The Implications of Energy Input Flexibility for a Resource Dependent Economy

Luise Röpke * Karen Pittel†

Abstract

The paper analyzes resource policies in an economy in which renewable and fossil resources are realistically assumed to be essential inputs to production. Also realistically, the two types of resources are imperfect substitutes whose degree of substitutability can, however, increase over time. The focus of the - analytical as well as numerical - analysis is on the impact of this rising substitutability on the extraction of the exhaustible resource. This is especially interesting in a setting in which the use of the fossil resource induces a market failure, e.g., in the form of an environmental externality (of which climate change is the most prominent example), and in which policies are introduced to internalize this market failure. It is shown that policies which aim to slow down resource extraction but whose design is determined from political rather than optimality considerations are likely to result in even faster resource extraction. We show that this effect - often labeled a 'Green Paradox' - can be accompanied by extraction-increasing effects of rising substitutability. More specifically, we find two types of flexibility effects that have opposing effects on the extraction path. The first effect speeds up ex-traction due to the expectation of higher flexibility in the future. This effect arises independently of whether the increase in substitutability is due to exogenous technological change or is endogenously driven. The second effect slows down extraction and arises when substitutability increases endogenously in accord with a changing input mix. Our results have several important implications for the design of policy measures. Specifically, a policy measure that induces flexibility-increasing technological progress must take into consideration the supply-side effects that result from the anticipation of increasing flexibility. The model also shows that for a policy to be effective, not only must flexibility effects be taken into account but the specific type of flexibility effect is also important.

JEL-Classification: Q32, O44, O30
Keywords: non-renewable resources, elasticity of substitution, climate policy, green paradox

*Ifo Institute - Leibniz Institute for Economic Research at the University of Munich, Poschinger Strasse 5, 81679 Munich, Germany, email: roepke@ifo.de.

†Ifo Institute - Leibniz Institute for Economic Research at the University of Munich and CESifo, Poschinger Strasse 5, 81679 Munich, Germany, email: pittel@ifo.de.

1 Introduction

In principle, all measures that are discussed to reduce anthropogenic CO_2 emissions boil down to two basis channels. The first is decreasing the amount of energy within goods produced and consumed. This reduction can stem from a variety of different sources: It may result from innovations that allow to produce existing goods with less energy; it can be induced by the innovation of new goods that substitute old, more energy-intensive varieties or it might – in the simplest case – result from a decrease in the level of production (or negative growth). The second basic option to decrease CO_2 emissions is by substituting fossil energy by renewable energy. It is this second option that our paper focuses on.

Accomplishing the climate goals efficiently is likely to require both, falling energy intensities as well as substitution by renewable energy. Yet, despite the growing political focus on renewable energy sources, much of the literature on resource scarcity, climate change, and growth still focuses on the challenge posed by the exhaustibility of fossil resources. Usually, technological progress is considered to lower energy intensity of production and thus to alleviate resource scarcity and reduce climate emissions per unit produced. Moreover, those papers that take renewable resources into account mostly assume that energy from fossil and renewable sources are perfect substitutes (see, e.g., Heal 1976; Hoel and Kverndokk 1996; van der Ploeg and Withagen 2011b). Realistically, renewables will, however, remain only imperfect substitutes for oil, coal, and gas, at least in the foreseeable future. While fossils and renewables are nearly perfect substitutes for some activities – take electricity generated from wind and coal, or cars powered by biofuels, electricity or gasoline, for example – substitution is less likely with respect to other activities. The usage of oil in the chemical industry is just one of these examples. So, it seems appropriate to assume that fossils and renewables are only imperfect substitutes and will remain imperfect substitutes at least in the foreseeable future.

Technological development has, however, already overcome some limits to substitution and thus increased the potential for and the degree of substitutability. Today, countries like Germany generate more than 20% of their electricity from renewable sources.[1] The largest share of this generation stems from highly intermittent sources like wind and solar. The more or less seamless integration of this volatile electricity production into the market has become possible due to the increased flexibility of the system. Another illustrative example of increasing flexibility, or substitutability, between exhaustible and renewable resources is the use of biofuels in Brazil. Starting in the 70s, the mandatory share of ethanol in transport fuels has constantly been increased over the years. In response to this, industry developed flexible-fuel motors for both gasoline and ethanol use. In 2013, those 'flex vehicles' reach a share of 95% "of all new cars and light vehicle sales" in Brazil (Singh 2013, p. 347; see also, e.g., Eisenthal 2013; UNEP 2013).

It can be expected that the substitutability between renewable and fossil energy will further

[1] In Germany, the share of renewable energies in final electricity consumption increased from 6.2% in 2000 to 23.5% in 2012 (BMU 2013).

increase in the future. Take the example of producing gas from wind or solar power ('power-to-gas'). While these technologies already exist today, their low degree of efficiency and thus high costs prevent production on a large scale. Yet, it is well conceivable that technological progress improves conversion efficiencies further while fossil energy prices rise such that 'power-to-gas' might become a viable alternative in the future. Other examples of increasing flexibility might encompass the installation of smart grids, the expansion of storage capacities or the intensified trade on electricity markets – or other technologies that are not even in their infancy today.

It is the increase in substitutability that the paper at hand focuses on. We assume that fossil and renewable energies are imperfect substitutes and also remain imperfect substitutes in the future although substitutability improves over time. Energy remains an essential input to production thus taking account of Daly and Farleys apt observation that "it is impossible to create something from nothing" (Daly and Farley 2004, p. 122). Moreover, we consider that burning fossil fuels, and thereby the emission of greenhouse gases, generates a market failure in the form of a negative environmental externality commonly referred to as climate change. Our specific interest is in the implications that a higher substitutability exerts on the level and speed of fossil fuel extraction. As to be expected, optimal extraction reacts on changes in substitutability. But, beyond this, we also consider the effect of higher flexibility on the effectiveness of climate policies. We specifically compare the results of a no policy scenario and a non-optimal policy scenario to the socially optimal case of a Pigou-like carbon tax. The non-optimal policies we consider are designed with an eye on the real world, i.e. with political rather than optimality considerations in mind. These policies are likely to be not even second-best. In this context, we analyze possible 'green paradox' outcomes, that is policy measures might speed up resource extraction and thus climate change rather than slowing it down.

We show that how resource owners react to climate policies also depends on whether or not the substitutability between energy inputs changes over time. Due to increasing flexibility, owners adjust their intertemporal extraction decision. We show that how resource extraction is affected by increasing substitutability depends crucially on the forces driving the increase in flexibility. We consider two alternative model specifications: First, substitutability rises exogenously over time - comparable to exogenous technological progress which is unaffected by the decisions made by consumers and resource owners. Second, we allow substitutability to develop endogenously, depending on the input mix chosen by output producers. We specifically assume the elasticity of substitution between fossil and renewable energy to rise when fossil energy becomes more scarce. This adjustment reflects learning on the side of the producers. The scarcer fossil energy becomes, the more technologies are adapted to accommodate a steadily rising share of renewables in the energy mix.

As a consequence of increasing flexibility, we find two types of flexibility effects that influence the extraction decisions of the resource owners. The first effect speeds extraction up due to the expectation of higher flexibility in the future. This effect arises independently of whether the increase in substitutability is due to exogenous technological change or is endogenously driven. The second effect slows extraction down and arises when substitutability increases endogenously in accord with a changing input mix. Our results have several important implications for the design of policy measures. Specifically, a policy measure that induces flexibility-increasing technological progress must take into consideration the supply-side effects that result from the anticipation of increasing flexibility. The model also shows that for a policy to be effective, not only must flexibility effects be taken into account but the specific type of flexibility effect is also important.

Considering that the elasticity of substitution is not constant but variable has a long tradition in economic theory. Sato and Hoffman (1968) already argue that it is more realistic to assume a variable instead of a constant elasticity of substitution. They develop different variable elasticity of substitution production functions in an attempt to generalize the standard constant elasticity of substitution production technology (CES). Around the same time, Lu and Fletcher (1968) also introduce a generalized function where the elasticity of substitution is a function of relative factor inputs and for which the CES function is a special case. Another generalization approach can be found in Revankar (1971). Then, Kadiyala (1972) finally incorporates all the above mentioned functions as special cases in an even more generalized set-up. Non of these papers, however, consider the potentially important implications that changes in the degree of substitutability could have in the context of exhaustible resources.

More recently, the new growth literature started to deal extensively with overcoming constraints to economic development that result from exhaustible resources. In this context, the role elasticity of substitution could play is, for example, acknowledged by Bretschger (2005, p. 150). He stresses that "[all] possibilities of substitution and, specifically, the effects technology exerts on promoting substitution, have to be studied." Moreover, authors started to relate the elasticity of substitution to the degree of economic development of an economy (e.g. Mansanjala and Papageorgiou 2004, Karagiannis et al. 2004). Karagiannis et al. (2004), for example, argue that, as the elasticity of substitution depends on economic development, unrestricted endogenous growth is possible even in the absence of exogenous technological progress and despite the existence of constrained production factors. In this context, De la Grandville points out that the elasticity of substitution is a "potent explanatory variable of economic growth" (De la Grandville 1989, p. 479).[2]

Growiec and Schumacher (2008) were, to our knowledge, the first to directly combine the issues of exhaustible resources and increasing elasticities of substitution. They show that (exogenous) technological progress which succeeds in increasing elasticity of substitution can be sufficient to overcome resource exhaustibility. They do, however, not take into account implications of externalities arising from exhaustible resource use and the possibility that changes in flexibility might not be exogenous but rather be determined by the decisions made by firms. By including disutility from climate change and endogenous changes in the elasticity of substitution into our analysis, we are able to derive more precise climate political implications. In contrast to

[2]As the focus of our paper is on the implications of flexibility between inputs and not on growth, the latter interpretation is, however, less important in our context.

Growiec and Schumacher, we limit our analysis to the case of renewable and fossil energy being complements – i.e. to an elasticity of substitution between the two inputs that is smaller than unity. With an elasticity that exceeds unity, exhaustible resources would not be essential inputs to production anymore and a positive level of production could, even in the absence of technological progress, be maintained forever – in other words, the exhaustibility of fossil resources would cease to be a problem.[3] However, as argued before, at least in the short- and medium-term the assumption that fossil energy will remain an essential factor of production seems sensible.

In our paper, we assume in a first step and based on the model of Growiec and Schumacher that the elasticity of substitution increases exogenously (*IES case*). In a second step, we apply the production function of Lu and Fletcher (1968) in which the elasticity of substitution endogenously depends on the energy input mix (*VES case*). We consider the constant elasticity of substitution (*CES case*) as a benchmark scenario. The CES, IES, and VES cases are compared with respect to optimal resource extraction and evaluated regarding the effects of (optimal, laissez-faire, and non-optimal) climate policies.

The structure of the paper is as follows: In the next section, we introduce the concept of an increasing elasticity of substitution and discuss technological progress in this context. Then, we provide the general model approach (normative and positive) in Section 3. The exogenous and endogenous elasticities of substitution are introduced in Sections 4 and 5. Based on that, in Section 6, we analyze and compare the extraction behavior of supply side for different policy scenarios and the underlying production technologies. In Section 7 follows a numerical illustration of the results. Finally, Section 8 concludes.

2 The Elasticity of Substitution, Flexibility and Technological Change

As the elasticity of substitution is a crucial component of our analysis, we (re-)introduce the underlying concept in the following and link changes in the elasticity of substitution to technological development. In this context, we discuss and refer to the different types of technological change that could overcome the potential scarcity of production factors.

Formally, the elasticity of substitution, σ,

$$\sigma = -\frac{d\frac{x_1}{x_2}}{d\frac{dx_1}{dx_2}} \frac{\frac{dx_1}{dx_2}}{\frac{x_1}{x_2}}, \tag{1}$$

measures the change in the relative factor input ratio for a change in the relative marginal rate of substitution (see, e.g., Allen 1938). The elasticity of substitution can be understood as a

[3] An analysis with energy goods becoming perfect substitutes is closely related to a backstop analysis. An example for this is Hoel (2008) who analyzes policy measures that promote a clean backstop technology, resulting in the fossil fuel stock being exhausted sooner and hence producing a green paradox outcome.

measure of flexibility or efficiency (see, for example, Arrow et al. 1961, De la Grandville 1989, or Growiec and Schumacher 2008) or "as a 'menu of choice' available to entrepreneurs" (Yuhn 1991, p. 344).[4]

The value of the elasticity of substitution can range from 0 to ∞. For $0 < \sigma < 1$, production factors are complements and for $1 < \sigma < \infty$, they are substitutes. If, in the case of complements, the input of one of the production factors goes to zero for a given technology, output inevitably converges toward zero. As we consider exhaustible fossil energy as one of the production factors, this is the situation we are facing in our paper. To overcome the dismal result of output going to zero, technological progress is required.

In general, three different types of technological progress can be distinguished that can succeed in overcoming the constraints set by the absolute scarcity of fossil energy. The first is factor-augmenting (directed) technological change. Models in which endogenous directed technological change drives growth have been discussed at length in the recent literature (see, e.g., Acemoglu 2002; Di Maria and Valente 2008; Pittel and Bretschger 2010; Acemoglu et al. 2012). The second type is factor-neutral technological change. In this case, technological progress enhances total factor productivity, i.e. it raises the productivity of all factors without changing their relative importance. Factor-neutral technological progress is usually considered when the mechanisms driving growth are of little importance for the research question posed. Finally, the third type is flexibility-enhancing technological change. Overcoming the exhaustibility of an input requires in this case that elasticity of substitution increases until the production factors become substitutes, i.e., until $\sigma > 1$.

Yet, as discussed in the previous section, while we consider realistically that elasticity of substitution increases over time, we do not believe that renewable and fossil energy will become complements in the foreseeable future. Consequently, without any additional technological progress, economic activity would cease in our model despite the increase in flexibility. For this reason, we additionally assume total factor productivity to grow at an exogenously given rate. This allows us to focus specifically on the impact of the rising elasticity of substitution on the input mix without blurring effects from, for example, factor-augmenting technological change.

Regarding elasticity of substitution and its development over time, it is plausible to assume that the elasticity of substitution is constant in the short-run. But when an essential input becomes more and more scarce, as it is the case for exhaustible resources, it is also plausible to assume that some kind of learning process will be initiated that increases flexibility. This process can, for example, result decentrally from market processes, or it could be induced by policy measures. The two alternative set-ups we consider in this paper can be related to these two forces. On the one hand, we assume increasing flexibility to be exogenously driven. This exogenous development could, for example, result from state-funded fundamental research. The resulting technological change would in this case be independent of the decisions of private agents.

[4]The value of the elasticity of substitution can range from 0 to ∞. For $0 < \sigma < 1$, production factors are complements and for $1 < \sigma < \infty$, they are substitutes.

On the other hand, for any given elasticity of substitution, the rising scarcity of fossil energy could induce substitution processes that change the energy mix. The stronger the adjustment, the more firms learn how to substitute the scarcer factor by the more abundant one. The elasticity of substitution thus becomes endogenous and a function of the energy mix.

3 The Model Framework

In our stylized model, final output is produced from a composite energy good for whose production exhaustible and renewable energy is used. The burning of fossil fuels results in emissions which cause pollution and therefore damages. In this section, we consider a general type of production function. Based on this framework, we derive the social costs of carbon in a social planner setting together with the socially optimal extraction decision and compare the solution with the market equilibrium. From this, we can derive the time path of the optimal carbon tax. Specific technologies will be introduced in the following sections in which we then explicitly consider exogenously and endogenously changing elasticities of substitution.

The production function of final output is given by

$$Y(t) = F(A(t), L(t), R(t)) = A(t)R(n(t), m). \tag{2}$$

$A(t)$ denotes total factor productivity which is assumed to grow at a constant rate g, $g > 0$, and the initial level of total factor productivity is set to unity, $A(0) = 1$, such that $A(t) = e^{gt}$. $R(t) = R(n(t), m)$ represents the composite energy good that is produced from fossil fuels, $n(t)$, and renewable energy, m, which is supplied in constant amount at each point in time. In the following sections, we will provide different specific formulations of this energy production function. At this point, it is only assumed that R fulfills the standard property of positive decreasing marginal products.[5]

Fossil fuels n are extracted from an exhaustible resource stock denoted by $S(t)$. As we assume that storage of the extracted resource is not possible, the dynamics of the resource stock are given by

$$\dot{S}(t) = -n(t). \tag{3}$$

$S(0) = S_0 > 0$ is the initial stock of the resource in situ and constitutes an upper bound to resource extraction ($S_0 \geq \int_0^\infty n(t)dt$).[6] For simplicity, we assume that there are no extraction costs for fossil fuels or production costs of renewable energy. As a consequence, the entire supply of m is always employed in production.

[5]Please note that we could alternatively assume a more general production function as Growiec and Schumacher (2008). They also include the input of a constant and inelastic supply of labor in a Cobb-Douglas type production function. This would, however, not affect the qualitative nature of our results.

[6]Throughout this paper we will use $\dot{x}$ to denote the time derivative of a variable $x(t)$, $\dot{x} = \frac{\partial x(t)}{\partial t}$.

Households derive utility, $U[\cdot]$, from consumption, C, while pollution causes disutility in the form of damages, D. The representative household maximizes discounted lifetime utility

$$\int_0^\infty U[C(t), D(t)]e^{-\rho t}dt \tag{4}$$

with respect to its intertemporal budget constraint. $\rho > 0$ is the discount rate with which households discount future utility. Since we abstract from capital accumulation as well as costs of input usage, $Y(t) = C(t)$ holds.

The instantaneous utility function is of the isoelastic type

$$U(C(t), D(t)) = U(C(t)) + U(D(t)) = \frac{C(t)^{1-\eta}}{1-\eta} - \frac{D(t)^{1-\omega}}{1-\omega} \tag{5}$$

as in Aghion and Howitt (1998) or Grimaud and Rougé (2005). Utility of consumption and disutility of pollution are additively separable and the parameters $\eta > 0$, respectively $\omega > 0$, determine the constant relative risk aversion (CRRA) with $\frac{1}{\eta}$, respectively $\frac{1}{\omega}$ being the elasticities of intertemporal substitution. As for this utility function $U'(C) > 0$, $U''(C) < 0$ hold for all positive levels of consumption and $\lim_{C \to 0} = \infty$, the function satisfies the Inada conditions and implies a (strictly) positive level of consumption over time. $U(D)$ has essentially the same properties with $U'(D) < 0$, $U''(D) > 0$. Since damages create disutility, D enters the utility function with a negative sign.

Damages result from pollution, i.e. $D(t) = D(P(t))$, where $P(t)$ is the stock of pollution. We assume damages to be convex in pollution ($D'(P(t)) > 0$ and $D''(P(t)) > 0$) for all positive pollution levels. Pollution accumulates as a consequence of the burning of fossil fuels:

$$\dot{P}(t) = h(n(t)), \qquad h_n > 0. \tag{6}$$

As a point of reference for the market solution and to understand the dynamics resulting from this model framework, we derive the optimal time paths of production and extraction as well as the social costs of carbon in the social planner solution in the following section.

3.1 Social Planner

The social planner maximizes the present value of the representative household's utility, (4) and (5), subject to the production technology, (2), and the resource and pollution dynamics, (3) and (6). The current value Hamiltonian of this optimization problem is given by

$$H = \frac{1}{1-\eta}F(A, n, m)^{1-\eta} - \frac{1}{1-\omega}D(P)^{1-\omega} - \mu_s n + \mu_P h(n) \tag{7}$$

where μ_S and μ_P denote the shadow values of resource extraction and emissions.[7] μ_P is the shadow cost associated with the damages from accumulated pollution. μ_S is the scarcity rent of the resource stock.

From this we get the following first-order conditions:

$$H_n = 0 \quad \Leftrightarrow \quad F^{-\eta} F_n = \mu_S - \mu_P h_n, \tag{8}$$

$$-H_{S_n} = \dot{\mu}_S - \rho\mu_S \quad \Leftrightarrow \quad \dot{\mu}_S - \rho\mu_S = 0 \rightarrow \mu_S = \mu_{S_0} e^{\rho t}, \tag{9}$$

$$-H_P = \dot{\mu}_P - \rho\mu_P \quad \Leftrightarrow \quad \dot{\mu}_P - \rho\mu_P = D^{-\omega} D_P. \tag{10}$$

(8) gives the condition for an optimal extraction of the fossil resource by equating the marginal utility of extracting and consuming the resource to the social marginal costs of extraction.[8] (9) and (10) implicitly describe the optimal time paths of the state variables S and P. (9) equalizes the growth rate of the social value of extracting a marginal unit of the resource to the discount rate and (10) describes the optimal dynamics of the social costs of carbon. Moreover, the transversality condition reads:

$$\lim_{t\to\infty} (\mu_S S - \mu_P P) e^{-\rho t} = 0. \tag{11}$$

Taking the time derivative of (8) and dividing the resulting expression by (8), we get the Ramsey-Hotelling condition that characterizes the interior solution of the present optimality problem:[9]

$$\hat{F}_n - \eta\hat{F} = \frac{\dot{\mu}_S - \dot{\mu}_P h_n - \mu_P \dot{h}_n}{\mu_S - \mu_P h_n}. \tag{12}$$

The LHS of (12) determines the growth rate of utility from consumption which must equal the RHS, the growth rate of the marginal social cost of an additional unit of the resource extracted plus the growth rate of the scarcity rent. Without pollution, the RHS of (12) would reduce to $\frac{\dot{\mu}_S}{\mu_S}$ which, in the case of no pollution, equals the rate of discount ρ (see (9)). Thus we would be back to the standard Hotelling rule that equates the growth rate of the marginal benefits from extracting the resource to the discount rate.[10]

By solving the differential equation (10) we can derive the social costs of carbon denoted by SC:

$$SC = -\mu_P = \int_t^\infty e^{-\rho(\tau - t)} \left(D^{-\omega} D_P\right) d\tau. \tag{13}$$

Following van der Ploeg and Withagen (2011a, p. 7), we define the social costs of carbon to be equal to "the shadow cost of atmospheric CO_2 [which] is positive, because it measures the value in welfare terms of having a smaller CO_2 stock."

[7] In the following, we will omit time coefficients if unambiguous.

[8] Throughout this paper, x_y denotes the partial derivative of a variable x with respect to a variable y, i.e. $x_y = \frac{\partial x}{\partial y}$.

[9] Throughout this paper, we will use $\hat{x}$ to denote the growth rate of a variable $x(t)$, i.e. $\hat{x} = \frac{\dot{x}}{x}$.

[10] The Hotelling rule is also referred to as the Solow-Stiglitz efficiency condition in the case of a social optimum.

3.2 Regulated Market Equilibrium

In the present paper, we assume pollution damages to be an externality to households, that is, households take the damages from pollution as exogenous to their optimization problem. In an unregulated market economy, the negative externality will thus not be internalized. Accordingly, a regulator can improve welfare by introducing a carbon tax.[11]

The optimization problem of a representative household is in general the same as the optimization problem of the social planner except for the externality from burning fossil fuels and the tax on resource extraction (τ). The household maximizes the present value of utility, (4) and (5), net of taxes. Thus, the intertemporal optimization problem of the household is given by

$$\max \int_0^\infty e^{-\rho t}[U(C,D) - \tau n]dt \tag{14}$$

subject to (2) and (3). The current value Hamiltonian now reads

$$H = \frac{1}{1-\eta}F(A,n,m)^{1-\eta} - \frac{1}{1-\omega}D(P)^{1-\omega} - \tau n - \lambda n \tag{15}$$

where – as already stated above – damages, D, are taken to be exogenous by the individual household. The modified first-order conditions for n and S are now given by

$$H_n = 0 \qquad \Leftrightarrow \qquad F^{-\eta}F_n = \tau + \lambda, \tag{16}$$

$$-H_S = \dot{\lambda} - \rho\lambda \qquad \Leftrightarrow \qquad \dot{\lambda} - \rho\lambda = 0 \rightarrow \lambda = \lambda_0 e^{\rho t} \tag{17}$$

and the transversality condition reads $\lim_{t\to\infty} \lambda S e^{-\rho t} = 0$. By proceeding as in the derivation of (12), we get the equivalent condition for the regulated market economy:

$$\hat{F}_n - \eta\hat{F} = \frac{\dot{\tau} + \dot{\lambda}}{\tau + \lambda}. \tag{18}$$

Without externality and carbon taxes, the RHS would reduce to $\hat{\lambda}$. Together with $\hat{\lambda} = \rho$ from (17), this determines the socially optimal extraction path. Otherwise, with the externality but in a still unregulated market economy ($\tau = 0$), the production level cannot be socially optimal. This situation will be evaluated later in the policy analysis.

To derive the optimal tax that internalizes the pollution externality, we compare the market solution presented in (18) with the socially optimal solution presented in (12). From (8) and (16) and considering that in the social optimum $\lambda = \mu_S$, the optimal tax rate, τ_O, is given by

$$\tau_O = -\mu_P h_n. \tag{19}$$

[11] Alternatively, the regulator could also introduce, for example, a tax on pollution. Although the resulting optimal tax rates would differ, the qualitative results would remain unchanged.

(19) shows that the social optimum can be reached by taxing resource use n since resource use and pollution are linked by $h(n)$. The tax equals the marginal damage from the extraction of an additional marginal unit of the resource. This damage is determined by the effect that another marginal unit of emissions has on pollution, h_n, and the present value of the damages that result from this additional pollution today and in the future, μ_P.[12]

So far, we have solely considered a general type of energy production function without taking a closer look at, for example, the elasticity of substitution between fossil and renewable energy – and its potential development over time. To do this, we will choose specific functional forms for the production function integrating different assumptions about the development of the elasticity of substitution. Based on this, we can analyze the resulting consequences for socially optimal extraction as well as for extraction in a market economy. A special focus will be on the implications of non-optimal carbon taxes in the presence of rising flexibility. Section 4 introduces a production function with an exogenously developing elasticity of substitution while Section 5 analyzes production for an endogenously adjusting substitution elasticity. Based on this, extraction paths resulting from the normative analysis of the social optimum as well as in the market solution for different policy scenarios are derived in Section 6.

4 The Exogenous Model

For the remainder of this paper, we assume the production technology of the composite energy good to have the same basic structure as a constant elasticity of substitution (CES) production function. The differences between a CES function and the production functions of this and the following section mainly stem from the differences in modeling the elasticity of substitution and its development over time.

In our first approach, the production function is given by:

$$R^X = \left(\psi n^{-\theta^X} + (1-\psi) m^{-\theta^X}\right)^{-\frac{1}{\theta^X}} \tag{20}$$

with $\theta^X(t) = \frac{1-\sigma^X(t)}{\sigma^X(t)}$ where θ^X is the elasticity parameter and σ^X denotes the elasticity of substitution as introduced in (1). This production function fulfills the standard properties of a CES function, i.e. positive and decreasing marginal products, linear-homogeneity of degree one, and constant returns to scale.

In contrast to the CES production function, we assume in this section that the elasticity of substitution increases exogenously over time according to

$$\sigma^X = \frac{\sigma_0^X + st}{1 + st} \tag{21}$$

[12] In case of a tax on pollution, the optimal tax rate would be $\tau_O^{pollution} = -\mu_P$.

where $s > 0$ and $0 < \sigma_0^X < 1$ are exogenously given.

In the following, we will refer to σ_0^X as the elasticity of substitution parameter and to s as the flexibility parameter. The production function resulting from (20) and (21) will be called, as already pointed out, the increasing elasticity of substitution (IES) production function.[13]

The parameter s determines the level of substitution elasticity at each point in time as well as the speed with which elasticity of substitution converges to unity. For $s = 0$, (21) reduces to $\sigma = \sigma_0$ and (20) becomes a standard CES function. The higher s, the higher the elasticity of substitution ($\frac{\partial \sigma}{\partial s} = \frac{t(1-\sigma_0)}{(1+st)^2} > 0$). The relation between the growth rate of the elasticity ($\hat{\sigma} = \frac{s(1-\sigma_0)}{(1+st)(\sigma_0+st)}$) and s is ambiguous but turns positive if s and t are sufficiently high ($\frac{\partial \hat{\sigma}}{\partial s} = \frac{(s^2t^2-\sigma_0)(\sigma_0-1)}{(1+st)^2(\sigma_0+st)^2}$). Regarding the elasticity of substitution parameter, a higher σ_0 raises σ ($\frac{\partial \sigma}{\partial \sigma_0} = \frac{1}{1+st} > 0$) but reduces the speed with which the elasticity grows ($\frac{\partial \hat{\sigma}}{\partial \sigma_0} = -\frac{s}{(st+\sigma_0)^2}$).

In the long-run, the elasticity of substitution converges to unity ($\lim_{t\to\infty} \sigma = 1$). For $0 < \sigma_0 < 1$ and $s > 0$, σ is smaller than unity at all times and approaches its long-run value from below. So, in contrast to Growiec and Schumacher (2008) and in line with our previous reasoning, n and m remain complements although flexibility increases and it becomes easier to substitute renewable energy for fossil fuels.[14] The convergence of σ to unity and the accompanying continuous decrease of its growth rate reflects common economic intuition: The higher the degree of flexibility that has already been reached, the more difficult it becomes to increase σ further. Please note that the complementarity of m and n implies that the elasticity parameter, θ, can take values between zero and infinity.

Due to the complementarity implication, the produced amount of the composite energy good declines continuously with the decreasing input of exhaustible energy. To keep output from falling, total factor productivity, A, has to increase over time. If g is sufficiently high, a positive growth rate of consumption can be maintained even if energy production converges toward zero.

In the IES specification, the elasticity of substitution and its dynamics are independent from the actual level of extraction. Flexibility increases exogenously and irrespective of the actual energy market conditions. However, one can also argue that whether the level of flexibility changes over time should, realistically, be linked to market conditions. Therefore, the next section introduces an energy production function where the elasticity of substitution is endogenized and a function of resource scarcity.

[13]As long as there is no confusion, we will write R instead of R^X, σ instead of σ^X, and σ_0 instead of σ_0^X.

[14]For $\sigma_0 > 1$, elasticity of substitution would converge to unity from above and fossil and renewable resources would always remain substitutes.

5 The Endogenous Model

In this second approach, we introduce an energy production technology where the elasticity of substitution, σ^N, depends endogenously on relative factor inputs. More concretely, we use a variable elasticity of substitution (VES) production function in the Lu and Fletcher (1968) tradition, a generalization of the CES production function. In our VES function, the elasticity of substitution increases with falling fossil resource inputs. To interpret this, one might think of technological progress as it has already been addressed in the introduction and Section 2. The new energy production function that replaces (20) reads:

$$R^N = \left(\psi n^{-\theta^N} \gamma \left(\frac{m}{n} \right)^{-z(1+\theta^N)} + (1-\psi) m^{-\theta^N} \right)^{-\frac{1}{\theta^N}} \tag{22}$$

with $\theta^N = \frac{1-\sigma_0^N}{\sigma_0^N} > 0$ (i.e. $0 < \sigma_0^N < 1$), $\gamma = \frac{1-\sigma_0^N}{1-\sigma_0^N - z}$ and $z > 0$.[15] This VES production function equals a standard CES production function except that $\psi n^{-\theta^N}$ is multiplied by the term $\gamma \left(\frac{m}{n} \right)^{-z(1+\theta^N)}$.

In analogy to the previous section, we now label z the flexibility parameter as it captures the sensibility of the production technology with respect to resource scarcity. For $z = 0$, the term $\gamma \left(\frac{m}{n} \right)^{-z(1+\theta^N)}$ collapses to unity and (22) to the standard CES function with $\sigma = \sigma_0^N$. For the contribution of the exhaustible resource to energy production to be positive, $\sigma_0^N + z < 1$ is assumed. As $\sigma_0^N > 0$ and $z > 0$ this implies $0 < z < 1$. Under these conditions, the VES production function shares some important properties with the CES production function, namely positive and decreasing marginal products as well as homogeneity of degree one.

The substitution elasticity of (22) can be derived as shown in (1) as the VES production function is homogenous of degree one.[16] The elasticity of substitution now equals

$$\sigma = \frac{\sigma_0}{1 - z(1 - \frac{dm}{dn}\frac{n}{m})} \tag{23}$$

where $\frac{dm}{dn} = -\frac{R_n}{R_m}$. Deriving R_n and R_m from (22) and inserting them into (23) gives after some manipulation

$$\sigma = \frac{\sigma_0}{1 - z \left(1 + \frac{\gamma(\theta - z(1+\theta))}{\theta \frac{1-\psi}{\psi} \left(\frac{m}{n} \right)^{z(1+\theta)-\theta} + \gamma z(1+\theta)} \right)}. \tag{24}$$

Again, we concentrate on the case of complements where, as exhaustible resources become scarcer, production flexibility and therefore σ increase but, analogously to the exogenous model, do not

[15] For a derivation of the VES production function, see Lu (1967) or Lu and Fletcher (1968).

[16] In this section and as long as there is no confusion, we will write R instead of R^N, σ instead of σ^N, θ instead of θ^N, and σ_0 instead of σ_0^N.

exceed unity. For $z = 0$, complementarity between m and n is given for $0 < \sigma_0 < 1$. As σ depends positively on the input ratio $\frac{m}{n}$ (i.e. $\frac{\partial \sigma}{\partial m/n} > 0$), the elasticity of substitution rises over time when the fossil input falls while the renewable input remains constant. For $n \to 0$, the elasticity converges to unity from below.

The (heuristic) intuition for the dynamics of the substitution elasticity is straightforward: An increase in the scarcity pressure of exhaustible resources as n converges toward zero induces learning efforts to improve production conditions and, as a consequence, flexibility increases.

With (22) and (24) and our additional assumptions regarding the parameter values, the general dynamics of σ and R under the VES specification resemble the dynamics of the IES case: Flexibility increases over time but exhaustible resources remain an essential production factor. As the energy inputs remain complements, energy output will again go to zero if the growth of total factor productivity is too low.

6 The Effects of Increasing Flexibility and Climate Policy on the Extraction Path

As already stated in the introduction, the main interest of our paper is on how an increasing elasticity of substitution between exhaustible and renewable energy affects the extraction decision of fossil resource owners and thereby climate as well as climate policies. So, in this section, we take a closer look at the implications on extraction and optimal climate policy. But beyond that, we also consider policies that are not first-best. As policies in the real world hardly fulfill the criteria of first-best and can, as pointed out by Sinn (2008) and others (e.g. Hoel and Jensen 2012), even lead to an increase in the speed of extraction, we are especially interested in the question how these non-optimal policies work under increasing flexibility. In addition to the standard intertemporal arbitrage effect, further effects from rising flexibility that influence the extraction decision of resource owners can be expected. More concretely, beside the standard intertemporal arbitrage effect, we find two flexibility effects in our IES and VES approaches. Before we start with the analytical derivation of the effects, their intuition will be provided in the following section.

6.1 The General Intuition

Extraction of fossil energy sources follows a Hotelling-like path that results from the intertemporal profit (resp. welfare) maximization of the resource owner (resp. social planner). This maximization implies that the present value of the resource is independent of when it is extracted. While it has been shown in a broad range of literature how a policy maker can introduce instruments that influence resource prices and thereby the extraction decision of a resource owner, in the present paper, we show that increasing substitutability between resource inputs also influences resource extraction. This section describes how increasing flexibility affects the intertemporal extraction decision. Afterward, we shortly recapitulate the effects that climate policy – in this case in the form of a carbon tax – has on extraction over time.

From the two production technologies analyzed in the present paper (IES and VES), two types of flexibility effects arise. In the IES case, the resource owner's extraction decision is mainly influenced by an additional effect that shifts extraction toward the present. We label this effect the *exogenous flexibility effect* as it arises in case flexibility cannot be influenced by the market participants (the exogenous flexibility effect is independent of the market participants' behavior). In the VES case, in which flexibility is endogenously induced, a second effect arises that we label the *endogenous flexibility effect*. This effect counteracts the first effect at least partially as it tends to slow down extraction. Both flexibility effects as well as the tax effect introduced below result from intertemporal arbitrage. Resource owners anticipate future changes in the market conditions and adjust the timing of their extraction accordingly.

The intuition behind the two flexibility effects is as follows. The exogenous flexibility effect arises as the resource owner anticipates that while over time scarcity of resources increases, the elasticity of substitution also increases. In the IES case, this is because elasticity of substitution is an increasing function of time (see Section 4), and in the VES case, it follows from the increasing scarcity of exhaustible resources that induces a flexibility-enhancing effect (see Section 5). In both cases, rising flexibility implies that production becomes less dependent on the input of fossil fuels such that the future value of the resource falls. This effect is exogenous to resource owners. Without an adjustment of the extraction path, this would mean that the present value of resources extracted in the future is lower than the present value of the resources extracted today. Resource owners react to this by extracting more resources today and less in the future until present values are equalized again.

In addition to the exogenous flexibility effect, the endogenous flexibility effect arises when resource owners can influence the speed with which elasticity of substitution changes. In the VES case, resource owners understand that the faster the resource is extracted, the higher is elasticity of substitution (due to rising scarcity) which leads to a faster decrease of the value of the resource. Anticipating this, resource owners have an incentive to flatten the extraction path in order to slow down the increase of the elasticity of substitution. Therefore, the endogenous flexibility effect counteracts, at least to some extent, the exogenous flexibility effect.

Similar to the flexibility effects, a tax alters the extraction path if it changes the present value of extraction differently at different points in time. This is the standard *tax effect*. If the tax rate is chosen optimally and reflects the social costs of carbon, the RHS of (12) and (18) are the same and resource owners reallocate their extraction over time such that the extraction path in the market economy equals the extraction path in the social optimum. But what happens if the tax is not chosen optimally?

In general, the extraction decision of a resource owner is determined by (18)

$$-\eta\hat{F} + \hat{F}_n = \frac{\dot{\tau} + \dot{\lambda}}{\tau + \lambda}$$

where the effect of the tax on extraction depends on the concrete realization of $\dot{\tau}$ and τ. In order to determine the effects of a specific tax on resource extraction and welfare, we have to compare the market outcome under taxation to the laissez-faire situation (i.e. without climate policy, $\tau = 0$).

In general whether or not taxation leads to an increase or a decrease in welfare depends on whether the tax succeeds in moving the extraction rate closer to the social optimum. As damages from emissions are exogenous to the individual agents in a market economy, they do not take account of the fact that earlier extraction leads to a faster accumulation of the pollution stock and thus a higher present value of damages. In comparison to the social optimum, resources are therefore extracted too fast in an unregulated market economy. Policies that improve welfare should therefore induce resource owners to postpone resource extraction.

In order to elucidate the pure effect of taxation on resource extraction in the absence of increasing flexibility, let us consider the case of a production function of the CES type as given in (20) with a constant elasticity of substitution $\sigma = \sigma_0$ and $\theta = \frac{1-\sigma_0}{\sigma_0}$. In this case the LHS of (12) can be shown to equal

$$-\eta\hat{F}^C + \hat{F}_n^C = (1-\eta)g - \vartheta\hat{n}^C \tag{25}$$

with $\vartheta > 0$ and where C refers to the constant elasticity of substitution case.[17] The expression on the RHS is positive if the intertemporal elasticity of consumption, $1/\eta$, is not too low. As $\hat{n}^C$ is negative, the value of $-\eta\hat{F}^C + \hat{F}_n^C$ decreases if the speed of resource exhaustion slows down. From (12), we see that this implies that taxation leads to slower extraction if $\frac{\dot{\tau}+\dot{\lambda}}{\tau+\lambda} < \frac{\dot{\lambda}}{\lambda}$. As $\hat{\lambda} = \rho$ holds, this implies that the speed of extraction falls if the growth rate of the tax is lower than the discount rate.[18] This result is well-known from resource economics (see, for example, Sinn 2008).

To summarize, in comparison to the unregulated market solution, a carbon tax unambiguously a) lowers welfare if the growth rate of the tax exceeds the discount rate, and b) increases welfare if the growth rate of the tax is lower than the discount rate (but higher than or equal to the socially optimal growth rate of τ).[19]

[17] To be precise: $\vartheta = \left[1 - (1-\eta)\psi\left(\frac{R}{n}\right)^{\theta^C}\right] + \theta^C\left[1 - \psi\left(\frac{R}{n}\right)^{\theta^C}\right]$ with $\psi\left(\frac{R}{n}\right)^{\theta^C} = \frac{1}{1+\frac{1-\psi}{\psi}\left(\frac{m}{n}\right)^{-\theta^C}} < 1$.

[18] This follows from $\frac{\dot{\lambda}+\dot{\tau}}{\lambda+\tau} = \rho + \frac{\tau}{\lambda+\tau}(g_\tau - \rho)$. From this expression, we also see that a carbon tax does not change the extraction speed if $\hat{\tau} = \hat{\lambda}$. If, however, $\hat{\tau} > \hat{\lambda}$, extraction speeds up due to taxation.

[19] If the growth rate of the tax is even lower than the socially optimal growth rate, welfare decreases again compared to the social optimum. In this case, given the level of impatience of the households, too much of resource use is shifted to the future. Given that $\hat{\tau}$ is low enough, welfare could even decrease compared to the unregulated market economy.

6.2 Extraction Paths

To obtain analytical expressions of the extraction paths for the model versions with the variable substitution elasticity, we employ the conditions derived for the general functional forms in the social optimum as well as in the market equilibrium (Section 3) and combine them with the specific functional forms described in Section 4 and 5. For the constant elasticity of substitution case, we employ (20) with $\sigma = \sigma_0$ as in the previous section.[20]

Proceeding as described, we get for the socially optimal extraction path

$$\hat{n}_O^i = -\left[\frac{\dot{\mu}_S^i - \dot{\mu}_P^i h_n - \mu_P^i \dot{h}_n}{\mu_S^i - \mu_P h_n^i} - (1-\eta)g - \Delta^i\right]\frac{1}{\Omega^i} \tag{26}$$

with $i = C, X, N$ and for the equilibrium condition of the regulated market economy

$$\hat{n}_M^i = -\left[\frac{\dot{\tau} + \dot{\lambda}^i}{\tau + \lambda^i} - (1-\eta)g - \Delta^i\right]\frac{1}{\Omega^i}. \tag{27}$$

In case no policy is conducted in a market economy (laissez-faire case), we get

$$\hat{n}_{LF}^i = -\left[\rho - (1-\eta)g - \Delta^i\right]\frac{1}{\Omega^i}. \tag{28}$$

where $\hat{\lambda} = \rho$ holds. Of course, the laissez-faire scenario is more a theoretical illustration than an actual phenomenon. Even if many countries have no explicit carbon taxes, they have energy taxes, command-and-control measures, or emission trading systems in place which could also be translated into carbon taxes.

While the functional forms of the optimal and market extraction paths are the same for all three model versions, Δ^i and Ω^i differ which then, consequently, also changes the shadow prices λ, μ_S, and μ_P. For the three models we get:

$$\Delta^C = 0 \tag{29}$$

$$\Delta^X = (1-\eta)\frac{s\theta^X}{1-\sigma_0}\ln(R) + \left[\left((1-\eta)+\theta^X\right)\chi\dot{\theta}^X - \theta^X\ln(n)\right]\hat{\theta}^X \tag{30}$$

$$\Delta^N = 0 \tag{31}$$

$$\Omega^C = 1 - \left(\epsilon^C\left((1-\eta)+\theta^C\right) - \theta^C\right) > 0 \tag{32}$$

$$\Omega^X = 1 - \left(\epsilon^X\left((1-\eta)+\theta^X\right) - \theta^X\right) > 0 \tag{33}$$

$$\Omega^N = 1 - \left(\epsilon^N\left((1-\eta)+\theta^N\right) - \theta^N\right)\gamma^{-1} > 0 \tag{34}$$

with $\epsilon^C = \psi\left(\frac{R}{n}\right)^{\theta^C}$, $\epsilon^X = \psi\left(\frac{R}{n}\right)^{\theta^X}$, $\epsilon^N = \psi\gamma\left(\frac{R}{n}\right)^{\theta^N}\left(\frac{n}{m}\right)^{z(1+\theta^N)}$, and $\chi = \psi\left(\frac{R}{n}\right)^{\theta^X}\ln(n) + (1-\psi)\left(\frac{R}{m}\right)^{\theta^X}\ln(m)$.

[20] For more details on the derivation of (26) to (28), see Appendix A.

It can easily be seen that for $s = 0$, resp. $z = 0$, the extraction rates for the IES and VES case coincide with the extraction paths of the standard CES case, i.e. $\Delta^C = \Delta^X = \Delta^N$ and $\Omega^C = \Omega^X = \Omega^N$.

With an optimal carbon tax, the extraction rates that result from (26) and (27) are obviously the same. In this case, the representative household chooses the socially optimal extraction path and the market outcome is therefore socially optimal. If the tax design is, however, not optimal, extraction in the market economy and the social optimum will differ. With respect to the current state of climate policies, it is fair to assume that none of these policies are designed optimally. Therefore, it is most relevant to consider how a non-optimal carbon tax influences extraction.

As can clearly be seen from (26) to (28), the extraction rates cannot be solved analytically. We will solely draw some general conclusions about the effects of flexibility in the next section and then calibrate and simulate the time paths of resource extraction numerically in Section 7.

6.3 Effects of Increasing Flexibility

As can easily be seen from (26) to (28), the effects that a production technology has on extraction are qualitatively the same across all policy scenarios. Whether the growth rate of extraction is higher or lower in the IES or VES cases compared to the CES technology depends on the signs and magnitude of the terms in (30), (31), (33), and (34) that are due to the increasing flexibility.

Due to the complexity of the analyzed differential equations and their dependency on endogenous variables (the shadow prices as well as the extraction rate itself), inferences from the analytical expressions still have to be tested numerically. Only then the endogeneity can fully be taken into account. Analyzing the analytical terms merely gives 'ceteris paribus' results, ignoring the reactions of the endogenous variables.

Let us first compare the CES and the IES case. Compared to the CES extraction rate, the additional term Δ^X appears in the IES case in the square brackets of (26), (27), and (28) while $\Delta^C = 0$. The functional forms of Ω^C and Ω^X are the same. The effect of the rising elasticity of substitution on extraction thus depends on the sign of Δ^X. Whether Δ^X is positive or negative depends, however, crucially on the scarcity of the fossil fuel input. Let us assume first that the exhaustible resource is relatively abundant. In this case, Δ^X is negative if σ converges to unity (see Appendix B). If fossil resources, however, become sufficiently scarce, Δ^X turns positive. So, the exogenous flexibility effect increases the speed of resource extraction the more, the scarcer the resource becomes.

Comparing the CES and VES extraction rates shows that while for both cases Δ^i is equal to zero, Ω^N differs from Ω^C by the term $\gamma^{-1} = (1 - z(1 + \theta^{N^{-1}}))$. We know that $\gamma > 0$ because per definition, $1 - \sigma_0 - z > 0$. Therefore, we have $0 < \gamma^{-1} < 1$. Then, Ω^N exceeds Ω^C and the speed of resource extraction is faster. This reflects the combination of the exogenous and endogenous flexibility effect which in sum still raises the speed of extraction.

7 Numerical Analysis

As stated before, the endogeneity and high complexity of the resource dynamics only allow to draw first tentative conclusions about the effect of increasing flexibility from the analytical expressions. Therefore, we derive the time paths of the endogenous variables numerically in this section. Based on the extraction paths, we can compare the development of the economy under the standard CES technology with the development in the IES and VES scenarios. This allows us to gain a better understanding of the direction and magnitude of the effects of interest. Moreover, we take a look on the resulting implications for climate policy and climate change. In a preliminary step to the numerical analysis, we specify functional forms for emissions, h, pollution, P, and damages, D.

Regarding the pollution dynamics and the flow of emissions, we assume the pollution stock to be non-degenerating over time and emissions to be a linear function of the fossil fuels burnt, i.e. $\dot{P} = h(n) = \varepsilon n$, with $\varepsilon > 0$ being a constant emission parameter. This specification can be interpreted as a very simplified representation of the climate system. The burning of fossil fuels leads (at least in the absence of carbon capture and storage) to a specific amount of emissions, determined by the carbon content of the fuel.[21] Emissions accumulate in the atmosphere and cause climate change. Climate physics estimates the effect of accumulated carbon on temperatures to be largely irreversible for about a thousand years – in contrast to the CO_2-stock which degenerates much faster (see Solomon et al. 2009). As with respect to the damages from climate change, it is the temperature that matters and not the carbon stock, it seems a good approximation of reality to assume pollution – interpreted in temperature terms – to be non-degenerating. Given this assumption, the stock of pollution is given by

$$P = \int_0^t \varepsilon n(\tau) d\tau. \tag{35}$$

Damages from pollution are a function of this pollution stock

$$D(P) = aP^2 \tag{36}$$

with $a > 0$ and where the social damages of carbon emissions are convex, i.e. D_P, $D_{PP} > 0$ (see, for example, van der Ploeg and Withagen 2011b).

The numerical analysis will be conducted in the following subsections for a parametrization that was chosen for the different scenarios and flexibility cases to have the highest possible degree of comparability. Throughout the analysis, parameters have the same values across policy scenarios as well as across production technologies (for example, $m^C = m^X = m^N$ or $\psi^C = \psi^X = \psi^N$).[22] As we are interested in the implications of rising flexibility under the different policy

[21] For simplicity, we abstract from the heterogeneity of fossil fuels in this paper.

[22] The specific parametrization chosen for the following analysis is $\eta = 1.8, \omega = 2$, $\sigma_0^X = \sigma_0^N = 0.2$, $s = z = 0.1$, $\varepsilon = 0.6$, $\rho = 0.05$, $m = 0.8$, $a = 0.6$, $\psi = 0.6$, and $g = 0.03$.

regimes, we consider the different policy scenarios – optimal policy, non-optimal policy and laissez-faire – successively in the following subsections. Since the level and dynamics of resource extraction are the decisive factors for emissions and climate change, n is always simulated first. The time paths of the other variables – energy production, total production, the elasticity of substitution, accumulated pollution, and climate damages – are then presented subsequently.

7.1 Optimal Policy Scenario

The optimal resource extraction path under the three technologies is described by (26), respectively (27), for the case of a regulated market equilibrium with a first-best carbon tax. The optimal resource extraction paths for the CES, IES, and VES production functions are depicted in Figure 1. The blue line shows the standard constant elasticity (CES), the red line the exogenously increasing (IES), and the green line the endogenously increasing (VES) elasticity of substitution case.

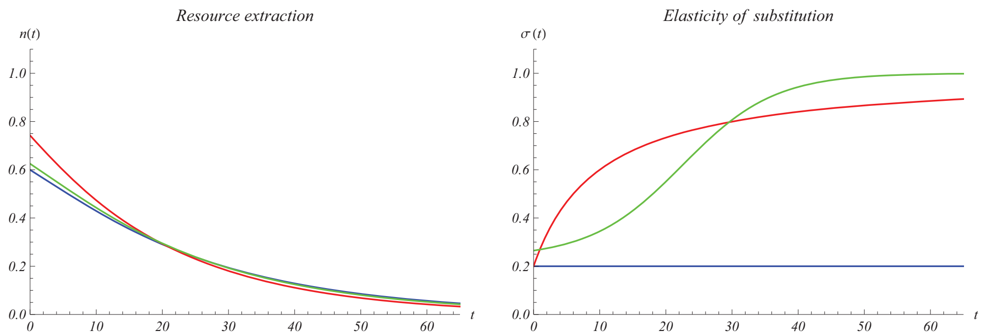

Figure 1: Time paths of resource extraction and elasticity of substitution in the optimal policy scenario
Note: Blue line: CES; Red line: IES; Green line: VES

From the LHS of Figure 1, we can immediately see that IES has the highest near term extraction with $n_0^X > n_0^N > n_0^C$. This is due to the exogenous flexibility effect. Moreover, from $n_0^N > n_0^C$ but $n_0^N < n_0^X$ we can see how the endogenous flexibility effect reduces the extraction-increasing effect of the exogenous flexibility effect. Intuition for these results is provided in detail in Section 6.1.

The driving force behind the different paths of resource extraction is the increasing flexibility in the IES and VES case. This increasing flexibility depends crucially on the parameters s and z. s is the flexibility parameter that determines how fast σ^X converges toward unity as t approaches infinity and z is the flexibility parameter that determines how sensitive σ^N reacts on the relative scarcity of n. The concrete realizations of s and z thus influence the speed with which σ^X and σ^N converge to unity and thereby also the speed of extraction and the initial resource extraction n_0^X and n_0^N.

Numerical simulations show very intuitive results: In the IES case, an increasing s increases the exogenous flexibility effect which tends to speed up extraction and thereby implies a higher n_0^X. The higher s, the faster σ^X converges toward unity and n_0^X converges toward an upper bound. An equivalent result can be found for the VES case as the elasticity of substitution is most sensitive to changes in the input mix when $z \to 1$.

On the RHS of Figure 1, we see the time paths of the elasticity of substitution that result from an exemplary value of $s = z = 0.1$. As a benchmark, the constant elasticity of substitution of the CES production function is depicted ($\sigma_0^{C,X,N} = 0.2$). We see that σ^X is a concave function of time and converges to unity. As in the IES case, in the VES case, σ^N converges toward unity, first with convex, than with concave shape. Comparing σ^X and σ^N, σ^N starts from a much higher level despite our assumption $\sigma_0^X = \sigma_0^N$. This is due to the fact that, in the VES case, the initial elasticity of substitution is given by (24) and not by σ_0^N. From (24), it follows directly that the underlying flexibility mechanism affects the level of σ^N even at $t = 0$ and due to the dependency of σ^N on relative factor scarcity, σ^N starts at a higher level than σ^X.

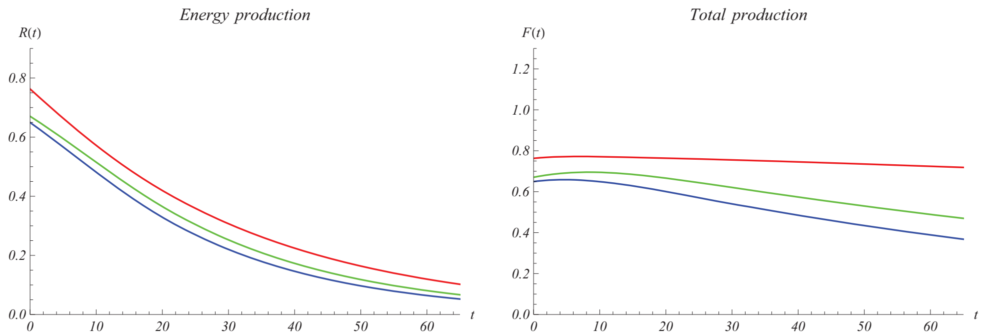

Figure 2: Time paths of energy and total production in the optimal policy scenario
Note: Blue line: CES; Red line: IES; Green line: VES

The LHS of Figure 2 shows the production of the composite energy good. As we assume renewable and fossil energy to remain imperfect substitutes, the increasing flexibility of IES and VES cannot compensate for exhaustibility. So, energy production always converges toward zero. The initial level and concrete time path of energy production, however, depend on the extraction path of n as well as on the production technology. In the CES case, production starts the lowest and also remains the lowest at all points in time. R^N starts higher than R^C, but lower than R^X. R^X starts the highest and also remains the highest over time. The total amount of energy produced in the IES and VES scenarios is higher than in the CES case although the amount of resources available for production is always the same. This result is of course attributable to the increasing elasticity of substitution. A higher elasticity of substitution can least partly relief the scarcity pressure of the exhaustible resource.

The second graph of Figure 2 shows final output production. Its long-term development depends crucially on the parameter choice for the exogenous rate of technological progress. The presented numerical example is deliberately chosen such that only in the IES case, the decrease of energy production is (almost) compensated by the exogenous technological progress. Of course, for sufficiently fast growth of factor productivity, the level of output in all three scenarios could grow over time - the ordering of growth rates would, however, be unaffected. We see that the decrease of production is slower for VES and IES than under CES. This result shows nicely that economies whose flexibility improves over time are less dependent on other types of technological progress.

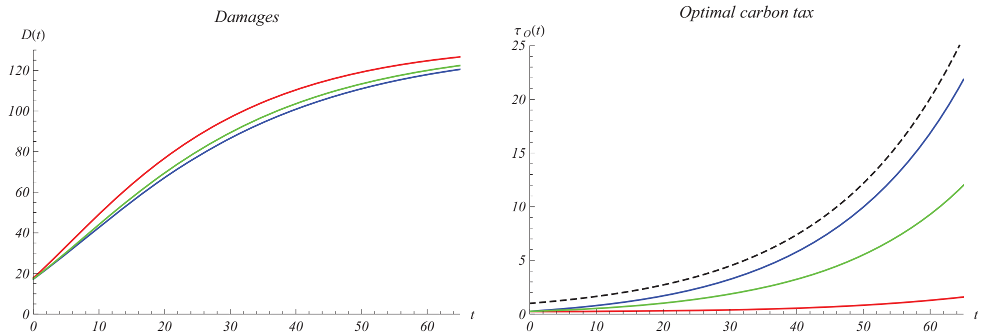

Figure 3: Time paths of damages and of the optimal carbon tax and discount rate in the optimal policy scenario
Note: Blue line: CES; Red line: IES; Green line: VES; Dashed line: Discount rate

Damages from fossil energy use (which are linked with pollution via (36)) are shown on the LHS of Figure 3. For any underlying production technology, pollution and therefore damages converge toward an upper limit. This limit is determined by the available stock of fossil energy resources, as in our economy the entire resource stock is exhausted ($\lim_{t\to\infty} P = \varepsilon S_0$, $\lim_{t\to\infty} D = a(\varepsilon S_0)^2$). The speed at which pollution and damages converge toward their maximum level, however, depends on the speed of resource extraction. The steeper the extraction path, the faster the convergence.

Let us finally take a look at the optimal tax rates for the different technology scenarios on the RHS of Figure 3. The optimal tax rate is determined by the social costs of carbon and the increase in pollution due to a marginal increase in extraction (see (19)). The tax rates are convex functions of the damages. Their time path results from the respective time path of pollution accumulation. Moreover, we see that the growth rates of the respective optimal tax are lower than the discount rate (see the black, dashed line). Therefore, the optimal carbon tax reduces, as expected, the speed of resource extraction (see Section 6.1). We can see that CES has the steepest τ_O and IES the flattest. Different reasons for this can be found. Once, in both flexibility cases, the extraction decision is more sensitive regarding future price changes. Second, in the CES case, dependency on the resource and therefore its value is higher. Consequently, the tax rate also has to be higher to reduce extraction to the optimal level.

First tentative conclusions about welfare can be drawn based on the underlying utility function, which weighs utility of consumption against the respective climate effects that produce disutility from pollution. We can observe a trade-off between higher consumption utility and higher pollution disutility in both the IES and VES case. In Figure 2, we can see the production gains of increasing flexibility. On the other hand, Figure 3 reveals the disutility-increasing effect of IES and VES compared to the CES technology. On a first sight, the consumption gains seem to outweigh increased damages. This is because regarding consumption we have flexibility gains that result in higher overall production, while with respect to the damages we only have an intertemporal relocation (higher short-term damages).

7.2 Laissez-Faire Scenario

Analogous to the previous section, the extraction paths for the laissez-faire scenario whose functional forms are given by (28) can be derived numerically. The results are presented in Figure 4. As before, extraction starts the highest in the IES case and the lowest in the CES case. Not surprisingly, we see steeper extraction paths and higher initial extraction in the laissez-faire case than in the optimal case for all our production functions (compare Figures 1 and 4). This has already been explained by the missing internalization of the negative externality of resource extraction resulting in too high extraction levels. The time paths of all other variables change of course as well but the ordering of the CES, IES and VES cases remains unaffected. For details on these results, see Appendix C (which also presents further comparisons between the optimal and the laissez-faire scenario of subsctions 7.1 and 7.2 and the non-optimal policy case of subsection 7.3).

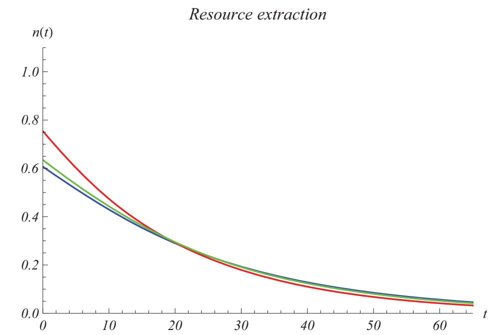

Figure 4: Time paths of resource extraction in the laissez-faire scenario
Note: Blue line: CES; Red line: IES; Green line: VES

7.3 Non-Optimal Policies

Assume now that the tax schedule is set exogenously and non-optimally by a policy maker. The time path of the tax rate is given by $\tau = \tau_M = \tau_{M0}e^{\pi t}$ where $\pi > 0$ is the exogenous growth rate of the tax. As can be seen in Figure 5, the tax schedule chosen in this section lies above the socially optimal tax rates for the different scenarios (see Figure 3) with a growth rate higher than the discount rate.

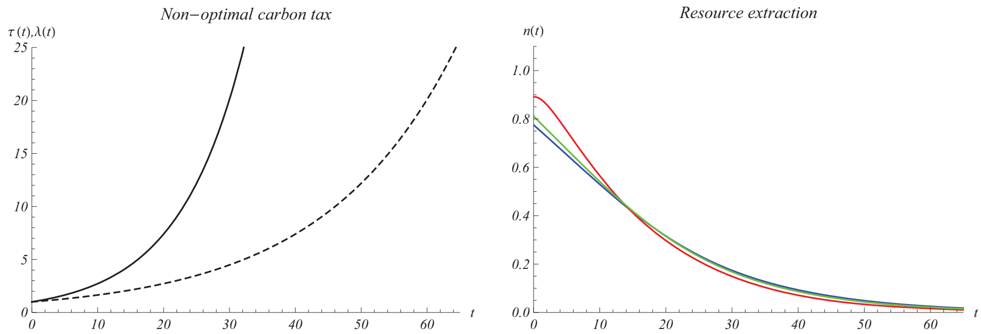

Figure 5: Time paths of the carbon tax and extraction of the fossil resource in the non-optimal policy scenario
Note: Blue line: CES; Red line: IES; Green line: VES; Solid line: Carbon tax; Dashed line: Discount rate

On the RHS of Figure 5, we see the resulting extraction paths of the exhaustible resource. Again, extraction in the IES and VES case is faster than in the CES case. Compared to the social optimum (resp. the optimal policy scenario), we see that the extraction paths are relatively steeper and start on a higher level - even compared to the laissez-faire situation. This is due to the fact that the non-optimal carbon tax not only increases at a faster rate than the optimal tax but the parameter values were also chosen such that $\rho < \frac{\dot{\tau}_M + \dot{\lambda}}{\tau_M + \lambda}$ which leads to $n_M(t = 0) > n_{LF}(t = 0)$. Therefore, in the chosen example, climate policy even increases further initial extraction and the extraction speed of resources. An explanation of the underlying intertemporal arbitrage effect has been provided in Section 6.1. Also here, the time paths of the other variables can be derived from the new extraction paths analogous to the optimal policy case (see Appendix C).

8 Results and Conclusions

In the present paper, we have analyzed the effects of increasing flexibility, or substitutability, in an energy market with both exhaustible and renewable energy goods that are used for production of a composite energy good. Burning exhaustible resources leads to carbon emissions and thereby produces a negative climate externality. Exhaustible and renewable energies are modelled as complements with an elasticity of substitution smaller than one, but over time, the elasticity increases and converges toward unity (therefore, the energy goods remain complements but their substitutability increases). This increase is modelled both exogenously and endogenously on the basis of a standard CES production function which is extended for the respective scenario. We analyze the effects of this flexibility increase both analytically and numerically and compare the results to the standard CES case. Moreover, we analyze the impact of three policy scenarios, optimal, laissez-faire, and non-optimal policy, in the context of increasing flexibility.

We find two flexibility effects that change the extraction decision of resource owners in case of increasing flexibility. One is the exogenous flexibility effect that arises under both specifications of increasing substitutability. This effect tends to increase short-term extraction and thereby steepens the extraction path indicating that resource owners anticipate future price decreases and thereby a smaller value of their resources due to higher flexibility. The endogenous flexibility effect arises only for an endogenously increasing elasticity of substitution. This effect tends to decrease initial extraction and flattens the extraction path as the resource owner anticipates the positive relation between resource scarcity and increasing flexibility.

Under the production specifications in our paper, extraction increases unambiguously under both, the exogenous and endogenous scenario. In the endogenous case, resource owners react to the knowledge that rising resource scarcity increases the elasticity of substitution and try to attenuate this effect by slowing down resource extraction (endogenous flexibility effect). However, as the exhaustible resource becomes inevitably scarcer, they cannot forestall the increase of flexibility completely. Thus the exogenous flexibility effect dominates and initial resource extraction rises.

The shift of resource extraction to the present due to increasing flexibility is comparable to the policy-induced green paradox that Sinn (2008) describes. Only, in our case we have a technology-induced rather than policy-induced Green Paradox.

With respect to policies that aim at increasing flexibility (recall the example of biofuels in the Brazilian transport sector), our results show clearly that these policy measures must be considered in the light of the intertemporal reallocation effect that results from the anticipation of rising flexibility. Moreover, feedback effects of policy on the development of the elasticity have to be taken into account if the elasticity evolves endogenously. In the case of our production function, policies affect resources extraction which in turns changes the elasticity of substitution.

From our results we can also draw first conclusions about welfare. Compared to the CES technology, we find welfare gains due to increased energy production in the increasing flexibility scenarios as well as welfare losses due to faster climate change. While consumption profits from level effects of higher flexibility that result in higher overall production, we only have an intertemporal relocation with respect to the damages (higher short-term damages). So, if the damages from climate change are not too strong, the consumption gains might outweigh increased damages.

The negative climate externality from exhaustible resource consumption demands for policy intervention which is analyzed here in form of a carbon tax imposed on resource consumption. This tax influences the resource owner's extraction decision due to the so-called tax effect. Resource owners anticipate the effects of taxation on the future value of the resources in situ and adjusts their extraction path accordingly. If the carbon tax is set optimally, the climate externality is internalized and resource extraction is at the socially optimal level. With respect to non-optimal climate policies, we confirm the result known from the literature that policies which aim to slow down resource extraction but whose design is determined from political rather than optimality considerations are likely to result in even faster resource extraction, i.e. a tax-induced green paradox still arises under increasing flexibility.

The paper reveals further research questions. For example, a complete endogenization of the flexibility increasing process seems to be an issue of high importance. Here, one might think of a separate R&D sector that can produce and sell patents for better integration of renewable energy into the energy market. The demand for those patents increases as resources become scarcer and energy prices increase. In this context, it is interesting to find and analyze the effects on resource extraction, but also on climate as well as climate policies. Moreover, even though the issue of increasing substitutability between different energy sources is of high importance in the present discussions about climate change and scarce resources, there is only little empirical research. Not only in the context of climate policy, but also in the context of determinants for long-term growth, this topic is of high importance.

References

Acemoglu, D. (2002), Directed technical change. Review of Economic Studies 69(4), 781-809.

Acemoglu, D., Aghion, P., Bursztyn, L. and Hemous, D. (2012), The environment and directed technical change. American Economic Review 102(1), 131-66.

Aghion, P. and Howitt, P.W. (1998), Endogenous Growth Theory. The MIT Press.

Allen, R.G.D. (1938), Mathematical Analysis for Economists. Macmillan Company.

Arrow, K.J., Chenery, H.B., Minhas, B.S. and Solow, R.M. (1961), Capital-Labor substitution and economic efficiency. The Review of Economics and Statistics 43(3), 225-250.

BMU (2013), Erneuerbare Energien in Zahlen. Nationale und internationale Entwicklung.

Bretschger, L. (2005). Economics of technological change and the natural environment: How effective are innovations as a remedy for resource scarcity?. Ecological Economics 54(2-3), 148-163.

Daly, H. and Farley, J. (2004), Ecological Economics - Principles and Applications. Island Press.

De La Grandville, O. (1989), In quest of the Slutzky Diamond. The American Economic Review 79(3), 468-481.

Di Maria, C. and Valente, S. (2008), Hicks meets Hotelling: The direction of technical change in capital-resource economies. Environment and Development Economics 13, 691-717.

Eisenthal, J. (2013), Building the blend: How Brazil grew its ethanol industry.

Grimaud, A. and Rougé, L. (2005), Polluting non-renewable resources, innovation and growth: Welfare and environmental policy. Resource and Energy Economics 27, 109-129.

Growiec, J. and Schumacher, I. (2008), On technical change in the elasticities of resource inputs. Resources Policy 33(4), 210-221.

Heal, G. (1976), The relationship between price and extraction cost for a resource with a backstop technology. The Bell Journal of Economics 7(2), 371-378.

Hicks, J.R. (1932), The Theory of Wages. Macmillan Company.

Hoel, M. (2008), Bush meets Hotelling: Effects of improved renewable energy technology on greenhouse gas emissions. CESifo Working Paper No. 2492.

Hoel, M. and Jensen, S. (2012), Cutting costs of catching carbon - intertemporal effects under imperfect climate policy. Resource and Energy Economics 34(4), 680-695.

Hoel, M. and Kverndokk, S. (1996), Depletion of fossil fuels and the impacts of global warming. Resource and Energy Economics 18(2), 115-136.

Kadiyala, K.R. (1972), Production Functions and Elasticity of Substitution. Southern Economic Journal 38(3), 281-284.

Karagiannis, G., Palivos, T. and Papageorgiou, C. (2005), Variable elasticity of substitution and economic growth: Theory and evidence. In C. Diebolt and C. Kyrtsou, eds, New Trends in Macroeconomics, Springer, pp. 2137.

Lu, Y. (1967), Variable elasticity of substitution production functions, technical change and factor shares. Unpublished PhD thesis, Iowa State University.

Lu, Y. and Fletcher, L. (1968), A generalization of the CES production function. The Review of Economics and Statistics 50(4), 449-452.

Masanjala, W. and Papageorgiou, C. (2004), The Solow model with CES technology: Nonlinearities and parameter heterogeneity. Journal of Applied Econometrics 19(2), 171201.

Pittel, K. and Bretschger, L. (2010), The implications of heterogeneous resource intensities on technical change and growth. Canadian Economic Review 43, 1173-1197.

Revankar, N.S. (1971), A class of variable elasticity of substitution production functions. Econometrica 39(1), 61-71.

Sato, R. and Hoffman, R.F. (1968), Production functions with variable elasticity of factor substitution: Some analysis and testing. Review of Economics and Statistics 50, 453-460.

Singh, B. (2013), Biofuel Crop Sustainability. Wiley-Blackwell.

Sinn, H.-W. (2008), Public policies against global warming: A supply side approach. International Tax and Public Finance 15(4), 360-394.

Solomon, S., Plattner, G.K., Knutti, R. and Friedlingstein, R. (2009), Irreversible climate change due to carbon dioxide emissions, carbon dioxide emissions. Proceedings of the National Academy of Sciences 106, 1704-1709.

UNEP (2013), Brazil's developing automotive fuel economy policy.

van der Ploeg, R. and Withagen, C. (2011a), Growth, renewables and the optimal carbon tax. OxCarre Research Paper 55.

van der Ploeg, R. and Withagen, C. (2011b), Growth and the optimal carbon tax: When to switch from exhaustible resources to renewables?. CEPR Discussion Papers 8215.

Yuhn, K. (1991), Economic growth, technical change biases, and the elasticity of substitution: A test of the De La Grandville Hypothesis. The Review of Economics and Statistics 73(2), 340-346.

Appendix

A. Analytical Derivation of Extraction Rates

Solving the CES and the IES Model: In the following, we derive the growth rates of resource extraction for the social optimum as well as the market economy. The analogous results for the CES model follow straightforwardly.

The socially optimal growth rate of resource extraction, $\hat{n}_O^X$, can be determined from (12). To solve for $\hat{n}$, we derive $\hat{F}$ and $\hat{F}_n$ from (2), (20) and (21) which gives

$$\hat{F} = g + \frac{s}{\sigma_0 - 1} \ln(R^{-\theta^X}) + \frac{1}{\theta^X} \left(\chi \dot{\theta}^X + \theta^X \epsilon^X \hat{n} \right), \tag{37}$$

$$\hat{F}_n = g + \frac{s}{\sigma_0 - 1} \ln(R^{-\theta^X}) - \left(\theta^X + 1\right) \hat{n} - \ln(n) \dot{\theta}^X + \left(\frac{1}{\theta^X} + 1 \right) (\chi \dot{\theta}^X + \theta^X \epsilon^X \hat{n}) \tag{38}$$

with $\epsilon^X = \psi \left(\frac{R}{n}\right)^{\theta^X}$, and $\chi = \psi \left(\frac{R}{n}\right)^{\theta^X} \ln(n) + (1-\psi) \left(\frac{R}{m}\right)^{\theta^X} \ln(m)$.[23]

Inserting these expressions into (12) gives the socially optimal time path of resource extraction:

$$\hat{n}_O^X = \frac{(\dot{\mu}_S - \dot{\mu}_P h_n - \mu_P \dot{h}_n)}{(\mu_S - \mu_P h_n) \left[\epsilon^X ((1-\eta) + \theta^X) - \theta^X - 1\right]} + \frac{(\mu_S - \mu_P h_n) \left[(1-\eta)(g + \frac{s}{\sigma_0 - 1} \ln(R^{-\theta^X})) + (\frac{1}{\theta^X}(1-\eta) + 1) \chi \dot{\theta}^X - \ln(n) \dot{\theta}^X \right]}{(\mu_S - \mu_P h_n) \left[\epsilon^X ((1-\eta) + \theta^X) - \theta^X - 1\right]}. \tag{39}$$

(18) together with equations (37) and (38) gives the growth rate of resource extraction in the market economy, $\hat{n}_M^X$:

$$\hat{n}_M^X = \frac{(\dot{\tau} + \dot{\lambda}) - (\tau + \lambda) \left[(1-\eta)(g + \frac{s}{\sigma_0 - 1} \ln(R^{-\theta^X})) + (\frac{1}{\theta^X}(1-\eta) + 1) \chi \dot{\theta}^X - \ln(n) \dot{\theta}^X \right]}{(\tau + \lambda) \left[\epsilon^X ((1-\eta) + \theta^X) - \theta^X - 1\right]}. \tag{40}$$

The equilibrium for the laissez-faire case can simply be obtained from (40) by setting $\tau = 0$.

The solution for the standard CES case with a constant elasticity of substitution results from (39) and (40) by setting $s = 0$. In this case, σ is given by σ_0 and $\dot{\theta}$ is equal to zero.

[23] Growiec and Schumacher (2008) interpret ϵ^X as the share of n on R based on the assumption of constant returns to scale in the underlying production function.

Solving the VES Model: The growth rate of n for the VES case can again be derived from (12). Using (2), (22) and (24) we get for $\hat{F}$ and $\hat{F}_n$

$$\hat{F} = g + \epsilon^N \left(1 - \frac{z(1+\theta^N)}{\theta^N}\right)\hat{n}, \tag{41}$$

$$\hat{F}_n = g + \hat{n}\left[(\theta^N - z(1+\theta^N))(\epsilon^N(\frac{1}{\theta^N}+1)-1)-1\right] \tag{42}$$

with $\epsilon^N = \frac{\gamma\psi n^{-\theta^N}\left(\frac{m}{n}\right)^{-z(1+\theta^N)}}{R^{-\theta^N}}$. Inserting these equations into (12) gives the socially optimal growth rate of extraction

$$\hat{n}^N_O = \frac{(\dot{\mu}_S - \dot{\mu}_P h_n - \mu_P \dot{h}_n) - (\mu_S - \mu_P h_n)g(1-\eta)}{(\mu_S - \mu_P h_n)\left[(\theta^N - z(1+\theta^N))(\epsilon^N(\frac{1}{\theta^N}(1-\eta)+1)-1)-1\right]} \tag{43}$$

where μ_P and μ_S as well as the respective time derivatives have been determined previously in Section 3.1.

Analogously, the growth rate of resource extraction in the market economy can be derived from (18):

$$\hat{n}^N_M = \frac{(\dot{\tau} + \dot{\lambda}) - (\tau+\lambda)g(1-\eta)}{(\tau+\lambda)[(\theta^N - z(1+\theta^N))(\epsilon^N(\frac{1}{\theta^N}(1-\eta)+1)-1)-1]}. \tag{44}$$

To obtain the laissez faire solution again set $\tau = 0$.

B. The Effect of Flexibility: The Case of Δ^X

From Section 6.2, Δ^X is given by

$$\Delta^X = (1-\eta)\frac{s\theta^X}{1-\sigma_0}\ln(R) + \left[\left((1-\eta)+\theta^X\right)\chi - \theta^X \ln(n)\right]\hat{\theta}^X$$

with $R = R^X = \left(\psi n^{-\theta^X} + (1-\psi)m^{-\theta^X}\right)^{-\frac{1}{\theta^X}}$ and $\chi = \psi\left(\frac{R}{n}\right)^{\theta^X}\ln(n) + (1-\psi)\left(\frac{R}{m}\right)^{\theta^X}\ln(m)$ gives

$$\Delta^X = (1-\eta)\frac{s}{1-\sigma_0}\ln\left(\psi n^{-\theta^X} + (1-\psi)m^{-\theta^X}\right) + \left[\left((1-\eta)+\theta^X\right)\left(\psi\left(\frac{R}{n}\right)^{\theta^X}\ln(n) + (1-\psi)\left(\frac{R}{m}\right)^{\theta^X}\ln(m)\right) - \theta^X\ln(n)\right]\hat{\theta}^X$$

Using the equation for R gives

$$\psi\left(\frac{R}{n}\right)^{\theta^X} = \psi\left(\frac{\left(\psi n^{-\theta^X}+(1-\psi)m^{-\theta^X}\right)^{-\frac{1}{\theta^X}}}{n}\right)^{\theta^X} = \frac{1}{1+\frac{1-\psi}{\psi}\left(\frac{m}{n}\right)^{-\theta^X}}$$

and analogously

$$(1-\psi)\left(\frac{R}{m}\right)^{\theta^X} = \frac{1}{1+\frac{\psi}{1-\psi}\left(\frac{n}{m}\right)^{-\theta^X}}.$$

Inserting these expressions into Δ^X we get

$$\begin{aligned}\Delta^X &= (1-\eta)\frac{s}{1-\sigma_0}\ln\left(\psi n^{-\theta^X}+(1-\psi)m^{-\theta^X}\right)\\ &+\left[\left((1-\eta)+\theta^X\right)\left(\frac{1}{1+\frac{1-\psi}{\psi}\left(\frac{m}{n}\right)^{-\theta^X}}\ln(n)+\frac{1}{1+\frac{\psi}{1-\psi}\left(\frac{n}{m}\right)^{-\theta^X}}\ln(m)\right)-\theta^X\ln(n)\right]\hat{\theta}^X\end{aligned}$$

where $\hat{\theta}^X = -\frac{s}{st+\sigma_0^X}$.

To see the effect that an increasing elasticity of substitution has on Δ^X, consider its limit when σ^X converges toward unity. In this case, the first term goes to zero while the term in square brackets, [...], is positive if fossil resources are not too scarce:

$$\lim_{\sigma\to 1}[...] = \left[\left((1-\eta)\right)\left(\psi\ln(n)+(1-\psi)\ln(m)\right)\right].$$

If, however, resource extraction converges also to zero, this term becomes negative. Given that the growth rate of θ^X is also negative, this implies the results on the extraction path laid out in Section 6.3.

C. Numerical Results of the Laissez-faire and Non-optimal Policy Scenario

The present section summarizes and compares the results of the numerical simulations for the laissez-faire and non-optimal policy scenarios. The time paths of elasticity of substitution in the laissez-faire scenario (non-optimal policy scenario) presented in the upper LHS of Figure 6 (Figure 7) are basically the same as in the optimal policy scenario. σ^C and σ^X remain unchanged, σ^N still converges towards unity. As explained in Sections 7.2 and 7.3, we have $\sigma^N_{LF}(t=0) < \sigma^N_O(t=0)$ ($\sigma^N_M(t=0) < \sigma^N_O(t=0)$) with $\sigma^N_M(t=0) < \sigma^N_{LF}(t=0)$. The upper RHS of Figure 6 (Figure 7) shows the damages resulting from the climate externality. The picture is similar to the optimal policy scenario depicted in Figure 3: Convergence is slowest in the CES case and fastest in the IES case. However, compared to the optimal policy scenario, pollution accumulation and therefore damages starts at a higher level due to the higher initial extraction in the laissez-faire scenario (non-optimal policy scenario) such that we have $D_O(t = 0) < D_{LF}(t = 0) < D_M(t = 0)$.

In accordance to the higher initial extraction level, energy production paths also start higher in both policy scenarios but the ordering of the production levels remains the same, see the LHS of Figure 6 (Figure 7). Compared to the optimal paths from Figure 2, production starts on a higher level due to the higher initial energy production with $R_O(t = 0) < R_{LF}(t = 0) < R_M(t = 0)$. The time paths of total production on the lower RHS of Figure 6 (Figure 7) can again directly be related to the energy production paths with exogenous growth in total factor productivity determining long-term development with $F_O(t = 0) < F_{LF}(t = 0) < F_M(t = 0)$. Moreover, in both the laissez-faire and the non-optimal policy scenario, we see that total factor productivity does not increase sufficiently to compensate for decreasing energy production such that total production decreases over time (in the non-optimal policy scenario even stronger than in the laissez-faire policy scenario).

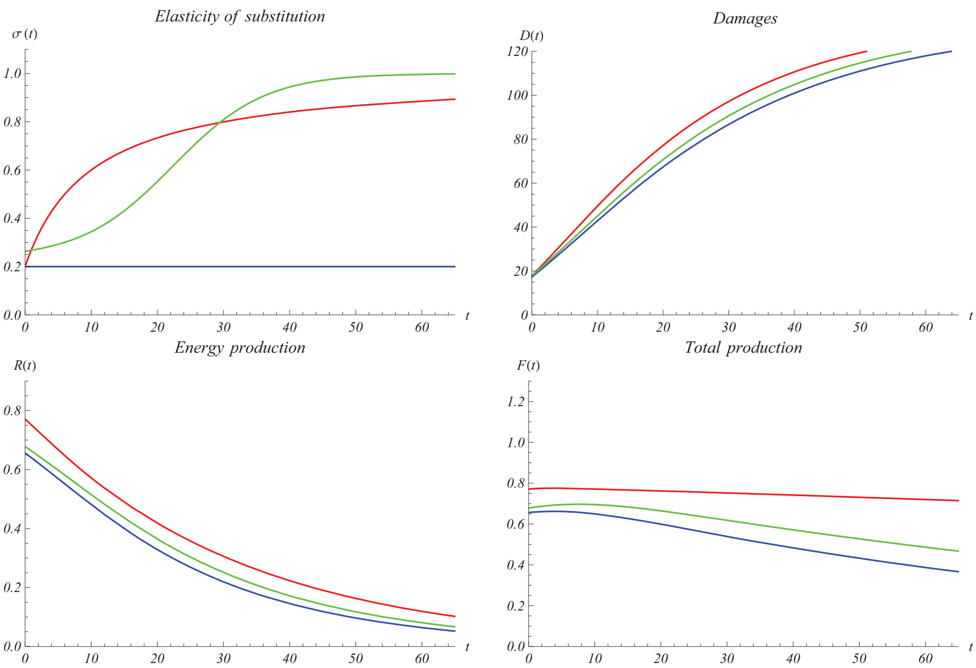

Figure 6: Time paths of elasticity of substitution and damages, energy and total production in the laissez-faire scenario
Note: Blue line: CES; Red line: IES; Green line: VES

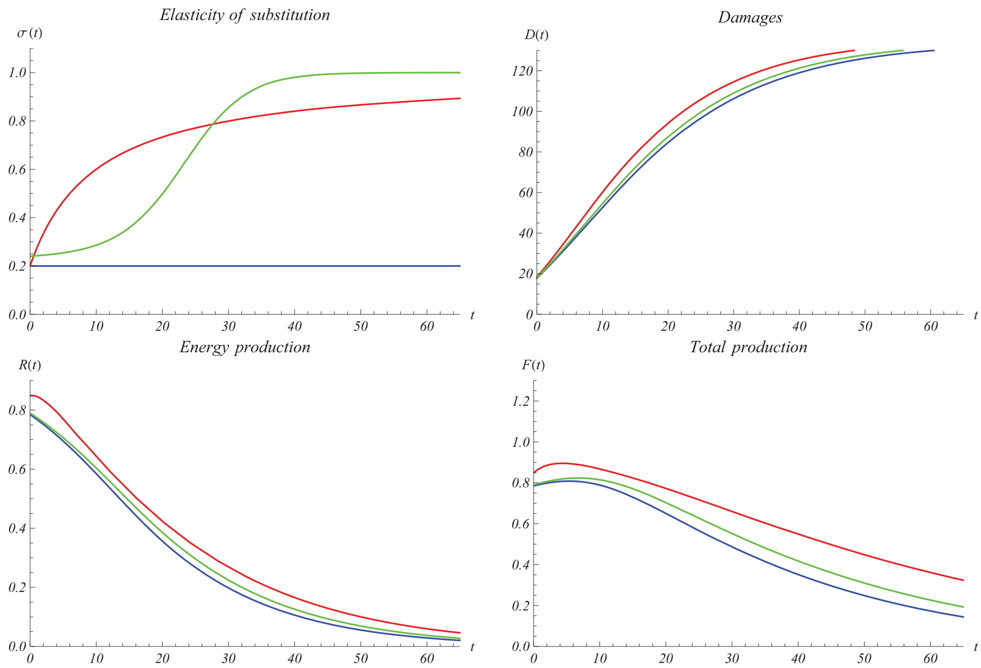

Figure 7: Time paths of resource extraction and damages, energy and total production in the non-optimal policy scenario
Note: Blue line: CES; Red line: IES; Green line: VES