Populism, Partisanship, and the Funding of Political Campaigns*

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Abstract

We define populism as a politician’s effort to appeal to a large group of voters with limited information regarding a policy-relevant state of nature. The populist motive makes it impossible for political candidates in an election to communicate their information to voters credibly. We show that the presence of special interest groups (SIGs) with partisan preferences can mitigate this effect and thereby improve policy. This does not happen because SIGs are better informed than policy makers. Instead, campaign contributions by SIGs allow politicians to insulate themselves from the need to adopt populist platforms. We show that a regime in which SIGs are allowed to contribute to political campaigns welfare-dominates (ex ante) regimes in which no such contributions are allowed, or where campaigns are publicly financed, or where they are funded by the candidates’ private wealth.

Keywords: Campaign finance, political advertising, privately informed candidates, pandering, populism, special interest politics.

JEL codes: D72, D82.

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1 Introduction

Political competition does not always yield desirable outcomes. A well-known failure of the democratic process arises when political candidates possess private policy-relevant information, such as expert information received from policy advisers. In this case, candidates may have an incentive to hide this information in their election campaigns. In particular, candidates interested in maximizing the probability of their election may choose to “pander” to the electorate and campaign on platforms that are popular, but not optimal given the candidate’s information, resulting in inefficient policies implemented in equilibrium.

In this paper, we study the relationship between politicians’ incentives to pander and campaign finance. Our framework is based on a canonical model of pandering by office-motivate candidates: The optimal policy from the perspective of the voters depends on an unknown state of nature. Political candidates receive private signals regarding the state of nature and set policy platforms to maximize their chance of election. The parameters of the model are such that it would be socially optimal for the two candidates to make their platforms responsive to their private information. For example, a candidate who receives information that a reduction in public spending is socially optimal should campaign on a platform of austerity. However, this does not happen in equilibrium, due to the incentive to pander: By campaigning on a “populist” platform that maximizes the voters’ ex ante expected utility (in our example, high public spending), a politician suppresses information that could indicate a different optimal policy. Voters therefore cannot learn from the politician’s platform, which in turn makes the populist’s policy attractive to voters.

We then introduce political advertising to this framework. To do so, we assume that a part of the electorate is “impressionable” and votes for a given candidate if exposed to sufficiently many ads backing this candidate or denigrating his opponent. Political ads are costly but do not convey any information regarding the candidates’ signals. Thus, voters learn no policy-relevant information from either the content or the number of political ads. However, under certain circumstances the presence of an uninformative advertising channel affects the candidates’ incentives when setting their campaign platforms, and thereby alters the informational properties of the election indirectly. This effect can improve the final welfare of voters considerably. We show that the crucial feature of advertising in our model is not what political ads say or do not say, but who pays for them.

1See, for example, Schultz (1995, 1996); Heidhues and Lagerlöf (2003); Martinelli (2001); Loertscher (2010); Felgenhauer (2010). A detailed literature review is provided in Section 2.
We consider three funding sources: Special interest groups, the candidates themselves, and the state. Special interest groups represent an arbitrarily small portion of the electorate, have state-independent “partisan” preferences, and possess no private information. Perhaps unexpectedly, the welfare of the rational (i.e., non-impressionable) voters can still increase if these groups are allowed to advertise in the election. The reason is that a candidate who campaigns on his private information instead of being populist may become less attractive to the uninformed voters, but also more attractive to one of the special interest groups. If the candidate can use donations from this group to increase his vote share through advertising—or, equivalently here, if the group itself advertises for the candidate—he can insulate himself from the need to adopt populist policies. As a consequence, electoral campaigns become more informative and voter welfare improves. Interestingly, a necessary condition for this to happen is that an asymmetry exists among the interest groups: Groups favoring policies not preferred by a majority of voters ex ante must have a sufficiently strong financial advantage over groups favoring more popular policies.

We also investigate whether a public funding system, as well as advertising funded by the candidates themselves, can have similar effects. Within the model we examine, the answer is negative. Consider, for example, a European-style system of public funding of elections in which candidates are compensated in proportion to their electoral success. Being populist now not only appeals to many voters, but also brings in the most funds. In fact, the monetary incentives a candidate faces in such elections are exactly the opposite of those provided by partisan special interests. Similarly, a candidate who spends his private wealth to advertise may win an election even with a non-populist platform, but will recognize that being populist is a less expensive way to win. It is the combination of the facts that special interest groups have extreme policy preferences, do not set their own campaigns, but can use their financial resources to support the campaigns of the politicians, that counteracts the populist motive and increases welfare.

The rest of the paper is organized as follows. In Section 2 we review the theoretical literature related to this paper. In Section 3 we specify all aspects of our model, with the exception of the supply of advertising funds. In Section 4 we characterize the policies that arise when advertising is not possible and show that equilibrium policies must entail a welfare loss due to the candidates’ incentive to pander. In Section 5 we introduce campaign funding by special interest groups and develop a necessary and sufficient condition under which special interest advertising improves voter welfare over the case of no advertising. We also show that as the asymmetry between groups grows, policies approach the first-best. Section 6 extends the analysis to public funding and funding by the candidates themselves. Section 7 concludes. Most proofs are in the Appendix.
2 Relation to the Literature

The main model and results of this paper are related to two strands of literature: The literature on elections with privately informed candidates, and the literatures on special interest politics. Each will be reviewed below. Research related to state-funded and candidate-funded campaigns will be reviewed in Section 6 later in the paper.

2.1 Elections with privately informed candidates

The idea that candidates may be better informed than voters in elections originated with Downs (1957). It has since motivated many contributions that examine the interplay of ideology, uncertainty, and information in elections (see Piketty (1999) for an overview).

Generally, truthful revelation of private information should not be expected when candidates are better informed than voters. For elections with office-motivated candidates (the case considered here), this is first demonstrated in Heidhues and Lagerlöf (2003), who show that on equilibrium of a two candidate election both candidates propose policies that are optimal given the uninformed prior. Our baseline model is largely based on their framework. Loertscher (2010) extends this analysis to a continuum of states and policies. Felgenhauer (2010) shows that introducing an uninformed third competitor changes the populism result and induces the informed candidates to set platforms according to their private information. Jensen (2010) introduces state-dependent candidate quality and shows that candidates who receive information that they are weaker than their opponent have an incentive to set contrary platforms. Laslier and Van der Straeten (2004) introduce informed voters. The results are now reversed, and in the unique equilibrium both candidates set platforms that maximize the expected utility of the voters. In our model, we assume that a fraction of the electorate is informed; however, a larger fraction is uninformed. In this case, politicians still pander to the uninformed by choosing populist policies (in the benchmark model without advertising).

For the case of privately informed policy-motivated candidates, similar information aggregation failures arise. This is first demonstrated in Schultz (1995, 1996) who derives a pooling equilibrium that does not reveal the candidates’ information. Martinelli (2001) shows that these results are weakened if voters receive some private information themselves. Martinelli and Matsui (2002) show that policy reversals may occur as a result of the candidates’ incentive to manipulate voters’ beliefs (e.g., the left-wing party implements policies to the right of those implemented by the right-wing party). Canes-Wrone, Herron, and Shotts (2001) and Schultz (2002) introduce reelection concerns, in which case the following tradeoff arises: Choosing an inferior policy before the election increases the policy maker’s chance of remaining in office, and choosing a better policy after the election. However, a longer term length lessens this distortion (Schultz 2008).
2.2 Special interest politics

A large literature examines the influence of special interest groups in democracies, and a good introduction to this literature is in Grossman and Helpman (2001). This paper concerns, specifically, the informational role of special interests on political competition.

Austen-Smith (1987) develops an early model in which interest groups invest in political campaigns after policies are set, and contributions are used to better inform voters of the candidates’ platforms. As in our model, outside contributions affect the politicians’ platforms. Unlike our result, however, the resulting distortion reduces the welfare of voters. In a model that contains impressionable voters, Baron (1994) shows that campaign contributions by special interest groups can create platform divergence if the benefits of a policy can be targeted to a particular interest group without affecting the other. Prat (2002) views advertising by special interest groups as a (credible) signal of the group’s private information regarding valence characteristics of candidates in an election. The group’s ability to signal to voters can be used to extract policy concessions from the candidates. A cap on advertising reduces its value as a signal but increases the degree to which policies are aligned with the voters’ preferences. Coate (2004) develops a model where partisan interest groups have a moderating effect on policy. The reason is that, in equilibrium, groups give to moderate candidates of the opposing part of the political spectrum, who can use these funds to advertise their position to voters. Capping contributions encourages the entry of partisan candidates, resulting in more partisan policies.

If special interest groups try to influence a policy maker who is already in office, we speak of lobbying or post-election influence. While our paper is not concerned with this case, the lobbying literature has identified a number of alternative cases in which special interest influence increases social welfare. Consider an interest group with private information concerning a policy-relevant state variable. Unless an interest group’s preferences are perfectly aligned with the policy maker’s, only coarse information can be revealed in the equilibrium of a cheap-talk communication game between the group and the policy maker (Crawford and Sobel 1982). In this case, allowing for monetary transfers between the interest group and the policy maker can overcome some of these credibility constraints. Potters and van Winden (1992) take a first step in this direction: In their model, the interest group’s choice of whether or not to send a costly message can be a discriminating signal that reveals the group’s information. Austen-Smith (1995) and Lohmann (1995) extend the signaling story by viewing campaign contributions as buying access to policy makers. In this case, whether a group wants to buy access can serve as a credible signal of its information. Ball (1995) shows that when monetary transfers from the sender to the receiver are allowed in the Crawford-Sobel model the interest group
is generally able to reveal all of its information credibly. Lohmann (1998) presents a model in which the interest group’s expert knowledge allows it to monitor the quality of a politician’s decision better than a voter would be able to. A politician who accepts money in exchange for favorable policies thus puts himself under enhanced scrutiny, and while political decisions are now biased they are also of higher quality.

Like some of the papers reviewed above, ours makes an argument that money spent by special interest groups can improve policy outcomes by changing information-related aspects of the policy making process. However, this works through a different—and, to our knowledge, novel—mechanism: A special interest group’s role is not to advise or monitor a policy maker, or to provide information to voters, but merely to counterbalance an informational problem in elections, namely the problem of populism.

3 The Model

Our model of political competition with privately informed candidates is based on the framework developed in Heidhuess and Lagerlöf (2003) and Laslier and Van der Straeten (2004). We add to this framework a channel through which costly (but uninformative) political advertising can influence election outcomes.

The timing is as follows. At the beginning of the game, nature chooses a state variable that determines the policy preferences of voters. Next, two political candidates and some voters receive partially informative signals about the state of nature. The candidates then set their campaign platforms, which the voters observe. After that, political advertising—funded by candidates, interest groups, or the state—takes place. Finally, an election is held and the winning candidate’s platform is implemented. In the following, we describes each of these elements, except for the funding of advertising.

3.1 Political environment

Society must choose a policy \( x \in X \equiv \{L, H\} \) (e.g., a low or high level of public spending, or a low or high degree of regulation of an industry). The effect of policy \( x \) depends on a state variable \( \theta \in \Theta \equiv \{l, h\} \), which is drawn by Nature according to

\[
Pr[\theta = h] = p > \frac{1}{2}
\]

There are two candidates for office, denoted 1 and 2. The candidates compete in the election by choosing policy platforms \( x_1 \in X \) and \( x_2 \in X \). Platform choices are made simultaneously and, once chosen, a candidate becomes committed to his platform. Candidates are purely office-motivated and maximize the probability of being elected. A
candidate wins if his vote share exceeds 1/2. If both candidates receive a vote share of exactly 1/2 then each wins with equal probability.

The electorate consists of a large number of voters, divided into three groups: Uninformed voters, who comprise a fraction $\gamma_U$ of the electorate; informed voters, who comprise a fraction $\gamma_I$; and impressionable voters, who make up the remaining fraction $\gamma_M = 1 - \gamma_U - \gamma_I$. We assume that none of these voter groups holds a majority, and there are more uninformed voters than informed voters:

**Assumption 1.** $\gamma_U, \gamma_I, \gamma_M < \frac{1}{2}$ and $\gamma_U > \gamma_I$.

The informed and uninformed voters as well as the candidates know the ex ante probabilities of the two possible states, $p$ and $1 - p$. After the state of the world is drawn, but before candidates and voters make their decisions, the candidates and the informed voters receive additional private signals. These signals are denoted $s_1$, $s_2$, and $s_I$, respectively, and can take on values in $\Theta$. We assume that for $i \in \{1, 2, I\}$, $s_i$ is drawn according to

$$Pr[s_i | \theta] = \begin{cases} 1 - \varepsilon & \text{if } s_i = \theta, \\ \varepsilon & \text{otherwise,} \end{cases}$$

where $0 < \varepsilon < 1 - p$. That is, the candidates’ and informed voters’ private signals inform these agents imperfectly about the state $\theta$ (however, signals are precise enough for the probability of state $l$, conditional on signal $l$, to exceed 1/2). All three signals are independent conditional on $\theta$, and the signal $s_I$ is common to all informed voters. The uninformed and impressionable voters do not receive any signals.

Uninformed and informed voters have state-dependent preferences. They receive a payoff that is high if the policy matches the state, and low otherwise:

$$u(x, \theta) = \begin{cases} 1 & \text{if } (x, \theta) = (H, h), (L, l), \\ 0 & \text{if } (x, \theta) = (H, l), (L, h) \end{cases}$$

These voters are sincere, in that they vote for the candidate whose platform offers the larger expected utility, computed using the information the voter possesses at the time of the election.\(^2\)

Impressionable voters do not maximize a utility function. Their voting behavior depends directly on the amount of political advertising for the candidates. Specifically, we assume that the fraction of impressionable voters voting for candidate 1 is

$$z(a_1, a_2) = \frac{1}{2} + a_1 - a_2,$$

\(^2\)Note that, up to this point, if $\gamma_U = 1$ then our model would be that of Heidhues and Lagerlöf (2003), and if $\gamma_I = 1$ then it would be that of Laslier and Van der Straeten (2004).
where and \(a_1 \geq 0\) and \(a_2 \geq 0\) represent the amount of advertising for candidate 1 and candidate 2, respectively.\(^3\)

Political advertising is assumed to be uninformative about a politician’s signal and may come from several sources: It may be funded privately by the candidates, through a public system, or by special interest groups. We will introduce all three possibilities later in the paper. Until then, we assume \(a_1 = a_2 = 0\).

### 3.2 Strategies and beliefs

A *campaign strategy* for candidate \(i = 1, 2\) is a mapping

\[
\chi_i : \Theta \rightarrow [0, 1]
\]

from \(i\)'s information set to probability distributions over policies (i.e., \(\chi_i(s_i)\) is the probability with platform \(H\) is chosen by candidate \(i\) given the candidate’s private signal \(s_i \in \{l, h\}\)). If \(\chi_i(s_i) \in \{0, 1\}\), we may simply write \(\chi_i(s_i) = L\) or \(\chi_i(s_i) = H\). The strategy \(\chi_i(h) = H\) and \(\chi_i(l) = L\) is called *truthful*. On the other hand, a strategy such that \(\chi_i(l) = \chi_i(h)\) is called *uninformative*.

Voting strategies for the uninformed and informed voters are mappings

\[
\nu_U : X^2 \rightarrow [0, 1],
\]

\[
\nu_I : X^2 \times \Theta \rightarrow [0, 1]
\]

from the voters’ information sets to probability distributions over candidates (i.e., \(\nu_U(x_1, x_2)\) is the probability with which an uninformed voter votes for candidate 1 if the campaign platforms are \(x_1\) and \(x_2\), and \(\nu_I(x_1, x_2, s_I)\) is the probability with which an informed voter votes for candidate 1 if the campaign platforms are \(x_1\) and \(x_2\) and the voters’ signal is \(s_I\)).\(^4\)

Beliefs are mappings from the agents’ information sets to probability distributions over states:

\[
\mu_i : \{l, h\} \rightarrow [0, 1] \quad (i = 1, 2),
\]

\[
\mu_U : X^2 \rightarrow [0, 1],
\]

\[
\mu_I : X^2 \times \{l, h\} \rightarrow [0, 1].
\]

\(^3\)Baron (1994) is the first paper to introduce impressionable voters in order to examine issues related to campaign advertising by politicians. There, these voters are called “uninformed voters.”

\(^4\)Note that we require that all uninformed voters play the same strategy \(\nu_U\), and all informed voters play the same strategy \(\nu_I\). This is without loss of generality: Any voting strategy that is asymmetric within a voter group can be recast as an appropriately chosen strategy that is symmetric within the group.
For example, \( \mu_I(x_1, x_2, s_I) \) is an informed voter’s belief that the state is \( \theta = h \) if the two platforms are \( x_1 \) and \( x_2 \) and the voters’ private signal is \( s_I \). Beliefs for candidates and uninformed voters are defined similarly. As usual, we assume that beliefs are Bayesian at all information sets that are reached with positive probability. Given beliefs \( \mu_U \) and \( \mu_I \), voting strategies \( \nu_U \) and \( \nu_I \) are sincere if they place positive weight on a candidate’s platform only if it offers a weakly larger expected utility as the opposing candidate’s platform. Note that voters prefer platform \( H \) over \( L \) if they believe state \( h \) to be more likely than state \( l \), and vice versa.

Our notion of equilibrium postulates that candidates maximize their chance of winning, voters vote sincerely, and beliefs are Bayesian:

**Definition 1.** A sincere Bayesian equilibrium in the game without advertising is a profile of strategies \( (\chi_1, \chi_2, \nu_U, \nu_I) \) and a profile of beliefs \( (\mu_1, \mu_2, \mu_U, \mu_I) \) such that the following conditions are satisfied:

(i) Campaign strategy \( \chi_i \) \((i = 1, 2)\) maximizes candidate \( i \)'s probability of winning, given \( \mu_i, \nu_U, \nu_I, \) and \( \chi_{-i} \).

(ii) The voting strategies \( \nu_U \) and \( \nu_I \) are sincere, given \( \mu_U \) and \( \mu_I \).

(iii) Beliefs \( \mu_1, \mu_2, \mu_U, \mu_I \) are derived from the strategies chosen by the players, as well as nature, through Bayes’ rule whenever possible.

Note that condition (iii) poses no restrictions on beliefs at unreached information sets. While our model always has equilibria in which all information sets are reached, it also has equilibria where this is not the case. When this happens, we will discuss the reasonableness of out-of-equilibrium beliefs as we go along.

### 3.3 First-best policy

The policy that maximizes the expected welfare of the voters, conditional on \((s_1, s_2, s_I)\), is called the full information, or first-best, policy and denoted \( x^{FI}(s_1, s_2, s_I) \). Note that the likelihood that the state is \( h \), conditional on \((s_1, s_2, s_I)\), is

\[
\mu(k) \equiv Pr[\theta = h|s_1, s_2, s_I] = \frac{p(1-\varepsilon)^k\varepsilon^{3-k}}{p(1-\varepsilon)^k\varepsilon^{3-k} + (1-p)\varepsilon^k(1-\varepsilon)^{3-k}},
\]

where \( k = \#\{s \in (s_1, s_2, s_I) : s = h\} \). The expected utility of an uninformed or informed voter from policy \( x \) is then either \( \mu(k) \) (for \( x = H \)) or \( 1 - \mu(k) \) (for \( x = L \)). Since \( \varepsilon < 1 - p \), \( \mu(k) > 1/2 \) if and only if \( k > 2 \). Thus, the full-information policy is set according to the

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\(^5\)When considering candidate \( i \in \{1, 2\} \) we adopt the usual convention of calling \( i \)'s opponent \( -i \).
majority of the three signals:

\[ x^{FI}(s_1, s_2, s_I) = \begin{cases} 
H & \text{if } \# \{ s \in (s_1, s_2, s_I) : s = h \} \geq 2, \\
L & \text{otherwise.} 
\end{cases} \]  

(2)

Of course, no single agent in our model knows all three signals: Information can flow from candidates to voters only via the candidates’ choice of campaign platforms, and from voters to candidates only through their voting behavior in the election, at which point candidates are already committed to their platforms. These communication constraints do not affect the implementability of the full information policy. To see this, suppose there is no campaign advertising and consider the following profile of strategies:

\[ \chi_i(s_i) = s_i \quad \forall i, \]  

(3)

\[ \nu_U(x_1, x_2) = \frac{1}{2} \quad \forall (x_1, x_2), \]  

(4)

\[ \nu_I(s_I, x_1, x_2) = \begin{cases} 
1 & \text{if } x_1 = s_I \neq x_2, \\
0 & \text{if } x_1 \neq s_I = x_2, \\
\frac{1}{2} & \text{otherwise.} 
\end{cases} \]  

(5)

In this profile, the candidates campaign truthfully and the uninformed voters split their vote across the two candidates equally. If the candidates offer different platforms, the informed voters vote for the candidate whose platform agrees with the informed voters’ signal. Since the impressionable voters also split their vote equally, the candidate who attracts the informed voters wins. Thus, the policy implemented under this profile always agrees with at least two signals.

Notice that voting strategy used by the uninformed voters, (4), is not sincere: If candidates use the truthful strategies given in (3), and each platform is offered by exactly one candidate, the uninformed voters’ Bayesian belief that \( \theta = h \) must be

\[ \mu_U(H, L) = \mu_U(L, H) = \frac{p(1-\varepsilon)\varepsilon}{p(1-\varepsilon)\varepsilon + (1-p)\varepsilon(1-\varepsilon)} = p > \frac{1}{2}. \]  

(6)

In this case, every uninformed voters strictly prefers \( H \) over \( L \), and thus must vote for the candidate whose platform is \( H \) with probability one.

Our next result shows that truthful candidate strategies are necessary (but not sufficient) for welfare maximization if voters are sincere:

**Lemma 1.** If the uninformed and informed voters are sincere, then for every strategy profile in which the candidate strategies are not truthful, there exists a number \( \omega > 0 \) such that the full information policy is implemented with probability less than \( 1 - \omega \). This is true regardless of the level or source of campaign advertising.
Furthermore, if \( a_1 = a_2 = 0 \), the following statement holds: If the candidates are truthful, then for every strategy profile in which voters are sincere, there exists a number \( \omega' > 0 \) such that the full information policy is implemented with probability less than \( 1 - \omega' \).

Lemma 1 identifies two possible sources of inefficient policies: Sincere voting and non-truthful campaign platforms. Sincerity, of course, is an assumption we make on voters’ behavior, which we will discuss in more detail in Section 4.1 below. On the other hand, campaign strategies are determined strategically by the candidates in order to maximize their chances of electoral success. Candidates will choose truthful strategies if and only if doing so is optimal for them, and the optimality of any particular campaign strategy in turn depends on what is assumed about voter behavior as well as campaign advertising. The rest of the paper examines this relationship between voting, advertising, and political campaigns.

4 Equilibrium Without Advertising

In this section, we characterize the sincere Bayesian equilibria of our model under the assumption that there is no advertising. In this case, half of the impressionable voters vote for candidate 1 and half vote for candidate 2. As there are more uninformed than informed voters, a politician is thus guaranteed to win if he attracts all uninformed voters.

4.1 The problem of populism

In principle, elections can aggregate the information held by politicians and voters into policies that are optimal conditional on the entirety of this information. As shown in Lemma 1, this requires truthful campaigns and, in the absence of advertising, non-sincere voting. Because we assume that voters are sincere, it is clear that voter welfare cannot be maximized in equilibrium without advertising.

We now examine if the second requirement for welfare maximization—truthful campaigns—can be satisfied. The next result shows that the answer is negative.

Proposition 2. (No truthful campaigns) In the game without advertising, there does not exist a sincere Bayesian equilibrium in which both candidates play truthful strategies.

The intuition for Proposition 2 is most easily seen in the case where signals are very precise (\( \varepsilon \) is very small). Assume that candidate 1 obtains private signal \( s_1 = l \). He must believe that, with a high likelihood, the state of nature is \( l \), and hence that candidate 2 also has private signal \( s_2 = l \). Assuming truthful candidate strategies, the platforms offered are then likely \( x_1 = x_2 = L \). Suppose candidate wins with probability 1/2 in this
case. If candidate 1 deviated to $H$ instead, the policy platforms would be $x_1 = H$ and $x_2 = L$, and the voters would infer that $s_1 = h$ and $s_2 = l$. In this event, the uninformed voters would believe that the state is $h$ with probability $p > 1/2$ (see (6)) and thus vote for candidate 1, who wins with probability 1.

We call the effect that prevents truthful campaigns populism, or pandering. It is a political failure that arises in all equilibria of our model without advertising. A politician who sets platform $H$, even when his private signal indicates otherwise, affects the uninformed voters in two ways: First, he manipulates information about his signal; second, he makes himself more attractive to the uninformed voters given their manipulated beliefs about the state. These two effects are closely linked: Policy $H$ would not be an attractive policy if the uninformed were sufficiently certain that the state of the world was $l$. But it is precisely the fact that the candidate offers $H$ that prevents the uninformed from learning too much about the state.

We emphasize that, for populist deviations from a truthful strategy to be profitable, the uninformed voters must be sincere. If they abstained from voting, or if each uninformed voter flipped a coin and voted for either candidate with probability 1/2 (as in strategy (5)), the candidates’ desire to appeal to the uninformed voters would be eliminated. Instead, they would want to attract the informed vote by choosing platforms that match their private signals. The first-best policy is then implemented with probability one, increasing also the uninformed voters’ welfare. The assumption of sincere voting is therefore necessary for the problem of populism to arise in our model.

This raises an obvious question: Why should voters be able to process information in a Bayesian way and at the same time fail to realize that, by not abstaining, they are actually making matters worse? We have two answers to this question. For one, it would not help if a single uninformed voter deviated from a sincere voting strategy and abstained instead. To change the election outcome, it is necessary that sufficiently many uninformed voters engage in a coordinated abstention. Therefore, sincere voting cannot be regarded as suboptimal behavior for any single voter, although it is clearly suboptimal in the aggregate. Second, voters may also cast sincere ballots as a way of expressing a point of view. It is hard to imagine that such voters would consider not voting because they are less well informed than others. The sincerity condition in our equilibrium definition can therefore be thought of as describing the behavior of “expressive voters,” who fail to overcome the swing voter’s curse (Feddersen and Pesendorfer 1996).

### 4.2 Equilibrium characterization

Proposition 2 implies that any sincere Bayesian equilibrium of the model without advertising generates the “wrong” policies sometimes, and hence induces a welfare loss. We now characterize the equilibria of the model and measure how large the welfare loss is.
First, the strategic incentive to pander suggests that candidates might simply choose to offer policy $H$, regardless of their signals. Because voters learn nothing from the campaign platforms, the a priori optimal policy $H$ is still optimal for the uninformed voters. These are indeed equilibrium strategies, and the resulting equilibrium can be called a “populist equilibrium.” There are, however, a number of other equilibria, as the following result shows:

**Proposition 3.** *(Equilibria without advertising)* In the game without advertising, the following are sincere Bayesian equilibrium strategies:

(i) Populist pooling equilibrium: Both candidates choose platform $H$ regardless of their signals.

(ii) Contrarian pooling equilibrium: Both candidates choose platform $L$ regardless of their signals.

(iii) Semi-separating equilibrium: A candidate with an $h$-signal chooses platform $H$ with probability one, and a candidate with an $l$-signal chooses platform $H$ with probability

\[
\chi_1(l) = \chi_2(l) = \frac{(2p - 1)\varepsilon(1 - \varepsilon)}{(1 - p)(1 - \varepsilon)^2 - p\varepsilon^2}.
\]

If two different platforms are offered, the informed voters vote for the candidate who offers $H$ if and only if $s_I = h$. The uninformed voters vote for the candidate who offers $H$ with probability

\[
\nu_U(H, L) = 1 - \nu_U(L, H) \approx \frac{1}{2} \left( 1 + \frac{\gamma_I}{\gamma_U} \right),
\]

where the approximation applies to the case of a large (but finite) electorate.

(iv) Asymmetric equilibrium: One candidate is truthful and wins with probability one, while the other plays any uninformative strategy and never wins.

Among the equilibria characterized in Proposition 3, we consider the semi-separating equilibrium the most reasonable, for the following reasons. In both pooling equilibria there are unreached information sets, in which case we are free to impose any beliefs that support the equilibrium. In a populist equilibrium, for example, enough uninformed voters must vote for $H$ should a candidate deviate and offer $L$. For this to be sincerely optimal, the uninformed voters must believe that $\theta = h$ with probability $1/2$ or higher in the event $L$ is offered. However, this belief does not satisfy forward induction criteria such as D1 (Cho and Kreps 1987). To see why, consider the populist pooling equilibrium and suppose candidate $i$ surprisingly chose platform $x_i = L$. The set of voting strategies for which $i$ has at least the same chance of winning as in

\[6\]
provided the strategy played by the pooling candidate is fully mixed (there will be no unreached information sets in this case). Nevertheless, we regard this equilibrium as unrealistic: Proposition 3 (iii) describes uncontested elections in which one candidate is essentially not competing, while the other candidate is assured to win. If we take seriously the idea of political competition, uncontested elections do not appear realistic.

This leaves the mixed strategy equilibrium, in which all information sets are reached with positive probability, and both candidates have an equal chance of winning. Note that the probability that a candidate with an \( l \)-signal sets platform \( H \), \( \chi_i(l) \), is strictly between zero and one for all \( \varepsilon \in (0, 1-p) \). The voters therefore learn from the candidates’ campaign platforms, but only imperfectly. If the signal noise is low, this equilibrium entails relatively little welfare loss, but as \( \varepsilon \) increases the equilibrium converges to the populist equilibrium of Proposition 3 (i), in which no information is transmitted and the enacted policy is incorrect with probability \( 1-p \).

\[ \text{Figure 1: Voter welfare without campaign advertising (} p = 0.75 \text{).} \]

\[ \text{equilibrium, given } s_i \text{, is strictly larger when } s_i = l \text{ than when } s_i = h. \text{ The reason is that informed voters with an } h \text{-signal would never vote for platform } L \text{, even if they were certain that } s_i = l. \text{ On the other hand, informed voters with an } l \text{-signal will vote for } L \text{ if they deem it sufficiently probable that } s_i = l. \text{ From the perspective of the candidate, an } l \text{-signal makes it more likely that the informed voters also have an } l \text{-signal. Thus, a candidate with an } l \text{-signals wants to deviate to } L \text{ whenever a candidate with an } h \text{-signal does, but not vice versa. D1 requires that, in such a case, voters must believe that candidate 1 has an } l \text{-signal with probability one. But then } Pr[\theta = h|s_i = l] = pe/[pe + (1-p)(1-\varepsilon)] < 1/2 \text{ (because } \varepsilon < 1 - p < 1/2). \]
Figure 1 depicts numerical estimates of the ex ante expected utility of an uninformed or informed voter across the equilibria, for $p = 0.75$ and $\varepsilon \in (0, 0.25)$. (The contrarian pooling equilibrium is not shown; its welfare is 0.25.) For comparison, welfare under the first-best policy is also shown, as is welfare under a hypothetical strategy profile of truthful campaigning and sincere voting. The semi-separating equilibrium has the highest voter welfare among the equilibria, but still falls short of the theoretical maximum. Part of this welfare loss is accounted for by the politicians' incentive to hide their information (the difference between the blue line and grey dashed line), and part of it is accounted for by the fact that voters are sincere (the difference between the grey dashed line and the green dashed line).

5 Advertising Funded by Special Interests

In the previous section, we examined the equilibria of our model in the absence of campaign advertising. We will now change this assumption and introduce political advertising.

The present section focuses on political advertising financed by special interest groups and is organized as follows. In Section 5.1, we formally introduce special interest groups to the model. In Section 5.2, we examine the advertising contest between the two groups, and in Section 5.3, we look at the candidates’ campaign incentives when they anticipate the advertising contest between the groups. In Section 5.4 we combine these two analyses and derive our main result, which provides conditions for an overall equilibrium in truthful campaigns. Finally, Section 5.5 contains a discussion of the result.

5.1 Special interest groups

We think of special interest groups (SIGs) as groups of citizens that are small, have preferences different from those of most voters, and are wealthy enough to influence elections by spending resources on political campaigns. To incorporate these characteristics, we assume the presence of two single voters, called SIG $H$ and SIG $L$. SIG $H$ receives a benefit of one if the policy is $H$, and zero otherwise. Likewise, SIG $L$ receives a benefit of one if the policy is $L$, and zero otherwise. These payoffs are independent of the state $\theta$. We therefore say that the groups have partisan preferences.

Special interest groups can influence electoral outcomes by providing political advertising in one of two ways: They can make contributions to a candidate’s campaign who then uses the donated funds to advertise (the case of traditional campaign contributions), or they can advertise for a candidate directly (the case of independent political expenditures, which are permitted in the United States). Our results will be the same in either case, and we assume that SIGs advertise directly.
The timing of our model with SIG-funded advertising is as follows: As before, nature chooses the state, the candidates and informed voters observe their signals, and the candidates choose their campaign platforms. At this point, the SIGs make simultaneous advertising choices. We let \( a_j^i \geq 0 \) denote the amount of advertising by SIG \( j \in \{H, L\} \) for candidate \( i \in \{1, 2\} \). Thus, the total amount of advertising bought by SIG \( j \) is \( a_j = a_j^1 + a_j^2 \), and the total amount of advertising for candidate \( i \in \{1, 2\} \) is \( a_i = a_i^H + a_i^L \). The variables \( a_1 \) and \( a_2 \) enter the function (1), which returns the share of impressionable voters who vote for candidate 1. Finally, the election takes place and the candidate who receives a majority of votes wins.\(^7\)

Interest group \( j \in \{H, L\} \) maximizes its expected payoff, that is, the probability of obtaining its preferred policy \( j \) minus its cost of advertising. We assume that the total cost of advertising by SIG \( j \) is \( \beta_j a_j \), with \( \beta_j > 0 \). Differences in the cost coefficients \( \beta_j \) reflect the possibility that one interest group may be less well funded, or less well organized, than the other. Alternatively, one group may be less efficient in producing campaign ads, or may be utilizing a less effective advertising channel.

Our equilibrium notion is now readily extended to include the activities by the interest groups. Note that SIG \( j \)'s strategy is a mapping

\[
(\alpha_{1}^{j}, \alpha_{2}^{j}) : X \times X \rightarrow [0, \infty) \times [0, \infty),
\]

where \( \alpha_{i}^{j}(x_1, x_2) \) denotes the advertising bought by SIG \( j \) for candidate \( i \) after observing campaign platforms \( x_1 \) and \( x_2 \). Note also that each SIG is has the same information as an uninformed voter, and its belief about the state \( \theta \) after observing platforms \( (x_1, x_2) \) is \( \mu_U(x_1, x_2) \).

Thus, in the extended model with special interest advertising, a sincere Bayesian equilibrium is a strategy profile \((\chi_1, \chi_2, \nu_U, \nu_I, \alpha^H, \alpha^L)\) and a belief profile \((\mu_1, \mu_2, \mu_U, \mu_I)\) that satisfy the previous conditions in Definition 1, as well as the new condition that \( \alpha^H \) and \( \alpha^L \) maximize the expected payoffs of SIG \( H \) and SIG \( L \), respectively.

### 5.2 The SIGs’ problem

In this section, we examine the advertising contest between the SIGs. To begin, we simplify the SIGs’ strategies as follows. If \( x_1 = x_2 \), the final policy does not depend on advertising, so we set \( \alpha_{1}^j(H, H) = \alpha_{2}^j(L, L) = 0 \) for \( i \in \{1, 2\} \) and \( j \in \{H, L\} \). On the other hand, if

---

\(^7\)Note that the SIGs do not spend money in order to influence the policy platforms of the candidates. Instead, they spend in order to help a candidate win the election once the policy platforms are chosen. Grossman and Helpman (2001) call the former motive the “influence motive” and the latter the “electoral motive.” In a model with the influence motive, SIGs commit to schedules specifying an amount of spending for each policy, to which the politicians react. On the other hand, in a model with the electoral motive (as is this), politicians commit to policies to which the SIG’s react. The electoral motive first appears in Austen-Smith (1987).
x_1 \neq x_2 the SIGs can influence the final policy outcome through their advertising choices. Because SIG H (L) cannot benefit from advertising for a candidate whose platform is L (H), we also have \( \alpha_1^H(L, H) = \alpha_2^H(H, L) = \alpha_1^L(H, L) = \alpha_2^L(L, H) = 0. \)

Thus, the only components of SIG j’s strategy that are possibly non-zero are \( \alpha_1^j(H, L) \) and \( \alpha_2^j(L, H) \), and because the continuation game at the platform pair \((H, L)\) is symmetric to the game at \((L, H)\) we may assume that \[ \alpha_1^j(H, L) = \alpha_2^j(L, H) \equiv a^j, \quad j \in \{H, L\}. \]

A single pair of numbers \((a^H, a^L)\) is hence sufficient to describe the SIGs’ behavior in our model.

Now if one candidate sets platform H and the other sets L, then the fraction of impressionable voters who vote for the candidate with platform H is \[ z(a^H, a^L) = \frac{1}{2} + a^H - a^L. \]

The advertising contest between the special interest groups is hence a handicapped version of the well-known all-pay auction (Hillman and Riley 1989; Baye, Kovenock, and de Vries 1996). The handicap comes into play because, if both groups advertise the same amount, the impressionable voters split their vote equally as is the case when there is no advertising. Assuming that politicians campaign truthfully—as they will in the final equilibrium—this implies that group H wins and obtains its preferred policy. Thus, in order to have a chance at winning, group L must advertise more than group H.

Formally, we have the following game between the SIGs: The groups simultaneously choose efforts \( a^L \geq 0 \) and \( a^H \geq 0 \) and pay costs \( \beta^L a^L \) and \( \beta^H a^H \), respectively. Group L wins a prize worth \( \pi > 0 \) if \( a^L - a^H > k \), and zero otherwise, where \( k > 0 \). Similarly, group H wins \( \pi \) if \( a^L - a^H \leq k \), and zero otherwise. The precise values for \( k \) and \( \pi \) will be derived from the parameters of the political environment later. We call this game the all-pay auction with a k-handicap on group L. The following result describes the Nash equilibrium of this game.

**Lemma 4.** The all-pay auction with a k-handicap on group L has a Nash equilibrium (possibly in mixed strategies) in which the groups allocate their advertising efforts as follows:

(i) If \( 1/\beta^L \geq k/\pi + 1/\beta^H \), then group L randomizes \( a^L \) uniformly on the interval \([k, k + \pi/\beta^H]\). Group H plays \( a^H = 0 \) with probability \( 1 - \beta^L/\beta^H \) and, with the remaining probability, randomizes \( a^H \) uniformly on the interval \([0, \pi/\beta^H]\). Group L wins with probability \( \Pr[a^L \geq a^H + k] = 1 - \frac{1}{2} \frac{\beta^L}{\beta^H} \).
(ii) If \( \frac{k}{\pi} < \frac{1}{\beta_L} < \frac{k}{\pi} + \frac{1}{\beta_H} \), then group \( L \) plays \( a^L = 0 \) with probability \( 1 - \frac{\beta_H}{\beta_L} + k \frac{\beta_H}{\pi} \) and, with the remaining probability, randomizes \( a^L \) uniformly on the interval \( [k, \frac{\pi}{\beta_L}] \). Group \( H \) plays \( a^H = 0 \) with probability \( k \frac{\beta_L}{\pi} \) and, with the remaining probability, randomizes \( a^H \) uniformly on the interval \( [0, \frac{\pi}{\beta_L} - k] \). Group \( L \) wins with probability

\[
\Pr[a^L \geq a^H + k] = \frac{1}{2} \frac{\beta_H}{\beta_L} \left[ 1 - \left( \frac{k \frac{\beta_L}{\pi}}{\beta_H} \right)^2 \right].
\]

(iii) If \( \frac{1}{\beta_L} \leq \frac{k}{\pi} \), then \( a^L = a^H = 0 \) and group \( L \) wins with probability zero.

Figure 2 plots the cumulative distribution functions for the groups’ advertising efforts, denoted \( F^H(a^H) \) and \( F^L(a^L) \), for cases (i) and (ii) in Lemma 4.

5.3 The candidates’ problem

Let us now turn to the candidates’ platform choices. These are made in anticipation of the advertising contest between the two interest groups, and we will derive conditions for truthful campaign strategies to be optimal.

Suppose that candidates are truthful and set policy platforms to match their private signals. If both set the same platform, we will assume that each wins with probability
Now consider the case where one candidate offers platform $H$ and the other platform $L$. In this case, the uninformed voters Bayesian belief is $\mu_U(H, L) = \mu_U(L, H) = p$, as shown in (6). The informed voters’ belief is

$$
\mu_I(H, L, h) = \mu_I(L, H, h) = \frac{p(1 - \varepsilon)}{p(1 - \varepsilon) + (1 - p)\varepsilon} > \frac{1}{2},
$$

$$
\mu_I(H, L, l) = \mu_I(L, H, l) = \frac{p\varepsilon}{p\varepsilon + (1 - p)(1 - \varepsilon)} < \frac{1}{2},
$$

where the last inequality is because $\varepsilon < 1 - p$. The uninformed voters will hence vote for the candidate whose platform is $H$, while the informed voters will vote for the candidate whose platform agrees with the informed voters’ signal.

Thus, if $(x_1, x_2) = (H, L)$ or $(x_1, x_2) = (L, H)$, platform $H$ wins with probability one if the informed voters receive signal $s_I = h$. If the informed voters receive signal is $s_I = l$, then platform $L$ may win, provided that the $L$-candidate’s vote share among the impressionable voters is large enough. That is, if

$$
\gamma_I + z(a^L, a^H)\gamma_M \geq \frac{1}{2} \iff z(a^L, a^H) \geq \frac{1}{2} - \frac{\gamma_I}{\gamma_M}.
$$

Let $X$ be the probability that the above inequality holds.

We can now compute the win probability of candidate $i$ who receives signal $s_i$ and sets platform $x_i$ against a truthful opponent. Denote this probability by $W_i(x_i|s_i)$. For $x_i = H$, we have

$$
W_i(H|s_i) = \mu_i(s_i)\left[Pr[s_{-i} = h|\theta = h]\frac{1}{2} + Pr[s_{-i} = l|\theta = h]\left(Pr[s_I = h|\theta = h] \right.ight.
$$

$$
\left. + Pr[s_I = h|\theta = h](1 - X)\right) \right] + (1 - \mu_i(s_i))\left[Pr[s_{-i} = h|\theta = l]\frac{1}{2}
$$

$$
+ Pr[s_{-i} = l|\theta = l]\left(Pr[s_I = h|\theta = l] + Pr[s_I = h|\theta = l](1 - X)\right)\right]
$$

$$
= \mu_i(s_i)\left[\frac{1 - \varepsilon}{2} + \varepsilon\left((1 - \varepsilon) + \varepsilon(1 - X)\right)\right]
$$

$$
+ (1 - \mu_i(s_i))\left[\frac{\varepsilon}{2} + (1 - \varepsilon)\left(\varepsilon + (1 - \varepsilon)(1 - X)\right)\right],
$$

where the Bayesian belief held by candidate $i$ is

$$
\mu_i(h) = \frac{p(1 - \varepsilon)}{p(1 - \varepsilon) + (1 - p)\varepsilon}, \quad \mu_i(l) = \frac{p\varepsilon}{p\varepsilon + (1 - p)(1 - \varepsilon)},
$$

(8)
The win probability for platform $L$ can be expressed similarly:

$$W_i(L|s_i) = \mu_i(s_i)\left[ Pr[s_{-i}=h|\theta=h]Pr[s_I=l|\theta=h]X + Pr[s_{-i}=l|\theta=h]\frac{1}{2} \right]$$

$$+ (1-\mu_i(s_i))\left[ Pr[s_{-i}=h|\theta=l]Pr[s_I=l|\theta=l]X + Pr[s_{-i}=l|\theta=l]\frac{1}{2} \right]$$

$$= \mu_i(s_i)\left[(1-\epsilon)\varepsilon X + \frac{\varepsilon}{2}\right] + (1-\mu_i(s_i))\left[\varepsilon(1-\epsilon)X + \frac{1-\varepsilon}{2}\right].$$

(9)

For truthful campaigns it is necessary that $W_i(L|h) \geq W_i(H|h)$ and $W_i(H|l) \geq W_i(L|l)$. Both conditions are linear inequalities in $X$. Solving the first condition (truthful revelation of a low signal) for $X$, we get

$$X \geq \frac{1}{2}\left(1 + \frac{\varepsilon(1-\varepsilon)}{pe^2 + (1-p)(1-\varepsilon)^2}\right).$$

(10)

Note that the right-hand side in (10) is larger than $1/2$, and under the assumption $\varepsilon < 1-p$ it is less than one. The second condition (truthful revelation of a high signal) is satisfied for all $X \in [0,1]$.\(^8\)

5.4 Equilibrium with truthful campaigns

We now bring the partial analyses of Sections 5.2 and 5.3 together, by replacing the variables $k$, $\pi$, and $X$ with expressions that are in terms of our model parameters.

Recall that SIGs only choose positive advertising efforts when the campaign platforms are $H$ and $L$, respectively. Assuming that these platforms were generated by truthful campaign strategies, each SIG’s belief that the state is $h$ is $\mu_U(H,L) = \mu_U(L,H) = p$. If the informed voters receive signal $s_I = h$, the candidate with platform $H$ will win the election regardless of the advertising chosen by the groups. For advertising to influence the final election outcome, the informed voters must receive signal $s_I = l$. If $\theta = h$ this has probability $\varepsilon$, and if $\theta = l$ this has probability $1-\varepsilon$. Thus, the “effective prize” over which the SIGs compete in the advertising contest is their value of obtaining their preferred policies (which equals one) multiplied by the probability that the election is not already decided by the uninformed and informed voters:

$$\pi = p\varepsilon + (1-p)(1-\varepsilon).$$

(11)

\(^8\)Both claims are formally shown in the Proof of Proposition 5.
Furthermore, conditional on \( s_I = l \), platform \( L \) is elected if \( \gamma_I + [1/2 + aL - aH] \gamma_M \geq 1/2 \).

This gives us the handicap on group \( L \) in the advertising contest:

\[
k = \frac{1}{2} - \frac{\gamma_I}{\gamma_M} - \frac{1}{2} = \frac{\gamma_U - \gamma_I}{2 \gamma_M}.
\]  

(12)

Finally, the probability \( X \) that platform \( L \) wins against \( H \) when \( s_I = l \) is the win probability of group \( L \) in the advertising contest. Replacing \( k \) and \( \pi \) in the win probabilities in Lemma 4 by the expressions in (11)–(12), we have

\[
X = \begin{cases} 
1 - \frac{\beta^L}{2 \beta^H} & \text{if } \frac{1}{\beta^L} \geq M + \frac{1}{\beta^H}, \\
\frac{\beta^H}{2 \beta^L} \left[ 1 - (\beta^L M)^2 \right] & \text{if } M < \frac{1}{\beta^L} < M + \frac{1}{\beta^H}, \\
0 & \text{otherwise},
\end{cases}
\]

(13)

where

\[
M = \frac{\gamma_U - \gamma_I}{2 \gamma_M} \frac{1}{p \varepsilon + (1-p)(1-\varepsilon)}.
\]

We are now ready to derive conditions for an equilibrium with truthful campaigns. For simplicity, suppose that the first condition in (13) holds, that is

\[
\frac{1}{\beta^L} - \frac{1}{\beta^H} \geq \frac{\gamma_U - \gamma_I}{2 \gamma_M} \frac{1}{p \varepsilon + (1-p)(1-\varepsilon)}.
\]

(14)

For truthful campaigning, condition (10) must hold. Using \( X = 1 - \beta^L/(2 \beta^H) \) from (13) in (10) and rearranging, we obtain

\[
\frac{\beta^L}{\beta^H} \leq \frac{1}{1 - \frac{\varepsilon(1-\varepsilon)}{p \varepsilon^2 + (1-p)(1-\varepsilon)^2}}.
\]

(15)

Together, (14) and (15) provide a sufficient condition for existence of equilibrium in which the politicians campaign truthfully in the presence of partisan interest groups. The right-hand sides of both inequalities contain the parameters of our basic model described in Section 3, while the left-hand sides contain parameters describing the interest groups. In particular, the conditions require that group \( L \) has sufficiently small advertising costs, both in absolute terms and relative to group \( H \)'s costs. This, in essence, is our main result, which is stated below.
Proposition 5. (*Special interest-funded campaigns*) In the model with political advertising funded by special interest groups, the following is true. If groups L’s advertising cost $\beta_L$ is not too large, and if $\beta_L/\beta_H < 1$ and sufficiently small, there exists a sincere Bayesian equilibrium in which both candidates campaign truthfully. Furthermore, as $\beta_L/\beta_H \to 0$, the probability that the full-information policy is implemented in this equilibrium approaches one, and the expected welfare of the informed and uninformed voters approaches the first-best welfare.

If (14) is violated but $\beta_L$ is not too large, then group L’s win probability in the contest is given by the second part of (13). In this case, condition (10) becomes

$$\frac{\beta_H}{\beta_L} \left[ 1 - \left( \frac{\gamma_U - \gamma_I}{2 \gamma_M \frac{\beta_L}{\beta_H}} \right)^2 \right] \geq 1 + \frac{\epsilon(1 - \epsilon)}{p\epsilon^2 + (1 - p)(1 - \epsilon)^2},$$

and if it holds an equilibrium with truthful campaigns again exists.

Figure 3 shows how all three inequalities (14)–(16) define the set of values for $\beta_L$ and $\beta_H$ such that truthful equilibria exist. (The figure is drawn for $p = 0.75$, $\epsilon = 0.1$, $\gamma_U = 0.35$, $\gamma_I = 0.3$, $\gamma_M = 0.35$). In the blue shaded region, defined by (14) and (15), the equilibrium in the advertising contest between the SIGs is characterized by part (i)
of Lemma 4. Note that in order to prove our main result, considering only this case was sufficient. In the smaller, green shaded region, the equilibrium in the advertising contest is characterized by part (ii) of Lemma 4.

Figure 4: Convergence to first-best with SIG-funded campaigns (\(p = 0.75\)).

Figure 4 plots the expected welfare of the uninformed and informed voters in a truthful campaign equilibrium with special interest groups, for \(p = 0.75\) and four different values for \(\beta^L/\beta^H\). Welfare in the best equilibrium without advertising (the semi-separating equilibrium) and the full-information benchmark are depicted as dashed curves. Given \(\beta^L/\beta^H\), an equilibrium with truthful campaigning exists for small enough \(\varepsilon\)-values, and if it exists expected welfare is very close to the first-best.

5.5 Discussion

In our baseline model without advertising, both candidates tried to attract the uninformed voters by offering policy \(H\), regardless of whether their signals indicated that this was the optimal policy for voters. In the model with advertising, each SIG tries to get the impressionable voters to vote for the candidate who offers the group’s preferred policy, also without regard for whether this is the correct policy for the uninformed or informed voters. Neither the candidates nor the SIGs have these voters’ welfare in mind when making their decisions. Yet, on balance, the politicians’ incentive to campaign on the
populist platform $H$ and the advertising advantage of group $L$ offset one another, resulting in policies that are more likely to be correct, given the state of nature.\(^9\)

Situations in which interest groups with a financial advantage are also groups favoring less popular policies often arise. Suppose that group $L$ is a business association while group $H$ is a labor union. Business associations often have deeper pockets, and spend more to influence elections, relative to unions. In U.S. elections, for instance, business interests have outspent labor interests in terms of campaign contributions to politicians as well as independent political expenditures (see, for example, Klumpp, Mialon, and Williams 2014). Under the assumption that labor-friendly policies are a priori more popular in the electorate, our result suggests that this imbalance in political spending may not be a bad thing. One implication of Proposition 5 is then that limiting advertising by special interest groups may have a detrimental effect on voter welfare, as a caps on advertising would first bind group $L$. This would make it less likely that the non-populist platform wins, and hence diminish the incentive of a candidate with an $l$-signal to campaign on this platform.

Our model is quite flexible and permits a number generalizations which we now briefly discuss. First, the main result would be unchanged if interest groups were prohibited from advertising for the candidates but allowed to donate funds to the candidates. Since the candidates are office-motivated, the would simply use these donations to advertise for themselves. Second, while election law may prohibit outside advertising for candidates, it may permit interest groups to spend money on “issue ads” that promote specific policy goals or causes without references to politicians. Since SIG $L$ only advertises for candidates whose campaign platform is $L$, and SIG $H$ only advertises for candidates whose campaign platform is $H$, the ads in our model could be interpreted as such issue ads. Third, if the interest groups’ preferences were the same as those of the voters, their presence could still provide politicians with an incentive to campaign truthfully. However, the mechanism would be quite different: If the candidates receive non-matching signals and set non-matching platforms, both interest groups would want to get a sufficient number of impressionable voters to vote for $L$ to neutralize the larger size of the uninformed voter group (which sincerely votes for $H$) and make the informed voters pivotal. Advertising for the non-populist policy would now be a public good among the interest groups, and the game between the two groups would be one of voluntary public good provision instead of an all-pay auction.

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\(^9\)This effect is somewhat reminiscent of the advocacy effect first established in Dewatripont and Tirole (1999). There, an agent charged with discovering decision-relevant information for the principal has an incentive to shirk, even if offered an optimal contract. Competition between two agents with opposing goals, neither of which aligned the principal’s interests, can improve the principal’s outcome by generating more information at a lesser cost.
6 Alternative Funding Systems

The previous section demonstrated that special-interest funded campaign advertising can improve voter welfare, by giving candidates an incentive to reveal their private information. In this section, we consider two alternative sources of campaign funds: The candidates and the state.

6.1 Private candidate wealth

Political candidates often use their own money to fund their campaigns, and the sums spent by wealthy politicians on their election campaigns can dwarf those spent by special interest groups. In the 2010 California governor’s race, for example, billionaire Republican candidate Meg Whitman spent more than $140 million of her own wealth on her election campaign, approximately $118 million of which was allocated to television advertising (California Watch 2011). We now examine the question whether wealthy candidates can “afford the luxury” of campaigning in their voters’ interest.

To do so, let us assume that candidates pay for advertising with their private resources, and that candidate $i = 1, 2$ has a marginal advertising cost of $\beta_i > 0$. A wealthy candidate would then be one with a very low $\beta_i$. Both candidates choose their advertising expenditures after observing each other’s platform choices.\footnote{Herrera, Levine, and Martinelli (2008) and Ashworth and Bueno de Mesquita (2009) study models with a similar timing of platform choice and advertising, but without private candidate information. Meirowitz (2008) assumes the opposite order of events.}

An advertising strategy for candidate $i$’s is a mapping

$$\alpha_i : X \times X \times \Theta \rightarrow [0, \infty),$$

where $\alpha_i(x_1, x_2, s_i)$ denotes advertising by candidate $i$ when the campaign platforms are $x_1$ and $x_2$ and the candidate’s private signal is $s_i$.

Our equilibrium notion will be that of sincere Bayesian equilibrium in Definition 1, with one added requirement: The candidates’ advertising strategies $\alpha_1$ and $\alpha_2$ form a Bayesian Nash equilibrium in the advertising contest for every $(x_1, x_2)$ and $(s_1, s_2)$, taken the strategies of uninformed and informed voters as given. This contest will have a mixed strategy equilibrium similar to the one described in Lemma 4. We can say the following:

**Proposition 6.** (Candidate-funded campaigns) Suppose that advertising is provided by the candidates. There does not exist a sincere Bayesian equilibrium in which the candidates set truthful campaigns, and voter welfare is bounded away from the first-best.
The intuition for the result is extremely simple. Note that the special interest groups from the previous section cared only about the policy outcome but not about the candidates. The candidates, on the other hand, care only about being elected but not about policy. It was precisely the interaction of these opposing forces, in combination with the cost asymmetry between the SIGs, that led to the truthful equilibrium policies in Proposition 5. If advertising competition between the politicians were to emulate competition between interest groups, then a candidate whose platform is $L$ would have to have a lower advertising cost than the candidate whose platform is $H$. But because either one of the two candidate should campaign on platform $L$ when receiving an $l$-signal, the required asymmetry between the candidates is impossible.\footnote{In terms of the parameter regions plotted in Figure 3, this can be visualized as follows: One would take the blue and green shaded region (which lies strictly below the 45°-line), mirror it around the origin (i.e., switch $\beta^H$ and $\beta^L$), and then intersect this mirror image with the original shaded region. This intersection would be empty.}

6.2 Public campaign financing

Next, we consider a European-style system of public funding of elections. That is, we imagine a pool of public funds of overall size $\Gamma$, to be awarded to the candidates after the election and in proportion to their vote share. Such a system is theoretically examined by Ortuno-Ortín and Schultz (2004), who show that it provides policy-motivated candidates with a strong incentive to set convergent platforms.

As in Ortuno-Ortín and Schultz (2004), we assume that both candidates have access to credit markets that allow them to borrow (at a zero interest rate) against public funds to be awarded after the election. Furthermore, candidates have access to actuarially fair insurance markets and can exchange any probability distribution over public funds received after the election for a fixed payment equal to the expected value of this distribution. Funds for the election are acquired on the credit and insurance markets after both candidates have set their platforms. Insurers have the same information as uninformed voters (in particular, candidates cannot credibly communicate their signals to them).

Our equilibrium notion will, once again, be sincere Bayesian equilibrium, with one added requirement. Denote by $\Gamma_i(x_1, x_2)$ the funds acquired on the financial market by candidate $i$ when the platforms are $(x_1, x_2)$. Because publicly provided campaign funds have no alternative uses, the advertising bought by candidate $i$ is $\Gamma_i(x_1, x_2)/\beta$, where $\beta > 0$ is the common advertising cost coefficient. For equilibrium, we impose that the following holds for all $(x_1, x_2)$:
\[ \Gamma_1(x_1, x_2) = \Gamma \times \mathbb{E}_{s_i} \left[ \nu_U(x_1, x_2) \gamma_U + \nu_I(x_1, x_2, s) \gamma_I \right. \\
+ \left. z \left( \frac{\Gamma_1(x_1, x_2)}{\beta}, \frac{\Gamma - \Gamma_1(x_1, x_2)}{\beta} \right) \gamma_M \right| x_1, x_2 \] (17)

This condition says that the funds available to a candidate, given platforms \((x_1, x_2)\), are a proportion of total funds equal to the expected vote share of the candidate, conditional on \((x_1, x_2)\).

Note that for every pair of platforms \((x_1, x_2)\) there exists \(\Gamma_1(x_1, x_2)\) that satisfies requirement (17). To see this, use the right-hand side of (17) to define a function \(T: [0, \Gamma] \rightarrow [0, \Gamma]\) (given \(x_1\) and \(x_2\)). Because the conditions of Brouwer’s Theorem hold, \(T\) has a fixed point, and an equilibrium therefore exists. We assume that this fixed point is unique. This is a mild assumption that says that the public funding system is deterministic and does not lead to multiple “self-enforcing” funding levels for the political candidates.\(^{12}\)

**Proposition 7. (Publicly funded campaigns)** Suppose that advertising is provided by a deterministic system of public election funding. There does not exist an equilibrium in which candidates campaign truthfully, and voter welfare is bounded away from the first-best.

The intuition for this result is similar to the reason behind the convergence result in Ortuno-Ortín and Schultz (2004). There, in a Hotelling-type setup, moving one’s platform closer to the median voter increases votes, which leads to a larger share of campaign funds awarded to the candidate, which in turn can be spent to attract more impressionable voters. Here, choosing a populist platform does the same: By the argument given in Section 4.1, the populist policy always results in a higher expected vote share than the non-populist policy, which leads to more campaign funds, which in turn can be spent to attract more impressionable voters.

### 7 Conclusion

This paper examined a model in which privately informed candidates compete in an election. We showed that the presence of partisan special interest groups can improve voter welfare. Unlike previous papers, ours does not require interest groups to possess private information, or that advertising messages are informative. Instead, our results relied on an “advocacy effect”: The combination of populism and partisanship resulted

\(^{12}\)A sufficient condition to rule out funding indeterminacies, given \((x_1, x_2)\), is that the right-hand side of (17) has slope less than one in \(\Gamma_1\). This is the case if \(\Gamma/\beta < 2\gamma_M\).
in balanced policies that were more responsive to the state of nature than equilibrium policies without advertising.

The result is interesting because the influence of large sums of money in elections, often provided by interest group, is typically seen as distorting the democratic process. In our model, this is the case as well. However, it is also true that democratic institutions often do not provide candidates running for elected office with strong incentives to campaign in a way that maximizes the electorate’s welfare. In the model presented here, the politicians’ incentive to campaign on populist policies is also a distortion of the democratic process. What our main result shows, then, is that the distortionary effect of populism can be corrected by the distortionary effect of special interest influence. At the same time, public funding or candidate funding fail to achieve the same result in our model.

Appendix

Proof of Lemma 1

First, consider campaign strategies for which

\[ Pr[x_1 = x_2 = H|s_1 = s_2 = l] + Pr[x_1 = x_2 = L|s_1 = s_2 = h] = \omega > 0. \]

In this case, the probability that the full-information policy is implemented is at most \(1 - \omega\). On the other hand, if

\[ Pr[x_1 = x_2 = H|s_1 = s_2 = l] = Pr[x_1 = x_2 = L|s_1 = s_2 = h] = 0, \]

either the candidates are both truthful or one candidate always offers \(H\) while the other always offer \(L\) (i.e., \(\chi_i(s_i) = 1\) and \(x_{-i}(s_{-i}) = 0\)). Without loss of generality, suppose candidate 1 offers \(H\) regardless of \(s_1\) and candidate 2 offers \(L\) regardless of \(s_2\). The uninformed voters’ Bayesian belief is then the same as their prior,

\[ \mu_U(H, L) = p > \frac{1}{2}, \]

while the informed voters’ Bayesian belief is

\[ \mu_I(H, L, s_I) = \begin{cases} \frac{p(1-\varepsilon)}{p(1-\varepsilon) + (1-p)\varepsilon} > \frac{1}{2} & \text{if } s_I = h, \\ \frac{p\varepsilon}{p\varepsilon + (1-p)(1-\varepsilon)} < \frac{1}{2} & \text{if } s_I = l \end{cases} \]

(the inequalities follow from \(0 < \varepsilon < 1-p < 1/2 < p < 1-\varepsilon < 1\)). Thus, the uninformed voters vote for candidate 1, and the informed voters vote for candidate 1 if and only
if $s_I = h$. This implies that, when $s_1 = s_2 = l$ and $s_I = h$, candidate 1 wins (because $\gamma_U + \gamma_I > 1/2$) and platform $H$ is implemented, while the full information policy is $x^{FI} = L$. This happens with probability $\omega = p\varepsilon^2(1 - \varepsilon) + (1 - p)(1 - \varepsilon)^2\varepsilon > 0$. It follows that when voters are sincere and candidates are non-truthful, the probability the full-information policy is not implemented is bounded away from one by some number $\omega > 0$ (which may depend on the strategy profile, but not on the level or source of advertising).

Now assume $a_1 = a_2 = 0$, and suppose $\chi_1$ and $\chi_2$ are truthful. Consider the signals $(s_1, s_2, s_I) = (l, h, l)$. The full information policy is $x^{FI}(l, h, l) = L$ and the policy platforms offered are $x_1 = L$ and $x_2 = H$. The uninformed beliefs are $\mu_U(L, H) = p > 1/2$, so that the uninformed voters strictly prefer policy $x_2 = H$ over policy $x_1 = L$ and thus sincerely vote for candidate 2. Since $\gamma_U > \gamma_I$, candidate 2 obtains more than half of the votes and wins, so policy $x_2 = H \neq x^{FI}$ is implemented. This happens with probability $\omega' = p\varepsilon^2(1 - \varepsilon) + (1 - p)(1 - \varepsilon)^2\varepsilon$. It follows that when voters are sincere, candidates are truthful, and there is no advertising, the probability that the full-information policy is implemented is bounded away from one by $\omega' > 0$.

**Proof of Proposition 2**

Suppose both candidates choose truthful campaigns, and consider the signals $s_1 = s_2 = l$, so that the campaign platforms are $x_1 = x_2 = L$. There must be at least one candidate who wins with probability strictly less than one. Without loss of generality, suppose candidate 1 wins with probability $\alpha < 1$ in this case. Let $\alpha' \geq 0$ be the probability that candidate 1 wins if the platforms offered are $(H, H)$. Now consider the pair of platforms $(L, H)$. Assuming that candidates choose truthful platforms, the uninformed voters’ Bayesian beliefs are given by (6), $\mu_U(L, H) = p > 1/2$. Thus, the uninformed voters prefer platform $H$ over $L$. All uninformed voters therefore sincerely vote for candidate 2, who then wins with probability one. Similarly, if $(x_1, x_2) = (H, L)$, all uninformed voters vote for candidate 1, who wins.

The following must then be true: If $x_1 = L$, candidate 1 wins with probability $\alpha < 1$ if $x_2 = L$ and with probability zero if $x_2 = H$. If $x_1 = H$, candidate 1 wins with probability one if $x_2 = L$ and with probability $\alpha' \geq 0$ if $x_2 = H$. Thus, against a truthful strategy by candidate 2, candidate 1 has a strictly larger chance of winning with platform $x_1 = H$ than with $x_1 = L$. An equilibrium in which both candidates set truthful campaigns hence cannot exist.

**Proof of Proposition 3**

We will prove parts (i)–(iv) of the result in sequence.
Part (i): Populist pooling equilibrium. Suppose both candidates set platform $H$ and voting strategies are symmetric; this implies that in equilibrium each candidate wins with probability $1/2$. Consider now a deviation by candidate 1 to platform $L$. This is an out-of-equilibrium event, and if the uninformed voters believed that $\mu_{U}(L,H) \geq 1/2$, it is optimal for all uninformed voters to vote for candidate 2, so candidate 1 loses as a result of the deviation. The same applies when candidate 2 deviates. Thus, it is possible to support the equilibrium by beliefs $\mu_{U}(L,H) \geq 1/2$ and $\mu_{U}(H,L) \geq 1/2$.

Part (ii): Contrarian pooling equilibrium. The proof is analogous to part (i) and omitted.

Part (iii): Semi-separating equilibrium. Suppose $\chi_{i}(h) = 1$ and $\chi_{i}(l) = q$ for $i = 1,2$. Consider the cases $(x_{1},x_{2}) = (H,L)$ or $(x_{1},x_{2}) = (L,H)$. The uninformed voters’ Bayesian beliefs must satisfy

$$
\mu_{U}(H,L) = \mu(L,H) = \frac{p\epsilon(q + (1-\epsilon))}{p\epsilon(q + (1-\epsilon)) + (1-p)(1-\epsilon)((1-\epsilon)q + \epsilon)}.
$$

In the mixed strategy equilibrium, the uninformed voters must be indifferent between voting for either candidate. This requires $\mu_{U}(H,L) = \mu_{U}(L,H) = 1/2$, which in turn implies

$$
q = \chi_{i}(l) = \frac{(2p-1)\epsilon(1-\epsilon)}{(1-p)(1-\epsilon)^{2} - p\epsilon^{2}}. \quad (18)
$$

(18) is the probability that a candidate sets platform $H$ after having received an $l$-signal, as stated in the result. Note that $\epsilon < 1 - p$ implies $\chi_{i}(l) \in (0,1)$. The informed voters’ belief can be written as

$$
\mu_{I}(H,L,h) = \mu_{I}(L,H,h) = \frac{1/2(1-\epsilon)}{1/2(1-\epsilon) + 1/2\epsilon} = 1 - \epsilon > \frac{1}{2},
$$

$$
\mu_{I}(H,L,l) = \mu_{I}(L,H,l) = \frac{1/2\epsilon}{1/2(1-\epsilon) + 1/2\epsilon} = \epsilon < \frac{1}{2}. \quad (19)
$$

Thus the informed voters vote according to their own signal $s_{I}$: $\nu_{I}(H,L,h) = \nu_{I}(L,H,l) = 1$ and $\nu_{I}(H,L,l) = \nu_{I}(L,H,h) = 0$.

Now suppose that when the two candidates offer the same platform each wins with probability $1/2$. Furthermore, suppose that the probability that platform $L$ wins against platform $H$ if $s_{I} = l$ is approximately $X$, and the probability that $L$ wins against $H$ if $s_{I} = h$ is approximately zero. Let $W_{i}(x_{i}|s_{i})$ be the win probability of candidate $i$ who receives signal $s_{i}$ and sets platform $x_{i}$, assuming that the opposing candidate uses the
semi-separating strategies. For \( x_i = L \), this can be expressed as follows:

\[
W_i(L|s_i) \approx \mu_i(s_i) \left[ Pr[s_{-i} = h|\theta = h]Pr[s_l = l|\theta = h]X + Pr[s_{-i} = l|\theta = h] \left( q\varepsilon X + \left( 1 - q \right) \frac{1}{2} \right) \right] + (1 - \mu_i(s_i)) \left[ Pr[s_{-i} = h|\theta = l] \left( Pr[s_l = l|\theta = l]X + \left( 1 - q \right) \frac{1}{2} \right) \right]
\]

\[
= \mu_i(s_i) \left[ (1 - \varepsilon)\varepsilon X + \varepsilon \left( q\varepsilon X + \left( 1 - q \right) \frac{1}{2} \right) \right] + (1 - \mu_i(s_i)) \left[ \varepsilon(1 - \varepsilon)X + (1 - \varepsilon) \left( q(1 - \varepsilon)X + \left( 1 - q \right) \frac{1}{2} \right) \right],
\]

where the Bayesian belief held by candidate \( i \) is

\[
\mu_i(h) = \frac{p(1 - \varepsilon)}{p(1 - \varepsilon) + (1 - p)\varepsilon}, \quad \mu_i(l) = \frac{p\varepsilon}{p\varepsilon + (1 - p)(1 - \varepsilon)}.
\]

The win probability for platform \( H \) can be expressed similarly:

\[
W_i(H|s_i) \approx \mu_i(s_i) \left[ Pr[s_{-i} = h|\theta = h] \left( \frac{1}{2} + (1 - q)Pr[s_l = h|\theta = h] \right) \right] + (1 - \mu_i(s_i)) \left[ Pr[s_{-i} = l|\theta = h] \left( \frac{1}{2} \right) + (1 - q)(Pr[s_l = h|\theta = l] + Pr[s_l = h|\theta = l](1 - X)) \right]
\]

\[
= \mu_i(s_i) \left[ \frac{1 - \varepsilon}{2} + \varepsilon \left( \frac{1}{2} + (1 - q)((1 - \varepsilon) + \varepsilon(1 - X)) \right) \right] + (1 - \mu_i(s_i)) \left[ \frac{\varepsilon}{2} + (1 - \varepsilon)(\frac{q}{2} + (1 - q)(\varepsilon + (1 - \varepsilon)(1 - X))) \right].
\]

For the equilibrium campaign strategies to be optimal, we need \( W_i(L|l) \approx W_i(H|l) \) and \( W_i(L|h) \leq W_i(H|h) \). The first condition allows us to compute

\[
X \approx \frac{1}{2} \frac{(1 - p)(1 - \varepsilon) + \varepsilon p}{(1 - p)(1 - 2\varepsilon) + \varepsilon^2}.
\]

It can be verified that \( X \in (0, 1) \), and that \( W_i(L|h) < W_i(H|h) \). Thus, the candidates’ strategies are optimal, under the assumption that the probability that \( L \) wins against \( H \) if \( s_I = l \) is approximately \( X \), and the probability that \( L \) wins against \( H \) if \( s_I = h \) is approximately zero.

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With a large but finite number of uninformed voters, these probabilities can be achieved through an appropriate mixed strategy $\nu_U$ of the uninformed voters. Let $v$ be the share of uninformed voters who vote for the candidate with platform $H$ if $(x_1,x_2) = (H,L)$. Thus, for equilibrium we need

\[ Pr[\gamma_I + (1-v)\gamma_U > v\gamma_U] \approx X, \]  
\[ Pr[(1-v)\gamma_U > v\gamma_U + \gamma_I] \approx 0. \]  

With a large number of voters, $v$ can be approximated by a normal distribution with mean $\nu_U(H,L)$ and negligible variance, so the (20) implies

$\gamma_I + (1 - \nu_U(H,L))\gamma_U \approx \nu_U(H,L)\gamma_U \iff \nu_U(H,L) \approx \frac{1}{2} \left( 1 + \frac{\gamma_I}{\gamma_U} \right),$

as stated in the result. Condition (21) then holds as well (because $v$ has almost zero variance). Finally, by symmetry, we have $\nu_U(L,H) = 1 - \nu_U(H,L)$. This completes the proof of part (iii).

**Part (iv): Pooling equilibrium.** Without loss of generality suppose candidate 1 is truthful and candidate 2 is uninformative. Then the uninformed beliefs are

$\mu_U(H,x_2) = Pr[\theta = h|x_1 = H] = \frac{p(1-\varepsilon)}{p(1-\varepsilon) + (1-p)\varepsilon} > \frac{1}{2}$

(the inequality follows from $p > 1/2$, $\varepsilon < 1/2$) and

$\mu_U(L,x_2) = Pr[\theta = h|x_1 = L] = \frac{p\varepsilon}{p\varepsilon + (1-p)(1-\varepsilon)} < \frac{1}{2}$

(the inequality follows from $\varepsilon < 1 - p$). Thus, the uninformed voters prefer platform $H$ over $L$ if $x_1 = H$, and platform $L$ over $H$ if $x_1 = L$. It is therefore optimal for the uninformed voters to always vote for candidate 1, so candidate 1 wins with probability one for all $(x_1,x_2)$ and cannot possibly improve his chance of winning by deviating to a non-truthful strategy. But this implies that candidate 2 wins with probability zero for all $(x_1,x_2)$, and so deviating to any other strategy is not profitable for candidate 2. \hfill $\square$

**Proof of Lemma 4**

The proof is similar to the standard derivation of mixed strategy equilibria in the all-pay auction. That is, we consider strategy profiles in which each group chooses zero advertising with some probability (which could be zero), and with the remaining probability randomizes uniformly over some interval of positive advertising levels. The
primary departure from the standard proof is our need to account for the handicapping of group $L$.

Note that the handicapped group $L$ will never choose an advertising level $0 < a^L < k$, as the chance of winning with $a^L < k$ is zero, while any positive $a^L$ has a positive cost. Thus, assume that group $L$ sets $a^L = 0$ with some probability $q^L$, and randomizes $a^L \geq k$ uniformly with density $s^L > 0$. Then group $H$’s expected payoff from advertising $a^H \geq 0$ is given by

$$E[u^H(a^H)] = [q^L + s^L(a^H + k)] \pi - \beta^H a^H.$$  

If group $L$ randomizes $a^L$ with density $s^L = \beta^L / \pi$, group $H$’s expected payoff function becomes flat at every $a^H$ for which its probability of winning is interior. In this case, group $H$ is indifferent among all such advertising levels, a necessary condition for randomization on part of group $H$.

To go the other way around, assume that group $H$ sets $a^H = 0$ with some probability $q^H$, and with the remaining probability randomizes $a^H > 0$ uniformly with density $s^H > 0$. Then group $L$’s expected payoff from advertising $a^L \geq k$ is given by

$$E[u^L(a^L)] = [q^H + s^H(a^L - k)] \pi - \beta^L a^L.$$  

If group $H$ randomizes $a^H$ with density $s^H = \beta^L / \pi$, group $L$’s expected payoff function becomes flat at every $a^L$ for which its probability of winning is interior. In this case, group $L$ is indifferent among all such advertising levels, a necessary condition for randomization on part of group $L$.

Now that we know the densities of the uniform part of the groups’ strategies, we can determine the probabilities $q^L$ and $q^H$ with which they choose zero advertising. Consider the following three cases:

**Case (i).** Suppose that $q^L = 0$, or equivalently, $L$ randomizes uniformly on the interval $[k, k + \pi / \beta^H]$. In this case, $H$ will never choose an advertising level $a^H > \pi / \beta^H$, as the chance of winning with $a^H = \pi / \beta^H$ is one already. Thus, if group $H$ randomizes, it randomizes with density $s^H = \beta^L / \pi$ on the interval $[0, \pi / \beta^H]$. This accounts for a total probability mass of $\beta^L / \beta^H$, so that the probability mass that $H$ puts on zero is $q^H = 1 - \beta^L / \beta^H$. This is precisely the strategy profile given in part (i) of the result.

Group $H$’s expected payoff in this profile is zero, and it cannot do better than this: Every $a^H \in [0, \pi / \beta^H]$ yields a zero expected payoff by construction, and setting $a^H > \pi / \beta^H$ yields a smaller (i.e., negative) payoff as the probability of winning is one already at $a^H = \pi / \beta^H$. To compute group $L$’s expected payoff, consider its payoff at
advertising level $k$:

$$E[u^L(k)] = q^H \pi - k \beta^L = \left(1 - \frac{\beta^L}{\beta^H}\right) \pi - k \beta^L.$$ 

This is non-negative if and only if

$$\frac{1}{\beta^L} \geq \frac{k}{\pi} + \frac{1}{\beta^H}. \tag{22}$$

If this condition is violated, group $L$ would want to deviate from its mixed strategy to a zero advertising level, as this guarantees a zero payoff. If (22) holds, on the other hand, group $L$ receives the same positive expected payoff from every advertising level $a^L \in [k, k + \pi/\beta^H]$, while for advertising levels below $k$ or above $k + \pi/\beta^H$, its payoff decreases. It follows that the strategy profile in part (i) is an equilibrium if and only if (22) is satisfied.

In this equilibrium, group $L$ wins with probability one if $a^H = 0$, which happens with probability $1 - \beta^L/\beta^H$. If group $H$ randomizes uniformly over $[0, \pi/\beta^H]$, which happens with probability $\beta^L/\beta^H$, then each group wins with probability $1/2$. Thus, the overall win probability for group $L$ is

$$1 - \frac{\beta^L}{\beta^H} + \frac{1}{2} \frac{\beta^L}{\beta^H} = 1 - \frac{1}{2} \frac{\beta^L}{\beta^H}.$$  

**Case (ii).** If (22) does not hold, the profile in part (i) cannot be an equilibrium because $L$’s expected payoff is negative. By adjusting $q^H$ and $q^L$, however, we can make the expected payoffs for both groups exactly zero. Consider first $L$’s payoff at its minimum positive advertising level, $k$:

$$E[u^L(k)] = q^H \pi - k \beta^L = 0 \Rightarrow q^H = \frac{\beta^L}{\pi} k.$$ 

When group $H$ randomizes uniformly, it still does so with density $s^H = \beta^L/\pi$, which means that the support of $H$’s mixed strategy adjusts to $[0, \pi/\beta^L - k]$. For group $L$, advertising levels $0 < a^L < k$ and $a^L > \pi/\beta^L$ are now strictly dominated by zero; thus, the support of the uniform part of $L$’s strategy adjusts to $[k, \pi/\beta^L]$. Furthermore, when $L$ randomizes uniformly it does so with density $s^L = \beta^H/\pi$, which accounts for a total probability mass of $(\pi/\beta^L - k) \cdot (\beta^H/\pi) = \beta^H/\beta^L - k \beta^H/\pi$. Thus, the probability mass that $L$ puts on zero is $q^L = 1 - \beta^H/\beta^L + k \beta^H/\pi$. This is precisely the strategy profile given in part (ii) of the result.
Note that \( q^H \) is always positive, and \( q^L \) is positive if and only if the inequality in (22) is reversed. Furthermore, both \( q^H \) and \( q^L \) are less than one if and only if \( 1/\beta^L > k/\pi \). It follows that the equilibrium exists if

\[
\frac{k}{\pi} < \frac{1}{\beta^L} < \frac{k}{\pi} + \frac{1}{\beta^H}.
\]

In this equilibrium, both groups get a zero expected payoff at every advertising level in the support of their strategies, and a negative expected payoff at all advertising levels outside the support of their strategies.

Group \( L \) wins with probability zero if \( a^L = 0 \), which happens with probability \( 1 - \beta^H / \beta^L + k \beta^H / \pi \). If it randomizes uniformly over \([k, \pi/\beta^L]\), which happens with probability \( \beta^H / \beta^L - k \beta^H / \pi \), it wins with probability one if \( a^L = 0 \) (which happens with probability \( k \beta^L / \pi \)), and with probability \( 1/2 \) if group \( H \) randomizes uniformly over \([0, \pi/\beta^L - k]\) (which happens with probability \( 1 - k \beta^L / \pi \)). Thus, the overall win probability for group \( L \) is

\[
\left( \frac{\beta^H}{\beta^L} - k \frac{\beta^H}{\pi} \right) \left[ k \frac{\beta^L}{\pi} + \left( 1 - k \frac{\beta^L}{\pi} \right) \frac{1}{2} \right] = \frac{1}{2} \beta^H \left[ 1 - \left( \frac{k \beta^L}{\pi} \right)^2 \right].
\]

Case (iii). Finally, if \( 1/\beta^L \leq k/\pi \), then \( a^L = a^H = 0 \) is an equilibrium. Note that group \( H \) wins with probability one while exerting zero effort, so it cannot possibly deviate and gain. Group \( L \) wins with probability zero and spends zero, for a zero payoff. In order to win with a positive probability, it would have to deviate to \( a^L \geq k \). Since \( 1/\beta^L \leq k/\pi \) (or \( \beta^L \geq \pi/k \), doing so must result in a non-positive payoff.

Proof of Proposition 5

The main argument was presented in Sections 5.2, 5.3, and 5.4. What is left is to establish that

1. \( W_i(H|h) \geq W_i(L|h) \) (truthful revelation of \( s_i = h \));

2. there always exist values for \( \beta^L \) and \( \beta^H \) such that conditions (14)–(15) are satisfied (so an equilibrium with truthful campaigns indeed exists for some parameter values);

and

3. and that voter welfare is maximized asymptotically as \( \beta^L / \beta^H \to 0 \).

This will be done in the corresponding steps 1–3 below.
Step 1. Note that if $W_i(H|h) \geq W_i(L|h)$ holds for $X$ then it holds for $X' < X$. Hence it suffices to show that it holds if $X = 1$, in which case the inequality becomes

$$\mu_i(h) \left[ \frac{1 - \varepsilon}{2} + \varepsilon(1 - \varepsilon) \right] + (1 - \mu_i(h)) \left[ \frac{\varepsilon}{2} + \varepsilon(1 - \varepsilon) \right] \geq \mu_i(h) \left[ (1 - \varepsilon)\varepsilon + \frac{\varepsilon}{2} \right] + (1 - \mu_i(h)) \left[ \varepsilon(1 - \varepsilon) + \frac{1 - \varepsilon}{2} \right]$$

or $\mu_i(h)(1 - 2\varepsilon) \geq (1/2)(1 - 2\varepsilon)$. This is satisfied if

$$\mu_i(h) = \frac{p(1 - \varepsilon)}{p(1 - \varepsilon) + (1 - p)\varepsilon} \geq \frac{1}{2} \quad \iff \quad p(1 - \varepsilon) \geq (1 - p)\varepsilon,$$

which is true since $\varepsilon < 1/2 < p$.

Step 2. Note that (14) can only hold if $\beta^L$ not be too large, that is,

$$\beta^L < \frac{2\gamma M}{\gamma U - \gamma I} (p\varepsilon + (1 - p)(1 - \varepsilon)).$$

If this is satisfied, then the condition will hold as long $\beta^L/\beta^H$ is sufficiently small. Similarly, condition (15) imposes an upper bound on the ratio $\beta^L/\beta^H$. This bound must be positive, that is

$$1 - \frac{\varepsilon(1 - \varepsilon)}{p\varepsilon^2 + (1 - p)(1 - \varepsilon)^2} > 0 \quad \iff \quad p\varepsilon^2 + (1 - p)(1 - \varepsilon)^2 - \varepsilon(1 - \varepsilon) > 0.$$

Note that $\partial[p\varepsilon^2 + (1-p)(1-\varepsilon)^2 - \varepsilon(1-\varepsilon)]/\partial p = \varepsilon^2 - (1-\varepsilon)^2 < 0$ (since $\varepsilon < 1/2$). Further, since we assume that $p < 1 - \varepsilon$, it suffices to show that $p\varepsilon^2 + (1-p)(1-\varepsilon)^2 - \varepsilon(1-\varepsilon) \geq 0$ at $p = 1 - \varepsilon$. This is indeed the case:

$$(1 - \varepsilon)\varepsilon^2 + \varepsilon(1-\varepsilon)^2 - \varepsilon(1-\varepsilon) = (1 - \varepsilon)\varepsilon [\varepsilon + 1 - \varepsilon - 1] = 0 \geq 0,$$

and it follows that the conditions for an equilibrium with truthful campaigns are satisfied for sufficiently low $\beta^L$ and sufficiently low $\beta^L/\beta^H$.

Step 3. The full-information policy is implemented in the equilibrium unless $s_1 \neq s_2$, $s_I = l$, and group $H$ wins the advertising contest. Using the expression in (13), this event has probability

$$\left[ 2p(1 - \varepsilon)\varepsilon^2 + 2(1 - p)(1 - \varepsilon)^2 \varepsilon \right] \cdot \frac{1}{2} \frac{\beta^L}{\beta^H} \to 0 \quad \text{as} \quad \frac{\beta^L}{\beta^H} \to 0.$$

Thus, as $\beta^L/\beta^H \to 0$ the probability that $x^{FI}$ is implemented approaches one. □
Proof of Proposition 6

Assume, contrary to the result, that the candidates’ strategies are truthful. Then, after observing each others’ platforms both candidates become informed about $s_1$ and $s_2$, but neither knows $s_I$.

We use the following notation. First, let $X_i$ be the probability that candidate $i$ wins if $x_i = L$, $x_{-i} = H$, and $s_I = l$. Second, let $Y_i$ be the probability that candidate $i$ wins if $x_i = x_{-i} = H$. Third, let $Z_i$ be the probability that candidate $i$ wins if $x_i = x_{-i} = L$.

Note that $X_i$ is the win probability in Lemma 4 with $\beta^L = \beta_i$, $\beta^H = \beta_{-i}$, $k = \frac{7u - \gamma l}{2\gamma M}$, $\pi = p\epsilon + (1 - p)(1 - \epsilon)$.

Note also that $Y_1 + Y_2 = 1$ and $Z_1 + Z_2 = 1$. (Unlike the SIGs, candidates will generally want to advertise when $x_i = x_{-i}$, which means that $Y_i$ and $Z_i$ are not necessarily equal to 1/2.)

Following steps analogous to those in Section 5.3, candidate $i$’s win probability with platform $x_i \in \{H, L\}$, given signal $s_i$, can be expressed as follows:

$$W_i(H|s_i) = \mu_i(s_i)\left[(1 - \epsilon)Y_i + \epsilon\left((1 - \epsilon) + \epsilon(1 - X_{-i})\right)\right]$$
$$+ (1 - \mu_i(s_i))\left[\epsilon Y_i + (1 - \epsilon)(\epsilon + (1 - \epsilon)(1 - X_{-i}))\right],$$

$$W_i(L|s_i) = \mu_i(s_i)\left[(1 - \epsilon)\epsilon X_i + \epsilon Z_i\right] + (1 - \mu_i(s_i))\left[\epsilon(1 - \epsilon)X_i + (1 - \epsilon)Z_i\right].$$

For truthful revelation of an $l$-signal, we need $W_i(L|l) \geq W_i(H|l)$ for $i = 1, 2$. Adding these inequalities for $i = 1, 2$ and using $\mu_2(l) = \mu_1(l)$, $Y_2 = 1 - Y_1$, $Z_2 = 1 - Z_1$, we have

$$(X_1 + X_2)\left[\epsilon(1 - \epsilon) + \mu_1(l)\epsilon^2 + (1 - \mu_1(l))(1 - \epsilon)^2\right] \geq 1.$$

Substituting (8) for the candidates’ belief, the above inequality can be written as

$$X_1 + X_2 \geq \frac{p\epsilon + (1 - p)(1 - \epsilon)}{p\epsilon^2 + (1 - p)(1 - \epsilon)^2} > 1. \quad (23)$$

Now denote by $P(c, d, k, \pi)$ a player’s win probability in an all-pay auction with $k$-handicap on this player, when the player’s own cost of effort is $c$, the opponent’s cost of effort is $d$, and the common prize is $\pi$. We make two observations. First, because in any given contest the two players’ win probabilities must sum to one,

$$P(\beta_1, \beta_2, k, \pi) + P(\beta_2, \beta_1, -k, \pi) = 1.$$
Second, by Lemma 4, \( P(c, d, k, \pi) \) is weakly decreasing in \( k \), so for \( k > 0 \) we have
\[
P(c, d, k, \pi) \leq P(c, d, -k, \pi).
\]
It follows that
\[
X_1 + X_2 = P\left(\beta_1, \beta_2, \frac{\gamma_U - \gamma_I}{2\gamma_M}, p\varepsilon + (1-p)(1-\varepsilon)\right) + P\left(\beta_2, \beta_1, \frac{\gamma_U - \gamma_I}{2\gamma_M}, p\varepsilon + (1-p)(1-\varepsilon)\right) \leq P\left(\beta_1, \beta_2, \frac{\gamma_U - \gamma_I}{2\gamma_M}, p\varepsilon + (1-p)(1-\varepsilon)\right) + P\left(\beta_2, \beta_1, -\frac{\gamma_U - \gamma_I}{2\gamma_M}, p\varepsilon + (1-p)(1-\varepsilon)\right) = 1,
\]
contradicting (23).

**Proof of Proposition 7**

We first show that if the funding system is deterministic and candidates are truthful, a candidate always wins with platform \( H \) against platform \( L \). Note that truthful campaign strategies imply that \( \nu_U(H, L) = \nu_I(H, L, h) = 1 \) and \( \nu_I(H, L, l) = 0 \), as shown in the proof of Proposition 5. Furthermore,
\[
Pr[s_I = h \mid x_1 = H, x_2 = L] = p(1-\varepsilon) + (1-p)\varepsilon.
\]
Also note that \( a_1 = a_2 \) implies \( z(a_1, a_2) = z(0, 0) = \frac{1}{2} \). Using the fact that \( \gamma_U > \gamma_I \), it follows that
\[
\Gamma \times \mathbb{E}_{s_I} \left[ \nu_U(H, L)\gamma_U + \nu_I(H, L, s_I)\gamma_I + z \left( \frac{\Gamma/2}{\beta}, \frac{\Gamma/2}{\beta} \right) \gamma_M \bigg| H, L \right] = \Gamma \times \left[ \gamma_U + \left[ p(1-\varepsilon) + (1-p)\varepsilon \right] \gamma_I + \frac{1}{2} \gamma_M \right] > \Gamma/2.
\]
Because the public funding system is deterministic, if \((x_1, x_2) = (H, L)\) the fixed point of the mapping (17) lies to the right of \( \Gamma/2 \). In turn, this implies that if \((x_1, x_2) = (L, H)\) then candidate 1 attracts all uninformed voters and receives more public funding than candidate 2. Thus, candidate 1 must win with probability one. A similar argument shows that if \((x_1, x_2) = (L, H)\) then candidate 2 must win with probability one. One can now proceed as in the proof of Proposition 2 to show that, if one candidate plays a truthful strategy, the other has an incentive to always set platform \( H \). Thus, an equilibrium with
truthful campaigns cannot exist, and Lemma 1 implies that voter welfare is bounded away from the first-best.

References


[7] California Watch. February 8, 2011. “Whitman Lost, but Payday was Sweet for Many.” californiawatch.org/dailyreport/whitman-lost-payday-was-sweet-many-8567.


