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Evolutionary Farsightedness in International Environmental Agreements

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Abstract: This paper proposes a dynamic game model of the process through which countries join international environmental agreements (IEAs). The model assumes that both the number of signatory countries and the stock of accumulated pollution evolve over time, as a result of countries' emission and membership decisions. The evolution of the number of signatory countries is described by a discrete-time replicator dynamics, while that of the stock of pollution results from feedback emission strategies. We show that evolutionary farsightedness, that is, the capacity of players to account for the impact of their decisions on the evolution of the number of signatory countries, is beneficial to the formation and stability of self-enforcing IEAs.

Key Words: International Environmental Agreements, Dynamic Games, Replicator Dynamics.

Résumé : Cet article propose un modèle du processus par lequel les pays adhèrent à un accord environnemental international sous la forme d'un jeu dynamique. Le modèle suppose que le stock de pollution et le nombre de signataires de l'accord évoluent dans le temps. L'évolution du nombre de signataires est décrite par une dynamique de réplication en temps discret, alors que celle du stock de pollution résulte des stratégies de production des joueurs. Nous montrons que le fait pour les joueurs de prendre en compte l'impact de leurs décisions sur le nombre de signataires est bénéfique pour la formation et la stabilité des accords environnementaux.

Mots clés : Accords environnementaux internationaux, jeux dynamiques, dynamique de réplication.

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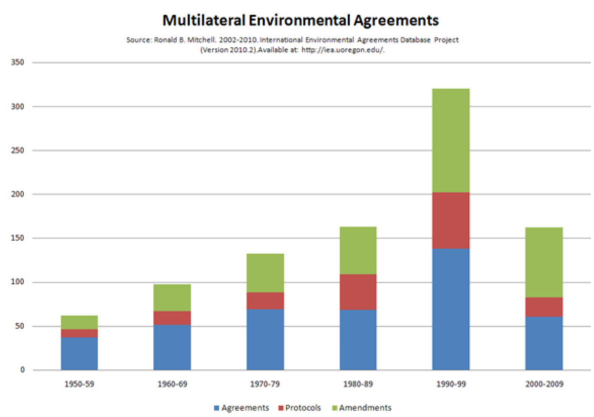
1 Introduction

Many environmental concerns are international in scope. Obvious examples include the reduction of anthropogenic greenhouse gases emissions and the preservation of biodiversity. In many cases, because the benefits of any costly action undertaken by a single country to preserve the environment are shared by all countries, the Tragedy of the Commons (Hardin 1968) predicts the ultimate depletion of the shared resource in the absence of coordination between countries. International environmental agreements (IEAs) are the usual way taken by countries to coordinate their use of environmental resources; the assumption underlying adhesion to an IEA is that coordination between countries ultimately enhances global welfare.

However, the negotiation of an IEA is challenging for two main reasons. The first one is sovereignty: participation of countries in IEAs is voluntary, and there is no supranational jurisdiction that can force adhesion or compliance of individual countries to such agreements. The second reason is the free-riding incentive: each country stands to gain from letting others join IEAs and bear the costs of compliance, while enjoying the global benefits. Despite of these difficulties, numerous IEAs have been signed and ratified by many countries during the last 60 years (see Figure 1).



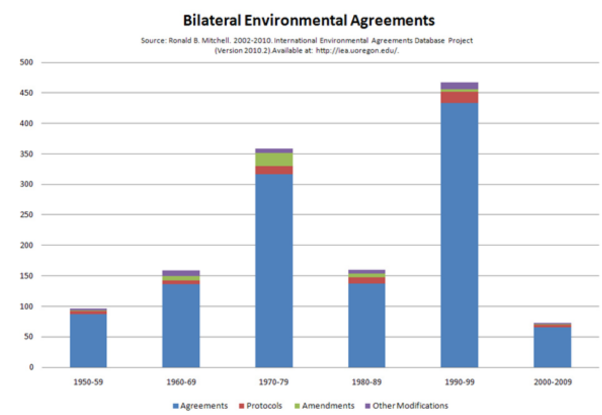


Figure 1: Number of countries ratifying environmental agreements since 1950.

In this paper, we consider the air pollution issue, more specifically the global stock of airborne pollutants (e.g. greenhouse gases). This stock is affected by the production activities of each country, and its negative impact is shared globally. For air pollution alone, Mitchell (2012) reports on 19 international agreements, of which the three best known are probably the UN Framework Convention on Climate Change (1992) leading to the Kyoto Protocol, the Montreal Protocol on Ozone Protection (1987) and the Convention on Long-Range Transboundary Air Pollution (1979). Figure 2 reports on the evolution, over time, of the number of countries that ratified each of these three multilateral agreements.

Some comments arise from the observation of existing IEAs. First, the number of participants in the agreement can evolve over time, as countries agree to join an increasingly large coalition, or as countries decide to walk out.[1] Second, the fact that protocols or agreements are considered to be legally binding cannot really prevent countries from reneging on previous commitments. Third, large coalitions can be sustained over time. Fourth, agreements can include punishment clauses for countries who are not complying.[2]

The aim of this paper is to propose a dynamic game model for the formation of IEAs and the evolution of the number of their participating countries. Two approaches have been used in the game theory literature to model participation in IEAs. The cooperative approach assumes that players, or a subset of them, are already committed to coordinating their actions, and concentrates on the collective optimization and benefits

[1]For instance, in December 2011, Canada renounced its participation in the Kyoto Protocol.

[2]For instance, in the case of the Kyoto Protocol, if the enforcement branch determines that an Annex I country is not in compliance with its emissions limitation, then that country is required to make up the difference during the second commitment period plus an additional 30%. In addition, that country will be suspended from making transfers under an emissions trading program.

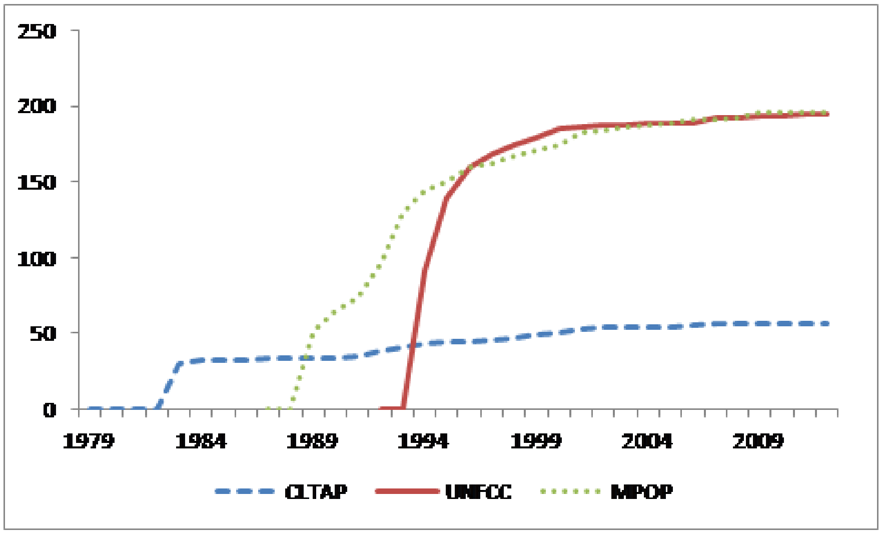


Figure 2: Number of countries who ratified the Convention on Long-Range Transboundary Air Pollution (CLTAP), the Montreal Protocol on Ozone Protection (MPOP) and the UN Framework Convention on Climate Change (UNFCC).

redistribution problems. Such an approach is not suited to analyzing the coalition formation process, since the existence of a coalition of players is taken for granted. On the other hand, the non-cooperative approach assumes that players act in their own best interest and cannot make binding agreements, so that stable coalitions can only form when agreements are self-enforcing in the sense of d'Aspremont et al. (1983). This approach concentrates on the existence and size of stable coalitions under various assumptions about the nature of the agreement between the players. This paper adopts the non-cooperative point of view, which allows to consider two main decision processes in an IEA : the adhesion process, and the emission (or, equivalently, the abatement) decisions by member and non-member countries.

In most of the literature on non-cooperative games and self-enforcing agreements, membership in IEAs is assumed to be stable over time. Indeed, the majority of papers use either a static framework, where membership and emission decisions are taken simultaneously, or a two-stage game process, where countries decide once and for all whether or not to become members of an IEA in the first stage, and then decide on their emission levels (possibly dynamically) in the second stage. Such models cannot account for the fact that countries can reconsider their membership decision at various points in time, especially if the state of the world (e.g. the economic impact or the level of the stock of pollutant) changes.

Papers that do consider in some way dynamic membership decisions include those of Germain et al. (2003), de Zeeuw (2008), Rubio & Ulph (2007) and Breton et al. (2010). In Germain et al. (2003), a transfer scheme is proposed such that full cooperation is maintained with the evolution of the stock of pollution, and consequently the number of IEA members does not change over time. In this case, as in the cooperative approach, the existence of the coalition is taken for granted, and the transfer scheme ensures its stability over time as the stock of pollutants evolves. In de Zeeuw (2008), it is shown that the farsightedness stability concept can ensure the stability of coalitions of various sizes in a dynamic setting, even when deviations are not detected immediately. In this case, members of a stable coalition are deterred from exiting at all time because they foresee that their defection will trigger others to do so, leading to a new stable situation reducing their individual welfare. Again in that case the existence of an initial stable coalition is taken for granted, and the farsightedness of its members ensures its stability over time.

Rubio & Ulph (2007) use a discrete-time model where both a membership and an emission game are solved at each decision date, so that the number of member countries changes with the pollution stock. Their model presumes that, in each period, the identity of the IEA members is randomly selected and an equilibrium is attained, making the welfare of all countries (members and non-members) equal. Solving a membership game at each time period amounts to assuming that countries are able to agree instantaneously on an IEA, and that as soon as the pollution stock changes, they have to negotiate again, and be shuffled into the two groups again. Notice that in this model, the mechanism by which countries reach an equilibrium is not specified; the number of IEA members evolves over time only because of changes in the pollution stock. Breton et al. (2010) also consider that the number of member countries evolves with time and the pollution stock, but they do not assume that equilibrium is attained instantly at each time period. Rather, countries may join or quit an agreement over time, following an evolutionary process based on individual welfare considerations. In their model, the evolution of the players' welfare over time depends both on the dynamics of the emissions and pollution stock and on the evolution of the number of members of the IEA, but players are myopic with respect to the impact of their decisions on the size of the coalition.

A grim result of the non-cooperative approach to the formation of IEAs, related to the free-riding incentive mentioned above, is that large coalitions cannot be sustained in general, except if additional considerations (apart from environmental issues) are taken into account. Examples include issue linkages (Botteon & Carraro 1998, Le Breton & Soubeyran 1997, Barrett 1997, Katsoulacos 1997, Carraro & Siniscalco 1997, 1998, Mohr & Thomas 1998), transfers (Carraro & Siniscalco 1993, Hoel & Schneider 1997, Germain et al. 2003), reputation effects (Hoel & Schneider 1997, Jeppensen & Andersen 1998, Cabon-Dhersin & Ramani 2006), and punishments (Bahn et al. 2009, Breton et al. 2010). Increased participation in IEAs can also be obtained under different equilibrium concepts, such as first-move advantage or farsightedness (Barrett 1994, Rubio & Ulph 2006, Diamantoudi & Sartzetakis 2006, de Zeeuw 2008), or by imposing constraints on actions by non-members (Rubio & Ulph 2007).

This paper contributes to the literature on the formation of IEA by explicitly modeling the mechanism of the adhesion process of members in an IEA with a punishment clause, as in Breton et al. (2010). In our model, however, players are aware of the impact of their decisions on the evolution of the size of the coalition. In that sense, our paper can also be related to the farsightedness literature, but, contrary to the farsightedness rational conjecture assumptions, we do not suppose that coalitions immediately break down whenever a defection is detected, or that stable coalitions do not change size as the state of the world evolves. Rather, we recognize that it takes time for an IEA to achieve stability. Following the spirit of evolutionary games, we assume that evolutionary pressures favor the group of countries that perform better, which is gradually joined by new players, where the speed at which countries switch groups is related to the difference in welfare. Contrary to evolutionary games, the players are aware of these evolutionary pressures and account for them in their strategic emission decisions. We name this particular awareness *evolutionary farsightedness*.

This paper investigates how the interaction between strategic production decisions by member and non-member countries and the evolutionary adjustment process in the size of coalitions affects the IEA stability results. In particular, we want to determine the impact of evolutionary farsightedness on the evolution of membership in IEAs, on the initial conditions ensuring that a stable IEA is eventually reached, and on the steady-state of the pollution stock and coalition size. We use a discrete-time dynamic game model where the state variables are the size of the coalition and the level of the pollution stock, and where players decide on their emission levels using feedback strategies. Members of an IEA agree to maximize their joint welfare and to inflict a (costly) punishment to non-members, who independently maximize their own individual welfare. We show that evolutionary farsightedness results in larger stable coalitions, lower long-term pollution stock, and higher welfare.

The paper is organized as follows. Section 2 presents the model and notation and defines the evolutionary farsightedness equilibrium concept. Section 3 proposes a value-iteration algorithm to approximate the equilibrium strategies and value functions and Section 4 characterizes the steady-state of the dynamic system. A numerical illustration is presented in Section 5, where our results are compared to those obtained when assuming that players are myopic with respect to the evolution of the size of an IEA. Section 6 is a conclusion.

2 Model

We consider n identical countries engaged in production activities generating benefits as well as pollution. Denote by $e_{jt} \geq 0$ the emissions generated by the production of country j in time period t. We suppose that the net revenue (i.e., gross revenue minus costs) derived from country j's production activity in a given period is a concave differentiable function of its emissions $R(e_{jt})$, with $R'(0) > 0$ and $\lim_{e_{jt}\to\infty} R(e_{jt}) < 0$. Countries suffer in each period an environmental damage cost $C(p_t)$, where $p_t \geq 0$ is the global stock of pollution at time t and where $C(\cdot)$ is assumed increasing and convex, with $C(0) = 0$.

Suppose that at time t, a fraction $s_t \in [0, 1]$ of countries, identified as "signatories," have agreed to participate in an IEA, while the remaining fraction $(1 - s_t)$ are identified as non-signatories or "defectors." We denote by S the set of signatories and by D the set of defectors; consequently, attributes related to the set of signatory countries will be indexed by S while those related to the set of defectors will be indexed by D. We make the usual assumption that each signatory country determines its production activity and corresponding emission level by maximizing the global welfare of the group of signatories, while each non-signatory country decides on its emission level by maximizing its own individual welfare. Moreover, as in Breton et al. (2010), we suppose that signatories inflict a punishment to each defector, proportional to the level of the pollution stock. The non-environmental cost incurred by a defector punished by ns_t signatories is given by

$$c^D(s_t, p_t) = ns_t\alpha p_t$$

where $\alpha \geq 0$ is the punishment parameter. Given that punishing a country also has a negative impact, we presume that each signatory country incurs a cost proportional to the punishment αp_t imposed to the $n\,(1 - s_t)$ defectors, given by

$$c^S(s_t, p_t) = n(1 - s_t)\tau\alpha p_t$$

where $\tau \geq 0$ is the punishment cost parameter. In a given period t, the welfare of a signatory and a defector country j is thus given respectively by

$$W_{jt}^S\,(e_{jt}, s_t, p_t) = R(e_{jt}) - C(p_t) - c^S\,(s_t, p_t)\,, \quad j \in S \tag{1}$$

$$W_{jt}^D\,(e_{jt}, s_t, p_t) = R(e_{jt}) - C(p_t) - c^D\,(s_t, p_t)\,, \quad j \in D. \tag{2}$$

Both the pollution stock and the number of signatories evolve over time. The evolution over time of the pollution stock is described by the discrete-time equation

$$p_{t+1} = p_t(1 - \delta) + \sum_{j=1}^{n} e_{jt} \tag{3}$$

where $\delta \in (0, 1)$ is the natural decay of the pollution stock. The evolution of the fraction of signatories is described by the following difference equation

$$s_{t+1} = s_t \frac{U_\theta^S(s_t, p_t)}{s_t U_\theta^S(s_t, p_t) + (1 - s_t)U_\theta^D(s_t, p_t)},\ s_t \in [\frac{1}{n}, 1] \tag{4}$$

where $U_\theta^S(\cdot)$ and $U_\theta^D\,(\cdot)$ are the welfare functions of signatories and defectors respectively, which will be defined more precisely below in (5)–(6). Equation (4) is the well-known replicator dynamics: players, observing the current welfare of countries in both groups, are enticed to join the group who is performing better. Moreover, the speed of the change in the fraction of signatories is related to the relative importance of the welfare differences between the two groups of countries.

Define the long-term welfare function at (s_t, p_t) of a given country j as the discounted sum, over an infinite horizon, of the periodic welfare of this country, that is,

$$U\,(j, s_t, p_t) = \sum_{k=t}^{\infty} \beta^{k-t} W_{jk}^I\,(e_{jk}, s_k, p_k)\,,\ j \in \{S, D\}\,, I = S, D,$$

where $\beta \in (0,1)$ is the periodic discount factor, and where $W^I_{jk}(\cdot)$ is given either by (1) or (2), depending on j being a signatory or a defector.

We now define a discrete-time feedback stationary emission strategy for a country j as a function $\theta_j : [0,1] \times \mathbb{R}^+ \to \mathbb{R}^+$ indicating the emission level of Country j during a given time period as a function of the fraction of signatories and of the pollution stock at the beginning of the period, and denote $\theta = [\theta_j]_{j=1,\dots,n}$ the vector of feedback strategies of all countries. Since countries are identical, we are interested in the set Θ of strategy vectors such that θ_j differs from θ_i only if j is a signatory and i is a defector, so that θ^S then denotes the emission strategy of a signatory, and θ^D the emission strategy of a defector. For a given strategy vector $\theta \in \Theta$, the long-term welfare functions U^S_θ and U^D_θ are then given by

$$U^S_\theta(s_t,p_t) = \sum_{k=t}^{\infty} \beta^{k-t} W^S_{jk}\left(\theta^S(s_k,p_k), s_k, p_k\right),\ j \in S \tag{5}$$

$$U^D_\theta(s_t,p_t) = \sum_{k=t}^{\infty} \beta^{k-t} W^D_{jk}\left(\theta^D(s_k,p_k), s_k, p_k\right),\ j \in D. \tag{6}$$

Finally, using the dynamics (3)–(4), the long-term welfare functions corresponding to a strategy vector $\theta \in \Theta$ satisfy the functional equations

$$U^S_\theta(s,p) = W^S_{jk}\left(\theta^S(s,p), s, p\right) + \beta U^S_\theta(s',p') \tag{7}$$

$$U^D_\theta(s,p) = W^D_{jk}\left(\theta^D(s,p), s, p\right) + \beta U^D_\theta(s',p') \tag{8}$$

$$p' = p(1-\delta) + ns\theta^S(s,p) + n(1-s)\theta^D(s,p) \tag{9}$$

$$s' = s\frac{U^S_\theta(s,p)}{sU^S_\theta(s,p) + (1-s)U^D_\theta(s,p)}. \tag{10}$$

Given (7)–(10), we seek a dynamic equilibrium in feedback strategies between the coalition of ns signatories and the remaining players.

Definition 1 *A strategy $\theta^* = \left[\theta^{S*}, \theta^{D*}\right] \in \Theta$ is an evolutionary farsighted (EF) equilibrium and $V^S = U^S_{\theta^*}$ (resp. $V^D = U^D_{\theta^*}$) is the corresponding equilibrium long-term welfare of a signatory (resp. defector) if the following conditions are satisfied for all $(s,p) \in [0,1] \times \mathbb{R}^+$:*

$$V^S(s,p) = \max_{e \geq 0}\{R(e) - C(p) - c^S(s,p) + \beta V^S(s',p')\} \tag{11}$$

$$s.t.$$

$$p' = p(1-\delta) + nse + n(1-s)\theta^{D*}(s,p) \tag{12}$$

$$s' = s\frac{V^S(s,p)}{sV^S(s,p) + (1-s)V^D(s,p)} \tag{13}$$

$$V^D(s,p) = \max_{e \geq 0}\{R(e) - C(p) - c^D(s,p) + \beta V^D(s',p')\} \tag{14}$$

$$s.t.$$

$$p' = p(1-\delta) + e + ns\theta^{S*}(s,e) + (n(1-s)-1)\,\theta^{D*}(s,e) \tag{15}$$

$$s' = s\frac{V^S(s,p)}{sV^S(s,p) + (1-s)V^D(s,p)}. \tag{16}$$

Given a strategy vector θ, denote θ^{-j} the vector containing the strategies of all the players except Player j. The EF equilibrium defined in 1 has the following characteristics. The stationary feedback strategy θ^{*D} belongs to the set of best responses, in the class of feedback strategies, of any defector country j to the vector θ^{*-j}: non-signatory countries cannot increase their long-term welfare by unilaterally changing their emission strategy, taking into account the dynamics of the pollution stock and the evolutionary dynamics of the number of signatories. Moreover, the vector of identical stationary strategies θ^{*S} belongs to the set of best collective response, in the class of feedback strategies, of the coalition of signatories when all defectors use strategy θ^{*D}, that is, maximizing the total welfare of the signatory countries : signatory countries cannot increase their total welfare by collectively changing their emission strategy, taking into account the dynamics of the pollution stock and the evolutionary dynamics of the number of signatories.

Notice that while the model (revenues, costs and dynamics) is very similar to that of Breton et al. (2010), the equilibrium concept is different. In Breton et al. (2010), even if players are able to observe the number of signatories, they do not account for the dynamics of the IEA membership when selecting their emission strategies. We will see below that this additional foresight is beneficial to the formation and stability of coalitions and to the long-term level of the pollution stock, and the more so when players put a high value on their future welfare.

3 Evolutionary farsighted equilibrium

If an EF equilibrium exists, equations (11)–(16) define two coupled infinite horizon dynamic programs that can be solved to yield the equilibrium strategy vector and long-term welfare functions. Because of the complexity of the dynamics (16), these dynamic programs do not admit analytical solutions. We first analyze the solution of an auxiliary static equilibrium problem.

Denote by U_p the partial derivative of a function $U(s,p)$ with respect to p. Consider two differentiable functions U^S and U^D such that $U_p^S < 0$, $U_p^D < 0$, $U_{pp}^S \leq 0$ and $U_{pp}^D \leq 0$. At a given (s,p), consider a one-stage game where players choose their emissions levels e_j, $j = 1, ...n$, receive an immediate payoff according to (1)–(2), and then move to a new state

$$\begin{aligned} s' &= s\frac{U^S(s,p)}{sU^S(s,p)+(1-s)U^D(s,p)} \\ p' &= p(1-\delta)+\sum_{j=1}^{n} e_j \end{aligned}$$

where they receive a final payoff described by the functions U^S and U^D.

First consider the optimization problem solved by a coalition of signatories when the total of emissions by the defectors is E^D :

$$\begin{aligned} e^S &= \arg\max_{e\geq 0}\{R(e)-C(p)-\tau\alpha n(1-s)p \\ &\quad +\beta U^S(s',p(1-\delta)+nse+E^D)\}. \end{aligned}$$

Under our assumptions on U^S, the optimization problem is concave in e and there is a unique solution $e^S \geq 0$ satisfying the first order conditions

$$\begin{aligned} R'(e^S)+\beta ns U_p^S(s',p(1-\delta)+nse^S+E) &\leq 0 \\ e^S\left(R'(e^S)+\beta ns U_p^S(s',p(1-\delta)+nse^S+E)\right) &= 0. \end{aligned}$$

According to our assumptions on R, there exists a unique e_0 such that $R'(e_0) = 0$, and consequently $0 \leq e^S < e_0$ since $U_p^S < 0$. Moreover, since $U_{pp}^S \leq 0$, $e^S\left(E^D\right)$ is non-increasing in E^D.

Now consider the optimization problem solved by a single defector when the total of emissions by the other defectors is E^d and the total of emissions by the signatories is E^S:

$$\begin{aligned} e^D &= \max_{e\geq 0}\{R(e)-C(p)-\alpha nsp \\ &\quad +\beta U^D(s',p(1-\delta)+e+E^d+E^S)\}. \end{aligned}$$

Again, the optimization problem is concave in e and e^D satisfies the first order conditions

$$\begin{aligned} R'(e^D) + \beta U_p^D(s', p(1-\delta) + e^D + E^d + E^S) &\leq 0 \\ e^D\left(R'(e^D) + \beta U_p^D(s', p(1-\delta) + e^D + E^d + E^S)\right) &= 0. \end{aligned}$$

Since all defectors are identical, the equilibrium solution e^D is symmetrical and therefore satisfies

$$\begin{aligned} R'(e^D) + \beta U_p^D(s', p(1-\delta) + n(1-s)e^D + E^S) &\leq 0 \\ e^D\left(R'(e^D) + \beta U_p^D(s', p(1-\delta) + n(1-s)e^D + E^S)\right) &= 0. \end{aligned}$$

Similarly as for the signatories, we obtain that $0 \leq e^D < e_0$ and that $e^D(E^S)$ is non-increasing in E^S.

Therefore, reaction functions $e^D(e^S)$ and $e^S(e^D)$ are both non-increasing over $[0, e_0]$. As a consequence, there exists at least one equilibrium solution $\left(e^{S*}, e^{D*}\right)$ such that $e^D(e^{S*}) = e^{D*}$ and $e^S(e^{D*}) = e^{S*}$. Moreover, the cobweb algorithm will converge to an equilibrium solution.

Finally, since optimal emissions of all players are bounded by e_0, then if $p \leq \frac{n}{\delta}e_0$

$$\begin{aligned} p' &= p(1-\delta) + \sum_{j=1}^{n} e_j \\ &\leq \frac{n}{\delta}e_0(1-\delta) + ne_0 \\ &= \frac{n}{\delta}e_0. \end{aligned}$$

Notice that e_0 corresponds to the optimal emission level when countries do not suffer any cost from the accumulation of pollution (in that case, the welfare of a single country is independent of the emission decisions by other countries). When looking for an equilibrium solution and corresponding value function, we can restrict the domain of possible decisions to the interval $[0, e_0]$ and the domain of the pollution stock to the interval $\left[0, \frac{n}{\delta}e_0\right]$.

Assume that an EF equilibrium exists for the dynamic game defined in Section 2 and that functions V^S and V^D satisfying (11)–(16) are differentiable, decreasing and concave in p on $\Gamma \equiv [0,1] \times \left[0, \frac{n}{\delta}e_0\right]$. Then at any given $(s,p) \in \Gamma$, there exists a $\left(e^S, e^D\right)$ such that

$$s' = s\frac{V^S(s,p)}{sV^S(s,p) + (1-s)V^D(s,p)} \tag{17}$$

$$p' = p(1-\delta) + n(1-s)e^D + nse^S \tag{18}$$

$$R'(e^S) + \beta ns V_p^S(s', p\prime) \leq 0 \tag{19}$$

$$e^S\left(R'(e^S) + \beta ns V_p^S(s', p')\right) = 0 \tag{20}$$

$$R'(e^D) + \beta V_p^D(s', p') \leq 0 \tag{21}$$

$$e^D\left(R'(e^D) + \beta V_p^D(s', p\prime\right) = 0 \tag{22}$$

$$\begin{aligned} V^S(s,p) &= R(e^S) - C(p) - \tau\alpha n(1-s)p + \beta V^S(s', p') \\ V^D(s,p) &= R(e^D) - C(p) - \alpha nsp + \beta V^D(s', p'). \end{aligned}$$

This suggests the following value-iteration algorithm: Given two functions $V^{S;k} : \Gamma \to \mathbb{R}$ and $V^{D;k} : \Gamma \to \mathbb{R}$ at iteration k, find $e^{S;k}(s,p)$, $e^{D;k}(s,p)$, $s'(s,p)$ and $p'(s,p)$ satisfying (17)–(22) on Γ, and then set for all $(s,p) \in \Gamma$

$$V^{S;k+1}(s,p) = R(e^{S;k}(s,p)) - C(p) - \tau\alpha n(1-s)p + \beta V^{S;k}(s'(s,p), p'(s,p)) \tag{23}$$

$$V^{D;k+1}(s,p) = R(e^{D;k}(s,p)) - C(p) - \alpha nsp + \beta V^D(s'(s,p), p'(s,p)). \tag{24}$$

Unfortunately, there is no way to ensure that, if $V^{S;k}$ and $V^{D;k}$ are decreasing and concave in p, then $V^{S;k+1}$ and $V^{D;k+1}$ are also decreasing and concave in p. However, if the value iteration algorithm converges, then the strategies $\theta^{S*} = e^{S;k}$ and $\theta^{D*} = e^{D;k}$ and the corresponding value functions $V^{S;k}$ and $V^{D;k}$ satisfy the conditions defining an EF equilibrium at all $(s,p) \in \Gamma$.

Since the domain Γ is continuous, it is not possible in a numerical implementation to solve (17)–(22) (23)–(24) for all $(s,p) \in \Gamma$. In our experiments, we use a discrete two-dimensional grid $\mathcal{G} \subset \Gamma$ and functions $\widehat{V}^{S;k} : \mathcal{G} \to \mathbb{R}$ and $\widehat{V}^{D;k} : \mathcal{G} \to \mathbb{R}$. Assume that functions $\widehat{V}^{S;k}$ and $\widehat{V}^{D;k}$ are known. At iteration k, define functions $V^{S;k}$ and $V^{D;k}$ as two-dimensional cubic spline interpolation functions of $\widehat{V}^{S;k}$ and $\widehat{V}^{D;k}$, twice continuously differentiable, such that $V^{S;k}$ coincides with $\widehat{V}^{S;k}$ and $V^{D;k}$ coincides with $\widehat{V}^{D;k}$ on $\mathcal{G}$. Notice that functions $V_p^{S;k}$ and $V_p^{D;k}$ are then known analytically on Γ. Algorithm (17)–(22) is then performed on $\mathcal{G}$, and we set, for all $(s,p) \in \mathcal{G}$,

$$\widehat{V}^{S;k+1}(s,p) = R(e^{S;k}(s,p)) - C(p) - \tau\alpha n(1-s)p + \beta V^{S;k}(s'(s,p), p'(s,p)) \tag{25}$$

$$\widehat{V}^{D;k+1}(s,p) = R(e^{D;k}(s,p)) - C(p) - \alpha nsp + \beta V^{D;k}(s'(s,p), p'(s,p)). \tag{26}$$

The process stops when $\left\|\widehat{V}^{S;k+1}, \widehat{V}^{S;k}\right\| < \varepsilon$ and $\left\|\widehat{V}^{D;k+1}, \widehat{V}^{D;k}\right\| < \varepsilon$. The detailed algorithm is provided in the appendix (Section 7).

4 Steady-state and trajectories

Assuming that the players use an equilibrium strategy $\theta^* = \left[\theta^{S*}, \theta^{D*}\right]$, a steady state of the dynamic system

$$s_{t+1} = s_t \frac{V^S(s_t,p_t)}{s_t V^S(s_t,p_t) + (1-s_t)V^D(s_t,p_t)} \tag{27}$$

$$p_{t+1} = p_t(1-\delta) + ns_t\theta^{S*}(s_t,p_t) + n(1-s_t)\theta^{D*}(s_t,p_t) \tag{28}$$

is a state $(\overline{s},\overline{p})$ such that both the stock of pollution and the size of the coalition are constant. A steady-state $(\overline{s},\overline{p})$ satisfies the following set of equations:

$$\delta\overline{p} = n\overline{s}\theta^{S*}(\overline{s},\overline{p}) + n(1-\overline{s})\theta^{D*}(\overline{s},\overline{p}) \tag{29}$$

$$V^S(\overline{s},\overline{p}) = V^D(\overline{s},\overline{p}). \tag{30}$$

Steady-states are of special interest, since they indicate the size of a stable IEA and the resulting equilibrium pollution stock. We are also interested in illustrating the formation of such stable IEAs over time, by analyzing the trajectories of the state variables from various initial conditions. When multiple steady-states exist, this analysis also yields the basin of attraction of these steady-states, that is, the initial conditions leading to one or the other.

In order to do so, we assume that equation (29) implicitly defines a function $p^{\#}(s) : [0,1] \to \mathbb{R}^+$ such that, to any $s \in [0,1]$ corresponds a unique $p^{\#}$ satisfying

$$\delta p^{\#} = ns\theta^{S*}\left(s,p^{\#}\right) + n(1-s)\theta^{D*}\left(s,p^{\#}\right). \tag{31}$$

Function $p^{\#}$ separates Γ into two half spaces according to the dynamic system (27)–(28):

$$\Gamma_1 = \{(s_t,p_t) : p_{t+1} > p_t\}$$
$$\Gamma_2 = \{(s_t,p_t) : p_{t+1} < p_t\}.$$

In the same way, we assume that equation (30) implicitly defines a function $p^\flat(s) : [0,1] \to \mathbb{R}^+$ such that, to any $s \in [0,1]$ corresponds a unique $p^\flat$ satisfying

$$V^S(s,p^\flat) = V^D(s,p^\flat). \tag{32}$$

Function $p^\flat$ separates Γ into two half spaces according to the dynamic system (27)–(28):

$$\begin{aligned}\Gamma_3 &= \{(s_t, p_t) : s_{t+1} > s_t\} \\ \Gamma_4 &= \{(s_t, p_t) : s_{t+1} < s_t\}.\end{aligned}$$

An inner steady-state is defined by the intersection of functions $p^\#$ and $p^\flat$ over $[0,1]$. Other steady-states may exist on the boundary of $[0,1]$, e.g. full defection ($s=0$) and full cooperation ($s=1$).

5 Numerical illustration

This section presents a numerical investigation of the behavior of EF equilibria and corresponding system trajectories and steady-states, and compares these steady-states with those attained when players are partially myopic (PM) as in Breton et al. (2010). (partially myopic players do not account for the dynamics of the number of signatories when deciding on their emission strategies). The numerical approach presented in Section 3 can be used to compute equilibrium strategies and steady-states of the dynamic system for any user-defined functional form for the revenue function R, environmental damage function C, and even punishment cost functions c^D and c^S. In our numerical illustration, we follow the usual modelling assumptions made in the literature on self-enforcing IEAs and consider a simple model with quadratic revenue and linear damage functions, $R(e) = e(b - \frac{1}{2}e)$ and $C(p) = dp$.[3] Base case parameter values are listed in Table 1.

Table 1: Parameter values, linear damage and quadratic revenue.

N	b	d	τ	$\alpha \times 10^6$	β	δ
200	100	0.08	0.3	38.7	0.9	0.4

5.1 Steady-state and trajectories

Figure 3 plots functions $p^\#$ (dashed) and $p^\flat$ (straight) in Γ. Region Γ_1 is below the graph of $p^\#$. Starting from any state in Γ_1, the pollution stock is increasing when players use their EF equilibrium strategy, and the reverse is true in Γ_2, above the graph of $p^\#$. Region Γ_3 is above the graph of $p^\flat$. In that region, the long-term welfare of signatories is higher than that of a defector, so that, starting from any state in Γ_3, the number of signatories is increasing when players use their EF equilibrium strategy, and the reverse is true in Γ_4, below the graph of $p^\flat$.

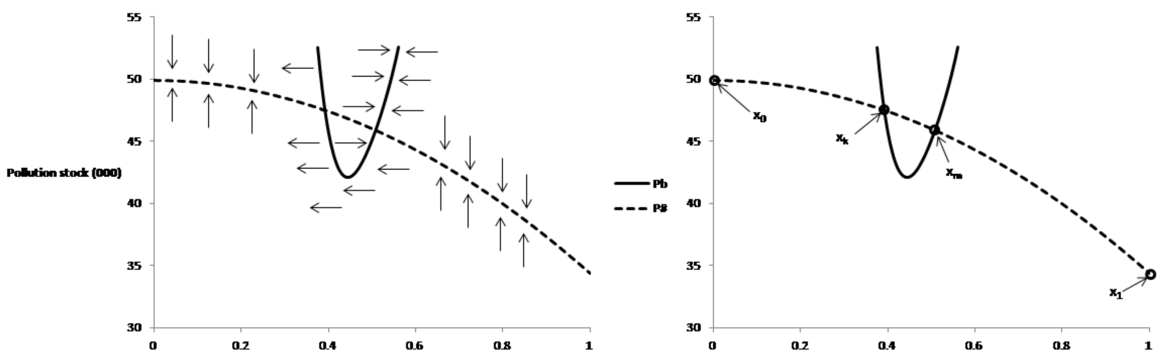


Figure 3: Plot of $p^\#$ and $p^\flat$ as a function of $s \in (0,1)$ for the base case; parameter values are given in Table 1.

Figure 3 illustrates a situation with two stable steady-states and two unstable steady-states. $x_0 = (0, p^\#(0))$ is stable: since $x_0 \in \Gamma_4$, there is no incentive for an increase in the number of signatories. On the other hand, $x_1 = (1, p^\#(1))$ is not stable, since $x_1 \in \Gamma_4$, so that there is an incentive to defect from full

[3] This choice allows for analytical solutions of static and myopic dynamic problems.

cooperation. In the same way, $x_k = (k, p^{\#}(k))$ is not stable: a small perturbation along $p^{\#}$ to the left of k leads into Γ_4 and a further decrease in s, while a small perturbation along $p^{\#}$ to the right of k leads into Γ_3 and a further increase in s. Finally, $x_m = (m, p^{\#}(m))$ is stable: a small perturbation along $p^{\#}$ to the left of m leads into Γ_3 and increasing s while a small perturbation along $p^{\#}$ to the right of m leads into Γ_4 and decreasing s. For the same example, Figure 4 illustrates typical system evolutions from various initial conditions, and the basins of attraction of the two stable steady-states.

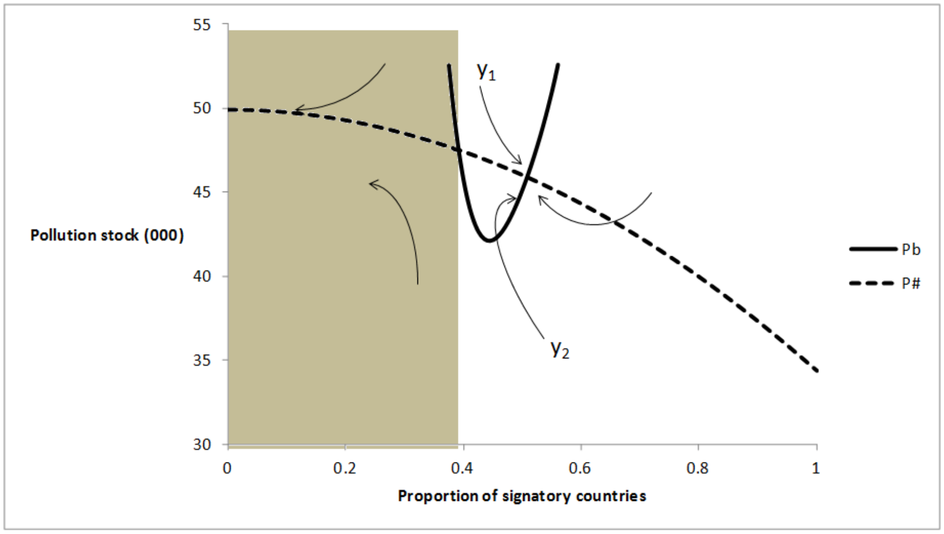


Figure 4: Trajectories corresponding to various initial conditions. The shaded area is the attraction basin of the full-defection stable steady-state.

We observe that various time-evolutions for the number of signatories are possible. For example, when the initial level of the pollution stock and the initial number of signatories are high enough (see Y_1), an initial coalition may consistently attract new members while the pollution stock is regularly reduced, until a stable coalition is reached. However, even when initial conditions are such that signatories fare better, it may happen (starting from the shaded area in Γ_3) that an initially growing coalition which succeeds in decreasing the pollution stock becomes less attractive and starts losing members, until the coalition vanishes. The reverse can also be true: starting from a low pollution stock and initial coalitions where members have incentive to defect (see Y_2), it may happen that the growth of the pollution stock changes the conditions in such a way that the coalition becomes attractive and a stable coalition is reached in the steady-state. The set of conditions eventually leading to full defection is represented by the shaded area. The complementary area contains the set of initial conditions leading to a stable coalition.

Changes in parameter values lead to qualitatively similar behavior of the steady-state functions $p^{\#}$ and $p^{\flat}$: $p^{\#}$ is generally concave decreasing in s while $p^{\flat}$ is U-shaped: for some given pollution stock p, two distinct values of s may achieve stability of the coalition, and it may also happen that there exist no coalition size at a given p achieving equality of payoffs.

Notice that the shape of the steady-state function $p^{\flat}$ is due to our assumption about the form of the punishment function: punishment suffered by defectors is increasing in s, while the cost of punishment suffered by signatories is decreasing in s. When there is no mechanism to favour coordination, the welfare of signatories generally deteriorates as their number increases, while the reverse happens to defectors (as s increases, both signatories and non-signatories emit less at equilibrium, reducing the common environmental damage, but the reduction in the emissions and revenues of signatories is much steeper).[4] In our model, the two outcomes combine to allow for large stable coalitions to form.

According to the parameter values and the relative positions of $p^{\#}$ and $p^{\flat}$ in Γ, three situations may arise, as illustrated in Figure 5. In the left panel, defectors are always better off than signatories, so that

[4]This is the reason why large coalitions cannot be sustained in general.

no stable coalition can form and the only steady-state is full defection. In the center panel, the steady-state may be either full defection or a stable coalition, depending on the initial conditions. Finally, the right panel corresponds to a case where there is no stable inner steady-state, so that the two possible outcomes are either full defection or full cooperation, depending on the initial conditions.

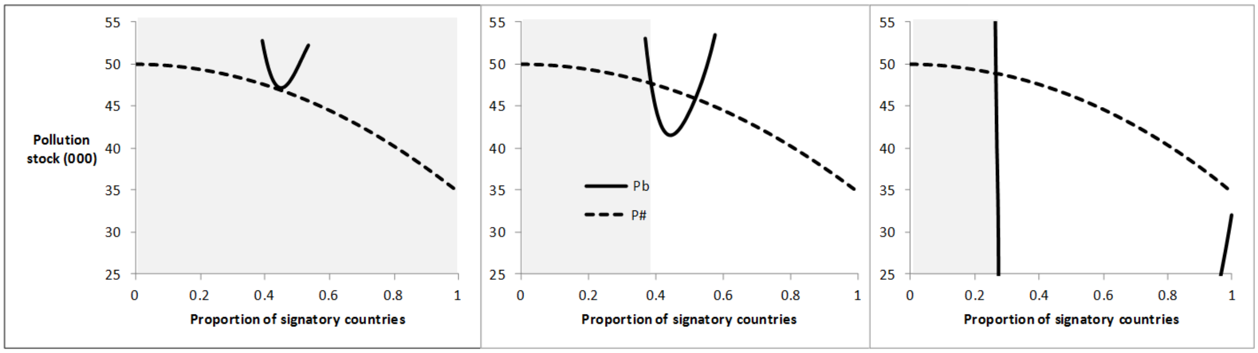


Figure 5: Three possible steady-state outcomes. The shaded area is the set of initial conditions leading to full defection. Left panel: $\beta = 0.89, \alpha = 36 \times 10^{-6}$. Center panel: $\beta = 0.89, \alpha = 37 \times 10^{-6}$. Right panel: $\beta = 0.89, \alpha = 68.5 \times 10^{-6}$. All other parameters as in Table 1.

5.2 Sensitivity to parameter values

As can be readily observed by Figure 3, increased participation and a larger set of initial conditions leading to cooperation can be achieved by moving $p^{\#}$ up, resulting in higher pollution stock at the steady-state, or by moving $p^{\flat}$ down, resulting in a lower steady-state pollution stock. Varying parameter values has an impact on both steady-state curves. Table 2 reports on the effect of increasing parameter values, within a reasonable range, on the steady-state equilibrium; numerical investigations show that the equilibrium strategy vector is much less sensitive to changes in parameter values than the relative welfare of players, so that the corresponding variation of $p^{\flat}$ is much more important than that of $p^{\#}$ for a given s.

Table 2: Impact of an increase in parameter values; s^* is the size of a stable coalition and p^* is the steady-state pollution stock if this coalition forms.

	N	b	d	τ	α	β	δ
$p^{\#}$	↑	↑	↓	↓	↓	↓	↓
$p^{\flat}$	↑	↓	↑	↑	↓	↑	↓
s^*	↓	↑	↓	↓	↑	↓	↑
p^*	↑	↑	↑	↑	↓	↑	↓

Increasing α, δ or b, or decreasing β, d or τ within the range of reasonable parameter values, all lead to larger stable coalition. However, if a partial coalition forms, the resulting steady-state pollution stock is lower with an increase in α or δ, or a decrease in the other parameter values. Indeed, increasing b raises the profitability of emissions, and even if the steady-state coalition is larger, the total equilibrium emissions are larger, resulting in a higher steady-state pollution stock. The U-shape of $p^{\flat}$ implies that when a set of parameter yields a larger stable coalition, then the set of initial conditions leading to this steady-state is also larger.

5.3 Impact of foresight

To assess the impact of our evolutionary farsightedness assumption, we now compare our results with the corresponding ones obtained when players act myopically with respect to the coalition dynamics. The PM equilibrium strategies and corresponding value functions can readily be obtained by replacing the transition (17) by $s' = s$ in Algorithm (17)–(24). Notice that in the simple linear quadratic case considered here, one can obtain the PM solution analytically (see Breton et al. 2010).

Figure 6 represents both the PM and the EF solutions for the illustrative case in Table 1. In all our numerical experiments, we find that functions $p^{\#}$ (long-dashed) are not very sensitive to the evolutionary farsightedness assumption, so that their graph almost coincide ($p^{\#}$ is on average slightly higher with the EF assumption, mostly when s is high). However, functions $p^{\flat}$ in the EF case (solid) are always below their counterpart in the PM case (short-dashed).

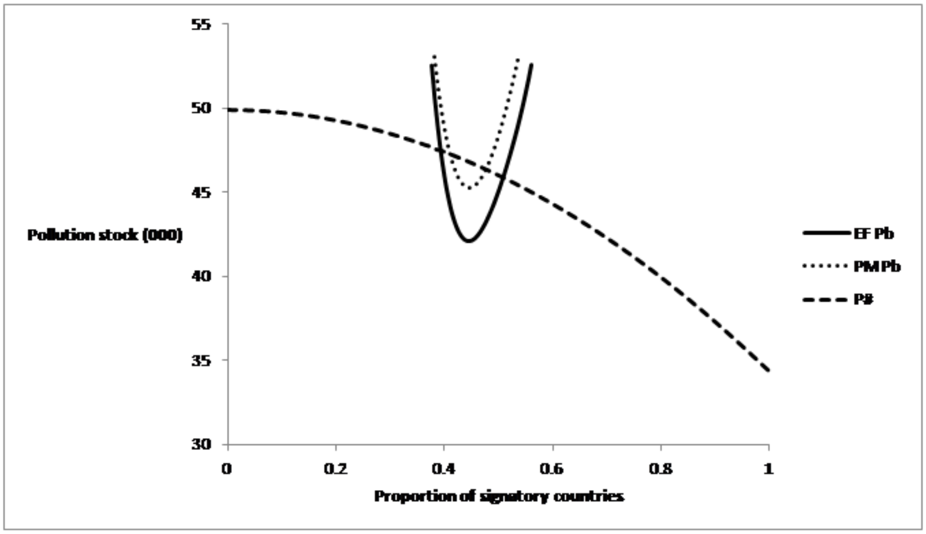


Figure 6: Comparison of the PM and EF steady-state solutions. Parameters are those of Table 1.

Our results imply that, when a stable coalition exists under the EF assumption, it contains a larger number of countries, has a larger basin of attraction, and results in lower steady-state pollution stock than a stable coalition (if it exists) under the PM assumption. The difference between the two solutions becomes increasingly significant as the discount factor increases, giving more importance to future welfare. As a consequence, for some set of parameter values, there exists a stable coalition under the EF assumption while the only stable steady-state is full defection under the PM assumption. Such a situation is represented in Figure 7 (for some extreme cases, one can even obtain full cooperation as an EF equilibrium and full defection as a PM equilibrium).

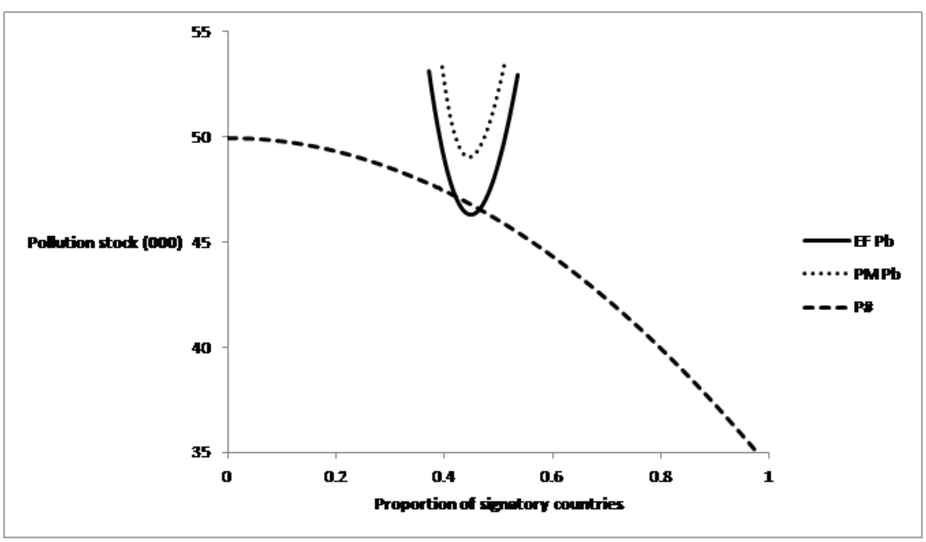


Figure 7: A case where a stable coalition only exists under the EF assumption. Parameters are as in Table 1 except for $\alpha = 38 \times 10^{-6}$.

It is interesting to remark that equilibrium emissions (and consequently the steady-state function $p^{\#}$) do not differ significantly according to the evolutionary farsightedness assumption. The increased stability of coalitions under the EF assumption is then almost exclusively due to a better evaluation of the welfare differences when the dynamics are taken into account by the players.

6 Conclusion

This paper examines the dynamics of membership in international environmental agreements, where we assume that countries are driven by welfare consideration. Accordingly, membership in IEAs increases when signatory countries have higher welfare than the others, decreases if the defectors have higher welfare than signatories, and a coalition is stable when the welfare are equal for both groups of players. Assuming that the membership in the IEA is described by a discrete-time replicator dynamics, this paper investigates the impact of evolutionary farsightedness on the steady-state of the system when the agreement includes a punishment clause.

Under the evolutionary farsightedness assumption, players are aware of the impact of their decisions on the evolution of both the pollution stock and the number of signatories. We characterize the EF equilibrium as the solution of a dynamic game with two state variables between signatories, acting as a single player, and all defectors . We use a value-iteration algorithm, coupled with spline interpolation of the game value function, to compute the equilibrium strategies, welfare, and steady-states. We then perform a comparison of the results obtained under the EF assumption with those obtained when one assumes that players are partially myopic and do not account for the impact of their decisions on the membership of the agreement.

We find that evolutionary farsightedness is beneficial to the formation of stable coalitions and to the reduction of the pollution stock. When a stable coalition exists in both cases, with evolutionary farsighted players the stable coalition size is larger and the steady-state pollution stock is smaller than in the myopic case. For some parameter values, a stable coalition of farsighted players can form while the myopic case leads to full defection. The difference between farsighted and myopic solution is increasingly significant as the players put more value on their future welfare.

7 Appendix: Detailed algorithm

Step 1. Initialization

Read the model parameters. Compute upper and lower bounds on the pollution stock. Define a grid $\mathcal{G} = \mathcal{S} \times \mathcal{P}$ on the state space. Define the tolerance ϵ.
Initialize $\widehat{V}^{S;0}$ and $\widehat{V}^{D;0}$ on $\mathcal{G}$. Set $k = 0$.

Step 2. Value iteration

2.1 Interpolate $\widehat{V}^{S;k}$ and $\widehat{V}^{D;k}$ by two-dimensional cubic splines $V^{S,k}$ and $V^{D,k}$ and define the corresponding derivatives $V_p^{S,k}$ and $V_p^{D,k}$

2.2 For all $(s, P) \in \mathcal{G}$

i) Find $(e^{S,k}(s,p), e^{D,k}(s,p))$ satisfying (20) and (22) using (17)–(18); check non-negativity of $e^{S,k}(s,p)$ and $e^{D,k}(s,p)$

ii) Compute $\widehat{V}^{S,k+1}(s,p)$ and $\widehat{V}^{D,k+1}(s,p)$ according to (25) and (26)

2.3 If $\left\|\widehat{V}^{S,k+1} - \widehat{V}^{S,k}\right\| < \epsilon$ and $\left\|\widehat{V}^{D,k+1} - \widehat{V}^{D,k}\right\| < \epsilon$, go to Step 3. Otherwise, set $k := k+1$ and go to Step 2.

Step 3. **Steady-state functions**

3.1 Interpolate $\widehat{V}^{S;k}$ and $\widehat{V}^{D;k}$ by two-dimensional cubic splines V^S and V^D

3.2 For each $s \in \mathcal{G}_1$, find the fixed point $p^{\#}(s)$ of 31 using emission strategies satisfying (20) and (22)

3.3 For each $s \in \mathcal{G}_1$, find the root $p^{\flat}(s)$ of $V^S(s,\cdot) - V^D(s,\cdot)$

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