How Much Would You Pay to Resolve Long-Run Risk?

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Abstract

Though risk aversion and the elasticity of intertemporal substitution have been the subjects of careful scrutiny when calibrating preferences, the long-run risks literature as well as the broader literature using recursive utility to address asset pricing puzzles have ignored the full implications of their parameter specifications. Recursive utility implies that the temporal resolution of risk matters and a quantitative assessment of how much it matters should be part of the calibration process. This paper gives a sense of the magnitudes of implied timing premia. Its objective is to inject temporal resolution of risk into the discussion of the quantitative properties of long-run risks and related models.

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1. Introduction

The long-run risks model of Bansal and Yaron (2004) has delivered a unified explanation of several otherwise puzzling aspects of asset markets.\footnote{See, for example, Piazzesi and Schneider (2007), Hansen et al (2008), Colacito and Croce (2011) and Chen (forthcoming). For elaboration and many additional references see Bansal (2008).} Since Mehra and Prescott (1985) posed the equity premium puzzle, it has been understood that the asset market puzzles are \textit{quantitative} and that an explanation must be consistent with observations in other markets and also with introspection. Imposing such discipline led Mehra and Prescott to exclude rationalization of the observed equity premium by levels of risk aversion exceeding their well known upper bound of 10. This bound on risk aversion has been largely respected since, including in long-run risk models. However, I suggest in this paper that quantitative discipline has been lax in another equally important aspect of the long-run risks model (henceforth LRR).

As a representative agent model, LRR has two key components - the endowment process and preferences. The former is modeled as having a persistent predictable component for consumption growth and its volatility; it will be described more precisely below. The representative agent has Epstein-Zin preferences (Epstein and Zin, 1989; Weil, 1990), which permit a partial disentangling of the elasticity of intertemporal substitution (EIS) and the coefficient of relative risk aversion (RRA). Denoting time \( t \) consumption by \( c_t \), continuation utilities \( U_t \) satisfy the recursion

\[
\begin{align*}
U_t &= \left\{ (1 - \beta) c_t^\rho + \beta \left[ E_t(U_{t+1}^\alpha) \right]^{\rho/\alpha} \right\}^{1/\rho} & \rho \neq 0 \\
\log U_t &= (1 - \beta) \log c_t + \beta \log \left[ E_t(U_{t+1}^\alpha) \right]^{1/\alpha} & \rho = 0
\end{align*}
\]

where \( \rho < 1 \), \( 0 \neq \alpha < 1 \) and \( 0 < \beta < 1 \). The utility of a deterministic consumption path is in the CES class with \( EIS = (1 - \rho)^{-1} \). Epstein and Zin (1989) show that \( 1 - \alpha \) is the measure of relative risk aversion for timeless wealth gambles and also for suitable gambles in consumption where all risk is resolved at a single instant, justifying thereby the identification \( RRA = 1 - \alpha \). The noted disentangling is possible because a decrease in \( \alpha \) increases risk aversion without affecting the attitude towards consumption smoothing over time given certainty. With these interpretations of \( \rho \) and \( \alpha \), parameter values in the LRR literature are specified with due care paid to evidence about the elasticity of substitution and
the degree of risk aversion. However, as is clear from the theoretical literature, $\rho$ and $\alpha$ affect also another aspect of preference in addition to the \textit{EIS} and \textit{RRA}. Clearly, judging the plausibility of parameter values requires that one consider their full quantitative implications for the nature of preference.

The above model of utility belongs to the recursive class developed by Kreps and Porteus (1978) in order to model nonindifference to the way in which a given risk resolves over time. For a simple example, suppose that consumption is fixed and certain in periods 0 and 1, and that it will be constant thereafter either at a high level or at a low level, depending on the outcome of the toss of an unbiased coin. Do you care whether the coin is tossed at $t = 1$ or at $t = 2$? According to the standard additive power utility model ($\rho = \alpha$), the time of resolution of the given risk is a matter of indifference. But not so more generally for recursive utility. For the specification (1.1), it is well known that early resolution of a given risk (here tossing the coin at $t = 1$) is always preferable if and only if

\[
(1 - \alpha) = RRA > EIS^{-1} = (1 - \rho).
\]

This condition is satisfied by the parameter values typically used in LRR models where both \textit{EIS} and \textit{RRA} are typically taken to exceed 1. Moreover, there is clear intuition that nonindifference to temporal resolution of risk might matter in matching asset market data: because long-run risks are not resolved until much later, they are treated differently, and penalized more heavily, than are current risks thus permitting a large risk premium to emerge even when shocks to current consumption are small. This begs the question whether the differential treatment required to match asset returns data is plausible, which is obviously a quantitative question and calls for evidence about the attitude towards temporal resolution.

I am not aware of any market-based or experimental evidence that might help with a quantitative assessment. Therefore, I suggest a simple thought experiment that through introspection may help to judge plausibility of the parameter values used in the LRR literature. Thought experiments and introspection play a role also in assessing risk aversion parameters (see, for example, Kandel and Stambaugh (1990, 1991) and Rabin (2000)). In the latter context one considers questions of the form “how much would you pay for the following hypothetical gamble?”. Here I ask instead “what fraction of your consumption stream would you give up in order for all risk to be resolved next month?”

For another thought experiment concerning the value of information that reflects on parameter values in the LRR model, see D’Addona and Brevik (2011) which in part stimulated this paper. In a continuous-time economy with produc-
tion, Ai (2007) considers the intrinsic (or noninstrumental) preference for more information from a quantitative perspective. Our starker thought experiment (where early resolution means that all risk is resolved next month) and the discrete-time exchange economy setting arguably permit a sharper focus and make it easier for introspection to operate.

2. How much would you pay?

Consider a consumption process of the following form:

\[
\begin{align*}
\log(c_{t+1}/c_t) &= m + x_{t+1} + W_{c,t+1} \\
x_{t+1} &= ax_t + W_{x,t+1}, \quad t \geq 0,
\end{align*}
\]

where \(0 < a < 1\) and \(W_{c,t}\) and \(W_{x,t}\) are Gaussian innovations, distributed as \(N(0, \text{var}_c)\) and \(N(0, \text{var}_x)\), mutually independent and i.i.d. over time. Here \(x_t\) is a persistently varying predictable component of the drift in consumption growth. Though \(\text{var}_x\) should be thought of as much smaller than \(\text{var}_c\), small innovations to \(x_t\) are important because they affect not only consumption prospects in the short run but also consumption for the indefinite future. The parameter \(a\) determines persistence of the expected growth rate process.

Bansal and Yaron (2004) assume that the volatility of consumption growth is also driven by a persistently varying predictable component. The importance of such persistence is emphasized by Bansal et al (2012). I shut off stochastic volatility in order to permit simple closed-form expressions below and because I see no reason that this simplification would alter the main message. The LRR literature also distinguishes between consumption and dividends and specifies a suitable process for the latter. But it is the consumption process as a whole, and not its components, that is important here in trying to understand the nature of preferences.

An unimportant difference between (2.1) and the LRR model as it is usually written is that the model is typically written with the lagged value \(x_t\) rather than \(x_{t+1}\) appearing on the right side. This amounts to an equivalent reparametrization. For example, letting \(y_t = ax_t\), then (2.1) implies

\[
\begin{align*}
\log(c_{t+1}/c_t) &= m + y_t + (W_{c,t+1} + aW_{x,t+1}) \equiv m + y_t + W'_{c,t+1} \\
y_{t+1} &= ay_t + aW_{x,t+1} \equiv ay_t + W'_{x,t+1}, \quad t \geq 0.
\end{align*}
\]

Assume the special case of Epstein-Zin preferences where \(EIS = 1\). The magnitude of \(EIS\), particularly whether it is less than or greater than 1, is a
source of debate. Bansal and Yaron argue for an elasticity larger than 1 (in fact, $EIS > 1$ is important for the empirical performance of their model). In light of the inequality (1.2), one would expect that raising the elasticity above 1 would have the effect of making early resolution more desirable. Therefore, the calculations to follow provide lower bounds for ‘timing premia’ and as such, are still revealing of the Bansal and Yaron parametrization.

In LRR models, a consumption process similar to the above is the endowment of a representative agent in a Lucas-style exchange economy. It is well known that there is limited theoretical justification for the assumption of a representative agent; here it requires that everyone have identical Epstein-Zin (hence homothetic) preferences. Regardless, I treat the representative agent as a real individual when introspecting about her preferences. The infinite horizon can be understood as arising from a bequest motive, or as a rough approximation to a long but finitely-lived individual.

Here is the thought experiment. You are facing consumption described by (2.1) for $t = 0, 1, ...$. In particular, the riskiness of consumption resolves only gradually over time ($c_t$ and $x_t$ are realized only at time $t$). How much would you pay at time 0 to have all risk resolved next period? More precisely, you are offered the option of having all risk resolved at time 1. The cost is that you would have to relinquish the fraction $\pi$ of both current consumption and of the consumption that is subsequently realized for every later period. What is the maximum value $\pi^*$ for which you would be willing to accept this offer? Call $\pi^*$ the timing premium for the consumption process in (2.1).\footnote{Utility admits an interpretation in terms of consumption perpetuities. For any consumption process $c$, its utility as defined in (1.1) equals that level of consumption which if received in every period and state would be indifferent to $c$. Thus $\pi^*$ can be described as the fraction of the consumption perpetuity that if relinquished would just offset the benefit of early resolution of risk.}

If utility is defined by (1.1) with $\rho = 0$, then there exists a closed-form expression for the timing premium. Continuation utilities of the consumption process in (2.1), with risk resolved gradually as indicated, solve a recursive relation. Guess and verify that utility is given by

$$\log U_0 = \log c_0 + \frac{\beta a}{1-\beta a} x_0 + \frac{\beta}{(1-\beta)} \left[ m + \frac{1}{2} a \left( var_c + \frac{1}{(1-\beta a)^2 var_x} \right) \right]. \quad (2.3)$$

Denote by $U_0^*$ the utility of the alternative process where all risk is resolved at
time 1. Then the continuation utility $U^*_1$ at time 1 is given by

$$
\log U^*_1 = (1 - \beta) \left[ \log c_1 + \beta \log c_2 + \beta^2 \log c_3 + \ldots \right] \\
= \log c_0 + \sum_{t=1}^{\infty} \beta^{t-1} \log (c_t / c_{t-1}).
$$

Therefore, from the time 0 perspective $\log U^*_1$ is normally distributed with mean

$$
\log c_0 + \frac{m}{1-\beta} + \frac{\alpha}{1-\beta \alpha} x_0
$$

and variance $\frac{1}{1-\beta^2} \left( \text{var}_c + \frac{1}{(1-\beta \alpha)^2} \text{var}_x \right).$  

Conclude that

$$
\log U^*_0 = (1 - \beta) \log c_0 + \beta \log \left[ \left( E_0 (U^*_1)^{\alpha} \right)^{1/\alpha} \right] \\
= \log c_0 + \frac{\beta m}{1-\beta} + \frac{\beta \alpha}{1-\beta \alpha} x_0 + \frac{1}{2} \beta \alpha \frac{1}{1-\beta^2} \left( \text{var}_c + \frac{1}{(1-\beta \alpha)^2} \text{var}_x \right).
$$

Accordingly, one arrives at the following expression for the timing premium:

$$
\log (1 - \pi^*) = \log \frac{U_0}{U^*_0} = \frac{1}{2} \alpha \left[ \text{var}_c + \frac{1}{(1-\beta \alpha)^2} \text{var}_x \right] \cdot \frac{\beta^2}{1-\beta^2}. \quad (2.4)
$$

The premium is positive, that is, early resolution is preferred, if and only if $
alpha < 0$, consistent with (1.2). In that case, the premium is increasing in $RRA$, $\text{var}_c$, $\text{var}_x$, $\beta$ and $\alpha$, as one would expect.

Table 1 gives a sense of the quantitative meaning of this formula.$^4$

<table>
<thead>
<tr>
<th>Table 1: Parameter Values and Premia</th>
<th>Bansal-Yaron</th>
<th>Hansen (2007, Ex. 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\text{var}_c)^{1/2}$</td>
<td>.0075</td>
<td>.0054</td>
</tr>
<tr>
<td>$(\text{var}_x)^{1/2}$</td>
<td>.0003</td>
<td>.0005</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>.9790</td>
<td>.9800</td>
</tr>
<tr>
<td>$\beta$</td>
<td>.998</td>
<td>.998</td>
</tr>
<tr>
<td>$RRA$</td>
<td>10 or 7.5</td>
<td>2</td>
</tr>
<tr>
<td>$EIS$</td>
<td>1 [BY take 1.5]</td>
<td>1</td>
</tr>
<tr>
<td>Timing premium $\pi^*$</td>
<td>22.4% or 15.8%</td>
<td>6.6%</td>
</tr>
<tr>
<td>Risk premium $\pi$</td>
<td>43.1% or 38.2%</td>
<td>23.9%</td>
</tr>
</tbody>
</table>

$^3$Compute from (2.1) that $\log (c_{t+1} / c_t) = m + \alpha t + 1 x_0 + \sum_{i=0}^{t} \alpha \text{var}_{x, t+1-i} + \text{var}_{c, t+1}$ and substitute into the expression for $\log U^*_1$.

$^4$The last row will be explained in the sequel.
Parameter values are ‘similar’ to those used in the LRR literature for a monthly frequency.\textsuperscript{5} The parameters adapted from Bansal and Yaron (2004) yield a timing premium of about 20\% while those appearing in Hansen (2007) yield the smaller premium of about 6\%. Would you give up 20\% of your lifetime consumption in order to have all risk resolved next month? what about 6\%?

3. Discussion and perspective

Consider some questions that provide perspective on the preceding.

Why pay a premium?
Keep in mind that it is risk about consumption that is at issue rather than risk about income or security returns. Thus early resolution does not have any apparent instrumental value. Kreps and Porteus (1978, 1979) suggest that an instrumental value might arise because of an unmodeled underlying planning problem. Essentially, there are more primitive preferences defined over deeper variables that are the ultimate source of satisfaction, utility defined on consumption is an indirect utility function, and early resolution has value for reasons familiar from Spence and Zeckhauser (1972), for example. This sounds plausible in theory, but one needs a more concrete story in order to believe that it could generate a sizable timing premium.\textsuperscript{6}

At a psychic level, early resolution of risk may reduce anxiety. However, anxiety is plausibly more important when risk must be endured for a long time. Then an individual’s ranking of lotteries can be expected to change over time, which is precluded when utility is recursive. Thus anxiety cannot be a rationale for a timing premium given the utility functions considered here. (This argument is due to Grant et al (2000), Caplin and Leahy (2001) and Epstein (2008).)

What is new here about \textit{RRA} and \textit{EIS}?
The timing premia calculated above give pause for assuming values of RRA as

\textsuperscript{5}That is not to say that these are the preferred parameter values of Bansal and Yaron, for example. As already discussed, they use a larger \textit{EIS}, and their preferred model includes stochastic volatility. The reparametrization using (2.1) rather than (2.2) requires only a small adjustment in the Bansal and Yaron variances.

\textsuperscript{6}Ergin and Sarver (2012) characterize behavior, in terms of choice between ‘lotteries over sets of lotteries’, that indicates (or can be represented via) a hidden planning problem. It remains to see if this work will help in assessing the magnitudes of timing premia.
large as 7.5, values that we already have reason to suspect as too high. However, the preceding gives an additional reason for being wary of large degrees of risk aversion. In addition, because of (1.2), timing premia caution also against large values for $EIS$, about which there is less of a consensus, and against values of $EIS$ much smaller than one, such as argued by Hall (1988) and Campbell (2003). In the latter case, one would have to justify, both conceptually and quantitatively, the implied preference that risk be resolved later.\footnote{Bansal and Yaron show that $EIS > 1$ is necessary for their model, including their specification of the endowment process, to deliver a positive relation between the price to dividend ratio and expected consumption growth. However, the point being made here is more general and is not tied to the LRR model.}

How is the timing premium related to the welfare cost of risk? Perspective on the timing premium is provided by examining also what the representative agent would be willing to pay to eliminate risk entirely. Lucas (1987) introduced such a calculation into macroeconomics as a way to measure the welfare costs of business cycle fluctuations. His conclusion that consumption risk has very small welfare costs stimulated many others to see how different model specifications might lead to larger costs. My interest here is less in the total cost of risk per se than in using the latter to provide further perspective on the size of the timing premium. Specifically, are the timing premia reported in Table 1 large relative to the total welfare cost of risk?

Consider the deterministic consumption process $\bar{c} = (\bar{c}_t)$ where, for every $t$, $\bar{c}_{t+1}/\bar{c}_t$ equals $E_0(c_{t+1}/c_t)$, the unconditional mean of consumption growth that is implied by the LRR process (2.1). The indicated mean is given by

$$\log E_0(c_{t+1}/c_t) = m + a^{t+1}x_0 + \frac{1}{2} \left[ var_c + (\sum_{i=0}^t a^{2i}) var_x \right].$$

Therefore let $\bar{c}$ be defined by $\bar{c}_0 = c_0$ and

$$\log(\bar{c}_{t+1}/\bar{c}_t) = m + a^{t+1}x_0 + \frac{1}{2} \left[ var_c + (\sum_{i=0}^t a^{2i}) var_x \right].$$

Its utility at time 0 is $U_0$,

$$\log U_0 = \log c_0 + \frac{\beta m}{1-\beta} + \frac{\beta a}{1-\beta} x_0 + \frac{1}{2} \left( var_c + \frac{1}{(1-\beta)^2} var_x \right).$$

Risk is costly ($U_0 > U_0$) and the cost may be measured by the risk premium $\pi$, where $1 - \pi = U_0/U_0$.\footnote{Lucas uses $\pi$ to measure the benefit of eliminating risk rather than $\pi$ to measure its cost. The difference between the two measures parallels the difference between the compensating variation (used here) and the equivalent variation (used by Lucas) of a policy change.}

The last row of Table 1 shows the welfare costs implied
by the LRR model. Even for Hansen’s parameters with relatively low risk aversion $RRA = 2$, an individual giving up almost 24% of her deterministic consumption $c_t$ in every period would still be no worse off than with the long-run risk process in (2.1).

It can be verified further that, as one would expect,

$$U_0 > U^*_0 > U_0 \text{ if } \alpha < 0.$$ 

This suggests the following decomposition:

$$\frac{U_0}{U_0} = \frac{U_0}{U_0} \times \frac{U^*_0}{U^*_0},$$

whereby the total cost of risk is decomposed into the cost of bearing risk that is resolved ‘late’ (after time $1$), and the cost of bearing risk all of which is resolved ‘early’ (at time $1$). The quantitative significance of the former cost is seen in Table 1: the ratio $\pi^*/\pi$ varies from slightly over $1/4$ to about $1/2$. Thus the cost of the late resolution of risk is a significant part of the total welfare cost of risk.

More can be said about the relation between $\pi^*$ and $\pi$; in particular, the following observations are robust in that they do not depend on the calibration of the LRR process (2.1), including the values assumed for the variances and the persistence parameter. The expressions derived for utility imply that

$$\log (1 - \pi^*) = \log (U_0/U^*_0) = \frac{\beta}{1+\beta} \frac{-\alpha}{1-\alpha} \log (U_0/U_0)$$

$$\simeq \frac{1}{2} \frac{-\alpha}{1-\alpha} \log (U_0/U_0) = \frac{1}{2} \frac{-\alpha}{1-\alpha} \log (1 - \pi),$$

where for simplicity I approximate $\frac{\beta}{1+\beta}$ by $\frac{1}{2}$. Thus $\pi^*$ can be seen as a function of $\pi$,

$$\pi^* = 1 - (1 - \pi) \frac{1}{2} \frac{-\alpha}{1-\alpha},$$ (3.1)

which implies that

$$\frac{\pi^*}{\pi} \geq \frac{1}{2} \frac{RRA - 1}{RRA}.$$ 

For example, $\pi^*/\pi$ is at least 25% if $RRA = 2$ and at least 40% if $RRA = 5$. The lower bound rapidly approaches $1/2$ as $RRA$ takes on values exceeding 10.\footnote{Use the fact that the noted function is increasing and convex on $[0, 1]$, maps 0 into 0 and has derivative at 0 equal to $\frac{1}{2} \frac{-\alpha}{1-\alpha}$. Note further that the inequality is in fact an equality up to the validity of the approximation $\log (1 - x) \simeq -x$ for $x = \pi^*$ and $\pi$.}

\footnote{Such large degrees of risk aversion violate the Mehra-Prescott guideline but they are sometimes adopted nonetheless in attempting to match asset market data (see Tallarini (2000), for example).}
Modelers may disagree about what ratios are plausible, but surely the relative magnitude of the timing premium is a significant feature of the quantitative properties of a model.

What About More General Risk Preferences?
In (1.1) and in the Kreps-Porteous model more generally, risk preferences are in the vNM class. Epstein and Zin (1989) describe a more general class of recursive utility functions in which risk preferences that are consistent with the Allais Paradox are also permitted. Some of these specifications have been used to address the equity premium and related puzzles (references are given below). Therefore, I explore briefly the quantitative implications of such generalizations for timing premia.

To preserve simplicity while generalizing preferences, I simplify the endowment process and assume that the (log) growth rate is i.i.d. Though I adopt it here for the convenience of closed forms and for the clarity of intuition delivered thereby, this is a workhorse model,\(^{11}\) it fits U.S. data well and is hard to distinguish statistically from the LRR process. Comparison of timing premia ensuing for the i.i.d. and LRR cases may offer perspective on the improved fit of asset market data provided by the latter.

Generalize (1.1) and consider utility defined by:
\[
\log U_t = (1 - \beta) \log c_t + \beta \log \mu(U_{t+1}).
\] (3.2)

Here \(\mu(\cdot)\) is the certainty equivalent of random future utility using its conditional distribution at time \(t\).\(^{12}\) Assume that \(\mu(x) = x\) for any deterministic random variable \(x\), that \(\mu\) respects first-order and second-order stochastic dominance, and that \(\mu\) is linearly homogeneous (constant relative risk aversion).

It is convenient to use the renormalized certainty equivalent \(\mu^*\), where for any positive random variable \(X\) and associated distribution,
\[
\mu^* (\log X) \equiv \log \mu (X).
\]

The preceding analysis assumes the special case
\[
\mu(U_{t+1}) = \left[E(U_{t+1}^\alpha)\right]^{1/\alpha}, \text{ or equivalently, } \mu^* (\log U_{t+1}) = \frac{1}{\alpha} \log E(U_{t+1}^\alpha).
\] (3.3)

\(^{11}\) An i.i.d. growth process for consumption is assumed, for example, in Campbell and Cochrane (1999), Calvet and Fisher (2007), where dividends are separated from consumption, and in Barberis et al (2001).

\(^{12}\) In general it depends on the information at \(t\), but with the i.i.d. assumption such time dependence can be safely suppressed.
Specialize the endowment process (2.1) by taking \( a = var_x = 0 \), so that \( \log (c_{t+1}/c_t) \) is i.i.d. \( N(m, var_c) \). Then argue as above to derive

\[
\log U_0 = \log c_0 + \beta \left[ \frac{1}{1-\beta} \mu^* (\log (c_1/c_0)) \right] \\
\log U_0^* = \log c_0 + \beta \left[ \mu^* (\Sigma^\infty \beta^t \log (c_{t+1}/c_t)) \right],
\]

and hence that the timing premium is given in closed-form by

\[
\pi^* = 1 - \exp (-\beta \Delta), \\
\Delta \equiv \mu^* (\Sigma^\infty \beta^t \log (c_{t+1}/c_t)) - (1 - \beta)^{-1} \mu^* (\log (c_1/c_0)).
\]

Note that the one-period gamble in \( \Delta \) is distributed as \( N(m, var_c) \), while the multiperiod gamble is distributed as \( N\left(m/(1-\beta), var_c/(1-\beta^2)\right) \), which has both a much larger mean and is less risky in the sense of having a smaller coefficient of variation by the factor \( [(1 - \beta) / (1 + \beta)]^{1/2} \approx 0.03 \) if \( \beta = .998 \). In particular, the two gambles are far apart, which points to the global properties of \( \mu^* \) as being important for the magnitude of the timing premium. This brings to mind the well known critique that expected utility functions are unable to model intuitive risk attitudes over a range of gamble sizes (Kandel and Stambaugh (1990, 91) and Rabin (2000)). It was this critique that in large part prompted the application of risk preferences that exhibit first-order risk aversion (Epstein and Zin (1990) and Bekaert et al (1997)). It is interesting to see the effect on timing premia of these more general certainty equivalents.

For a benchmark, I compute timing premia assuming Epstein-Zin utility.

### Table 2: Timing Premia for IID Growth Rate

<table>
<thead>
<tr>
<th>RRA</th>
<th>10</th>
<th>5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Timing premium ( \pi^* )</td>
<td>8.1%</td>
<td>3.6%</td>
<td>0.9%</td>
</tr>
</tbody>
</table>

Table 2 assumes \( EIS = 1, \beta = .998 \), mean \( m = .0015 \) and variance \( var_c = .00007 \). These latter values are roughly consistent with the annual mean (1.8%) and standard deviation (2.9%) for real per-capita consumption growth used by Bansal and Yaron (2004) to calibrate their model. The timing premia follow from (3.4) and the value of \( \Delta \), written now \( \Delta_{EZ} \), given by

\[
\Delta_{EZ} = \frac{1}{2} \frac{\beta}{1-\beta^2} (-\alpha) var_c.
\]
Comparison with Table 1 shows that timing premia are considerably larger for the LRR model.

Next I consider the following disappointment aversion certainty equivalent:

Fix $0 < \gamma \leq 1$, and for any positive random variable $X$ (with distribution $P$), define $\mu_{da}(X)$ implicitly by

$$\log \mu_{da}(X) = E \log (X) - (\gamma^{-1} - 1) \int_{X \leq \mu_{da}} (\log \mu_{da} - \log X) dP,$$

or equivalently, (let $Y = \log X$ and $Q$ its induced distribution),

$$\mu_{da}^*(Y) = EY - (\gamma^{-1} - 1) \int_{Y \leq \mu_{da}^*} (\mu_{da}^*(Y) - Y) dQ. \quad (3.5)$$

The interpretation is that outcomes of $X$ that are disappointing because they fall below the certainty equivalent are penalized relative to $E \log (X)$. If $\gamma = 1$, then $\mu(X) = E \log (X)$ and, when substituted into (3.2), one obtains the expected utility model where $RRA = EIS = 1$. Accordingly, nonindifference to timing arises herein only from the disappointment factor when $\gamma < 1$. Because the latter adds to risk aversion, the effective degree of risk aversion is greater than 1. I compare this way of increasing risk aversion to using Epstein-Zin utility with $\alpha < 0$.

I show in the appendix that the two certainty equivalent values appearing in (3.4) are related by the equation

$$(1 - \beta^2)^{1/2} \mu_{da}^* \left( \sum_{t=0}^{\infty} \beta^t \log (c_{t+1}/c_t) \right) - \mu_{da}^* (\log (c_1/c_0)) \quad (3.6)$$

As a result, the difference $\Delta$ in (3.4), written now $\Delta_{da}$, can be expressed in the form

$$\Delta_{da} = \frac{m - \mu_{da}^* (\log (c_1/c_0))}{1 - \beta} \left( 1 - \left[ \frac{1 - \beta}{1 + \beta} \right]^{1/2} \right), \quad (3.7)$$

which expression involves the certainty equivalent of the single period gamble only.

Compare $\Delta_{EZ}$ and $\Delta_{da}$ to see the differing implications for timing premia of the expected utility versus disappointment aversion risk preferences. A meaningful

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comparison requires that the respective parameters $\alpha$ and $\gamma$ be suitably related. For example, suppose that the two certainty equivalents assign the same value to the distribution of $\log (c_1/c_0)$. Then substitute $\mu_{da}^*(\log (c_1/c_0)) = m + \frac{1}{2} \alpha \cdot \var c$ into (3.7) to deduce that

$$\Delta_{da} = \left(1 - \left[\frac{1-\beta}{1+\beta}\right]^{1/2}\right)^{1+\beta} \Delta_{EZ} \simeq 2\Delta_{EZ}. \tag{3.8}$$

Roughly, disappointment aversion implies timing premia twice as large as those reported in Table 2 when $\gamma$ is calibrated as described to $\alpha = -9, -4, -1$.

An alternative calibration is to assume that the two certainty equivalents assign the same value to the distribution of $\Sigma_0^\infty \beta^t \log (c_{t+1}/c_t)$. Then similar reasoning leads to the relation

$$\Delta_{da} = \frac{1}{\beta} \left[\left(\frac{1+\beta}{1-\beta}\right)^{1/2} - 1\right] \Delta_{EZ} \simeq 30\Delta_{EZ}, \tag{3.9}$$

and hence to much larger timing premia under disappointment aversion. (For example, the timing premium for $\gamma$ that corresponds to $\alpha = -1$ is about 25%. Thus with either calibration, timing premia are larger than with Epstein-Zin utility.

Equations (3.8) and (3.9) are valid more broadly than indicated thus far. The derivation in the appendix shows that they rely only on lognormality and on the fact that $\mu_{da}^*$ satisfies: for all $\lambda \geq 0$,

$$\mu_{da}^* (Y + \lambda) = \mu_{da}^* (Y) + \lambda \text{ and } \mu_{da}^* (\lambda Y) = \lambda \mu_{da}^* (Y), \tag{3.10}$$

that is, it exhibits both CARA (constant absolute risk aversion) and CRRA (constant relative risk aversion). The preceding comparative analysis applies as is to any certainty equivalent function $\mu^*$ satisfying these properties. For example, it applies also to the following generalization of (3.5):

$$\mu_{gda}^* (Y) = EY - \left(\gamma^{-1} - 1\right) \int_{Y \leq \mu_{gda}^*} (\delta \mu_{gda}^* (Y) - Y) dQ,$$

where $0 < \delta \leq 1$. Here outcomes are disappointing if they are smaller than the fraction $\delta$ of the certainty equivalent. This generalization of disappointment aversion (which corresponds to the special case $\delta = 1$) is in the spirit of that provided by Routledge and Zin (2010).\footnote{In our setting, their model would take the form $\mu_{RZ}^* (Y) = EY - \left(\gamma^{-1} - 1\right) \int_{Y \leq \log \delta + \mu_{RZ}^*} (\log \delta + \mu_{RZ}^* (Y) - Y) dQ$, which violates the second condition in (3.10).}
4. Concluding remarks

Though risk aversion and the elasticity of intertemporal substitution have been the subjects of careful scrutiny when calibrating preferences, the long-run risks literature and the broader literature using recursive utility to address asset pricing puzzles have ignored the full implications of their parameter specifications. Recursive utility implies that the temporal resolution of risk matters and a quantitative assessment of how much it matters should be part of the calibration process. This paper is not intended to provide an exhaustive or definitive assessment of parameters used in the literature. Its objective is to give a sense of the magnitudes of implied timing premia and, by adding to Ai (2007) and D’Addona and Brevik (2011), to inject temporal resolution of risk into the discussion of the quantitative properties of LRR and related models.

An important alternative to models based on recursive utility is the external habits model of Campbell and Cochrane (1999). Corresponding scrutiny of that model seems in order. Thus far plausibility of the habits formation process assumed for the representative agent has been judged solely by how it helps to match asset market data. The discipline urged by Mehra and Prescott (1985) suggests that at least one should examine also whether it seems plausible based on introspection about the quantitative effects of past consumption on current preferences. The difficulty of finding other market-based evidence concerning external habits, or about timing premia, does not justify leaving them as ‘free parameters.”

A. Appendix

A proof of (3.6) is provided here. Take \( \log \left( \frac{c_{t+1}}{c_t} \right) \) to be i.i.d. \( N(m, \text{var}_c) \).

Let \( Y = \log \left( \frac{c_1}{c_0} \right) \) and \( Y' = \sum_0^\infty \beta^t \log \left( \frac{c_{t+1}}{c_t} \right) \). They are distributed as \( N(m, \text{var}_c) \) and \( N \left( \frac{m}{1-\beta}, \frac{\text{var}_c}{1-\beta^2} \right) \) respectively. Therefore,

\[
Y'' \equiv \left( 1 - \beta^2 \right)^{1/2} Y' - m \left( \left[ \frac{1+\beta}{1-\beta} \right]^{1/2} - 1 \right) \quad \text{is} \quad N(m, \text{var}_c).
\]

Because \( \mu_{da}(Y'') \) and \( \mu_{da}(Y) \) depend only on the distributions of \( Y'' \) and \( Y \), they must be equal. Thus (3.6) follows from the homogeneity properties in (3.10).
References


