Sustainable Transmission Planning in Imperfectly Competitive Electricity Industries: Balancing Economic Efficiency and Environmental Outcomes

Afzal S. Siddiqui[a] **Makoto Tanaka**[b] **Yihsu Chen**[c]

[a]University College London, Stockholm University, and HEC Montréal, e-mail address: afzal.siddiqui@ucl.ac.uk

[b]National Graduate Institute for Policy Studies and Research Institute of Economy, Trade and Industry

[c]University of California at Santa Cruz

11 November 2016

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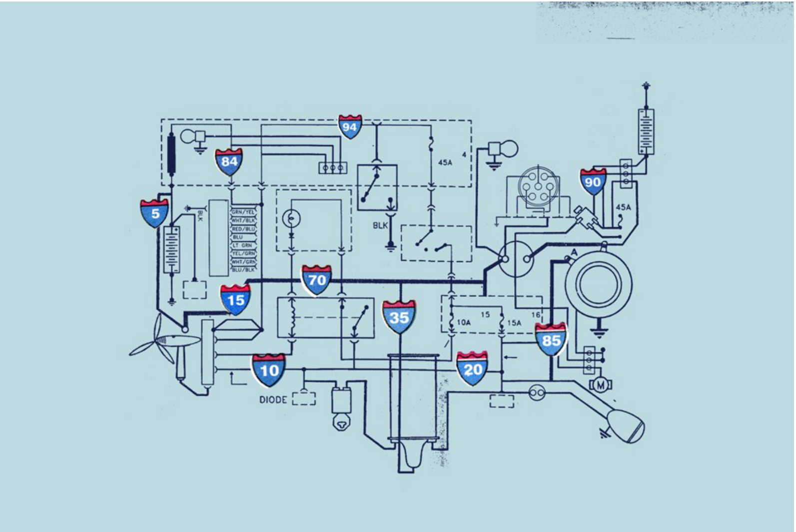
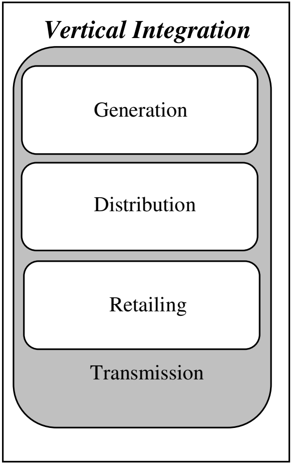
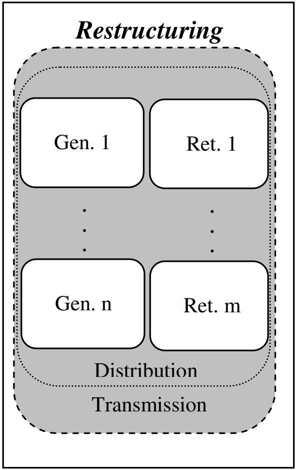
Introduction

“The Energy Interstate,” The Atlantic, June 2016



Evolving Paradigms





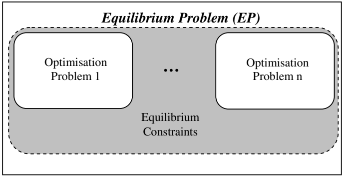
Deregulation and Decarbonisation

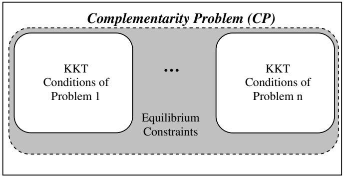
- **Regulated paradigm**
  - **Single decision maker**
  - **Single-level optimisation model for planning**
- **Post-restructuring**
  - **Imperfect competition**
  - **Endogenous price formation**
  - **Greater focus on sustainable energy**
- **Policymaker's dilemma**
  - **Comply with international treaties, e.g., EU 20-20-20 by 2020**
  - **Yet, cannot be seen to interfere with industry**
  - **Delicate balance between providing incentives to guide a sustainable energy transition and blunting the market**
  - **Need to understand the implications of strategic behaviour when designing markets or setting carbon taxes**

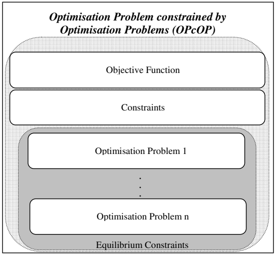
Transmission Planning in Deregulated Electricity Industries

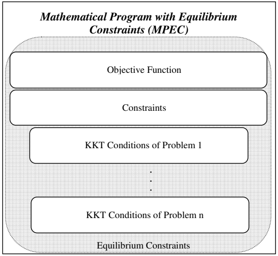
- **Borenstein et al. (2000) demonstrate the potential for enhanced competition from transmission expansion in a two-node Cournot duopoly model**
- **Sauma and Oren (2009) similarly examine the impact of financial transmission rights (FTRs)**
- **Sauma and Oren (2006, 2007) have tri-level models that illustrate the complexity for the transmission system operator (TSO) to obtain politically feasible transmission expansion**
- **Hobbs (2012) explores policy dilemmas involved with integration of renewable energy (RE)**
- **Bi-level transmission planning with RE investment: Baringo and Conejo (2013) and Maurovich-Horvat et al. (2015)**
- **Environmental economics literature examines the efficiency of policies under market power (Barnett, 1980; Requate, 2005)**

Playing Games









Research Objective and Findings

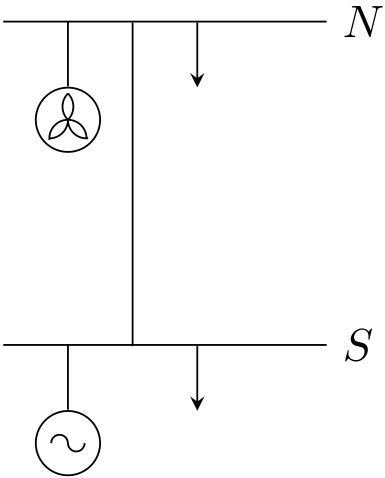
- **We address the problem of a welfare-maximising TSO that internalises the cost of damage from emissions**
- **Allow for strategic behaviour (Cournot oligopoly) or not (perfect competition) by lower-level producers**
- **Compare different market settings: central planner (CP), perfect competition (PC), and Cournot oligopoly (CO)**
- **Sustainable transmission investment is curtailed under PC**
- **A full carbon tax imposed on industry under PC results in a first-best solution**
- **However, a carbon tax under CO actually worsens welfare** *vis-à-vis* **doing nothing**

Mathematical Formulation and Analytical Insights

Assumptions

- **TSO's problem**
  - Set transmission line capacity $k \geq 0$ at upper level at cost $C_T k$
  - Maximise SW internalising cost of emissions damage $\frac{1}{2} D y_S^2$, $D \geq 0$
- **Industry's profit-maximisation problems**
  - Power output y_N and y_S by RE and NRE sectors at the lower level, respectively
  - Linear inverse demand function $p_j(x_j) = A_j - B_j x_j$ at node $j = N, S$ with $A_S > A_N$ and $B_j > 0$
  - Linear long-run costs, i.e., $c_j(y_j) = C_j y_j$ with $C_N > C_S$
  - Assume $A_S > A_N > C_N > C_T > C_S > 0$ and market clearing by welfare-maximising ISO
- **Market settings: denote $\bar{\cdot}$, $\hat{\cdot}$, and $\cdot^*$ as the optimal values for decision variables in CP, PC, and CO, respectively**

Network



Single-Level Quadratic Programming Problem

- **CP's SW maximisation problem**

$$
\begin{aligned}
\max_{y_N \geq 0, y_S \geq 0, t, k \geq 0} \quad & \left[A_S\left(y_S - t\right) - \frac{1}{2}B_S\left(y_S - t\right)^2\right] \\
& + \left[A_N\left(y_N + t\right) - \frac{1}{2}B_N\left(y_N + t\right)^2\right] \\
& - \sum_j C_j y_j - C_T k - \frac{1}{2} D y_S^2 \qquad (1) \\
\text{s.t.} \quad & \left(\lambda^-\right) - k \leq t \leq k \left(\lambda^+\right) \qquad (2) \\
& y_N + t \geq 0 \left(\beta_N\right) \qquad (3) \\
& y_S - t \geq 0 \left(\beta_S\right) \qquad (4)
\end{aligned}
$$

Single-Level Quadratic Programming Problem

- **CP's SW maximisation problem**

$$\max_{y_N \geq 0, y_S \geq 0, t, k \geq 0} \left[A_S (y_S - t) - \frac{1}{2} B_S (y_S - t)^2 \right] + \left[A_N (y_N + t) - \frac{1}{2} B_N (y_N + t)^2 \right] - \sum_j C_j y_j - C_T k - \frac{1}{2} D y_S^2 \quad (1)$$

s.t.

$$(\lambda^-) \; -k \leq t \leq k \; (\lambda^+) \quad (2)$$

$$y_N + t \geq 0 \; (\beta_N) \quad (3)$$

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CP's KKT Conditions

$$0 \leq y_S \perp -[A_S - B_S(y_S - t)] + C_S - \beta_S + Dy_S \geq 0 \quad (5)$$

$$0 \leq y_N \perp -[A_N - B_N(y_N + t)] + C_N - \beta_N \geq 0 \quad (6)$$

$$0 \leq k \perp C_T - \lambda^+ - \lambda^- \geq 0 \quad (7)$$

$$-A_S + B_S(y_S - t) + A_N - B_N(y_N + t)$$

$$-\lambda^+ + \lambda^- + \beta_N - \beta_S = 0 \text{ with } t \text{ u.r.s.} \quad (8)$$

$$0 \leq \lambda^+ \perp k - t \geq 0 \quad (9)$$

$$0 \leq \lambda^- \perp k + t \geq 0 \quad (10)$$

$$0 \leq \beta_N \perp y_N + t \geq 0 \quad (11)$$

$$0 \leq \beta_S \perp y_S - t \geq 0 \quad (12)$$

Solution to CP Problem

- **Case 1:** $\bar{k} > 0,\ \bar{y}_N = 0,\ \bar{y}_S > 0,\ \bar{t} > 0$

$$\bar{y}_S(\bar{k}) = \frac{A_S - C_S + B_S\bar{k}}{B_S + D} \tag{13}$$

$$\bar{k} = \frac{D(A_N - A_S - C_T) + B_S(A_N - C_S - C_T)}{(B_S + B_N)D + B_N B_S} \tag{14}$$

- **Case 2:** $\bar{k} > 0,\ \bar{y}_N > 0,\ \bar{y}_S > 0,\ \bar{t} > 0$

$$\bar{y}_S(\bar{k}) = \frac{A_S - C_S + B_S\bar{k}}{B_S + D} \tag{15}$$

$$\bar{y}_N(\bar{k}) = \frac{A_N - C_N - B_N\bar{k}}{B_N} \tag{16}$$

$$\bar{k} = \frac{(C_N - C_T)(B_S + D) - B_S C_S - A_S D}{B_S D} \tag{17}$$

- **Case 3:** $\bar{k} = 0,\ \bar{y}_N > 0,\ \bar{y}_S > 0,\ \bar{t} = 0$
- **Case 4:** $\bar{k} > 0,\ \bar{y}_N > 0,\ \bar{y}_S > 0,\ \bar{t} < 0$

$$\bar{y}_S(\bar{k}) = \frac{A_S - C_S - B_S\bar{k}}{B_S + D} \tag{18}$$

$$\bar{y}_N(\bar{k}) = \frac{A_N - C_N + B_N\bar{k}}{B_N} \tag{19}$$

$$\bar{k} = \frac{-(C_N + C_T)(B_S + D) + B_S C_S + A_S D}{B_S D} \tag{20}$$

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Properties of CP Solution

Proposition

Under CP, ceteris paribus increases in D result in:

Case 1: $\frac{\partial \bar{k}}{\partial D} < 0$, $\frac{\partial \bar{y}_S}{\partial D} < 0$.

Case 2: $\frac{\partial \bar{k}}{\partial D} < 0$, $\frac{\partial \bar{y}_S}{\partial D} < 0$, $\frac{\partial \bar{y}_N}{\partial D} > 0$.

Case 3: $\frac{\partial \bar{y}_S}{\partial D} < 0$, $\frac{\partial \bar{y}_N}{\partial D} = 0$.

Case 4: $\frac{\partial \bar{k}}{\partial D} > 0$, $\frac{\partial \bar{y}_S}{\partial D} < 0$, $\frac{\partial \bar{y}_N}{\partial D} > 0$.

Proposition

Under CP, ceteris paribus increases in D result in decreases in the damage cost from emissions in Cases 2 and 4.

Lower Level: Producers' Problems and MCP

- **Each optimisation problem is convex:**

$$\max_{y_S \geq 0} \quad p_S y_S - C_S y_S \tag{21}$$

$$\max_{y_N \geq 0} \quad p_N y_N - C_N y_N \tag{22}$$

$$\max_t \quad [A_N - B_N y_N]\, t - [A_S - B_S y_S]\, t - \frac{1}{2}(B_N + B_S)\, t^2 \tag{23}$$

s.t. **Equations** (2)–(4)

- **Problems** (21)–(23) **may be replaced by their KKT conditions**

$$0 \leq y_S \perp -[A_S - B_S (y_S - t)] + C_S \geq 0 \tag{24}$$

$$0 \leq y_N \perp -[A_N - B_N (y_N + t)] + C_N \geq 0 \tag{25}$$

$$-A_N + B_N y_N + A_S - B_S y_S + B_N t + B_S t + \lambda^+ - \lambda^- - \beta_N + \beta_S = 0 \text{ with } t \text{ u.r.s.} \tag{26}$$

Equations (9)–(12)

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Upper Level: TSO's MPEC and NLP

- Replacing the lower level via its MCP enables us to convert the bi-level problem into an MPEC:

$$\max_{\{k\geq 0\}\cup\Omega\cup\Xi} \quad \text{Equation (1)} \tag{27}$$
$$\text{s.t.} \quad \text{Equations (9)–(12), (24)–(26)}$$

$$\Omega \equiv \{y_N \geq 0, y_S \geq 0, t\},\ \Xi \equiv \{\lambda^+ \geq 0, \lambda^- \geq 0, \beta_N \geq 0, \beta_S \geq 0\}$$

- Using the solutions $\hat{y}_N(k)$, $\hat{y}_S(k)$, and $\hat{t}(k)$, we can next replace sets Ω and Ξ in the MPEC, thereby turning it into an unconstrained non-linear programming (NLP) problem

$$\begin{aligned}\max_{k\geq 0} \quad & A_S\left(\hat{y}_S(k)-\hat{t}(k)\right)-\frac{1}{2}B_S\left(\hat{y}_S(k)-\hat{t}(k)\right)^2-C_S\hat{y}_S(k)\\ & +A_N\left(\hat{y}_N(k)+\hat{t}(k)\right)-\frac{1}{2}B_N\left(\hat{y}_N(k)+\hat{t}(k)\right)^2-C_N\hat{y}_N(k)\\ & -C_Tk-\frac{1}{2}D\hat{y}_S(k)^2\end{aligned} \tag{28}$$

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$$\Omega \equiv \{y_N \geq 0, y_S \geq 0, t\},\ \Xi \equiv \{\lambda^+ \geq 0, \lambda^- \geq 0, \beta_N \geq 0, \beta_S \geq 0\}$$

- **Using the solutions $\hat{y}_N(k)$, $\hat{y}_S(k)$, and $\hat{t}(k)$, we can next replace sets Ω and Ξ in the MPEC, thereby turning it into an unconstrained non-linear programming (NLP) problem**

$$\begin{aligned} \max_{k\geq 0} \quad & A_S\left(\hat{y}_S(k) - \hat{t}(k)\right) - \frac{1}{2}B_S\left(\hat{y}_S(k) - \hat{t}(k)\right)^2 - C_S\hat{y}_S(k) \\ & + A_N\left(\hat{y}_N(k) + \hat{t}(k)\right) - \frac{1}{2}B_N\left(\hat{y}_N(k) + \hat{t}(k)\right)^2 - C_N\hat{y}_N(k) \\ & - C_T k - \frac{1}{2}D\hat{y}_S(k)^2 \end{aligned} \tag{28}$$

Solution under PC

- **Case 1:** $\hat{k} > 0,\ \hat{y}_N = 0,\ \hat{y}_S > 0,\ \hat{t} > 0$

$$\hat{y}_S(\hat{k}) = \frac{A_S - C_S + B_S \hat{k}}{B_S} \tag{29}$$

$$\hat{k} = \frac{D(C_S - A_S) + B_S(A_N - C_S - C_T)}{B_S(D + B_N)} \tag{30}$$

- **Case 2:** $\hat{k} > 0,\ \hat{y}_N > 0,\ \hat{y}_S > 0,\ \hat{t} > 0$

$$\hat{y}_S(\hat{k}) = \frac{A_S - C_S + B_S \hat{k}}{B_S} \tag{31}$$

$$\hat{y}_N(\hat{k}) = \frac{A_N - C_N - B_N \hat{k}}{B_N} \tag{32}$$

$$\hat{k} = \frac{D(C_S - A_S) + B_S(C_N - C_S - C_T)}{B_S D} \tag{33}$$

- **Case 3:** $\hat{k} = 0,\ \hat{y}_N > 0,\ \hat{y}_S > 0,\ \hat{t} = 0$

Solution under PC

- **Case 1:** $\hat{k} > 0,\ \hat{y}_N = 0,\ \hat{y}_S > 0,\ \hat{t} > 0$

$$\hat{y}_S(\hat{k}) = \frac{A_S - C_S + B_S\hat{k}}{B_S} \quad (29)$$

$$\hat{k} = \frac{D(C_S - A_S) + B_S(A_N - C_S - C_T)}{B_S(D + B_N)} \quad (30)$$

- **Case 2:** $\hat{k} > 0,\ \hat{y}_N > 0,\ \hat{y}_S > 0,\ \hat{t} > 0$

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$$\hat{k} = \frac{D(C_S - A_S) + B_S(C_N - C_S - C_T)}{B_S D} \quad (33)$$

- **Case 3:** $\hat{k} = 0,\ \hat{y}_N > 0,\ \hat{y}_S > 0,\ \hat{t} = 0$

Properties of PC Solution

Proposition

Under a solution for PC, ceteris paribus increases in D result in:

Case 1: $\frac{\partial \hat{k}}{\partial D} < 0$, $\frac{\partial \hat{y}_S}{\partial D} < 0$.

Case 2: $\frac{\partial \hat{k}}{\partial D} < 0$, $\frac{\partial \hat{y}_S}{\partial D} < 0$, $\frac{\partial \hat{y}_N}{\partial D} > 0$.

Case 3: $\frac{\partial \hat{y}_S}{\partial D} = 0$, $\frac{\partial \hat{y}_N}{\partial D} = 0$.

Proposition

Under a solution for PC, ceteris paribus increases in D result in a decrease and an increase in the damage cost from emissions in Cases 2 and 3, respectively.

Lower Level: Producers' Problems and MCP

- **Each optimisation problem is convex:**

$$\max_{y_S \geq 0} \quad [A_S - B_S (y_S - t)]\, y_S - C_S y_S \tag{34}$$

$$\max_{y_N \geq 0} \quad [A_N - B_N (y_N + t)]\, y_N - C_N y_N \tag{35}$$

- **ISO's problem is as before, and problems** (34)–(35) **may be replaced by their KKT conditions**

$$0 \leq y_S \perp -[A_S - B_S (2y_S - t)] + C_S \geq 0 \tag{36}$$

$$0 \leq y_N \perp -[A_N - B_N (2y_N + t)] + C_N \geq 0 \tag{37}$$

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$$0 \leq y_N \perp -[A_N - B_N (2y_N + t)] + C_N \geq 0 \tag{37}$$

Upper Level: TSO's MPEC and NLP

- Replacing the lower level via its MCP enables us to convert the bi-level problem into an MPEC:

$$\max_{\{k\geq 0\}\cup\Omega\cup\Xi} \quad \text{Equation (1)} \tag{38}$$
$$\text{s.t.} \quad \text{Equations (9)–(12), (26), (36)–(37)}$$

- Using the solutions $y_N^*(k)$, $y_S^*(k)$, and $t^*(k)$, we can next replace sets Ω and Ξ in the MPEC, thereby turning it into an unconstrained non-linear programming (NLP) problem

$$\begin{aligned}\max_{k\geq 0} \quad & A_S\left(y_S^*(k) - t^*(k)\right) - \frac{1}{2}B_S\left(y_S^*(k) - t^*(k)\right)^2 - C_S y_S^*(k) \\ & + A_N\left(y_N^*(k) + t^*(k)\right) - \frac{1}{2}B_N\left(y_N^*(k) + t^*(k)\right)^2 - C_N y_N^*(k) \\ & - C_T k - \frac{1}{2}D y_S^*(k)^2 \end{aligned} \tag{39}$$

Upper Level: TSO's MPEC and NLP

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$$\max_{\{k\geq 0\}\cup\Omega\cup\Xi} \quad \textbf{Equation } (1) \tag{38}$$
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- **Using the solutions $y_N^*(k)$, $y_S^*(k)$, and $t^*(k)$, we can next replace sets Ω and Ξ in the MPEC, thereby turning it into an unconstrained non-linear programming (NLP) problem**

$$\max_{k\geq 0} \quad A_S\left(y_S^*(k)-t^*(k)\right)-\frac{1}{2}B_S\left(y_S^*(k)-t^*(k)\right)^2-C_S y_S^*(k)$$
$$+A_N\left(y_N^*(k)+t^*(k)\right)-\frac{1}{2}B_N\left(y_N^*(k)+t^*(k)\right)^2-C_N y_N^*(k)$$
$$-C_T k-\frac{1}{2}D y_S^*(k)^2 \tag{39}$$

Solution under CO

- **Case 1:** $k^* > 0,\ y_N^* = 0,\ y_S^* > 0,\ t^* > 0$

$$y_S^*(k^*) = \frac{A_S - C_S + B_S k^*}{2B_S} \quad (40)$$

$$k^* = \frac{4A_N - A_S - 3C_S - 4C_T - \frac{D}{B_S}(A_S - C_S)}{B_S + 4B_N + D} \quad (41)$$

- **Case 2:** $k^* > 0,\ y_N^* > 0,\ y_S^* > 0,\ t^* > 0$

$$y_S^*(k^*) = \frac{A_S - C_S + B_S k^*}{2B_S} \quad (42)$$

$$y_N^*(k^*) = \frac{A_N - C_N - B_N k^*}{2B_N} \quad (43)$$

$$k^* = \frac{A_N + 3C_N - A_S - 3C_S - 4C_T - \frac{D}{B_S}(A_S - C_S)}{B_S + B_N + D} \quad (44)$$

- **Case 3:** $k^* = 0,\ y_N^* > 0,\ y_S^* > 0,\ t^* = 0$
- **Case 4:** $k^* > 0,\ y_N^* > 0,\ y_S^* > 0,\ t^* < 0$

$$y_S^*(k^*) = \frac{A_S - C_S - B_S k^*}{2B_S} \quad (45)$$

$$y_N^*(k^*) = \frac{A_N - C_N + B_N k^*}{2B_N} \quad (46)$$

$$k^* = \frac{A_S + 3C_S - A_N - 3C_N - 4C_T + \frac{D}{B_S}(A_S - C_S)}{B_S + B_N + D} \quad (47)$$

Solution under CO

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Properties of CO Solution

Proposition

Under a solution for CO, ceteris paribus increases in $\boldsymbol{D}$ result in:

Case 1: $\frac{\partial k^*}{\partial D} < 0$, $\frac{\partial y_S^*}{\partial D} < 0$.

Case 2: $\frac{\partial k^*}{\partial D} < 0$, $\frac{\partial y_S^*}{\partial D} < 0$, $\frac{\partial y_N^*}{\partial D} > 0$.

Case 3: $\frac{\partial y_S^*}{\partial D} = 0$, $\frac{\partial y_N^*}{\partial D} = 0$.

Case 4: $\frac{\partial k^*}{\partial D} > 0$, $\frac{\partial y_S^*}{\partial D} < 0$, $\frac{\partial y_N^*}{\partial D} > 0$.

Proposition

Under a solution for CO, ceteris paribus increases in $\boldsymbol{D}$ result in an increase in the damage cost from emissions in Case 3.

Proposition

Under solutions for CP, PC, and CO, ceteris paribus increases in $\boldsymbol{D}$ result in decreases in social welfare.

Carbon Tax: Perfect Competition

- The NRE sector's problem under a carbon tax of rate $0 \leq E \leq 1$:

$$\max_{y_S \geq 0} \quad p_S y_S - C_S y_S - \frac{1}{2} E D y_S^2$$
$$\Rightarrow \quad 0 \leq y_S \perp -[A_S - B_S(y_S - t)] + C_S + E D y_S \geq 0 \quad (48)$$

- The TSO's MPEC is now:

$$\max_{\{k \geq 0\} \cup \Omega \cup \Xi} \quad \text{Equation (1)} \quad (49)$$
$$\text{s.t.} \quad \text{Equations (9)–(12), (25)–(26), (48)}$$

Proposition

If a full carbon tax, i.e., $E = 1$, is imposed under PC, then the TSO's problem is equivalent to that of CP.

Proposition

Under a solution for PC with $E \in (0, 1)$, a more stringent carbon tax leads to an increase (decrease) in the transmission capacity, i.e., $\frac{\partial \hat{k}}{\partial E} > 0$ ($\frac{\partial \hat{k}}{\partial E} < 0$) for Cases 1 and 2 (Case 4).

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Carbon Tax: Cournot Oligopoly

- The NRE sector's problem under a carbon tax of rate $0 \leq E \leq 1$:

$$\max_{y_S \geq 0} \quad [A_S - B_S(y_S - t)]\, y_S - C_S y_S - \frac{1}{2} E D y_S^2$$
$$\Rightarrow \quad 0 \leq y_S \perp -[A_S - B_S(2y_S - t)] + C_S + EDy_S \geq 0 \quad (50)$$

- The TSO's MPEC is now:

$$\max_{\{k \geq 0\} \cup \Omega \cup \Xi} \quad \text{Equation (1)} \quad (51)$$
$$\text{s.t.} \quad \text{Equations (9)–(12), (26), (37), (50)}$$

Proposition

Under a solution for CO with $E \in (0,1)$, if $E > 1 - \frac{B_S}{D}$, then a more stringent carbon tax leads to a decrease (increase) in the transmission capacity, i.e., $\frac{\partial k^*}{\partial E} < 0$ ($\frac{\partial k^*}{\partial E} > 0$) for Cases 1 and 2 (Case 4), and vice versa.

Carbon Tax: Cournot Oligopoly

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$$\Rightarrow \quad 0 \leq y_S \perp -\left[A_S - B_S\left(2y_S - t\right)\right] + C_S + E D y_S \geq 0 \quad (50)$$

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$$\max_{\{k \geq 0\} \cup \Omega \cup \Xi} \quad \text{Equation (1)} \quad (51)$$
$$\text{s.t.} \quad \text{Equations (9)–(12), (26), (37), (50)}$$

Proposition

Under a solution for CO with $E \in (0,1)$, if $E > 1 - \frac{B_S}{D}$, then a more stringent carbon tax leads to a decrease (increase) in the transmission capacity, i.e., $\frac{\partial k^*}{\partial E} < 0$ ($\frac{\partial k^*}{\partial E} > 0$) for Cases 1 and 2 (Case 4), and vice versa.

Welfare Analysis of a Carbon Tax

Proposition

Under a solution for PC with $E \in (0, 1)$, a more stringent carbon tax leads to an increase in social welfare, i.e., $\frac{\partial s\hat{w}}{\partial E} > 0$.

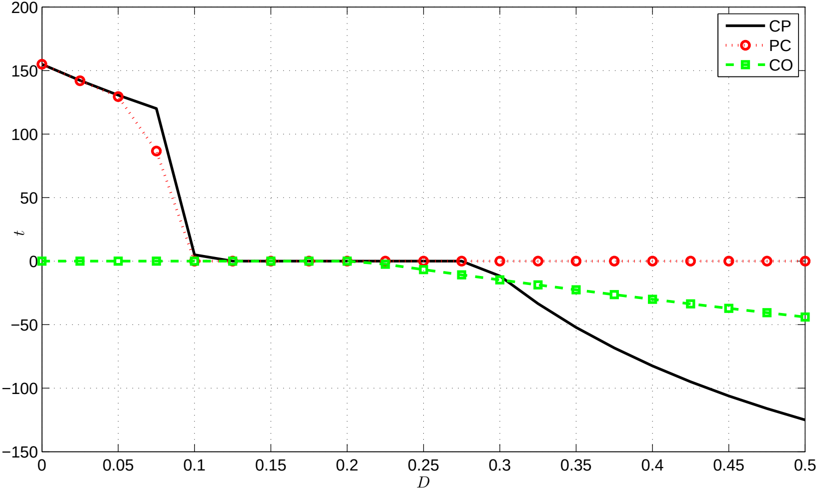
Proposition

Under a solution for CO with $E \in (0, 1)$, if $E > 1 - \frac{B_S}{D}$, then a more stringent carbon tax leads to a decrease in social welfare, i.e., $\frac{\partial sw^*}{\partial E} < 0$, and vice versa.

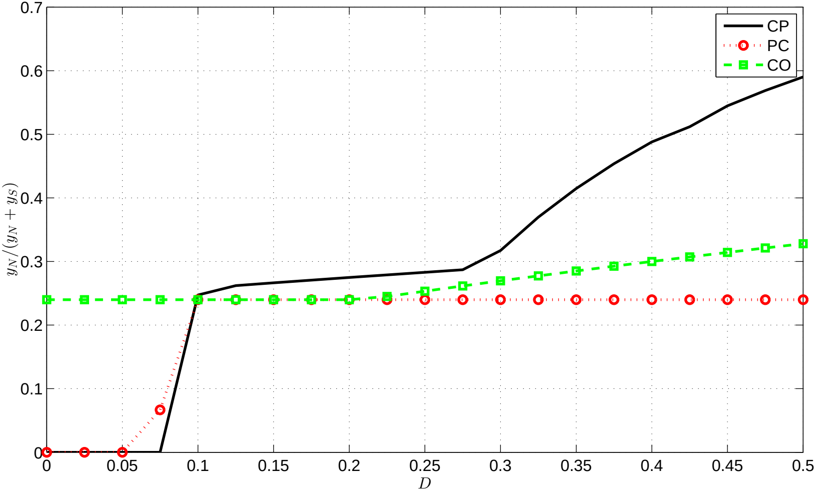
Numerical Examples

Optimal Net Import at Node N

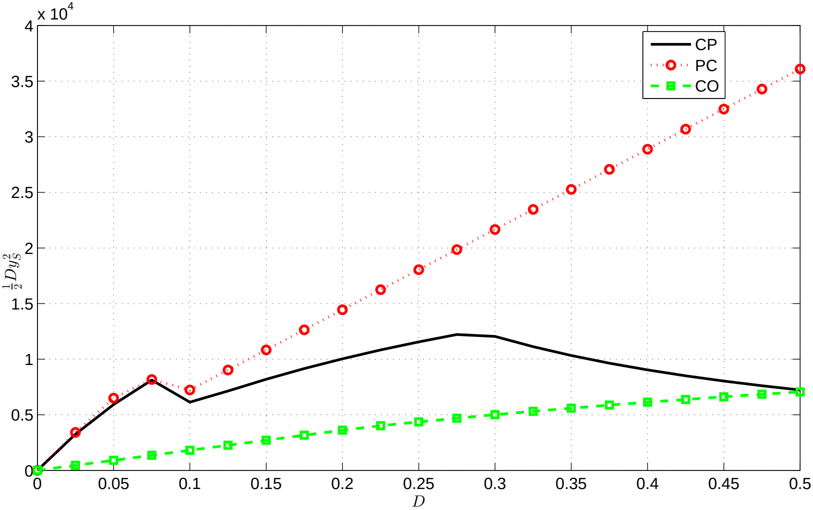
$C_S = 20$, $C_T = 25$, $C_N = 80$, $A_S = 400$, $A_N = 200$, $B_S = 1$, $B_N = 1$, $D \in [0, 0.5]$



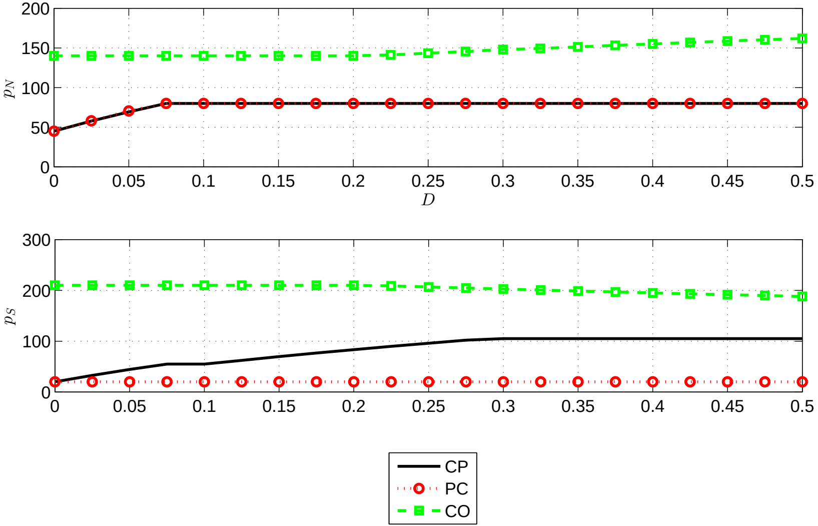
Fraction of Renewable Energy



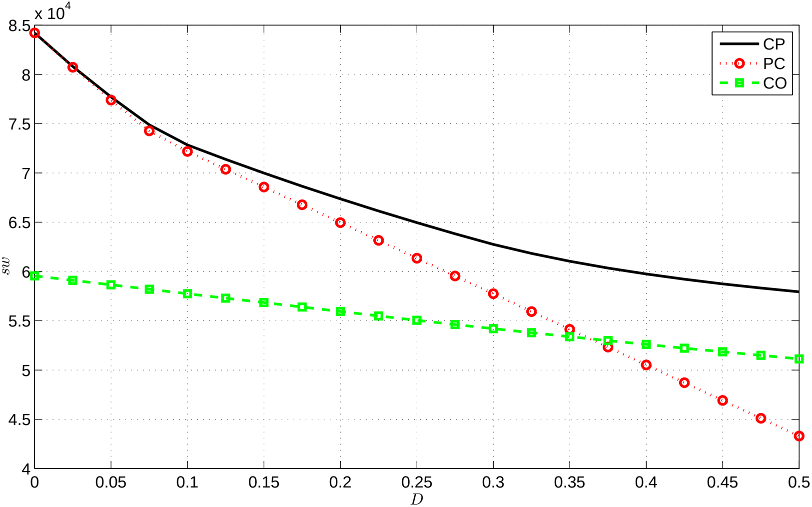
Equilibrium Damage Cost



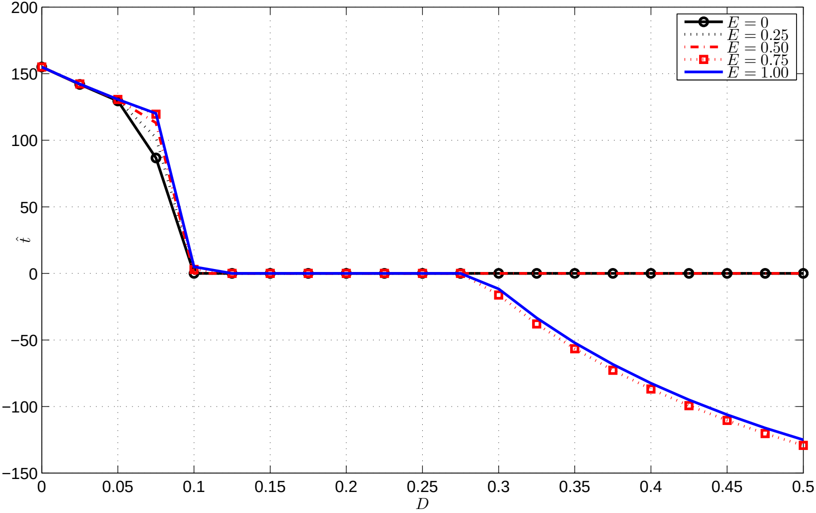
Equilibrium Nodal Prices



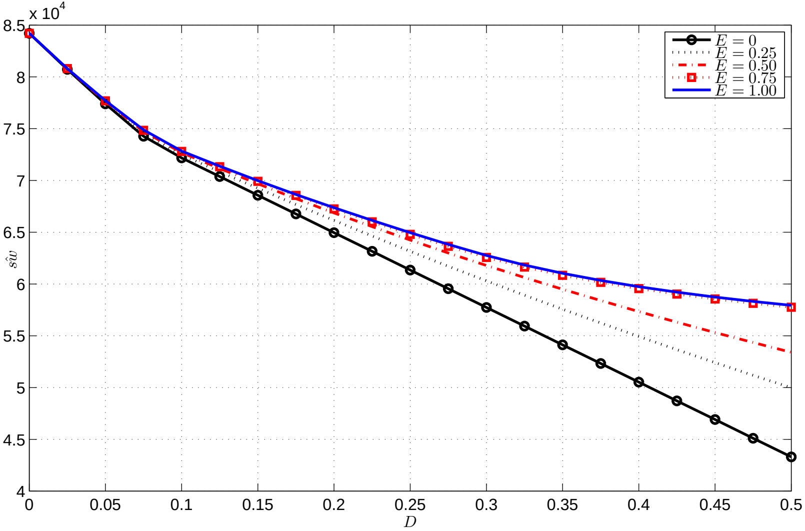
Maximised Social Welfare



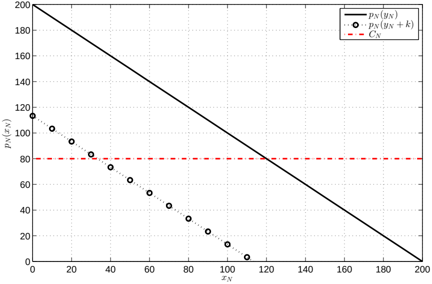
Optimal Net Import at Node N under PC



Maximised Social Welfare under PC

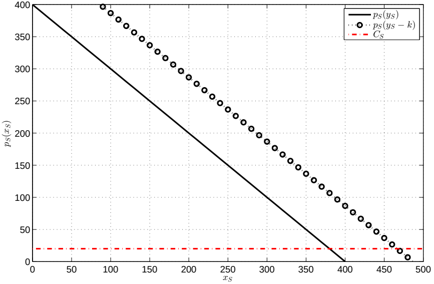


Power Sectors' Incentives without a Carbon Tax under PC

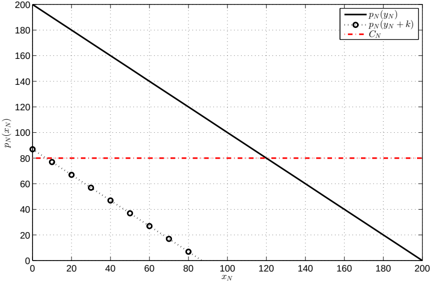


$$A_S(\hat{y}'_S(k)-1) - B_S(\hat{y}_S(k) - k)(\hat{y}'_S(k)-1) - C_S\hat{y}'_S(k) - C_N\hat{y}'_N(k) - C_T - D\hat{y}_S(k)\hat{y}'_S(k) = 0$$

$$\Rightarrow C_N = C_T + C_S + D\hat{y}_S(k)$$

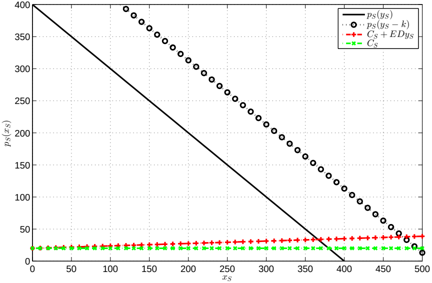


Power Sectors' Incentives with $E = 0.50$ Carbon Tax under PC

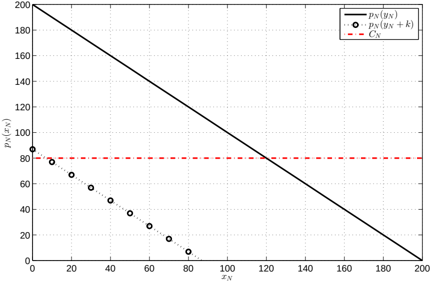


$$A_S(\hat{y}'_S(k) - 1) - B_S(\hat{y}_S(k) - k)(\hat{y}'_S(k) - 1) - C_S\hat{y}'_S(k) - C_N\hat{y}'_N(k) - C_T - D\hat{y}_S(k)\hat{y}'_S(k) = 0$$

$$\Rightarrow C_N = C_T + C_S\hat{y}'_S(k) + D\hat{y}_S(k)\hat{y}'_S(k) - (\hat{y}'_S(k) - 1)\hat{p}_S$$

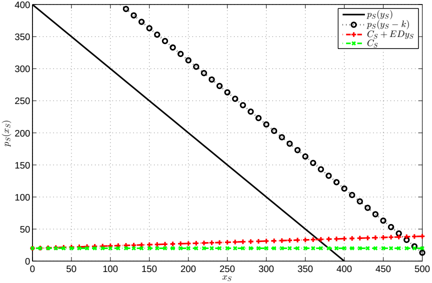


Power Sectors' Incentives with $E = 0.50$ Carbon Tax under PC

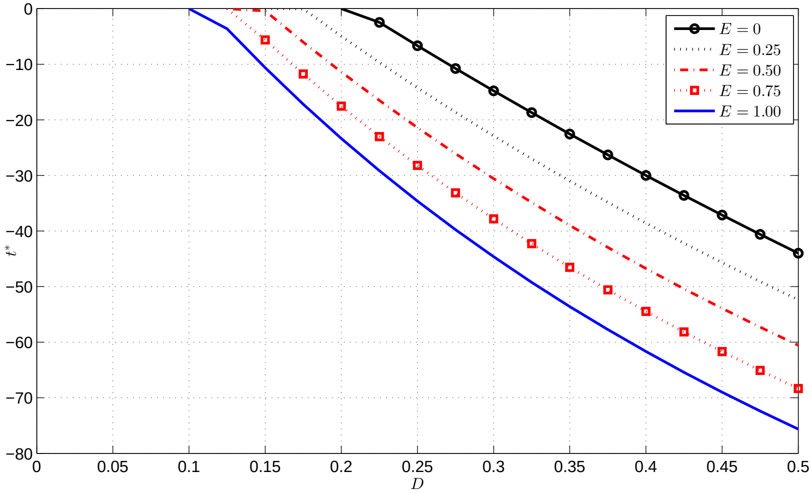


$$A_S(\hat{y}'_S(k) - 1) - B_S(\hat{y}_S(k) - k)(\hat{y}'_S(k) - 1) - C_S\hat{y}'_S(k) - C_N\hat{y}'_N(k) - C_T - D\hat{y}_S(k)\hat{y}'_S(k) = 0$$

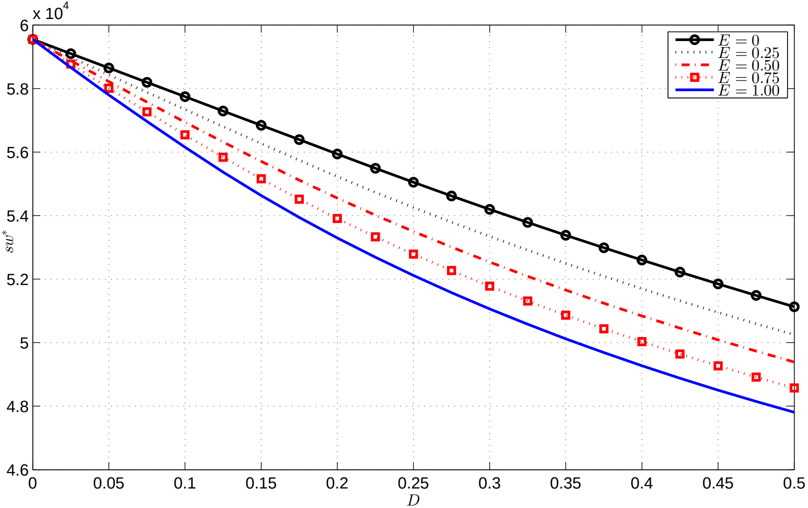
$$\Rightarrow C_N = C_T + C_S\hat{y}'_S(k) + D\hat{y}_S(k)\hat{y}'_S(k) - (\hat{y}'_S(k) - 1)\hat{p}_S$$



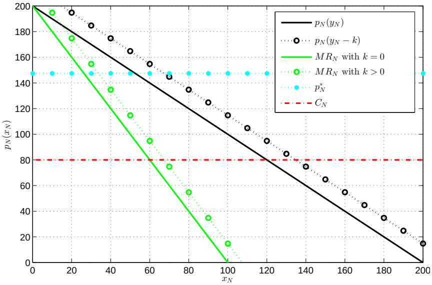
Optimal Net Import at Node N under CO



Maximised Social Welfare under CO

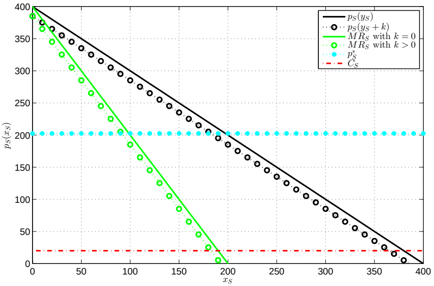


Power Sectors' Incentives without a Carbon Tax under CO

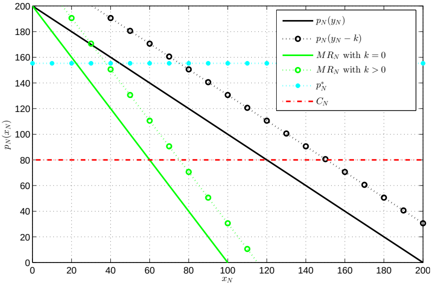


$$A_S(y_S^{*\prime}(k)+1) - B_S(y_S^*(k)+k)(y_S^{*\prime}(k)+1) - C_S y_S^{*\prime}(k) + A_N(y_N^{*\prime}(k)-1) - B_N(y_N^*(k)-k)(y_N^{*\prime}(k)-1) - C_N y_N^{*\prime}(k) - C_T - D y_S^*(k) y_S^{*\prime}(k) = 0$$

$$\Rightarrow \frac{p_S^*}{2} + \frac{C_S}{2} + \frac{D y_S^*(k)}{2} = C_T + \frac{p_N^*}{2} + \frac{C_N}{2}$$

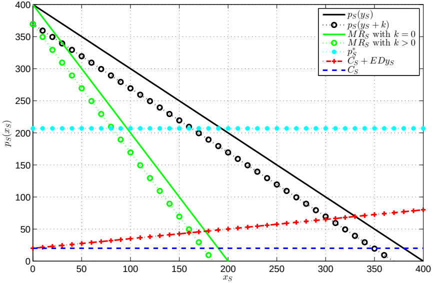


Power Sectors' Incentives with $E = 0.50$ Carbon Tax under CO

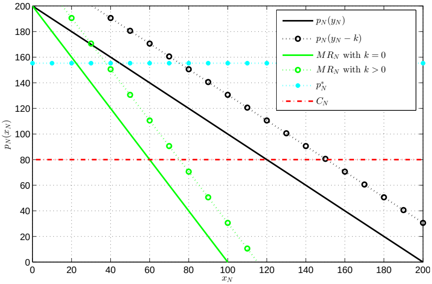


$$A_S(y_S^{*\prime}(k)+1) - B_S(y_S^*(k)+k)(y_S^{*\prime}(k)+1) - C_S y_S^{*\prime}(k) + A_N(y_N^{*\prime}(k)-1) - B_N(y_N^*(k)-k)(y_N^{*\prime}(k)-1) - C_N y_N^{*\prime}(k) - C_T - D y_S^*(k) y_S^{*\prime}(k) = 0$$

$$\Rightarrow p_S^*(y_S^{*\prime}(k)+1) - C_S y_S^{*\prime}(k) - D y_S^*(k) y_S^{*\prime}(k) = C_T + \frac{p_N^*}{2} + \frac{C_N}{2}$$

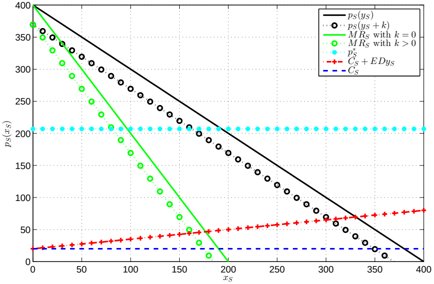


Power Sectors' Incentives with $E = 0.50$ Carbon Tax under CO



$$A_S(y_S^{*\prime}(k)+1) - B_S(y_S^*(k)+k)(y_S^{*\prime}(k)+1) - C_S y_S^{*\prime}(k) + A_N(y_N^{*\prime}(k)-1) - B_N(y_N^*(k)-k)(y_N^{*\prime}(k)-1) - C_N y_N^{*\prime}(k) - C_T - D y_S^*(k) y_S^{*\prime}(k) = 0$$

$$\Rightarrow p_S^*(y_S^{*\prime}(k)+1) - C_S y_S^{*\prime}(k) - D y_S^*(k) y_S^{*\prime}(k) = C_T + \frac{p_N^*}{2} + \frac{C_N}{2}$$



Conclusions

Summary

- Complementarity approach to compare CP, PC, and CO settings in analysing sustainable transmission expansion
  - CP matches the most efficient resource with demand but may not curb emissions via RE until cost of damage is high
  - PC: TSO's inability to induce consumption reduction leads to smaller line and relatively more RE generation initially
  - CO: power sectors' market power is used by the TSO to induce a reversal in the prevalent flows
  - A full carbon tax results in perfect alignment of incentives under PC: increasing E holding k fixed reduces $\hat{y}_S(k)$, and reducing $\hat{y}_S(k)$ increases maximised SW by exactly $(1 - E)D\hat{y}_S(k)$
  - However, the same regulation worsens outcomes under CO: reducing $y_S^*(k)$ decreases maximised SW by exactly $B_S y_S^*(k)$ while also increasing maximised SW by exactly $(1 - E)Dy_S^*(k)$
- Future work: stochastic model, endogenous carbon pricing, energy storage