Should we extract more shale gas?
The effect of climate and financial constraints

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Abstract

In the context of the deep contrast between the shale gas boom in the United States and the recent ban by France of exploration and exploitation of shale gas, this paper addresses the question of a potential arbitrage between shale gas development and the transition to clean energy, when environmental damages, both local and global, are taken into account. We construct a Hotelling-like model where electricity may be produced by three perfectly substitutable energy sources: an abundant dirty resource (coal), a non-renewable less polluting resource (shale gas), and an abundant clean resource (solar). The three resources differ by their carbon contents and hence their potential danger for the climate, and the local damages their extraction causes. The costs of electricity generation by coal, gas or solar-fired power plants also differ. Exploration and development allow to build the quantity of shale gas reserves that will be extracted. A fixed development cost must be paid before solar production begins. It is decreasing in time due to exogenous technical progress. Climate policy takes the form of a ceiling on atmospheric carbon concentration. We show that tightening climate policy always leads to bringing forward the transition to clean energy. When the local damage caused by shale gas extraction is high, it also leads to an increase of the quantity of shale gas developed, at the expense of coal. However, when the local damage is small, it may be the case that a more stringent climate policy leads to reduce the quantity of shale gas developed, when the advantage of shale gas over coal in terms of carbon emissions is not large enough. We finally study how these results are

modified when the social planner has to comply to the climate constraint without increasing energy expenditures.

1 Introduction

In France, the Jacob law of July 13th, 2011 banned hydraulic fracturing ("fracking"): "*En application de la Charte de l'environnement de 2004 et du principe d'action préventive et de correction prévu à l'article L. 110-1 du code de l'environnement, l'exploration et l'exploitation des mines d'hydrocarbures liquides ou gazeux par des forages suivis de fracturation hydraulique de la roche sont interdites sur le territoire national."* Moreover, the exploration licences held by companies like the American Schuepbach or the French Total were cancelled. Schuepbach complained to the court that this law was unfair and unconstitutional, but the Constitutional Court confirmed the ban on October 8th, 2013, saying that the Jacob law conforms to the constitution and is not disproportionate. By the same time, French President François Hollande said France will not allow exploration of shale gas as long as he is in office.

This position, although supported by a majority of the population[1], may seem puzzling, at a time where France is trying to reduce its reliance on nuclear energy whilst containing the increase of the consumer electricity price. Besides, France is the only one of the European Union's 28 countries besides Bulgaria to ban shale gas. However, the ban is grounded on two types of strong environmental arguments, that need to be examined closely. First, fracking is considered as dangerous and environmentally damaging. It pumps water, sand and chemical under high pressure deep underground to liberate the gas that is trapped in the rock. The main dangers are for surface water (through the disposal of the fracturing fluids) and groundwater (through the accidental leakage of fracking fluids from the pipe into potable aquifers). Also, seismic vibrations caused by the injection of water underground is feared. Finally, there are concerns over landscape, as the number of wells is very important and their layout very dense. Second, it is argued that what should be done in the face of global warming is to reduce drastically the use of fossil fuels, not to find new ones, which will have the effect of postponing the transition to clean

[1] IFOP survey, Sept. 13th, 2012: 74% of the respondents are opposed to shale gas exploitation; BVA survey, Oct. 2nd, 2014: 62%. Note that this is greater than the opposition to nuclear energy, which provides most of France's electricity.

renewable energy[2]. To these arguments, shale gas supporters answer that natural gas is less polluting than other fossil fuels (oil, and particularly coal), and that its substitution to coal and oil should be encouraged on environmental grounds. Indeed, it seems impossible to fight global warming effectively without substantially reducing the use of coal, what shale gas could allow. According to the International Monetary Fund (2014), "*Natural gas is the cleanest source of energy among other fossil fuels (petroleum products and coal) and does not suffer from the other liabilities potentially associated with nuclear power generation. The abundance of natural gas could thus provide a "bridge" between where we are now in terms of the global energy mix and a hopeful future that would chiefly involve renewable energy sources.*"

The contrast between the position held by France and the situation of the United States is stunning. United States is at date the first natural gas producer in the world. Shale gas has risen from 2% of domestic energy production a decade ago to nearly 40% today (IMF, 2014). It has profoundly modified the energy mix: shale gas is gradually replacing coal for electricity generation. Coal-fired power plants produced more than half of the total electricity supply in 1990, and natural gas-fired power plants 12%; in 2013, the figures are respectively 29% and 27% (Energy Information Administration, 2014). Shale gas supporters in the US put forward the facts that it has allowed to create jobs, relocate some manufacturing activities, lower the vulnerability to oil shocks, and impact positively the external balance (IMF, 2014).

This paper pretends neither to examine all aspects of this complex problem nor to prove the positions of France or the United States right. Our objective is to address the question of a potential arbitrage between shale gas development and the transition to clean energy, when environmental damages, both local and global, are taken into account, and financial constraints as well. To do so, we construct a Hotelling-like model where electricity may be produced by the means of three perfectly substitutable energy sources: an abundant dirty resource, a non-renewable less polluting resource, and an abundant clean resource (the clean backstop), provided that appropriate fixed costs are paid for. The three resources differ by their carbon contents and hence their potential danger for the climate, and the local damages their extraction causes. The costs of electricity generation by the three resources also differ. The dirty resource is typically coal. It is supposed to be abundant. The less dirty non-renewable resource is shale

[2]What is more difficult to explain is why it is not only exploitation of shale gas that is banned, but also exploration of potential reserves. Answering this question goes far beyond the present research.

gas. Exploration and development allow to build the quantity of shale gas reserves that will be extracted (Gaudet and Lasserre, 1988). Any quantity of shale gas can be developed, provided that the cost is paid for: physical scarcity is not a problem either. The clean backstop energy is typically solar energy. A fixed development cost must be paid before solar production begins. It is decreasing in time due to exogenous technical progress (Dasgupta *et al.*, 1982). Following Chakravorty *et al.* (2006a, 2006b), climate policy takes the form of a ceiling under which atmospheric CO_2 concentration must be kept. Agents derive their utility from the consumption of electricity. The social planner seeks to maximize the intertemporal welfare, taking account of the climate constraint.

We show that whatever the magnitude of the local damage caused by shale gas extraction, tightening climate policy always leads to bringing forward the transition to clean energy. When the local damage is high, it also leads to increase the quantity of shale gas developed, at the expense of coal. However, when the local damage is small, it may be the case that a more stringent climate policy leads to reduce the quantity of shale gas developed, when the advantage of shale gas over coal in terms of carbon emissions is not large enough.

We then compel the social planner to meet the ceiling imposed by climate policy without increasing total energy expenditures, compared to their level absent this policy. The primary effect of this constraint is to increase the monetary costs associated to the energy mix (production and investment costs), while the non-monetary costs, that is the environmental costs (the local and global damages) remain unchanged. Environmental matters becomes less important compared to costs. Two conflicting effects appear. On the one hand, the loss of importance of the local damage is an incentive to develop more shale gas and extract it earlier; on the other hand the loss of importance of the global damage reduces the advantage of shale gas in terms of carbon emissions, and thus has the opposite effect.

The remaining of the paper is as follows. Section 2 presents the model and the optimal solution. Section 3 shows the results of a comparative dynamics exercise performed to see how the optimal solution is modified when environmental policy becomes more stringent. Section 4 introduces the financial constraint. Section 5 presents an illustrative numerical exercise. Finally, Section 6 concludes.

2 The model

2.1 Assumptions

We consider an economy where electricity is initially produced by coal-fired power plants, and where two other energy sources, shale gas and solar, may be developed and used in electricity generation as well. Coal is supposed to be abundant but very polluting. Shale gas is non-renewable, and also polluting but to a lesser extent. Solar is abundant and clean. The three resources are perfect substitutes in electricity generation[3].

The label d for "dirty" stands for the dirty resource, namely coal. The pollution intensity of coal is θ_d: the extraction and use of one unit of coal leads to the emission of θ_d unit of CO_2 ("carbon" thereafter). The marginal long term production cost of electricity with coal is c_d. It is supposed to be constant. This cost includes the extraction cost of coal, but also capital costs and operating and maintenance costs[4]. The extraction rate of coal is $x_d(t)$.

The label e for "exhaustible" stands for shale gas. Its pollution intensity is θ_e, with $\theta_e \leq \theta_d$. Indeed, Heath *et al.* (2014), performing a meta-analysis of the literature to date, obtain that emissions from shale gas-generated electricity are approximately half that of coal-generated electricity. The long term marginal production cost of electricity using shale gas is c_e. As for coal, this includes the fuel extraction cost, other operating and maintenance costs and capital costs. We make the assumption that $c_e < c_d$ (see Energy Information Administration, 2014a and Table 1). The extraction of shale gas causes a local

[3]The assumption of perfect substitutability of the energy sources is valid as far as electricity generation is concerned. It is not the case at the moment in transport, which justifies our focus on electricity generation.

[4]This cost is in fact the levelized cost of electricity generated by coal-fired power plants. According to the US Energy Information Administration, "levelized cost of electricity (LCOE) is often cited as a convenient summary measure of the overall competiveness of different generating technologies. It represents the per-kilowatthour cost (in real dollars) of building and operating a generating plant over an assumed financial life and duty cycle. Key inputs to calculating LCOE include capital costs, fuel costs, fixed and variable operations and maintenance (O&M) costs, financing costs, and an assumed utilization rate for each plant type. The importance of the factors varies among the technologies. For technologies such as solar and wind generation that have no fuel costs and relatively small variable O&M costs, LCOE changes in rough proportion to the estimated capital cost of generation capacity. For technologies with significant fuel cost, both fuel cost and overnight cost estimates significantly affect LCOE." (EIA, 2014a).

	levelized capital cost	fixed O&M	variable O&M including fuel	transmission investment	total
conventional coal	60	4.2	30.3	1.2	95.6
natural gas-fired combined cycle	14.3	1.7	49.1	1.2	66.3
solar PV	114.5	11.4	0	4.1	130
solar thermal	195	42.1	0	6.0	243

Table 1: US average levelized cost of electricity (2012 \$/MWh). Source: EIA, 2014a

marginal damage d, supposed to be constant. This damage is due primarily to the technology employed to extract shale gas, namely hydraulic fracturing. Before beginning to extract shale gas, it is necessary to incur an upfront exploration cost. The total quantity of reserves X_e available after exploration and development is endogenous, and proportional to the exploration investment: $X_e = f(I)$, with $f'(.) > 0$ and $f''(.) < 0$. This can also be written $I = E(X_e)$, with $E'(X_e) > 0$ and $E''(X_e) > 0$, as in Gaudet and Lasserre (1988). We suppose that the exploration cost must be paid at the beginning of the planning horizon, even though the actual extraction of shale gas may be postponed to a later date[5]. The extraction rate of shale gas is $x_e(t)$.

The label b for "clean backstop" stands for solar energy. The long term marginal production cost of electricity with solar is c_b. We make the assumption $c_b > \max(c_e + d, c_d)$. Solar-fired power plants can be developed at a R&D cost $CF(t)$. It is supposed to be decreasing in time, because of (exogenous) technical progress: $CF'(t) < 0$ (Dasgupta *et al.*, 1982). The production rate of solar energy is $x_b(t)$.

The combustion of the two polluting resources generates carbon emissions that accumulate in the atmosphere. $Z(t)$ is the atmospheric concentration of carbon. Its change over time is given by:

$$\dot{Z}(t) = \theta_e x_e(t) + \theta_d x_d(t)$$

meaning that carbon concentration can only increase, as soon as fossil fuels are used for electricity generation. In other words, we suppose that there is no natural decay of carbon, as in van der Ploeg and

[5]This assumption is technical. It allows to get rid of problems of concavity of the value function appearing when exploration and exploitation of shale gas reserves are performed at the same date.

Withagen (2012) and Coulomb and Henriet (2014)[6].

Finally climate policy is modelled as a cap on the atmospheric carbon concentation $\overline{Z}$, following the strand of litterature initiated by Chakravorty *et al.* (2006a, 2006b).

Electricity produced at date t is $x(t) = x_d(t) + x_e(t) + x_b(t)$. Agents derive their utility directly from the consumption of electricity. Let $u(x(t))$ be the utility function at date t, with u twice continuously differentiable, strictly increasing and strictly concave, and ρ the social discount rate, assumed to be constant. The social planner chooses the extraction and production rates $x_d(t)$, $x_e(t)$, $x_b(t)$, the amount of shale gas developed X_e, and the date T_b at which the R&D investment for solar energy is made which maximize:

$$\int_0^\infty e^{-\rho t}\left[u\left(x_d(t) + x_e(t) + x_b(t)\right) - c_d x_d(t) - (c_e + d)x_e(t) - c_b x_b(t)\right]dt - E(X_e) - CF(T_b)e^{-\rho T_b}$$

under the constraints:

$$\int_0^\infty x_e(t)dt \leq X_e, \quad X_e(0) = X_e \text{ given} \tag{1}$$

$$\int_0^\infty (\theta_d x_d(t) + \theta_e x_e(t))dt \leq \overline{Z} - Z_0, \quad Z(0) = Z_0 \text{ given} \tag{2}$$

$$x_d(t) \geq 0,\ x_e(t) \geq 0,\ x_b(t) \geq 0 \tag{3}$$

In order to solve the general problem, we first assume that T_b and X_e are given, and we compute the constrained optimal price path. We obtain the value of the problem for each price path, and we maximize this value over T_b and X_e.

[6]Our model is close to the one in Henriet and Coulomb (2014) to other respects as well. However, they do not introduce fixed costs and local damages, which are key ingredients of our model.

2.2 Constrained optimal price path

The first order necessary conditions of optimality are, with $\lambda(t)$ the scarcity rent associated to the stock of shale gas and $\mu(t)$ the carbon value:

$$u'(x_d(t)) \leq c_d + \theta_d \mu(t) \quad (4)$$

$$u'(x_e(t)) \leq c_e + d + \lambda(t) + \theta_e \mu(t) \quad (5)$$

$$u'(x_b(t)) \leq c_b \quad (6)$$

with equality when the energy is actually used, and

$$\dot{\lambda}(t) = \rho\lambda(t) \quad (7)$$

$$\dot{\mu}(t) = \rho\mu(t) \text{ before the ceiling} \quad (8)$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)X_e(t) = 0 \quad (9)$$

$$\lim_{t\to\infty} e^{-\rho t}\mu(t)Z(t) = 0 \quad (10)$$

Following Chakravorty *et al* (2006a, 2006b) and the subsequent literature, it is easy to see that at the optimum:

- X_e is exhausted;
- the ceiling is reached at date T_b;
- the three energy sources are used successively – there is no phase of simultaneous use;
- R&D costs $CF(t)$ are paid when the clean backstop starts to be used, i.e. at date T_b (Dasgupta *et al.*, 1982).

We have supposed that the marginal cost of production of electricity with shale gas is lower than the one with coal: $c_e < c_d$. However, because of the existence of the local damage caused by shale gas extraction, the full marginal production cost for shale gas $c_e + d$ may be lower or higher than the marginal production cost for coal c_d. We successively study the two cases of a large and a small marginal local damage.

2.2.1 Large local damage

By large local damage we mean that the local damage more than compensates the gain in terms of production cost due to the use of shale gas instead of coal in electricity generation: $d > c_d - c_e$. Hence if the total marginal cost is taken into account, coal is cheaper than shale gas. However, shale gas has an advantage over coal as regards carbon emissions. We suppose that the local damage is not large enough to make solar cheaper than shale gas.

The price[7] path is potentially composed of three phases (see for instance Chakravorty *et al.*, 2006a, 2006b or Coulomb and Henriet, 2014):

- Phase 1: coal is used in quantity $X_d = \frac{\bar{Z}-Z_0-\theta_e X_e}{\theta_d}$, between dates 0 and T_e. Its price can be written:

$$p_d(t) = c_d + \theta_d \mu_0 e^{\rho t} \tag{11}$$

  with μ_0 such that: $\int_0^{T_e} x_d(t)dt = \int_0^{T_e} D\left(p_d(t)\right)dt = \frac{\bar{Z}-Z_0-\theta_e X_e}{\theta_d}$, where $D(.) = u'^{-1}(.)$ is the demand function.

- Phase 2: shale gas is used in quantity X_e, between dates T_e and T_b. Its price can be written:

$$p_e(t) = c_e + d + (\lambda_0 + \theta_e \mu_0)e^{\rho t} \tag{12}$$

  with λ_0 such that: $\int_{T_e}^{T_b} x_e(t)dt = \int_{T_e}^{T_b} D\left(p_e(t)\right)dt = X_e$. T_e, the date of the switch from coal to shale gas, is endogenously determined by the continuity of the energy price at date T_e: $p_d(T_e) = p_e(T_e)$, i.e.

$$c_d + \theta_d \mu_0 e^{\rho T_e} = c_e + d + (\lambda_0 + \theta_e \mu_0)e^{\rho T_e} \tag{13}$$

- Phase 3: the clean backstop is used at the constant price:

$$p_b(t) = c_b \tag{14}$$

  from date T_b onwards.

[7] Of course, "price" is used here simply but inaccurately to denote marginal utility.

One (or two) of these phases may not exist. For instance, in the absence of any constraint on the atmospheric carbon concentration (when $\overline{Z} \to \infty$), CO_2 emissions do not matter and, as coal is available in infinite amount and is the cheapest source of energy ($c_d < c_e + d < c_b$), it will be used alone forever. As soon as $\overline{Z}$ is finite however, there will be a switch to solar at some point. But is it useful to introduce shale gas as well? Clearly, if θ_e is close to θ_d, shale gas, which is more costly than coal, because of the local damage and the upfront development cost, and equally polluting, will never be used. On the other hand, if θ_e is close to zero and the ceiling constraint very tight, it may happen that shale gas is exploited from the beginning of the trajectory at the expense of coal.

To sum up, when the local damage due to shale gas extraction is large, shale gas does not replace coal immediately in electricity generation, unless its advantage in terms of carbon emissions is large and climate policy stringent enough to compensate its disadvantage in terms of local damage.

2.2.2 Small local damage

In this case, $d < c_d - c_e$. The advantage of shale gas in terms of production costs dominates. Shale gas is also less polluting. It will be used immediately in electricity generation. But it may be the case that we return to coal, more costly and more polluting than shale gas, later on, because shale gas is scarce while coal is abundant.

Again, the price path is potentially composed of 3 phases:

- Phase 1: shale gas is used in quantity X_e, between dates 0 and T_d. Its price is given by (12), with $(\lambda_0 + \theta_e \mu_0)$ such that: $\int_0^{T_d} x_e(t)dt = \int_0^{T_d} D\left(p_e(t)\right) dt = X_e$.

- Phase 2: coal is used in quantity $X_d = \frac{\bar{Z} - Z_0 - \theta_e X_e}{\theta_d}$, between dates T_d and T_b. Its price is given by (11), with μ_0 such that: $\int_{T_d}^{T_b} x_d(t)dt = \int_{T_d}^{T_b} D\left(p_d(t)\right) dt = \frac{\bar{Z} - Z_0 - \theta_e X_e}{\theta_d}$. T_d, the date of the switch from shale gas to coal, is endogenously determined by $p_e(T_d) = p_d(T_d)$, i.e.

$$c_e + d + (\lambda_0 + \theta_e \mu_0) e^{\rho T_d} = c_d + \theta_d \mu_0 e^{\rho T_d} \tag{15}$$

- Phase 3: the clean backstop is used at price c_b (see (14)) from date T_b onwards.

Here again, one of these phases may not exist. For instance, absent climate policy ($\overline{Z} \to \infty$) shale gas, the cheapest source of energy, is used first, then coal is used forever. Solar is never developed. As in the previous case, as soon as some climate policy is introduced, solar will be used at some point.

2.3 Solution

We now find the optimal quantity of shale gas to be developed X_e and the optimal date of the switch from the polluting energy to solar in electricity generation T_b.

2.3.1 Large local damage

When $d > c_d - c_e$, the optimal quantity of shale gas developed, X_e, and the optimal date of the switch from shale gas to solar, T_b, solve:

$$\lambda_0 = E'(X_e) \tag{16}$$

$$\left[u\left(x_e(T_b)\right) - (c_e + d + (\lambda_0 + \theta_e\mu_0)e^{\rho T_b})x_e(T_b)\right] - \left[u\left(x_b\right) - c_b x_b\right] = CF'(T_b) - \rho CF(T_b) \tag{17}$$

Equation (16 states that costs of exploration for finding shale gas reserves must be paid up to the point where the exploration cost of a marginal unit of reserve $E'(X_e)$ is equal to the value of this reserve under the ground, which is the initial scarcity rent λ_0. Equation (17) shows that at the optimal date of the switch from shale gas to solar the marginal benefice of the switch is equal to its marginal cost (Dasgupta *et al.*, 1982). It shows that the electricity price jumps downwards at the date of the switch, the size of the jump being proportional to the marginal cost of delaying R&D in the backstop technology.

Equations (1), (2), (13), (16) and (17) characterize the optimal solution when the sequence of energy use is coal (from 0 to T_e), shale gas (from T_e to T_b) and solar, i.e. when the three phases identified above exist.

We want now to check the conditions under which one of the two first phases does not exist, given that the last phase (solar) always exists as soon as some climate policy is introduced.

- Notice first that coal is never used alone to get to the ceiling if $c_d + \theta_d \frac{c_e + d - cd}{\theta_d - \theta_e} < c_b$. Indeed, if $c_d + \theta_d \frac{c_e + d - cd}{\theta_d - \theta_e} < c_b$, shale gas price and coal price necessarily cross at a price lower than c_b so that shale gas is always used at some date, as long as there is a constraint on the stock of pollution. On the other hand, if $c_d + \theta_d \frac{c_e+d-cd}{\theta_d-\theta_e} \geq c_b$, coal can be used alone to get to the ceiling if $\overline{Z}$ is high enough. This can also be written $d \geq (c_d - c_e) + \frac{\theta_d-\theta_e}{\theta_e}(c_b - c_e)$: the local damage must be very high. If it is the case and coal is used alone to get to the ceiling, then the values of μ_0 and T_b must solve the following system:

$$\theta_d \int_0^{T_b} x_d(t)dt = \overline{Z} - Z_0 \tag{18}$$

$$\left[u\left(x_d(T_b)\right) - (c_d + \theta_d\mu_0)e^{\rho T_b})x_d(T_b)\right] - \left[u\left(x_b\right) - c_b x_b\right] = CF'(T_b) - \rho CF(T_b) \tag{19}$$

where equation (18) is the combination of equations (1) and (2) for $X_e = 0$, and equation (19) is equation (17) in the case $X_e = 0$. Moreover, we must make sure that there is no incentive to extract shale gas: the final price of coal $p_d(T_b)$ must be lower than the price of the first unit of shale gas that could be extracted at date T_b, $c_e + d + \theta_e\mu_0 e^{\rho T_b}$. Hence we must have:

$$(\theta_d - \theta_e)\mu_0 e^{\rho T_b} \leq c_e + d - c_d \tag{20}$$

meaning that the marginal gain in terms of pollution of switching from coal to shale gas, evaluated at the carbon value at date T_b, is smaller than the marginal cost of the switch. If the solution of the above system is such that this condition is satisfied, then shale gas is never extracted. There exists a threshold value of the ceiling $\overline{Z}_1$, such that if $\overline{Z} \geq \overline{Z}_1$ shale gas is not developped. $\overline{Z}_1$ is solution of the system composed of equations (18), (19) and (20), this last equation being written as an equality.

- If shale gas is used alone, and coal is left under the ground, then the values of λ_0, μ_0, T_b and X_e must solve the system composed of equations (1), (16), (17) and

$$\theta_e X_e = \overline{Z} - Z_0 \tag{21}$$

which replaces (2). Moreover, to ensure that there exists no incentive to introduce coal at date 0, the initial price of shale gas $p_e(0)$ must be below the initial price of coal, $p_d(0)$, i.e. we must have

$$(\theta_d - \theta_e)\mu_0 \geq c_e + d - c_d + E'(X_e) \tag{22}$$

If the solution of the above system is such that this condition is satisfied, then shale gas is used alone to get to the ceiling. There exists a threshold value of the ceiling $\overline{Z}_2$ under which only shale is used. It is solution of the system composed of equations (1), (16), (17), (21) and (22), this last equation being taken as an equality.

- For an intermediate ceiling $\overline{Z}$ such that $\overline{Z}_1 > \overline{Z} > \overline{Z}_2$, the three phases exist.

2.3.2 Small local damage

When $d < c_d - c_e$, the optimal quantity of shale gas developed, X_e, and the optimal date of the switch from coal to solar, T_b, solve:

$$\lambda_0 = E'(X_e) \tag{23}$$

$$\left[u\left(x_d(T_b)\right) - (c_d + \theta_d \mu_0 e^{\rho T_b}) x_d(T_b)\right] - \left[u\left(x_b\right) - c_b x_b\right] = CF'(T_b) - \rho CF(T_b) \tag{24}$$

The interpretation of these equations is similar as in the case of a large local damage.

Equations (1), (2), (15), (23) and (24) characterize the optimal solution when the sequence of energy use is shale gas (from 0 to T_d), coal (from T_d to T_b) and solar (from T_b onwards).

- As shale gas is cheaper and less polluting than coal, necessarily $c_e + d + \theta_e \mu_0 < c_d + \theta_d \mu_0 \ \forall \mu_0$. Hence $\exists \lambda_0 > 0$ s.t. $p_e(0) < p_d(0)$, meaning that there always exists scope for shale gas exploration and extraction.

- Now, it is possible to switch direcly from shale gas to solar, and leave coal forever in the ground? If shale is used, alone, to get to the ceiling, then λ_0, μ_0, T_b and X_e must solve the system composed of equations (1), (21), (23) and:

$$\left[u\left(x_e(T_b)\right) - (c_e + d + (\lambda_0 + \theta_e \mu_0) e^{\rho T_b}) x_e(T_b)\right] - \left[u\left(x_b\right) - c_b x_b\right] = CF'(T_b) - \rho CF(T_b) \tag{25}$$

  Moreover, the final price of shale gas $p_e(T_b)$ must be lower than the price of the first unit of coal that could be extracted at date T_b, $p_d(T_b)$, i.e. we must have:

$$(\theta_d - \theta_e)\mu_0 e^{rT_b} > c_e + d - c_d + E'(X_e) e^{rT_b} \tag{26}$$

meaning that the cost in terms of pollution of switching to coal instead of going directly to solar is higher than the advantage in terms of production costs. It happens for values of the ceiling below $\bar{Z}_3$ defined by (1), (21), (23), (25) and (26) taken as an equality.

- For $\overline{Z} > \overline{Z}_3$, the three resources are used.

To sum up, Fig. 1 represents the optimal succession of energy sources in electricity generation as a function of the stringency of climate policy. When the local damage is very large and climate policy lenient, coal is used alone to get to the ceiling. It is not optimal in this case to explore and develop shale gas. When environmental policy becomes more stringent, shale gas replaces coal at some point before the ceiling. For an even more stringent environmental policy, coal is completely evicted by shale gas. When the local damage is small shale gas is always developed, and its extraction begins immediately. If climate policy is lenient, shale gas is replaced by coal at some point before the ceiling, because it is abundant whereas shale gas is scarce and costly to develop. However, if climate policy is stringent, coal is never extracted.

< Fig. 1 about here >

3 Comparative dynamics

We now perform exercises of comparative dynamics to see precisely how the optimal solution is modified when environmental policy becomes more stringent. In particular, we wonder whether climate policy justifies developing more shale gas, and making the transition to solar earlier.

3.1 Large local damage

We show in Appendix A that in this case:

$$\frac{\partial \lambda_0}{\partial \overline{Z}} < 0, \qquad \frac{\partial \mu_0}{\partial \overline{Z}} < 0, \qquad \frac{\partial T_e}{\partial \overline{Z}} > 0, \qquad \frac{\partial T_b}{\partial \overline{Z}} > 0, \qquad \frac{\partial X_e}{\partial \overline{Z}} < 0$$

When the marginal local damage of shale gas is large, with a lenient environmental policy few shale gas –if any– is extracted. Electricity is generated before the ceiling mainly by coal-fired power plants. However, as environmental policy becomes more stringent, the use of shale gas becomes more interesting because of its lower carbon content. This advantage on the climate point of view overcomes more and more the local damage drawback and the exploration cost. It is then optimal to use shale gas earlier and to develop it in a greater amount.

A more severe climate policy also makes the switch to solar energy happen earlier.

Clearly, in this case, the effect of a more stringent climate policy is to partially or even totally evict coal, and to replace it by more shale gas before the ceiling, and to make the transition to clean energy happen sooner.

3.2 Small local damage

Likewise, a comparative dynamics exercise yields in the case of a small local damage (see Appendix B):

$$\frac{\partial \mu_0}{\partial \overline{\overline{Z}}} < 0, \qquad \frac{\partial T_d}{\partial \overline{\overline{Z}}} < 0, \qquad \frac{\partial T_b}{\partial \overline{\overline{Z}}} > 0$$

Remember that in this case it is optimal to develop shale gas first. Then, quite intuitively, when environmental policy becomes more stringent, the date of the switch to coal is postponed while the date of the switch to solar is brought forward. However, the effect of a more stringent climate policy on shale gas extraction depends on its relative carbon content. We show in Appendix B that the two polar cases where shale gas is not polluting at all and shale gas is as polluting as coal lead to very different outcomes:

$$\text{if } \theta_e = 0, \ \frac{\partial \lambda_0}{\partial \overline{\overline{Z}}} < 0 \text{ and } \frac{\partial X_e}{\partial \overline{\overline{Z}}} < 0$$

$$\text{if } \theta_e = \theta_d, \ \frac{\partial \lambda_0}{\partial \overline{\overline{Z}}} > 0 \text{ and } \frac{\partial X_e}{\partial \overline{\overline{Z}}} > 0$$

When shale gas is not polluting at all, the more stringent climate policy is, the more shale gas is developed. The total marginal variable cost of shale gas is smaller than the one of coal because the marginal local damage is small; furthermore, shale gas is not polluting. The only reason why coal is not completely evicted is the costly initial exploration investment needed to develop shale gas. However, when shale gas is as polluting as coal, only the variable cost argument remains in favour of shale gas, but it is not enough: the more stringent climate policy is, the less shale gas is developed.

3.3 The cost of climate policy

What is the cost of a more stringent climate policy? This question is clearly of a great practical importance, since the cost argument is prominent in the fact that countries are reluctant to tighten climate policy, even if it is optimal from a welfare point of view.

Let A_0 be the present value of total energy expenditures:

$$A_0 = \int_0^\infty e^{-\rho t} \left[c_d x_d(t) + c_e x_e(t) + c_b x_b(t) \right] dt + E(X_e) + CF(T_b) e^{-\rho T_b} \tag{27}$$

A comparative dynamics exercise shows that $\frac{\partial A_0}{\partial \overline{Z}} > 0$ for high values of $\overline{Z}$, and $\frac{\partial A_0}{\partial \overline{Z}} < 0$ for low values of $\overline{Z}$, whatever the value of the marginal local damage. A stringent environmental policy is costly because it requires that expensive investments for shale gas exploration and solar plants installation are made, and that the transition to clean energy happens earlier. However, a lenient one may come with a decrease of energy expenditures, due to the decrease of energy consumption, which dominates the cost effect.

4 Constraint on energy expenditures

In order to get more insights on the arbitrage between the development of the clean backstop, the development of shale gas and the cost of energy consumption, we add a constraint on total energy expenditures. The constraint says that total expenditures that are related to energy consumption cannot exceed energy expenditures absent any environmental policy. This constraint can be seen as a political constraint. The problem is the same as the original one except that we add the following constraint:

$$A_0 \leq A_0^{\text{ref}} \tag{28}$$

where A_0^{ref} is the present value of energy expenditures when there is no climate policy. The objective is to see whether the previous results are modified when we force climate policy to be costless.

We have seen that the reference situation absent climate policy differs, depending on the value of the marginal local damage. If it is large, the reference path is a path where coal is used alone, from the origin onwards. Then $x_d(t) = D(c_d)$ and $A_0^{\text{ref}} = c_d D(c_d)/\rho$. If it is small, shale gas is used first (from 0 to T_d), then coal (from T_d onwards), and solar is never developed. Then:

$$A_0^{\text{ref}} = \int_0^{T_d} e^{-\rho t} c_e x_e(t) dt + \int_{T_d}^\infty e^{-\rho t} c_d x_d(t) dt - E(X_e)$$

with

$$x_e(t) = D(c_e + d + \lambda_0 e^{\rho t})$$
$$x_d(t) = D(c_d)$$

and where λ_0, X_e and T_d are solution of the following system:

$$\int_0^{T_d} x_e(t)dt = X_e$$
$$\lambda_0 = E'(X_e)$$
$$c_e + d + \lambda_0 e^{\rho T_d} = c_d$$

Let α be the Lagrange multiplier associated with constraint (28). The solutions are the same as the solutions without constraint, where c_e, c_d and c_b are replaced by $(1+\alpha)c_e$, $(1+\alpha)c_d$ and $(1+\alpha)c_b$, and $E(X_e)$ and $CF(T_b)$ are replaced by $(1+\alpha)E(X_e)$ and $(1+\alpha)CF(T_b)$. More precisely:

$$u'(x_d(t)) \leq (1+\alpha)c_d + \theta_d\mu_0 e^{\rho t}$$
$$u'(x_e(t)) \leq (1+\alpha)c_e + d + (\lambda_0 + \theta_e\mu_0)\, e^{\rho t}$$
$$u'(x_b(t)) \leq (1+\alpha)c_b$$

plus the complementarity slackness condition:

$$\alpha(A_0^{\text{ref}} - A_0) = 0,\ \alpha \geq 0,\ A_0^{\text{ref}} - A_0 \geq 0$$

and

- if $d > (1+\alpha)(c_d - c_e)$:

$$(1+\alpha)c_d + \theta_d\mu_0 e^{\rho T_e} = (1+\alpha)c_e + d + (\lambda_0 + \theta_e\mu_0)e^{\rho T_e}$$
$$\lambda_0 = (1+\alpha)E'(X_e)$$
$$\left[u\left(x_e(T_b)\right) - ((1+\alpha)c_e + d + (\lambda_0 + \theta_e\mu_0)e^{\rho T_b})x_e(T_b)\right] - \left[u\left(x_b\right) - (1+\alpha)c_b x_b\right]$$
$$= (1+\alpha)\left[CF'(T_b) - \rho CF(T_b)\right]$$

- if $d < (1+\alpha)(c_d - c_e)$:

$$(1+\alpha)c_e + d + (\lambda_0 + \theta_e\mu_0)e^{\rho T_d} = (1+\alpha)c_d + \theta_d\mu_0 e^{\rho T_d}$$

$$\lambda_0 = (1+\alpha)E'(X_e)$$

$$\left[u\left(x_d(T_b)\right) - ((1+\alpha)c_d + \theta_d\mu_0 e^{\rho T_b})x_d(T_b)\right] - \left[u\left(x_b\right) - (1+\alpha)c_b x_b\right] = (1+\alpha)\left[CF'(T_b) - \rho CF(T_b)\right]$$

When the financial constraint is binding, $\alpha > 0$. The primary effect of the constraint is to increase the monetary costs associated to electricity generation (extraction, investment and O& M costs), while the non-monetary costs, that is the environmental costs (local damage d and global damages $\theta_d\mu_0$ and $\theta_e\mu_0$) remain unchanged. Environmental matters become less important compared to costs. We may then expect that two conflicting effects appear. On the one hand, the declining importance of the local damage d is an incentive to develop more shale gas and extract it earlier; on the other hand the declining importance of the global damage reduces the advantage of shale gas in terms of carbon emissions, and thus has the opposite effect.

5 Simulations

We perform in this section illustrative simulations. We use standard functional forms: a quadratic utility function, a solar R&D cost decreasing at a constant rate due to exogenous technical progress, and a quadratic shale gas exploration cost:

$$u(x) = ax - \frac{x^2}{2} \Longrightarrow D(p) = a - p$$

$$CF(t) = CF_0 e^{-\gamma t}$$

$$E(X_e) = \frac{\varepsilon}{2}X_e^2$$

Parameters are given in Table 2.

Fig. 2 shows iso–X_e curves in the plane $(\overline{Z}, d)$. For the parameters given above, the local marginal damage is small if $d < 0.5$, large otherwise. Follow for instance the iso–X_e curve for $X_e = 10$ from the right to the left. First, the climate constraint is lenient and the local damage small. Shale gas is used

c_d	c_e	c_b	CF_0	θ_d	θ_e	ρ	γ	ε	a
1	0.5	3	50	0.5	0.3	0.02	0.03	0.05	5

Table 2: Parameters

first in electricity generation, then coal then solar. As we move to the left on Fig. 2, the same quantity of shale gas developed corresponds to a more and more stringent climate constraint and an increasing level of the local damage. The quantity of coal used is lower and lower, and the switch to solar occurs earlier and earlier. When the threshold $\overline{Z}_3$ is met, coal is completely evicted, and the economy switches directly from shale gas to solar. When the local damage reaches the threshold value of 0.5, coal is used again in electricity generation, now before shale gas. As we move further to the left, coal may be evicted by solar.

We now compare the results of simulations performed with and without the constraint on energy expenditures, in order to see which of the previous effects dominates and in what circumstances.

5.1 Large local damage

Fig. 3 represents how X_e changes with $\overline{Z}$, in the reference case (solid line) and the constrained case (dotted line), for the baseline value of θ_e ($\theta_e = 0.3$) and also for $\theta_e = 0$. When $\theta_e = 0.3$, the quantity of shale gas extracted is, for most values of the ceiling, larger in the constrained case than in the reference case. However, for very low values of the ceiling, the quantity of shale extracted is lower when the constraint on energy expenditures is binding. Things are very different when shale gas is not polluting at all ($\theta_e = 0$) : in this case, the quantity of shale gas extracted is smaller with the constraint on energy expenditures than without.

For both values of θ_e, date T_b of development of the clean backstop is postponed compared to the reference scenario (see Fig. 4).

< Fig. 3 about here >

< Fig. 4 about here >

5.2 Small local damage

Fig. 5 represents the result of then same exercise in the case of a small local damage. Now, for $\theta_e = 0.3$, the quantity of shale extracted is not significantly different in the constrained and the reference cases, whereas it is smaller when $\theta_e = 0$. On the other hand, date T_b of development of the backstop is postponed compared to the reference scenario for both values of θ_e (see Fig. 6).

$<$ Fig. 5 about here $>$

$<$ Fig. 6 about here $>$

The constraint on energy expenditures actually modifies the arbitrage between the different energy sources. The development of the clean backstop is always postponed, whereas the effect on the quantity of shale gas extracted is ambiguous. On the one hand, as local damages are not monetary costs, the relative total variable cost of shale decreases compared to those of coal and solar. On the other hand, as coal and shale gas extraction costs are not paid at the same date, it can also be optimal to increase coal extraction and decrease shale gas extraction in order to comply with the expenditure constraint.

6 Conclusion

We have shown in this paper that tightening climate policy always leads to bringing forward the transition to clean energy. When the local damage caused by shale gas extraction is high, the quantity of shale gas developed increases at the expense of coal, and the date at which shale gas extraction begins is brought forward. However, when the local damage is small, it may be the case that a more stringent climate policy leads to reduce the quantity of shale gas developed, when the advantage of shale gas over coal in terms of carbon emissions is not large enough. We have also studied how these results are modified when the social planner has to comply to the climate constraint without increasing energy expenditures. The financial constraint increases the weight of the monetary costs associated to electricity generation (extraction, investment and O& M costs) compared to the weight of the non-monetary costs, that is the environmental costs (local and global damages), in the decision-making process. Two conflicting effects appear. On the one hand, the declining importance of the local damage is an incentive to develop more shale gas and extract it earlier; on the other hand the declining importance of the global damage reduces the advantage of shale gas in terms of carbon emissions, and thus has the opposite effect. What effect prevails depends on the relative carbon contents of shale gaz and coal, on the magnitude of the local damage and on the stringency of climate policy.

A lot of aspects of the shale gas question are worth studying, among which:

- the impact of the subsoil property rights regime on the decision to develop shale gas;
- the NIMBY effects of shale gas extraction in densely populated areas;
- the reasons why in France, not only the *exploitation* of shale gas is banned, but also the *exploration* of potential reserves. Does the *knowledge* of the size of shale gas deposits present in the French subsoil make the exploitation ban impossible to enforce?

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Appendix

A Large local damage

In this case, equations (1) and (2) may be written as:

$$\int_{T_e}^{T_b} x_e(t)dt = X_e \tag{29}$$

$$\int_0^{T_e} \theta_d x_d(t)dt + \int_{T_e}^{T_b} \theta_e x_e(t)dt = \overline{Z} - Z_0$$

Using (29), this last equation reads:

$$\int_0^{T_e} x_d(t)dt = \frac{1}{\theta_d}\left(\overline{Z} - Z_0 - \theta_e X_e\right) \tag{30}$$

Totally differentiating system (29), (30), (13), (17) and (16) yields:

$$x_e(T_b)dT_b - x_e(T_e)dT_e + \int_{T_e}^{T_b} dx_e(t)dt = dX_e$$

$$x_d(T_e)dT_e + \int_0^{T_e} dx_d(t)dt = \frac{1}{\theta_d}\left(d\overline{Z} - \theta_e dX_e\right)$$

$$[\theta_d\mu_0 - (\lambda_0 + \theta_e\mu_0)]\,\rho dT_e + (\theta_d - \theta_e)d\mu_0 - d\lambda_0 = 0$$

$$\left[u'\left(x_e(T_b)\right)dx_e(T_b) - (c_e + d + (\lambda_0 + \theta_e\mu_0)e^{\rho T_b})dx_e(T_b) - ((d\lambda_0 + \theta_e d\mu_0) + (\lambda_0 + \theta_e\mu_0)\rho dT_b)e^{\rho T_b}x_e(T_b)\right]$$
$$= \left(CF''(T_b) - \rho CF'(T_b)\right)dT_b$$

$$d\lambda_0 = E''(X_e)dX_e$$

As

$$x_d(t) = D(p_d(t)) \Rightarrow dx_d(t) = D'(p_d(t))dp_d(t) = D'(p_d(t))\theta_d e^{\rho t}d\mu_0$$
$$x_e(t) = D(p_e(t)) \Rightarrow dx_e(t) = D'(p_e(t))dp_e(t) = D'(p_e(t))e^{\rho t}\left(d\lambda_0 + \theta_e d\mu_0\right)$$

the first 2 equations read equivalently:

$$x_e(T_b)dT_b - x_e(T_e)dT_e + \left[\int_{T_e}^{T_b} D'(p_e(t))e^{\rho t}dt\right](d\lambda_0 + \theta_e d\mu_0) = dX_e$$

$$x_d(T_e)dT_e + \left[\int_0^{T_e} D'(p_d(t))e^{\rho t}dt\right]\theta_d d\mu_0 = \frac{1}{\theta_d}\left(d\overline{Z} - \theta_e dX_e\right)$$

Besides,

$$\dot{D}(p_d(t)) = D'(p_d(t))\dot{p}_d(t) = D'(p_d(t))\theta_d\mu_0\rho e^{\rho t}$$
$$\Rightarrow \int_0^{T_e} D'(p_d(t))e^{\rho t}dt = \frac{1}{\theta_d\mu_0\rho}\int_0^{T_e}\dot{D}(p_d(t)dt = \frac{1}{\theta_d\mu_0\rho}\left[D(p_d(T_e)) - D(p_d(0)\right] = \frac{x_d(T_e) - x_d(0)}{\theta_d\mu_0\rho}$$

and

$$\int_{T_e}^{T_b} D'(p_e(t))e^{\rho t}dt = \frac{x_e(T_b) - x_e(T_e)}{(\lambda_0 + \theta_e\mu_0)\rho}$$

Hence the first 2 equations read:

$$-x_e(T_e)dT_e + x_e(T_b)dT_b - dX_e + \frac{x_e(T_b) - x_e(T_e)}{(\lambda_0 + \theta_e\mu_0)\rho}\left(d\lambda_0 + \theta_e d\mu_0\right) = 0$$

$$x_d(T_e)dT_e + \frac{\theta_e}{\theta_d}dX_e + \frac{x_d(T_e) - x_d(0)}{\mu_0\rho}d\mu_0 = \frac{1}{\theta_d}d\bar{Z}$$

Using the equality between marginal utilities, the fourth equation simplifies, and we obtain easily:

$$A \times \begin{pmatrix} dT_e \\ dT_b \\ dX_e \\ d\lambda_0 \\ d\mu_0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\theta_d} \\ 0 \\ 0 \\ 0 \end{pmatrix} d\overline{Z}$$

with

$$A = \begin{pmatrix} -x_e(T_e) & x_e(T_b) & -1 & \frac{x_e(T_b)-x_e(T_e)}{(\lambda_0+\theta_e\mu_0)\rho} & \theta_e\frac{x_e(T_b)-x_e(T_e)}{(\lambda_0+\theta_e\mu_0)\rho} \\ x_e(T_e) & 0 & \frac{\theta_e}{\theta_d} & 0 & \frac{x_e(T_e)-x_d(0)}{\mu_0\rho} \\ \left[\lambda_0+(\theta_e-\theta_d)\mu_0\right]\rho & 0 & 0 & 1 & \theta_e-\theta_d \\ 0 & (\lambda_0+\theta_e\mu_0)\rho x_e(T_b)+z_1 & 0 & x_e(T_b) & \theta_e x_e(T_b) \\ 0 & 0 & -z_2 & 1 & 0 \end{pmatrix}$$

where

$$z_1 = \left(CF''(T_b) - \rho CF'(T_b)\right) e^{-\rho T_b} > 0$$
$$z_2 = E''(X_e) > 0$$

Hence:

$$\begin{aligned}
&\rho\theta_d\mu_0(\lambda_0+\theta_e\mu_0)\det A \\
&= \theta_d\left[\underbrace{(x_e(T_e)-x_e(T_b))}_{>0}x_d(0)\theta_d\mu_0 + \underbrace{(x_d(0)-x_e(T_e))}_{>0}x_e(T_b)\left(\lambda_0+\theta_e\mu_0\right)\right]z_1z_2 \\
&+ \rho\left\{\left[\underbrace{(\theta_e x_e(T_b)-\theta_d x_d(0))}_{<0}\theta_e\mu_0 - x_d(0)\theta_d\lambda_0\right]\underbrace{(\lambda_0+(\theta_e-\theta_d)\mu_0)}_{<0}+x_e(T_e)\theta_d\lambda_0^2\right\}z_1 \\
&+ \rho\theta_d x_d(0)x_e(T_e)x_e(T_b)\theta_d\mu_0(\lambda_0+\theta_e\mu_0)z_2 \\
&+ \rho^2\theta_d(\lambda_0+\theta_e\mu_0)x_e(T_b)\left[x_e(T_e)\lambda_0^2 - x_d(0)(\lambda_0+\theta_e\mu_0)\underbrace{(\lambda_0+(\theta_e-\theta_d)\mu_0)}_{<0}\right]
\end{aligned}$$

i.e. $\det A > 0$.

$$A^{-1} \times \begin{pmatrix} 0 \\ \frac{1}{\theta_d} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\rho\theta_d\mu_0(\lambda_0+\theta_e\mu_0)\det A} \times$$

$$\begin{pmatrix} \mu_0\left(\lambda_0+\theta_e\mu_0\right)\left[\frac{\theta_d}{\lambda_0+\theta_e\mu_0}\left(x_e(T_e)-x_e(T_b)\right)z_1z_2+\rho z_1(\theta_d-\theta_e)+\rho x_e(T_b)\left(x_e(T_e)z_2\theta_d+\rho(\theta_d-\theta_e)\left(\lambda_0+\theta_e\mu_0\right)\right)\right] \\ -\rho x_e(T_b)\mu_0(\lambda_0+\theta_e\mu_0)\left[-x_e(T_e)\theta_d z_2+\rho\theta_e\underbrace{\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right)}_{<0}\right] \\ -\rho\mu_0\left[-x_e(T_b)z_1\theta_e\underbrace{\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right)}_{<0}+x_e(T_e)\theta_d\lambda_0\left(z_1+\rho x_e(T_b)(\lambda_0+\theta_e\mu_0)\right)\right] \\ -z_2\rho\mu_0\left[-x_e(T_b)z_1\theta_e\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right)+x_e(T_e)\theta_d\lambda_0\left(z_1+\rho x_e(T_b)(\lambda_0+\theta_e\mu_0)\right)\right] \\ -\rho\mu_0\left(\lambda_0+\theta_e\mu_0\right)\left[\begin{array}{c}\frac{\theta_d\mu_0}{\lambda_0+\theta_e\mu_0}(x_e(T_e)-x_e(T_b))z_1z_2+x_e(T_b)z_1z_2-\rho z_1\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right) \\ -\rho x_e(T_b)\left[-x_e(T_e)\theta_d\mu_0z_2+\rho(\lambda_0+\theta_e\mu_0)\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right)\right]\end{array}\right] \end{pmatrix}$$

As $\det A > 0$, we deduce:

$$\frac{\partial T_e}{\partial \overline{Z}} > 0, \quad \frac{\partial T_b}{\partial \overline{Z}} > 0, \quad \frac{\partial X_e}{\partial \overline{Z}} < 0, \quad \frac{\partial \lambda_0}{\partial \overline{Z}} < 0, \quad \frac{\partial \mu_0}{\partial \overline{Z}} < 0$$

B Small local damage

In this case, equations (1) and (2) may be written as:

$$\int_0^{T_d} x_e(t)dt = X_e \tag{31}$$

$$\int_{T_d}^{T_b} x_d(t)dt = \frac{1}{\theta_d}\left(\overline{Z} - Z_0 - \theta_e X_e\right) \tag{32}$$

Totally differentiating system (31), (32), (15), (17) and (16) yields:

$$x_e(T_d)dT_d + \frac{x_e(T_d) - x_e(0)}{(\lambda_0+\theta_e\mu_0)\rho} = dX_e$$

$$x_d(T_b)dT_b - x_d(T_d)dT_d + \frac{x_d(T_b) - x_d(T_d)}{\theta_d\mu_0\rho} = \frac{1}{\theta_d}\left(d\overline{Z} - \theta_e dX_e\right)$$

$$-((d\lambda_0 + \theta_e d\mu_0) + (\lambda_0 + \theta_e \mu_0)\rho dT_d)e^{\rho T_d} x_e(T_d) + \theta_d(d\mu_0 + \mu_0 \rho dT_d)e^{\rho T_d} x_d(T_d) = 0$$

$$-\theta_d(d\mu_0 + \rho dT_b)e^{\rho T_b} x_d(T_b) = (CF''(T_b) - \rho CF'(T_b))\, dT_b$$

$$d\lambda_0 = E''(X_e) dX_e$$

Using $x_e(T_d) = x_d(T_d)$, we obtain:

$$A \times \begin{pmatrix} dT_d \\ dT_b \\ dX_e \\ d\lambda_0 \\ d\mu_0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\theta_d} \\ 0 \\ 0 \\ 0 \end{pmatrix} d\overline{Z}$$

with

$$A = \begin{pmatrix} x_d(T_d) & 0 & -1 & \frac{x_d(T_d) - x_e(0)}{(\lambda_0 + \theta_e \mu_0)\rho} & \theta_e \frac{x_d(T_d) - x_e(0)}{(\lambda_0 + \theta_e \mu_0)\rho} \\ -x_d(T_d) & x_d(T_b) & \frac{\theta_e}{\theta_d} & 0 & \frac{x_d(T_b) - x_d(T_d)}{\mu_0 \rho} \\ \left[-\theta_d \mu_0 + (\lambda_0 + \theta_e \mu_0)\right] \rho & 0 & 0 & 1 & -(\theta_d - \theta_e) \\ 0 & y_1 & 0 & 0 & \theta_d x_d(T_b) \\ 0 & 0 & -E''(X_e) & 1 & 0 \end{pmatrix}$$

where

$$y_1 = \left(CF''(T_b) - \rho CF'(T_b)\right) e^{-\rho T_b} + \rho x_d(T_b)\theta_d \mu_0 > 0$$

Let's denote

$$y_2 = E''(X_e) \left[x_d(T_d)\theta_d \mu_0 + x_e(0) \left(\lambda_0 + (\theta_e - \theta_d)\, \mu_0\right)\right]$$

According to (15), we have:

$$\lambda_0 + (\theta_e - \theta_d)\, \mu_0 = (c_d - (c_e + d))\, e^{-\rho T_d} > 0$$

which implies that y_2 is also positive.

We have

$$
\begin{aligned}
& -\rho\theta_d\mu_0(\lambda_0+\theta_e\mu_0)\det A \\
& = \rho x_d(T_b)^2\theta_d^2\mu_0\left\{\rho(\lambda_0+\theta_e\mu_0)(\lambda_0+(\theta_e-\theta_d)\mu_0)+E^{''}(X_e)\left[x_d(T_d)\theta_d\mu_0+x_e(0)(\lambda_0+(\theta_e-\theta_d)\mu_0)\right]\right\} \\
& + y_1\rho\left\{x_d(T_d)\theta_d\lambda_0^2+x_e(0)\theta_e^2\mu_0(\lambda_0+(\theta_e-\theta_d)\mu_0)-x_d(T_b)\theta_d(\lambda_0+\theta_e\mu_0)(\lambda_0+(\theta_e-\theta_d)\mu_0)\right\} \\
& + y_1E^{''}(X_e)\theta_d\left\{x_e(0)(\lambda_0+\theta_e\mu_0)\left(x_d(T_d)-x_d(T_b)\right)+x_d(T_b)\theta_d\mu_0(x_e(0)-x_d(T_d))\right\}
\end{aligned}
$$

It is straightforward that the terms of the first and third lines are positive. Let look at the term of the second line:

$$
y_1\rho\left\{x_d(T_d)\theta_d\lambda_0^2+x_e(0)\theta_e^2\mu_0(\lambda_0+(\theta_e-\theta_d)\mu_0)-x_d(T_b)\theta_d(\lambda_0+\theta_e\mu_0)(\lambda_0+(\theta_e-\theta_d)\mu_0)\right\}
$$

Dividing by $y_1\rho>0$, it has the sign of:

$$
\begin{aligned}
& \lambda_0^2(\theta_d x_d(T_d)-\theta_d x_d(T_b)) \\
& + \lambda_0\mu_0(\theta_e^2x_e(0)+\theta_d^2x_d(T_b)-2\theta_e\theta_d x_d(T_b)) \\
& + \mu_0^2\theta_e(\theta_e^2x_e(0)+\theta_d^2x_d(T_b)-\theta_e\theta_d x_d(T_b)-\theta_e\theta_d x_e(0))
\end{aligned}
$$

It is straightforward that $\lambda_0^2(\theta_d x_d(T_d)-\theta_d x_d(T_b))>0$. Moreover

$$
\lambda_0\mu_0(\theta_e^2x_e(0)+\theta_d^2x_d(T_b)-2\theta_e\theta_d x_d(T_b))=\lambda_0\mu_0x_d(T_b)(\theta_d-\theta_e)^2+\lambda_0\mu_0\theta_e^2(x_e(0)-x_d(T_b)) \tag{33}
$$

and

$$
\mu_0^2\theta_e(\theta_e^2x_e(0)+\theta_d^2x_d(T_b)-\theta_e\theta_d x_d(T_b)-\theta_e\theta_d x_e(0))=\mu_0^2\theta_e(\theta_d-\theta_e)(\theta_d x_d(T_b)-\theta_e x_e(0)) \tag{34}
$$

so that regrouping the last two terms (33) and (34), one gets :

$$
\begin{aligned}
& \lambda_0\mu_0\left(\theta_e^2x_e(0)+\theta_d^2x_d(T_b)-2\theta_e\theta_d x_d(T_b)\right)+\mu_0^2\theta_e\left(\theta_e^2x_e(0)+\theta_d^2x_d(T_b)-\theta_e\theta_d x_d(T_b)-\theta_e\theta_d x_e(0)\right) \\
& = \lambda_0\mu_0x_d(T_b)(\theta_d-\theta_e)^2+\lambda_0\mu_0\theta_e^2(x_e(0)-x_d(T_b))+\mu_0^2\theta_e(\theta_d-\theta_e)(\theta_d x_d(T_b)-\theta_e x_e(0)) \\
& = \lambda_0\mu_0x_d(T_b)(\theta_d-\theta_e)^2+\lambda_0\mu_0\theta_e^2(x_e(0)-x_d(T_b))+\mu_0^2\theta_e(\theta_d-\theta_e)((\theta_d-\theta_e)x_d(T_b)-\theta_e(x_e(0)-x_d(T_b))) \\
& = \lambda_0\mu_0x_d(T_b)(\theta_d-\theta_e)^2+\lambda_0\mu_0\theta_e^2(x_e(0)-x_d(T_b))+\mu_0^2\theta_e(\theta_d-\theta_e)^2x_d(T_b)-\mu_0^2\theta_e^2(\theta_d-\theta_e)(x_e(0)-x_d(T_b)) \\
& = \mu_0x_d(T_b)(\theta_d-\theta_e)^2(\lambda_0+\theta_e\mu_0)+\mu_0\theta_e^2(x_e(0)-x_d(T_b))(\lambda_0+\mu_0(\theta_e-\theta_d))
\end{aligned}
$$

which is positive. As a result:

$$\det A < 0$$

We also obtain:

$$A^{-1} \times \begin{pmatrix} 0 \\ \frac{1}{\theta_d} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\theta_d(\lambda_0 + \theta_e\mu_0)\det A} \begin{pmatrix} y_1 \left[E''(X_e)(x_e(0) - x_d(T_d))\theta_d + \rho\left(\theta_d - \theta_e\right)\left(\lambda_0 + \theta_e\mu_0\right)\right]/\rho \\ -x_d(T_b)\theta_d \left[\rho(\lambda_0 + \theta_e\mu_0)\left(\lambda_0 + \left(\theta_e - \theta_d\right)\mu_0\right) + y_2\right] \\ y_1 \left[x_d(T_d)\theta_d\lambda_0 - x_e(0)\theta_e\left(\lambda_0 + \left(\theta_e - \theta_d\right)\mu_0\right)\right] \\ y_1 E''(X_e)\left[x_d(T_d)\theta_d\lambda_0 - x_e(0)\theta_e\left(\lambda_0 + \left(\theta_e - \theta_d\right)\mu_0\right)\right] \\ y_1\left[\rho(\lambda_0 + \theta_e\mu_0)\left(\lambda_0 + \left(\theta_e - \theta_d\right)\mu_0\right) + y_2\right] \end{pmatrix}$$

As $\det A < 0$, we deduce:

$$\frac{\partial T_d}{\partial \overline{Z}} < 0, \quad \frac{\partial T_b}{\partial \overline{Z}} > 0, \quad \frac{\partial X_e}{\partial \overline{Z}} \text{ ambiguous}, \quad \frac{\partial \lambda_0}{\partial \overline{Z}} \text{ ambiguous}, \quad \frac{\partial \mu_0}{\partial \overline{Z}} < 0$$

$\frac{\partial X_e}{\partial \overline{Z}}$ and $\frac{\partial \lambda_0}{\partial \overline{Z}}$ have the same sign as $x_e(0)\theta_e\left(\lambda_0 + \left(\theta_e - \theta_d\right)\mu_0\right) - x_d(T_d)\theta_d\lambda_0$. It is negative when $\theta_e = 0$, and positive when $\theta_e = \theta_d$.

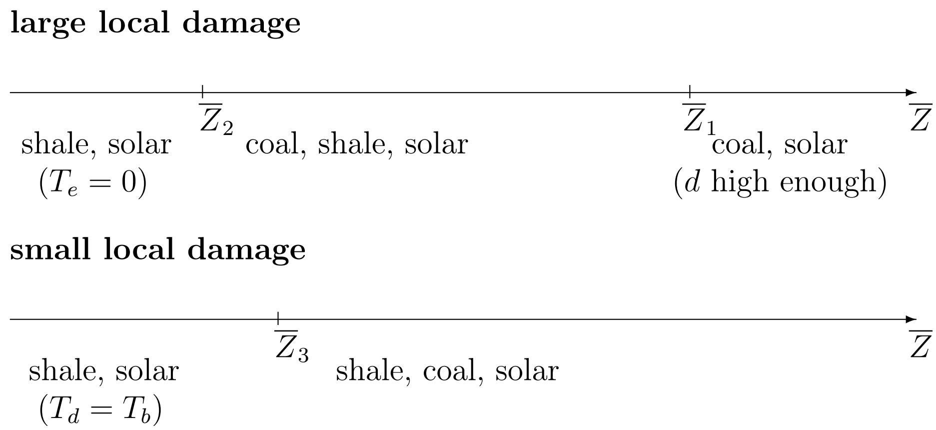

Figure 1: Optimal succession of energy sources as a function of the stringency of climate policy

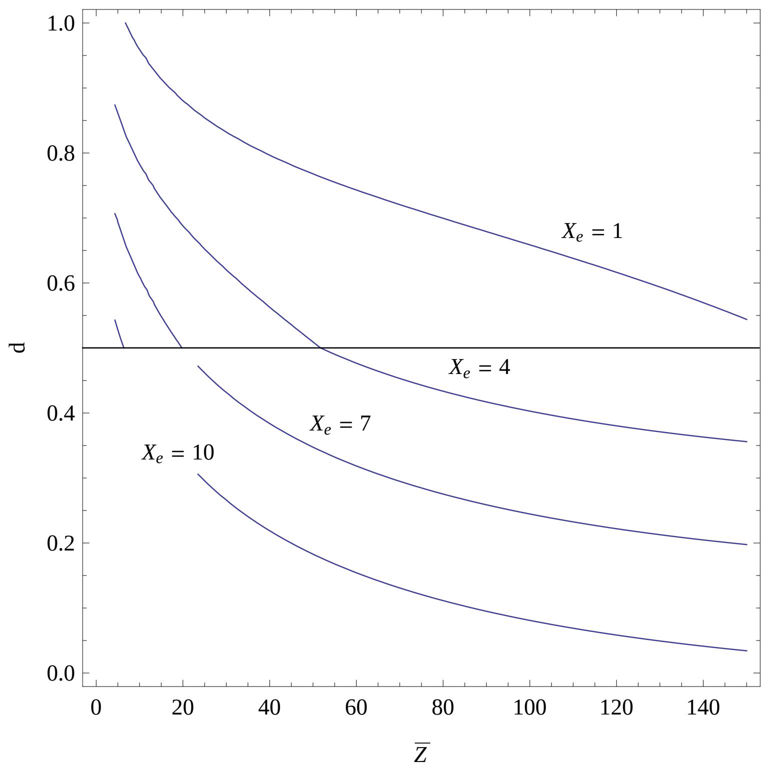

Figure 2: Iso-X_e lines

(a) $\theta_e = 0.3$

(b) $\theta_e = 0$

Figure 3: Quantity of shale extracted as a function of the value of the ceiling in the reference case (solid line) and the constrained case (dotted line) when the marginal local damage is large

(a) $\theta_e = 0.3$

(b) $\theta_e = 0$

Figure 4: Dates T_e (blue) and T_b (green) in the reference situation (solid line) and in the constrained case (dotted line) when the marginal local damage is large

(a) $\theta_e = 0.3$ (b) $\theta_e = 0$

Figure 5: Quantity of shale extracted as a function of the value of the ceiling in the reference case (solid line) and the constrained case (dotted line) when the marginal local damage is small

(a) $\theta_e = 0.3$ (b) $\theta_e = 0$

Figure 6: Dates T_d (black) and T_b (green) in the reference situation (solid line) and in the constrained case (dotted line) when the marginal local damage is small