr>
<td>Interest income tax $\tau^i$</td>
<td>0.34</td>
</tr>
<tr>
<td>Capital gains tax $\tau^g$</td>
<td>0.15</td>
</tr>
<tr>
<td>Pre-tax interest rate $\bar{R}$</td>
<td>0.07</td>
</tr>
<tr>
<td>Equity markdown $\xi$</td>
<td>0.82</td>
</tr>
</tbody>
</table>

are likely to overstate slightly the fiscal cost of equity.

5.2 Shareholder payouts

We assume that the present value of shareholder payout, $q$, is proportional to the firm’s cash flow and capital holdings at date $t + 1$, $\pi(k_{t+1}; \sigma_{t+1})$:

$$q(k_{t+1}, \sigma_{t+1}) = \frac{1}{1 - \beta} \pi(k_{t+1}; \sigma_{t+1}).$$

While admittedly ad-hoc, our specification aims to be parsimonious representation of shareholder payout policies. Total payout depends positively on the firm’s performance and tangible assets on the current period. As discussed in Section 4, the key property that makes equity valuable to the firm is the positive comovement of the firm’s net worth and shareholder payouts. In particular, we should note that the precautionary motive would remain even if we had specified equity as a full state-contingent contract; firms would still tolerate some residual risk because of the additional cost of equity $\xi < 1$. In our specification the linear relationship with cash flows further limits the insurance properties of equity. We later make sure that $\pi(k_{t+1}; \sigma_{t+1}) \geq 0$.

5.3 Technology, preferences, and entry parameters

We first discuss the parameters governing technology, which are set to match standard values in the literature. We postpone the calibration of the productivity process for the next subsection. We start with the parameterizations of the production function. We set $\eta$ to equate the entrepreneurs’ rents to the share of dividends over GDP, roughly 12 percent. Parameter $\nu$ is set to .3. Assuming entrepreneur rents are split 50-50 between capital and labor income accounts, this results in the standard total capital income share of 36 percent. We normalize the wage to

\[31\] The proportionality between payouts and $\pi$ is irrelevant as long as it is constant. Recall that $q$ is the present value per share, and thus any scaling of $q$ simply results in a change of units for shares. Our choice renders shares comparable to infinitely-lived assets.
\((\eta + \nu)/(1 - \eta - \nu)\) so employment is equal to net revenues. The depreciation rate is set to 10 percent.

For the household preferences we use an utility function of the form \(u(c - h(l))\) such that the labor supply is given simply by \(h'(l) = w\). This implies that the computation of the stationary equilibrium does not require specifying \(u\) and \(h\), and the wage rate can be normalized to 1 without any loss of generality. The discount rate \(\beta\) is pinned down by our earlier choice of the interest rate. The resulting value .96 is standard.

Next we turn to our calibration of the entry parameters. As we work with a stationary distribution, the entry rate in the model also serves as exit rate. In the data there is a slight upward trend in the number of firms, so the entry rate is slightly above the exit rate. We set our exit/entry parameter at 5 percent, closer to the exit rate in the data. For the net worth distribution of entrants we use a Pareto distribution with curvature parameter \(\varsigma\) equal to 1.3, which matches the relative capital holdings of entrants to incumbents. The entry cost \(f_e\) is set to match the 10th percentile of the distribution of NFA over capital\(^{32}\). Table 3 summarizes the parameter choices reported in this subsection.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor (\beta)</td>
<td>0.96</td>
</tr>
<tr>
<td>Entrepreneur rent (\eta)</td>
<td>0.14</td>
</tr>
<tr>
<td>Depreciation rate (\delta)</td>
<td>0.10</td>
</tr>
<tr>
<td>Capital elasticity (\nu)</td>
<td>0.20</td>
</tr>
<tr>
<td>Exit rate (\kappa)</td>
<td>0.05</td>
</tr>
<tr>
<td>Entry distribution (\varsigma)</td>
<td>1.3</td>
</tr>
<tr>
<td>Entry cost (f_e)</td>
<td>5.15</td>
</tr>
</tbody>
</table>

### 5.4 Productivity process

The productivity process is the other crucial aspect of the calibration. As our primary interest lies in the corporate finance decisions, it is important that we match the firms’ observed financing needs. Looking at the data, we identify two key drivers of the firms’ financing needs: negative cash flows and large investment expenses.

First, we observe that a substantial fraction of firms experience a negative cash flow. In any given year during the 2000-2007 period, about 25 percent of the firms in our sample had

\(^{32}\)Parameters \(\varsigma\) and \(f_e\) are matched to moments that require us to evaluate the full model, and thus it would be more correct to say that they are jointly calibrated with the productivity process. However the relationship between the parameters and the moments is very tight, so we feel comfortable linking them at this point.
a negative cash flow, defined as operating income before depreciation expenses. The transition rate from positive to negative cash flow is also quite high at 6 percent. Firms must balance the operating loss with either a decrease in assets or an increase in liabilities. In particular, cash flow shortfalls will provide the basis of the precautionary demand for financial assets.\footnote{Lins et al. (2010) document that CFOs use cash to guard against future negative cash flow shocks. Lines of credit, due to financial covenants, are not a good substitute, as documented by Sufi (2009).}

Second, firms occasionally have opportunities to expand their operations, perhaps by acquiring a foundering competitor or by upgrading their production process because a new technology has become available. These opportunities often present themselves without any relationship to the contemporaneous cash flow of the firm and usually require investment expenditures that are larger than the firm’s net revenues. For the period 2000-2007, we find that about 22% of the firms with positive cash flow incurred investment expenditures in excess of their cash flow in a given year. Among those, more than half had investment expenditures totaling 150% or more of their cash flow. Firms that want to take advantage of these opportunities need to finance their increase in assets without having the benefit of an immediate increase in cash flows.

Unfortunately, we find that the standard specification used in the literature does not allow either for operational losses or for forward-looking investment opportunities and thus does not generate a realistic level of financing needs. Under the usual autoregressive process, firms’ investment is driven by contemporaneous positive productivity shocks. Investment can then be easily financed from the firm’s own net revenues, since the latter also increase with the productivity shock. In short, it is quite easy for firms to self-finance under the usual productivity specifications, as financing needs arise only when the firm is experiencing a cash-flow windfall.

We instead propose a productivity process that directly incorporates the possibility of operational losses and investment opportunities, and it is thus capable of generating realistic levels of financing needs in the model. More precisely, productivity is modeled as a ladder where investment opportunity shocks lead a firm to move up the ladder, while operational losses lead a firm to drop off the ladder. We assume productivity takes one of \( n \) levels, \( \{z_1, z_2, \ldots, z_n\} \). We capture operational losses with state \( n = 1 \), setting \( z_1 = 0 \), so for simplicity there are zero net revenues in that state, and cost expenses \( c^I(k, z_1) \) are such that

\[
\pi(k, z_1) = 0
\]

for all \( k \). Note that this still implies that a firm experiencing operational loss has a negative cash flow. We set \( c^I(z, k) = 0 \) for all other states and levels of investment, thus ensuring that net revenues are non-negative everywhere but in state 1. The probability of operational losses is \( \phi > 0 \), which we assume to be i.i.d. across firms. Our specification for operational losses, while stark, is very parsimonious and keeps the portfolio decision in the firm’s problem simple. It also
implies that the no-default borrowing constraint is constant across firms, as it suffices to show that the firm can repay the outstanding debt in the event of operational losses.

Investment opportunities are modeled as a movement along the productivity ladder. Each period a fraction \( \iota \) of firms receive an investment opportunity shock. These firms will either transition to operational losses (with probability \( \phi \)) or will upgrade their productivity by one level. That is, a firm with productivity level \( z_t = z_i \) that receives an investment opportunity will transition to productivity level \( z_{t+1} = z_{i+1} \) next period with probability \( 1 - \phi \), or \( z_{t+1} = z_1 \) with probability \( \phi \). A firm without an investment opportunity remains at the same productivity level, \( z_{t+1} = z_i \) next period with probability \( 1 - \phi \), or \( z_{t+1} = z_1 \) with probability \( \phi \). \[34\]

Finally, we set productivity levels \( z_2, z_3, \ldots, z_n \) to be equally log-spaced, with growth rate \( \gamma_z \), that is, \( z_i = \gamma_z^{i-2} z_2 \). This guarantees that there is no hard-wired relationship between firm size and growth rates. \[35\]

Let us now discuss our numerical choices for the productivity parameters \( \phi, \iota, \gamma_z \). First we set the transition probability into operational losses \( \phi \) to 6%, which is the transition rate from positive to negative cash flows that we observe in the data. Since the operational losses shock is i.i.d., the calibration slightly underestimates the fraction of firms reporting a negative cash flow on a given date. \[36\]

For the investment opportunities, \( \iota \), we match the share of firms with investment expenditures exceeding their cash flow, about 22 percent of firms with positive cash flow. The resulting parameter value is \( \iota = .28 \). Both rates do not exactly coincide because investment is an endogenous variable in the model. In particular, firms with very low net worth may not be able to expand investment significantly due to their higher cost of capital.

Finally we set the growth rate of productivity, \( \gamma_z \), to reproduce an average growth rate in revenues of about 5 percent among firms with positive cash flow. The level \( z_2 \) is normalized to 1. We use six states for the productivity process, enough to generate a right tail in revenues, yet keep the computational time in check. \[37\]

Table 4 reports all the parameter choices concerning productivity.

\[34\] Firms at state \( z_1 \) automatically have an investment opportunity, so they transition to \( z_2 \) unless they suffer operational losses again. Firms with the highest productivity level, \( z_n \), do not receive further investment opportunities.

\[35\] In Appendix C we consider a simple extension that relates both firm size and growth rates to age. This can explain some interesting facts regarding NFA positions in the years immediately after an IPO, but has little effect on the overall fit of the model.

\[36\] We explored relaxing the i.i.d. assumption, which allows the model to match the hazard rate of operational losses as a function of size as well as the persistence of operational losses. The extension did not significantly alter the model’s aggregate implications, so we decided to keep the productivity specification to a minimal structure.

\[37\] We should note that our interest in firms’ financing choices necessitates the use of cash flows, as opposed to revenues or value added, when calibrating the productivity process. However, in Section 6 we show that with our calibration the model generates the distribution of revenues that is very close to the data. Overall, we believe our
Lastly, we want to emphasize that since we are targeting facts for publicly traded firms, we look only at firms in our model that have a positive probability of issuing equity. In our model firms with very high net worth can rely exclusively on self-financing for investment—and thus have no need to tap outside investors. We consider these firms to be private equity and drop them from our sample.\(^{38}\)

6 Results

6.1 Net financial assets, NFA

Does our model replicate the positive level of NFA observed during 2000-2007? Yes, it does. Table 5 reports the model predictions along with the corresponding data moments. Our model reproduces the large fraction of firms with a positive NFA position, 43.5 percent in the data versus 42.9 percent in the model. The model’s performance regarding the central moments is also very good. The median NFA to capital is just a tad below the data, and the mean is matched exactly.\(^{39}\)

The model does a remarkable job at matching the full distribution of NFA over K in the data. The standard deviation in the model and in the data is very close, so we are confident that our simple productivity process is capable of generating enough variation in corporate finance portfolios. Both the first and third quartiles are very close to the data.\(^{40}\) We overshoot the 90th percentile, albeit not by a large margin.

Figure 3 presents the histogram of the NFA to capital as generated by the model. As in the data, the distribution is skewed to the right and features a long right tail, with a small number of firms having large positive NFA. Our calibration is broadly consistent with Midrigan and Xu (2010).\(^{38}\) Note the model’s sample includes all firms with debt. Thus the censoring from the model does not help to generate positive NFA in the sample. The fraction of firms dropped is usually very small, less than 5 percent.\(^{39}\) We compute the moments from a simulation of 50,000 firms drawn from the stationary distribution. To ensure consistency we treat the simulated data as we treated the data in Section 2.\(^{40}\) Recall we used the fixed entry parameter \(f_e\) to directly target the 10th percentile, although this has surprisingly little effect on the overall shape of the distribution.
Table 5: Model and Data - Net financial assets to Capital

<table>
<thead>
<tr>
<th></th>
<th>2000s</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>mean</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>median</td>
<td>-0.07</td>
<td>-0.08</td>
</tr>
<tr>
<td>$Pr(NFA &gt; 0)$</td>
<td>43.6%</td>
<td>42.9%</td>
</tr>
<tr>
<td>std dev</td>
<td>0.65</td>
<td>0.64</td>
</tr>
<tr>
<td>10pct</td>
<td>-0.51</td>
<td>-0.50</td>
</tr>
<tr>
<td>25pct</td>
<td>-0.31</td>
<td>-0.35</td>
</tr>
<tr>
<td>75pct</td>
<td>0.35</td>
<td>0.31</td>
</tr>
<tr>
<td>90pct</td>
<td>1.38</td>
<td>1.88</td>
</tr>
</tbody>
</table>

Figure 3: NFA to capital histogram, model

of firms having very large NFA holdings relative to their productive assets. The model, though, does not generate a left tail and the overall distribution resembles a power law distribution. The reason is twofold. First, all firms have the same borrowing constraint and thus all firms close to the borrowing constraint have very similar NFA positions. Second, firms are at or close to the borrowing constraint only if they suffer operational losses or receive an investment opportunity while their net worth is very low. In both cases, they end up with similar low investment levels. Both factors combine to create a large mass of firms with very similar NFA to capital ratios in the lower end. We are thus not able to generate enough dispersion among firms that rely heavily on debt.

We should emphasize that our model can rationalize the corporate sector as a net lender
only through the mechanism highlighted in Section 4. No productivity process would generate positive NFA if we were to drop the borrowing constraint or the positive covariance between equity payouts and cash flows in the model. Without a borrowing constraint firms would finance only with debt, as it is the cheaper finance source. Without equity payouts providing partial insurance, only firms at debt capacity would resort to equity, and we would not observe firms with positive NFA actively relying on equity. We show below that the latter is an important feature of the data. Conversely, if equity had no cost and provided full insurance, all firms would spurn debt.

Quantitatively, though, the key to the model’s fit is our specification for productivity. After exploring several alternatives, we realized that it is necessary to generate realistic levels of financing needs in the data in order to match the level and dispersion of NFA. Motivated by the data, we modeled operational losses and investment opportunities as the two key drivers of the firms’ demand for finance. We imposed a minimal structure with a very parsimonious specification and calibrated the parameters to the frequency of operational losses and large investment expenditures—so we did not target any moment of the NFA distribution. The fact that the model performs very well suggests that the link between financing needs and balance sheets is very tight, and that operational losses and investment opportunities effectively capture the relevant shocks for corporate finance purposes.

6.2 Non-NFA variables

Since our calibration focuses on the firms’ financing needs, we next check how the model performs in matching variables other than NFA. Table 6 reports various unconditional moments for investment, revenues, employment, cash flow and equity in the model and data. Panel (a) reports the mean of a given variable relative to the mean capital, both in the model and in the data, while panel (b) does the same for standard deviations. Panel (c) presents the autoregressive coefficient for various variables. While none of these moments were targeted in the calibration, the model does a reasonable job in matching them.

Model’s performance for investment is very good: the persistence of investment generated by the model is very close to that observed in the data. The mean and standard deviation of investment predicted by the model are somewhat higher than in the data. We show below that this is mainly due to high investment rates by the unconstrained (positive NFA) firms.

Since our process for productivity is admittedly non-standard, it is important to check its implications for variables that are typically used in the literature to calibrate productivity process, such as employment and total revenues. Our model predicts firm-level employment that is highly persistent and dispersed, with an autocorrelation coefficient of 0.98 and a log-employment standard deviation of 1.32. These moment match almost exactly those in the data. Figure 4
Table 6: Model and Data - Other variables

<table>
<thead>
<tr>
<th></th>
<th>ratio of means</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
<td></td>
</tr>
<tr>
<td>investment/K</td>
<td>0.17</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>revenues/K</td>
<td>0.67</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>(ΔBE)/K</td>
<td>0.16</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>ratio of std dev</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
<td></td>
</tr>
<tr>
<td>investment/K</td>
<td>0.21</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>revenues/K</td>
<td>0.72</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td>(ΔBE)/K</td>
<td>0.37</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>cash flow/K</td>
<td>0.25</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>log employment</td>
<td>1.32</td>
<td>1.27</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>autocorrelation</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
<td></td>
</tr>
<tr>
<td>investment</td>
<td>0.62</td>
<td>0.74</td>
<td></td>
</tr>
<tr>
<td>revenues</td>
<td>0.98</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>ΔBE</td>
<td>0.32</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>cash flow</td>
<td>0.55</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>employment</td>
<td>0.98</td>
<td>0.99</td>
<td></td>
</tr>
</tbody>
</table>

reports the histogram for employment in the model. As in the data, the employment distribution is skewed to the right, although we fail to generate a left tail. Total revenues in the model are as persistent as employment, but are less dispersed, with the standard deviation of 0.72. Importantly, these moments are again very close to those found in the data. As for the mean revenues, the model somewhat underpredicts their values relative to the data.

Cash flow is another variable that determines firm’s financing needs. The model matches properties of cash flow in the data quite well. It predicts cash flow that are significantly less volatile and less persistent than total revenues, in line with the ranking observed in the data.

The model also makes a number of predictions for equity. We measure equity in the model at the book value (BE) from the firm’s balance sheet. This corresponds the closest to book equity measure we have in the Compustat’s balance sheet statements. To obtain flows to equity we compute changes in book equity positions (ΔBE) between period $t$ and $t-1$ in both the model and data. Based on this measure, the model predicts substantial reliance on equity financing with the average amount raised being around 16 percent of the average capital. These flows to book equity are not very persistent, with the autoregressive coefficient of 0.32, and not very

\footnote{It is equal to the total stockholders’ equity.}
volatile, with the standard deviation of 0.37. In the data, the amounts raised are somewhat smaller, less volatile and less persistent.

6.3 Which firms have positive net savings?

We have seen the model does a good job at matching the distribution of NFA across firms in the data as well as the aggregate moments regarding several other variables such as investment, equity, and revenues. However, does the model predict the right joint distribution of these variables and NFA? To answer this question we revisit the model’s predictions conditional on NFA and compare them with the data.

Let us start with a quick look at the model predictions. Figure 5 plots the policy functions for NFA and capital, as function of net worth, for a firm in state $z_3$ without an investment opportunity. We have also included book equity, and the ratio between NFA and capital.

Firms with low net worth are net borrowers and their investment is low. As a result, these firms also have low book equity and revenues (not shown). Their smaller scale reflects their higher cost of external finance. Note that even firms with very low net worth are not at the borrowing constraint, that is, $a > -\alpha$. As discussed in Section 4, this allows the firm to improve its net worth in the event of operational losses, hedging their investment.

\footnote{State $z_3$ roughly corresponds to the median productivity in the model. All the policy functions are qualitatively very similar across states. We only display the lower half of the support for net worth where most firms lay.}
As firms build their net worth, they increase both capital and NFA roughly at the same pace, and eventually become net savers. The latter clearly have more capital and book equity, and thus more revenue. Since both NFA and capital are increasing as a function of net worth, it is an open question whether NFA to capital increases with net worth. The lower-right plot displays the ratio of NFA to capital, which is clearly increasing and turns positive for sufficiently high levels of net worth. Summarizing, the model predicts that higher-NFA firms have higher revenues, investment, and book equity.

Figure 5: Policy functions

Figure 6 plots the previous policy functions together with the policy functions of a firm in the same productivity state \( z_3 \) but with an investment opportunity available. This allows us to see how firms adjust their positions, and how this adjustment is different depending on whether the firm has enough net worth to have accumulated net savings or not. Not surprisingly, firms react to an investment opportunity by increasing investment, drawing from their net savings or borrowing, and possibly raising some additional equity. Note how firms with low and high net worth differ in their capacity to take advantage of the investment opportunity. Firms with high net worth are capable of boosting their investment further as they have more spare borrowing capacity or even net savings available. This translates into higher revenue growth rates for firms with positive net savings. The latter also build their net worth much faster, which translates into higher equity growth as well.

Table 7 compares the quantitative predictions of the model with the data regarding the characteristics of firms with positive net savings. We computed the mean and median of several variables separately for firms with positive and negative NFA: the table reports the ratio. Let us first focus on investment, revenues, and book equity. In the model as in the data, firms with
positive net savings are significantly larger and more valuable, and invest more. Quantitatively, the model slightly overshoots the differences in investment, as the NFA position is a very strong proxy for whether a firm is financially constrained, while we suspect in the data there would be other factors influencing the firms’ cost of capital and thus mixing the split due to NFA. The model is spot on regarding revenues, providing further evidence that our calibration of the productivity process is indeed tuned in with the data. The model also tends to understate the differences in book equity values.

Table 7: Model and Data - Conditional moments

<table>
<thead>
<tr>
<th>Ratio X</th>
<th>NFA &gt; 0 to X</th>
<th>NFA ≤ 0 :</th>
<th>Model means</th>
<th>medians</th>
<th>Data means</th>
<th>medians</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>investment/K</td>
<td>1.868</td>
<td>1.242</td>
<td>1.282</td>
<td>1.308</td>
<td></td>
</tr>
<tr>
<td></td>
<td>revenues/K</td>
<td>1.408</td>
<td>1.666</td>
<td>1.314</td>
<td>1.615</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BE/K</td>
<td>2.190</td>
<td>1.800</td>
<td>2.999</td>
<td>2.583</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(∆BE)/K</td>
<td>1.757</td>
<td>2.460</td>
<td>1.214</td>
<td>2.911</td>
<td></td>
</tr>
</tbody>
</table>

Revenues growth rate:
NFA > 0 | 0.085 | 0.097 | 0.123 | 0.084
NFA ≤ 0 | 0.067 | 0.000 | 0.106 | 0.056

The bottom panel of Table presents mean and median revenues growth rates in the model and in the data. The model predicts that positive NFA firms grow faster – a pattern that also
characterizes firms growth rates in the data. Similarly, firms with positive NFA see their equity increase at a more rapid pace (last line of the top panel). As discussed before, investment shocks are key in the model to generate these differences. That said, operational losses and the inherent non-linearities of the law of motion for net worth also contribute to the disparity in growth rates and equity adjustment.

We thus find that firms with positive net savings in the model look very much alike in the data. We view this as strong evidence that we captured the key determinants of NFA positions in the data with a very parsimonious model.

6.4 Evidence on balance-sheet linkages

Our model is akin to a portfolio problem where the entrepreneur chooses the firm’s capital, net savings, and equity given their joint returns. We have seen the model does a good job at reproducing the observed distribution of these positions, and their correlation with revenues and other non-financial variables. Here we turn our attention to how firms adjust their asset and liability positions in the model and in the data, gathering support to the linkages implied by our theory. To this end we decompose the movements in the firm’s balance sheet. The latter can be written as $BE_t = NFA_t + K_t$, where $BE_t$ as before refers to book equity in period $t$, while $K_t$ is period $t$ capital stock. The balance sheet identity shows that increases in equity will have their counterpart in investment in physical capital and/or in acquisitions of financial assets.

While the relationship between firm value and investment is well-known, our model emphasizes the relationship between equity and NFA. In section 4 we showed that firms facilitate precautionary accumulation of financial assets by raising equity. Such strategy allows them to increase internal funds available to the firm when operational losses strike. We next examine two decompositions of the adjustments in the balance sheet that allows us to evaluate this aspect of the model performance.

We first decompose the changes in book equity in the model into changes in NFA and changes in $K$ as follows:

$$Var(\Delta BE) = Cov(\Delta BE; \Delta BE) = Cov(\Delta BE; \Delta NFA) + Cov(\Delta BE; \Delta K),$$

where $\Delta$ refers to the first-difference of a variable between period $t$ and $t-1$. Panel A of Table 8 reports how the different components contribute to the volatility of book equity changes in the model and in the data. To control for firm size, in our calculations we scale the changes in all variables by the beginning-of-period $t$ capital stock.

Table 8 shows that both changes in NFA and changes in $K$ covary positively with changes in book equity in the model. That is, a unit increase in firm’ book equity is split between net
investment and an increase in NFA, with a larger fraction allocated towards an accumulation of physical capital. Importantly, these predictions are consistent with the data, where among Compustat firms in the 2000s each dollar increase in book equity coincided with about two-thirds of a dollar increase in net investment, and a one-third of a dollar increase in net financial assets.

Book equity increases are due to both retained earnings and share issuances. The latter provides a particular direct observation on the use of costly equity to fund financial assets. The model predicts a positive correlation between changes in the NFA and equity issuances equal to 0.16 – almost spot on with the correlation of 0.17 in the data. The correlation is low but not insignificant: while retained earnings are the main source, equity issuances are used to build financial assets actively.

We can gain some further insights into the joint dynamics of NFA and investment by considering an alternative decomposition of book equity as the sum of all covariances of its components:

\[ Var(\Delta BE) = Var(\Delta NFA) + Var(\Delta K) + 2Cov(\Delta NFA; \Delta K). \]

Panel B of Table 8 presents each component of this decomposition scaled by \( Var(\Delta BE) \). As before, the changes in all variables are measured relative to the beginning-of-period capital stock to account for firm size.

The model predicts a negative covariance between investment and changes in NFA. Consistent with this prediction, the covariance is also negative in our sample of Compustat firms.

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Table 8: Model and Data - Decompositions

<table>
<thead>
<tr>
<th>Decomposition 1</th>
<th>Cov(ΔBE,ΔNFA)</th>
<th>Cov(ΔBE,ΔK)</th>
<th>Cov(ΔBE)</th>
<th>Cov(ΔBE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.352</td>
<td>0.648</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>0.140</td>
<td>0.858</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Decomposition 2</th>
<th>Var(ΔNFA)</th>
<th>Var(ΔK)</th>
<th>Cov(ΔNFA,ΔK)</th>
<th>Var(ΔBE)</th>
<th>Var(ΔBE)</th>
<th>Cov(ΔBE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.562</td>
<td>0.858</td>
<td>-0.421</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>0.364</td>
<td>1.082</td>
<td>-0.444</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

43Equity issuance is derived from the firms’ cash flow statements and is equal to firms’ sales of common and preferred stock less purchases of common and preferred stock. This measure captures proactive changes in firms’ equity outstanding and thus corresponds more closely to the model. We also considered changes in the number of common shares issued and common shares outstanding, and also found a positive correlation with changes in NFA. A formal regression analysis confirmed the significance of these correlations.
matching its value in the model almost exactly. The decomposition also shows that net investment and changes in book equity are about equally volatile, while changes in NFA exhibit the lowest volatility.

These interactions between the components of firms’ balance sheets are to a large extent shaped by firms’ cash flows variations. When cash flows are high, all else equal, firms are better equipped to satisfy their financing needs. However, when operational losses strike, firms’ cash flows turn negative, leading them to rely heavily on external finance. Indeed, in the model firms in operational losses have no retained earnings and must issue outside equity. At the same time, firms with profits also increase their book equity, although through retained earnings, rather than outside equity issuances. Overall, the model predicts a positive correlation between changes in book equity and cash flow, although the components of book equity driving this positive correlation are quite distinct for the firms at the opposite ends of the cash flow distribution. We find that in the data such correlation is also positive equal to 0.34 (compared to 0.30 in the model). Furthermore, firms with losses and profits differ significantly in their reliance on outside equity. We find that both in the model and in the data firms in losses are more likely to issue outside equity than firms in profits.\footnote{The model, however, underpredicts the size of these probabilities.}

Overall, we interpret all this evidence as providing support for the linkages that the model puts forward.

7 The cost of equity and corporate net savings in the 1970s

According to our theory, the relative cost of equity to debt is the key determinant of the corporate sector net savings position. The 1970s provide us with an opportunity to contrast the quantitative predictions of the model under higher equity costs. There is a broad consensus that the fiscal and regulatory burden on equity was substantially higher in the 1970s. As the cost of equity relative to debt increases, raising equity to fund financial assets becomes more expensive. This makes firm’s finances more volatile—leading firms to cut investment and slow down the accumulation of net financial assets. The model prediction is born out in the data. As reported in Section\footnote{2} the corporate sector was a net debtor in the 1970s, with much fewer firms with positive NFA positions.

To derive the quantitative predictions of the model we need an estimate of the cost of equity in the 1970s. There have been two main forces behind the easing of the fiscal and regulatory burden on equity over the past 40 years. First, there were significant cuts in top marginal income tax rates in the 1980s and, starting in 2003, dividend income was taxed separately from income
and at a rate significantly below income tax rates. The second force has been emphasized by McGrattan and Prescott (2005), who argue that changes in regulation have had an important impact on the effective marginal tax rates by increasing the share of equity held by fiduciary institutions that pay no taxes on dividend income or capital gains. We recognize that there are a number of other changes over time that are likely to have affected the costs and benefits of equity. We choose to maintain our focus on fiscal costs, and dividend taxes in particular, as they provide us with a straightforward way to quantify the change in the relative cost of equity.

We rely on Poterba (1987) for effective tax rate estimates and set the dividend tax rate \( \tau_d \) corresponding to the 1970s at 0.28. Our baseline calibration for the 2000s used a tax rate of \( \tau_d = 0.15 \), the statutory rate for most of the period. There is no statutory rate for the 1970s, since dividend income was not taxed separately. The effective tax rate is instead estimated from marginal income tax rates and the distribution of income across households. Thus according to our calibration, the decline in dividend taxation during the 1980s and 1990s, up to the Jobs and Growth Tax Relief Reconciliation Act of 2003, halved the effective dividend tax rate. We recompute our markdown parameter for the 1970s with the higher tax rate, which renders equity more expensive relative to debt, \( \xi = 0.69 \). The estimates for the effective dividend tax in the 1970s from McGrattan and Prescott (2005) are even higher.

We keep all the remaining parameters of the model unchanged. We should mention that tax rates on capital gains and interest income were also slightly higher in the 1970s. However, the effect of these two tax rates in the relative cost of equity to debt is quite small, and we feel comfortable abstracting from them and focusing on dividend taxes.

Table 9 reports the moments from the distribution of NFA to capital from the model evaluated at \( \tau_d = 0.28 \) and compares them with the data. The shift toward debt is remarkably close to the data. The model predicts the mean NFA to capital in the 1970s at \(-0.10\) – a dramatic drop relative to the 2000s – while the corresponding number in the data is \(-0.12\). Similarly, just above 32 percent of the firms in the model have a positive NFA in the 1970s, down from the 44 percent in the 2000s, and very close to the 27 percent in the data in the 1970s.

For such a stark exercise as ours, the overall fit of the distribution is surprisingly good. The 10th percentile is within range, and the third quartile is very close to the data. The model correctly predicts that the overall dispersion in balance sheets is lower with the higher dividend tax. However, the observed standard deviation for the 1970s is significantly lower than predicted.

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45 The public finance literature has documented this shift extensively as early as in Poterba (1987). The latter change was brought up by the Jobs and Growth Tax Relief Reconciliation Act of 2003, which spurred a large literature that we cannot hope to summarize here.

46 See Rydqvist et al. (2011) for cross-country evidence on the role of tax policies on the decline of direct stock ownership by households.

47 See Poterba (2002) for further details and an updated time series.

48 For the exercise, we treat the borrowing constraint as a parameter. As the support for the net worth distribution changes, we also adjust the entry distribution to replicate the entrants’ characteristics in the 2000s.
by the model. This is mainly driven by a longer right tail in the model than in the data, as one can see by comparing the 90th percentile. At the same time the model overshoots the shift in the data regarding the median NFA to capital. Clearly, the distribution is too skewed to the right compared with the data.

Of course we did not expect the model to generate a perfect fit to the distribution of NFA in the 1970s given that many other changes that likely affect firms’ capital structure and investment decisions took place in the last 40 years, i.e. the incidence of operational losses have changed over time, as well as the occurrences of investment opportunities, etc.. However, this simple exercise illustrates the power of the mechanism in the model, as it shows how an increase in the relative cost of equity to debt is capable of reproducing the shift in firms’ NFA position from a net lender to a net borrower.

8 Conclusions

In this paper we documented the positive net financial position of the U.S. corporate sector and publicly-traded firms, in particular, in the last decade. To explain this fact we develop a model capable of generating simultaneous demand for equity and net savings, despite the fiscal advantages associated with debt. Our hypothesis emphasizes the risk considerations firms face in their capital structure decisions. In particular, demand for net savings is driven by a precautionary motive as firms seek to avoid being financially constrained in future periods. Simultaneously, firms value equity as it provides partial insurance against investment risk. We showed that our model can match quantitatively the net lender position of the corporate sector for the period of 2000-2007 and replicates the overall distribution of NFA during that period very well. We then provided extensive evidence in support of the main mechanism of the model.
Going forward, we have several questions in mind. First, we would like to set the changes in the saving behavior of the corporate sector in the broader context of the whole economy. For example, the rise of corporate net savings broadly coincides with a fall in the personal savings rate for U.S. households and, more recently, with an increase in the current account deficit. How are these phenomena related? What are the implications for aggregate savings and investment?

We would also like to provide an in-depth exploration of the forces behind an increase in corporate savings over the past 40 years. We have conducted a simple check of model mechanism by allowing for a change in the relative cost of equity to debt through the tax channel and showing that it can account for the changes in NFA over time. No doubt there are other costs associated with equity, and it is possible that they have changed over the last 40 years as well. Other factors, such as firm-level uncertainty, and the availability of investment opportunities, etc. have also changed over time. We hope to explore the relative importance of these various factors in future work.

\[\text{Examples are issuance cost, adverse selection, loss of control, etc.}\]
References


Niskanen, Mervi and Tensie Steijvers, “Managerial ownership effects on cash holdings of private family firms,” Working Papers, University of Eastern Finland 2010.


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Appendix

A Data

In this section we describe our data work in more detail. Our firm-level analysis uses the Compustat data set for the 1970-2007 period. As in Hennessy and Whited (2005), Gourio and Miao (2010) we use the following criteria to restrict our working sample. First, we focus only on U.S. firms whose capital is above 50,000 USD, whose equity is non-negative, and whose sales are positive. Second, we exclude firms that according to Standard Industry Classification (SIC) belong to finance, insurance and real estate sector (SIC classification is between 6000 and 6799); regulated utilities (SIC classification is between 4900 and 4999); and information technology and telecommunication services firms (SIC classification of 7370-7379, 4800-4899, and 3570-3579).

If the SIC classification is not available, we then use North American Industry Classification System (NAICS) to exclude the firms belonging to the above three industries. In particular, finance, insurance and real estate firms are identified as those under NAICS sector codes 52 and 53; utilities are those with NAICS sector code 22; while information technology and telecommunication services are identified with sector code 51. If both SIC and NAICS classification codes were missing, we allocated the firm into sectors according to its Global Industry Classification Standard (GICS). Thus, we excluded firms with GICS classification of 40 (Financials); 55 (Utilities); 45 and 50 (Information Technology and Telecommunication Services, respectively).

We begin by summarizing the properties of the aggregate net financial assets (NFA) to capital ratio in the Compustat data set. We construct NFA as the difference between financial assets and liabilities. Financial assets are composed of cash and short-term investments, other current assets, and account receivables (trade and taxes). Liabilities are computed as the sum of debt in current (due within one year) liabilities and other current liabilities; long-term debt; and account payable (trade and taxes). Capital stock is obtained as the sum of the firm’s gross value of property, plant and equipment; its total investment and advances; unamortized value of intangible assets; and total inventories. Equity is obtained as the value of common and preferred shareholders’ equity. All our variables of interest are measured as a ratio of capital.

50Detailed analysis of the size of the Compustat sample, its industry composition, computation of capital-output
Figure A1 summarizes our findings. It plots two ratios: the ratio of average NFA to average capital; and the ratio of median NFA to median capital. We must keep in mind that while the ratio of means gives us a measure of NFA to capital that is closest to the Flow of Funds calculation, it is also heavily influenced by the outliers – firms with large capital and/or NFA. It is easy to see from Figure A1 that these large firms are borrowing, on net, 25 percent of their capital, and that this level has remained relatively stable over time. Contrasting this with the Flow of Funds pattern for corporate NFA suggests several possibilities. First, small and medium-sized firms in the Compustat sample are behind the rise in NFA. We verify this conjecture by looking at the median NFA to median capital, which allows us to control for the outliers in both variables. Indeed the ratio of medians exhibits a clear upward trend over time.

NFA are rising steadily over time, although they do not turn positive in the 2000s as the Flow of Funds series does. Furthermore, when we explicitly contrast the levels of NFA to capital for small and medium-sized firms with those of large firms (see Figure A4), we find clear support for the idea that small and medium-sized firms are responsible for the increase in NFA to capital over the past 40 years.

The second possibility is that private firms, which are not in the Compustat sample, contribute to the increase in NFA to capital. The balance sheet data for private firms, however, is limited, but the recent work by Gao et al. (2010) suggests that these firms may not have contributed much to the rise in NFA to capital in the U.S. corporate sector. In particular, Gao ratios, and in-depth decompositions of NFA in both Flow of Funds and Compustat data, etc. are provided in the online Appendix available at http://faculty.arts.ubc.ca/vhmatkovska/research.htm.

Figure A1: U.S. non-financial, non-utilities, non-technology corporate NFA to K

For this reason, our preferred aggregate measure of NFA in the Compustat sample is the mean and median of the ratio, which we reported in Figure 2 in the main text.
et al. (2010) using a sample of U.S. public and private firms during the 2000-2008 period show that on average private firms hold less than half as much cash as public firms do. While this work primarily concerns firms’ cash holdings, rather then NFA, it is still informative since, as we show later, an increase in cash holdings and other short-term investments contributed the most to the increase in NFA.

Next, we investigate the gross positions of the firms and their components. Our goal is to isolate the components of financial assets and liabilities that are behind the rise in NFA. Figure A2 shows our results. Panel (a) of that figure presents median financial assets and their components such as cash and short-term investments, other assets, and account receivables, all as a ratio to median capital. Panel (b) presents median liabilities and their components such as short-term and long-term debt and account payables, also as ratios to median capital. From the figures it is easy to see that both assets and liabilities are rising over time, but the increase in assets is more pronounced. Most of the rise in assets is due to higher cash and equivalent holdings of U.S. firms. Other asset categories have been going up as well, but at a much slower pace. Finally, account receivables have declined from about 28 percent of the median capital level in the 1970s to less than 20 percent in the 2000s.

![Figure A2: Gross positions and their components](image)

On the liability side, long-term debt and account payables have both fallen over time, while short-term debt has shown a slight increase. Overall, these decompositions suggest a shift in firms’ balance sheets away from long-term assets and liabilities toward their short-term counterparts, but with the share of account receivables and payables in the short-term assets and liabilities.

Niskanen and Steijvers (2010) using a sample of private family firms in Norway find that an increase in firm size is associated with a decrease in cash holdings, a feature that we also document for NFA in our data set of public U.S. firms.
liabilities falling over time. In the model we do not distinguish the maturity structure of debt, and thus in what follows, we focus on the overall NFA position.

Which firms are behind the rise in corporate NFA? We turn to this question next and study NFA positions conditional on firm industry, size, age and entry cohort.

Figure A3 plots the ratio of median NFA to median capital in five industries: Agriculture and Mining; Manufacturing; Trade, Transportation and Warehousing; Services; and Construction. Several notable features of the data stand out. First, the increase in NFA to capital is characteristics of all industries, with the exception of construction, which shows a clear break in the series in the late 1980s-early 1990s. However, we have few observations for this industry and thus do not argue that this is a robust finding. Manufacturing and Services sectors, on the other hand, show the most pronounced increase in NFA over our sample period.

Second, there is some heterogeneity in the level of NFA to capital across industries. For instance, firms in the Trade, Transportation and Warehousing industry have consistently had the lowest level of NFA to capital during the 1970-2007 period. Firms in the Manufacturing sector (the largest sector in our sample) have exhibited one of the highest levels of NFA to capital throughout the sample period and, in fact, have seen their NFA positions turn positive in the 2000s. Finally, agriculture and mining, and services, demonstrate similar levels and dynamics in their NFA to capital ratios during the 1970-2007 period.
Overall, these results suggest that the rise of corporate net savings is characteristic of all industries.

Next we turn to firm-level characteristics and relate them to the rise in NFA. First, we study NFA for firms of different size, as measured by their employment level. Figure A4 reports the median NFA to capital ratio for different employment percentiles, separately for the 1970s and 2000s. It is easy to see that firms of all sizes were net borrowers in the 1970s. In the 2000s the relationship between the NFA to capital ratio and employment became clearly decreasing, with smaller and medium size firms turning into net creditors in that decade. At the same time, larger firms, while increasing their net savings a bit, have remained net debtors. A similar pattern applies at the industry level as well, especially for firms in manufacturing, services, and construction. The increase experienced by agricultural and mining firms, as well as the firms in trade, transportation and warehousing is characteristic of all firms in their respective industries, but is more muted.

Second, we study NFA to capital separately for entrants into Compustat and incumbents for each decade. Table A1 summarizes mean and median of NFA to capital for entrants and incumbents in the 1970s and 2000s. A firm is defined as an entrant in a given decade if it appeared in Compustat in any year of that decade.

Our results indicate that entrants tend to have higher NFA to capital ratios relative to

Figure A4: NFA to capital by firm size

These results are available from the authors upon request.
Table A1: NFA to capital: Entrants and incumbents

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incumbents, and that this tendency has become more pronounced over time. The majority of the differential in NFA to capital ratios between incumbents and entrants is due to the larger cash holdings and short-term investments of the latter. Over time, both cohorts have increased their holdings of cash and short-term investments, but entrants have done so at a significantly faster pace.

Are the differences between entrant and incumbent firms all due to their age differential, or is there an independent cohort effect? We use the number of years since the IPO as a measure of the firm’s age. Figure A5 plots median NFA to capital as a function of age, separately for the 1970s and 2000s.

![Figure A5: NFA to capital by firm age](image)

The figure suggests no association between NFA to capital with age in the 1970s, but the relationship turns negative in the 2000s. The fact that younger firms tend to save more relative to older firms in the 2000s is not surprising given our earlier finding of a negative association

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54 Only in the 1970s is the median NFA to capital ratio for entrants somewhat below that for incumbents.

55 These results are available from the authors upon request.
of the NFA to capital ratio with size, and the fact that age and size are positively correlated in our sample.

Finally, we investigate the role of all the factors discussed above jointly through a panel regression. In our benchmark specifications that pools firms in Compustat during the 1970-2007 period, we find that after accounting for employment and age, as well as industry and cohort fixed effects, NFA to capital has increased over time and significantly so.

\[56\]

B Model

B.1 Feasible investment

We first focus on the set of feasible investment choices, \(\Gamma(\omega, z)\), for any given values for \(\alpha\) and \(\omega^k\). Given a choice of investment \(k'\), there are enough resources to ensure non-negative consumption if and only if

\[ \omega + p(k', z) + \alpha \geq +k', \quad (A1) \]

that is, net worth, plus maximum equity issuance \(s' = 1\) and debt \(a' = -\alpha\), is sufficient to finance investment. The set \(\Gamma(\omega, z) \subset \mathbb{R}_+\) is thus all the investment \(k'\) such that \((A1)\) is satisfied for given values of \(\omega\) and \(z\).

To characterize the set, let

\[ \psi(k', z) \equiv p(k', z) - k'. \]

This is the maximum amount of equity funds available, net of investment. It can possibly be negative if the firm is not able to raise enough equity to finance all investment. We can then re-write \((A1)\) as

\[ \omega + \psi(k', z) \geq -\alpha. \quad (A2) \]

Function \(\psi(k', z)\) is not monotone in \(k'\). It is easy to check that \(\psi(0, z) = 0\), \(\psi(k', z)\) is increasing at first with \(k'\) and has a maximum at point \(\tilde{k}(z) > 0\) where

\[ p_k(\tilde{k}(z), z) = 1. \]

Function \(\psi(k', z)\) decreases from then on, eventually crossing zero again. Thus we can characterize the set of feasible investments as

\[ \Gamma(\omega, z) = \{k' \geq 0 : \psi(k', z) \geq -\alpha - w\omega\}. \]

\[56\] The time effect remains positive and significant for the 2000s when we include firm-level fixed effects in the panel regression. These results are available from the authors upon request.
Thus the set $\Gamma(\omega, z)$ is a closed interval, which guarantees that $\Gamma(w, z)$ is convex and compact, and the resulting recursive problem is well-behaved. However, for arbitrary choice of $\alpha$ and $\omega^b$, the set may be empty. In the next subsection, we ensure that there is always a feasible level of investment.

**B.2 No default condition**

We now derive the value of $\alpha$ that ensures there is no default with probability 1. This is equivalent to saying that at all times there is a feasible level of investment compatible with non-negative consumption—that is, $\Gamma(\omega, z)$ is not empty. Clearly $\Gamma(\omega, z) \subset \Gamma(\omega', z)$ if $\omega < \omega'$ and $\Gamma(\omega, z) \neq \emptyset$. We thus evaluate $\Gamma(\omega, z)$ at the lower bound $\omega^b$. Clearly $\Gamma(\omega^b, z)$ is not empty if

$$\max_{k' \geq 0 \psi(k', z) \geq -\alpha - \omega^b}.$$

The right-hand side does not depend on the state; thus, a sufficient condition for $\Gamma(\omega^b, z)$ to be non-empty at all states is

$$\bar{\psi} \equiv \min_{z \in Z} \max_{k' \geq 0} \psi(k', z) \geq -\alpha - \omega^b.$$  \hfill (A3)

Note that this is only saying that the firm must be able to raise enough equity and debt, net of investment, to finance its negative net worth position.

Recall that the lower bound $\omega^b$ is achieved at $-\kappa - R\alpha$. Evaluating (A3) with equality, and substituting for the lower bound, we obtain

$$\alpha = \frac{\bar{\psi} - \kappa}{R - 1}.$$

It is, of course, possible to set the borrowing constraint at arbitrary values lower than $\alpha$ and there would be no default with probability 1.

**B.3 Taxes and equity markdown**

We provide here the derivation of the equity markdown due to fiscal considerations. Consider a representative household that has access to risk-less corporate debt, which pays a nominal interest rate $\tilde{R}$, and perfectly-diversified equity holdings at price $P_t$, paying dividends $D_t$ and appreciating $(1 + \gamma_a)P_t$, in nominal terms. The household is subject to dividend, capital-gains, and interest-income taxes, denoted $\tau^d$, $\tau^g$, and $\tau^i$, respectively.

The first-order necessary condition associated with the decision to hold corporate debt is:

$$u_t^c = \beta u_{t+1}^c \frac{1 + (1 - \tau^i)\tilde{R}}{1 + \gamma_p}$$
where $\gamma_p$ is the growth rate of the nominal price level. The corresponding optimality condition to equity holdings is

$$P_t u_t^e = \beta u_{t+1}^e \frac{(1 - \tau_d) D_{t+1} + P_{t+1} - \tau^g (P_{t+1} - P_t)}{1 + \gamma_p}$$

where we decomposed equity payouts into capital gains and dividends. We also assume, for simplicity, that accrued, rather than realized, capital gains are taxed. Let $d$ and $p$ be the dividend and asset price, in real terms. Combining the above expressions we obtain the arbitrage condition between debt and equity:

$$\frac{1 + (1 - \tau^i) \tilde{R}}{1 + \gamma_p} = (1 - \tau^d) \frac{d_{t+1}}{p_t} + \frac{1 + \gamma_a (1 - \tau^g)}{1 + \gamma_p}.$$ 

The left-hand side is the after-tax return on debt; the right-hand side is the after-tax return on equity. Thus the equity price must satisfy

$$p = \frac{(1 - \tau^d) d}{1 - \tau^i \tilde{R} - (1 - \tau^g) \gamma_a \frac{1 + \gamma_a}{1 + \gamma_p}}$$

where we dropped time subscripts assuming a constant dividend-to-price ratio. This is the equity price that the household will demand from the firm to remain indifferent between investing in debt or equity. For the equity price to be positive, it must be that $(1 - \tau^i) \tilde{R} - (1 - \tau^g) \gamma_a > 0$. Otherwise the asset price appreciation would, by itself, pay a higher return than debt.

Next we derive the cost of debt and equity for the firm. The cost of debt, per dollar borrowed, is

$$\rho^b = \frac{1 + (1 - \tau^e) \tilde{R}}{1 + \gamma_p},$$

where we have taken in account that interest payments are deducted from the corporate-tax liabilities. Each dollar raised from equity must be repaid at rate $(D_{t+1} + P_{t+1})/P_t$ or, in real terms,

$$\rho^e = \frac{d}{p} + \frac{1 + \gamma_a}{1 + \gamma_p}.$$ 

The markdown $\xi$ is the relative cost of debt to equity for the firm, that is, $\rho^b = \xi \rho^e$. If $\xi < 1$, debtors demand a lower rate than shareholders, and we say debt has a fiscal advantage. Substituting the formulas for $\rho^b$ and $\rho^e$, as well as the equity price derived in A4, we obtain

$$\xi = \frac{(1 - \tau^d) \left((1 - \tau^e) \tilde{R} - \gamma_a\right)}{(1 - \tau^i) \tilde{R} - (1 - \tau^g) \gamma_a}. $$

\[57\] We need to specify the equity distributions in order to correctly compute their effective tax, as dividend income and capital-gains have been historically taxed at different rates.
Note the dividend $d$ cancels, so the markdown is independent of the unit of account of the shares. While the inflation rate does not enter the expression explicitly either, $\tilde{R}$ is the nominal interest rate and thus the relative cost of equity will vary with the level of expected inflation.

C Net Savings and Age

In Section 5 we assumed operational losses and investment opportunities were i.i.d. across firms. This parsimonious approach was ideally suited to test whether the model would replicate the observed distribution of NFA over capital. However, the i.i.d. specification is too stark to capture some interesting relationships in the data, such as that between NFA and firms’ age. We briefly discuss here a simple extension of our model that sheds light on the dynamics of the firms’ balance sheets in the immediate years after an IPO. Throughout this section we focus on the period 2000-2007.

As mentioned in Section 2, we find that firms have relatively large net financial holdings immediately after an IPO but adjust their NFA position downward in the following years. We also find that younger firms are more prone to suffer operational losses and grow faster than older firms. This is in line with previous research, which has found that younger firms experience higher, more volatile growth rates.

Are the dynamics of NFA related to the specific dynamics of younger firms? We put this question to our model. With a simple extension we are able to match the pattern of operational losses and employment growth in the years after an IPO. We then check the model predictions regarding NFA positions for those same years.

We extend the state space, defining two firm types, labeled young and old. Entrants start as young firms and transition to old with probability $\mu$. This simple specification allows us to introduce relationships with age without overburdening the model with a complete age-dependent specification. We preserve the productivity structure of the baseline model, but we allow the parameters to differ across young and old firms.

The probability of operational losses for young and old firms, $\phi_y, \phi_o$, is set jointly with the transition probability $\mu$ to match the decreasing pattern of operational losses between ages 1 to 10. We find that setting $\phi_y = .11, \phi_o = .055$, and $\mu = .15$ delivers the best fit. The transition probability value implies an average duration of youth of about 7 years.

We use the remaining productivity parameters for young firms to fit the positive relationship

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58 In the model we pin down the IPO to entry.
59 The stochastic transition also captures the fact that firms’ actual age is not pinned down by the initial IPO in the data.
60 The unconditional probability of operational losses in the model also lines up with the data, at around 6 percent.
between size and age. In particular, we target the ratio of the median employment of ages 3 and 14, roughly the 10th and 90th percentiles in the age distribution. In the data, the ratio is just below 4, 3.8 to be precise. The model reproduces the ratio by setting $z_2 = .7, \gamma^y_2 = 1.22$ and $\iota = .4$. While we get the gradient between age and size right, the model slightly overestimates the unconditional correlation between age and size: .4 in the model versus .27 in the data.\footnote{The correlation is reported for the period 2000-2007. Using the full sample, 1970-2007, we obtain a much higher correlation, .35.}

Overall, the calibration lines up with what we know about younger firms. Due to the increased chance of investment opportunities, young firms have a higher growth rate than old firms in the model. Their revenues are also lower and more volatile, as brought up by the higher incidence of operational losses. The remaining parameters are as reported previously.

We now check the model’s implications regarding NFA over the early years of a firm. We find that, in the model as in the data, younger firms initially deplete their financial assets. Given that we just put the question to the model, that is, we did not directly target the balance sheet dynamics in the calibration, this is strongly suggestive that the different productivity process for younger firms can indeed explain life-cycle facts regarding corporate finance choices.

What is the mechanism behind this result? The main determinant is the higher incidence of shocks on younger firms, both leading to operational losses and to investment opportunities. In response to both shocks, firms resort to available NFA, whether to finance the shortfall in cash flow or the expansion of investment, respectively. Note that equity is particularly valuable to younger firms, as they have relatively lower net worth and higher volatility. This combination makes the partial insurance properties of equity very attractive. Relying on equity would suggest that younger firms are able to accumulate NFA at a fast rate. However, the smaller cash flow and the higher rate of shocks act in the opposite direction, slowing down NFA accumulation.

Note that new entrants have relatively high NFA over capital, yet they actually start with much lower net worth than older firms. The reason is that younger firms have lower productivity initially and thus start with small investments. Thus their NFA positions tend to be large relative to their capital holdings.

While this simple extension of the model matches qualitatively the pattern of NFA as firms age, it somewhat underperforms quantitatively. In particular, firms in the model rebound too fast and start building up NFA earlier than in the data. The median ten-year firm in the model already accumulates positive NFA. Also the initial NFA to capital ratios are too low for both median and mean new entrants. We conjecture that the lack of fit here is due to the lack of adjustment costs in investment: firms, as they enter the stock market, usually carry large holdings resulting from the IPO, and only over time are the desired investment plans carried out.