Sustainable Transmission Planning in Imperfectly Competitive Electricity Industries: Balancing Economic Efficiency and Environmental Outcomes

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Introduction
Evolving Paradigms

**Vertical Integration**
- Generation
- Distribution
- Retailing
- Transmission

**Restructuring**
- Gen. 1
- Ret. 1
- Gen. n
- Ret. m
- Distribution
- Transmission
Deregulation and Decarbonisation

- **Regulated paradigm**
  - Single decision maker
  - Single-level optimisation model for planning

- **Post-restructuring**
  - Imperfect competition
  - Endogenous price formation
  - Greater focus on sustainable energy

- **Policymaker’s dilemma**
  - Comply with international treaties, e.g., EU 20-20-20 by 2020
  - Yet, cannot be seen to interfere with industry
  - Delicate balance between providing incentives to guide a sustainable energy transition and blunting the market
  - Need to understand the implications of strategic behaviour when designing markets or setting carbon taxes
Transmission Planning in Deregulated Electricity Industries

- Borenstein et al. (2000) demonstrate the potential for enhanced competition from transmission expansion in a two-node Cournot duopoly model.
- Sauma and Oren (2009) similarly examine the impact of financial transmission rights (FTRs).
- Sauma and Oren (2006, 2007) have tri-level models that illustrate the complexity for the transmission system operator (TSO) to obtain politically feasible transmission expansion.
- Environmental economics literature examines the efficiency of policies under market power (Barnett, 1980; Requate, 2005).
Playing Games

**Equilibrium Problem (EP)**

- Optimisation Problem 1
- ... 
- Optimisation Problem n
- Equilibrium Constraints

**Complementarity Problem (CP)**

- KKT Conditions of Problem 1
- ... 
- KKT Conditions of Problem n
- Equilibrium Constraints

**Optimisation Problem constrained by Optimisation Problems (OPcOP)**

- Objective Function
- Constraints
- Optimisation Problem 1
- ... 
- Optimisation Problem n
- Equilibrium Constraints

**Mathematical Program with Equilibrium Constraints (MPEC)**

- Objective Function
- Constraints
- KKT Conditions of Problem 1
- ... 
- KKT Conditions of Problem n
- Equilibrium Constraints
We address the problem of a welfare-maximising TSO that internalises the cost of damage from emissions.

Allow for strategic behaviour (Cournot oligopoly) or not (perfect competition) by lower-level producers.

Compare different market settings: central planner (CP), perfect competition (PC), and Cournot oligopoly (CO).

Sustainable transmission investment is curtailed under PC.

A full carbon tax imposed on industry under PC results in a first-best solution.

However, a carbon tax under CO actually worsens welfare vis-à-vis doing nothing.
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Mathematical Formulation and Analytical Insights
Assumptions

- **TSO’s problem**
  - Set transmission line capacity $k \geq 0$ at upper level at cost $C_T k$
  - Maximise SW internalising cost of emissions damage $\frac{1}{2} D y_S^2$, $D \geq 0$

- **Industry’s profit-maximisation problems**
  - Power output $y_N$ and $y_S$ by RE and NRE sectors at the lower level, respectively
  - Linear inverse demand function $p_j(x_j) = A_j - B_j x_j$ at node $j = N, S$ with $A_S > A_N$ and $B_j > 0$
  - Linear long-run costs, i.e., $c_j(y_j) = C_j y_j$ with $C_N > C_S$
  - Assume $A_S > A_N > C_N > C_T > C_S > 0$ and market clearing by welfare-maximising ISO

- **Market settings**: denote $\bar{\cdot}$, $\hat{\cdot}$, and $\cdot^*$ as the optimal values for decision variables in CP, PC, and CO, respectively
Single-Level Quadratic Programming Problem

- CP’s SW maximisation problem

\[
\max_{y_N \geq 0, y_S \geq 0, t, k \geq 0} \left[ A_S (y_S - t) - \frac{1}{2} B_S (y_S - t)^2 \right] \\
+ \left[ A_N (y_N + t) - \frac{1}{2} B_N (y_N + t)^2 \right] \\
- \sum_j C_j y_j - C_T k - \frac{1}{2} D y_S^2
\]  

(1)

s.t. \[(\lambda^-) - k \leq t \leq k (\lambda^+)\]  

(2)

\[y_N + t \geq 0 (\beta_N)\]  

(3)

\[y_S - t \geq 0 (\beta_S)\]  

(4)
Central Planner

Single-Level Quadratic Programming Problem

- CP’s SW maximisation problem

\[
\max_{y_N \geq 0, y_S \geq 0, t, k \geq 0} \left[ A_S (y_S - t) - \frac{1}{2} B_S (y_S - t)^2 \right] \\
+ \left[ A_N (y_N + t) - \frac{1}{2} B_N (y_N + t)^2 \right] \\
- \sum_{j} C_j y_j - C_T k - \frac{1}{2} D y_S^2 
\]  
\tag{1}

s.t.  
\( (\lambda^-) - k \leq t \leq k (\lambda^+) \) \tag{2}  
\( y_N + t \geq 0 (\beta_N) \) \tag{3}  
\( y_S - t \geq 0 (\beta_S) \) \tag{4}
CP’s SW maximisation problem

\[
\max_{y_N \geq 0, y_S \geq 0, t, k \geq 0} \left[ A_S (y_S - t) - \frac{1}{2} B_S (y_S - t)^2 \right] \\
+ \left[ A_N (y_N + t) - \frac{1}{2} B_N (y_N + t)^2 \right] \\
- \sum_j C_j y_j - C_T k - \frac{1}{2} Dy_S^2 \]  
\tag{1}

s.t. 
\begin{align*}
(\lambda^-) - k & \leq t \leq k (\lambda^+) \quad (2) \\
y_N + t & \geq 0 (\beta_N) \quad (3) \\
y_S - t & \geq 0 (\beta_S) \quad (4)
\end{align*}
Central Planner

Single-Level Quadratic Programming Problem

- CP’s SW maximisation problem

\[
\max_{y_N \geq 0, y_S \geq 0, t, k \geq 0} \left[ A_S (y_S - t) - \frac{1}{2} B_S (y_S - t)^2 \right] \\
+ \left[ A_N (y_N + t) - \frac{1}{2} B_N (y_N + t)^2 \right] \\
- \sum_j C_j y_j - C_T k - \frac{1}{2} D y_S^2
\]  

\[ (\lambda^-) - k \leq t \leq k (\lambda^+) \]  
\[ y_N + t \geq 0 (\beta_N) \]  
\[ y_S - t \geq 0 (\beta_S) \]
CP’s KKT Conditions

\[0 \leq y_S \perp - [A_S - B_S (y_S - t)] + C_S - \beta_S + Dy_S \geq 0 \quad (5)\]
\[0 \leq y_N \perp - [A_N - B_N (y_N + t)] + C_N - \beta_N \geq 0 \quad (6)\]
\[0 \leq k \perp C_T - \lambda^+ - \lambda^- \geq 0 \quad (7)\]
\[-A_S + B_S (y_S - t) + A_N - B_N (y_N + t)\]
\[-\lambda^+ + \lambda^- + \beta_N - \beta_S = 0 \quad \text{with } t \text{ u.r.s.} \quad (8)\]
\[0 \leq \lambda^+ \perp k - t \geq 0 \quad (9)\]
\[0 \leq \lambda^- \perp k + t \geq 0 \quad (10)\]
\[0 \leq \beta_N \perp y_N + t \geq 0 \quad (11)\]
\[0 \leq \beta_S \perp y_S - t \geq 0 \quad (12)\]
Solution to CP Problem

- **Case 1:** $\bar{k} > 0$, $\bar{y}_N = 0$, $\bar{y}_S > 0$, $\bar{t} > 0$
  \[ \bar{y}_S(\bar{k}) = \frac{A_S-C_S+B_S\bar{k}}{B_S+D} \]  
  \[ \bar{k} = \frac{D(A_N-A_S-C_T)+B_S(A_N-C_S-C_T)}{(B_S+B_N)D+B_NB_S} \]  

- **Case 2:** $\bar{k} > 0$, $\bar{y}_N > 0$, $\bar{y}_S > 0$, $\bar{t} > 0$
  \[ \bar{y}_S(\bar{k}) = \frac{A_S-C_S+B_S\bar{k}}{B_S+D} \]  
  \[ \bar{y}_N(\bar{k}) = \frac{A_N-C_N-B_N\bar{k}}{B_N} \]  
  \[ \bar{k} = \frac{(C_N-C_T)(B_S+D)-B_SC_S-A_SD}{B_SD} \]  

- **Case 3:** $\bar{k} = 0$, $\bar{y}_N > 0$, $\bar{y}_S > 0$, $\bar{t} = 0$

- **Case 4:** $\bar{k} > 0$, $\bar{y}_N > 0$, $\bar{y}_S > 0$, $\bar{t} < 0$
  \[ \bar{y}_S(\bar{k}) = \frac{A_S-C_S-B_S\bar{k}}{B_S+D} \]  
  \[ \bar{y}_N(\bar{k}) = \frac{A_N-C_N+B_N\bar{k}}{B_N} \]  
  \[ \bar{k} = \frac{-(C_N+C_T)(B_S+D)+B_SC_S+A_SD}{B_SD} \]
Solution to CP Problem

- **Case 1:** \( \bar{k} > 0, \bar{y}_N = 0, \bar{y}_S > 0, \bar{t} > 0 \)
  \[
  \bar{y}_S(\bar{k}) = \frac{A_S - C_S + B_S \bar{k}}{B_S + D} \quad (13)
  \]
  \[
  \bar{y}_N(\bar{k}) = A_N - C_N - B_N \bar{k} \quad (14)
  \]
  \[
  \bar{k} = \frac{D(A_N - A_S - C_T) + B_S(A_N - C_S - C_T)}{(B_S + B_N)D + B_N B_S} \quad (14)
  \]

- **Case 2:** \( \bar{k} > 0, \bar{y}_N > 0, \bar{y}_S > 0, \bar{t} > 0 \)
  \[
  \bar{y}_S(\bar{k}) = \frac{A_S - C_S + B_S \bar{k}}{B_S + D} \quad (15)
  \]
  \[
  \bar{y}_N(\bar{k}) = A_N - C_N - B_N \bar{k} \quad (16)
  \]
  \[
  \bar{k} = \frac{(C_N - C_T)(B_S + D) - B_S C_S - A_S D}{B_S D} \quad (17)
  \]

- **Case 3:** \( \bar{k} = 0, \bar{y}_N > 0, \bar{y}_S > 0, \bar{t} = 0 \)
- **Case 4:** \( \bar{k} > 0, \bar{y}_N > 0, \bar{y}_S > 0, \bar{t} < 0 \)
  \[
  \bar{y}_S(\bar{k}) = \frac{A_S - C_S - B_S \bar{k}}{B_S + D} \quad (18)
  \]
  \[
  \bar{y}_N(\bar{k}) = A_N - C_N + B_N \bar{k} \quad (19)
  \]
  \[
  \bar{k} = \frac{-(C_N + C_T)(B_S + D) + B_S C_S + A_S D}{B_S D} \quad (20)
  \]
Solution to CP Problem

- **Case 1:** $\bar{k} > 0$, $\bar{y}_N = 0$, $\bar{y}_S > 0$, $\bar{t} > 0$
  
  $$\bar{y}_S(\bar{k}) = \frac{A_S - C_S + B_S \bar{k}}{B_S + D}$$  \hspace{1cm} (13)

  $$\bar{k} = \frac{D(A_N - A_S - C_T) + B_S(A_N - C_S - C_T)}{(B_S + B_N)D + B_N B_S}$$  \hspace{1cm} (14)

- **Case 2:** $\bar{k} > 0$, $\bar{y}_N > 0$, $\bar{y}_S > 0$, $\bar{t} > 0$
  
  $$\bar{y}_S(\bar{k}) = \frac{A_S - C_S + B_S \bar{k}}{B_S + D}$$  \hspace{1cm} (15)

  $$\bar{y}_N(\bar{k}) = \frac{A_N - C_N - B_N \bar{k}}{B_N}$$  \hspace{1cm} (16)

  $$\bar{k} = \frac{(C_N - C_T)(B_S + D) - B_S C_S - A_S D}{B_S D}$$  \hspace{1cm} (17)

- **Case 3:** $\bar{k} = 0$, $\bar{y}_N > 0$, $\bar{y}_S > 0$, $\bar{t} = 0$

- **Case 4:** $\bar{k} > 0$, $\bar{y}_N > 0$, $\bar{y}_S > 0$, $\bar{t} < 0$
  
  $$\bar{y}_S(\bar{k}) = \frac{A_S - C_S - B_S \bar{k}}{B_S + D}$$  \hspace{1cm} (18)

  $$\bar{y}_N(\bar{k}) = \frac{A_N - C_N + B_N \bar{k}}{B_N}$$  \hspace{1cm} (19)

  $$\bar{k} = \frac{-(C_N + C_T)(B_S + D) + B_S C_S + A_S D}{B_S D}$$  \hspace{1cm} (20)
Properties of CP Solution

**Proposition**

*Under CP, ceteris paribus increases in $D$ result in:*

**Case 1:** \[ \frac{\partial \bar{k}}{\partial D} < 0, \quad \frac{\partial \bar{y}_S}{\partial D} < 0. \]

**Case 2:** \[ \frac{\partial \bar{k}}{\partial D} < 0, \quad \frac{\partial \bar{y}_S}{\partial D} < 0, \quad \frac{\partial \bar{y}_N}{\partial D} > 0. \]

**Case 3:** \[ \frac{\partial \bar{y}_S}{\partial D} < 0, \quad \frac{\partial \bar{y}_N}{\partial D} = 0. \]

**Case 4:** \[ \frac{\partial \bar{k}}{\partial D} > 0, \quad \frac{\partial \bar{y}_S}{\partial D} < 0, \quad \frac{\partial \bar{y}_N}{\partial D} > 0. \]

**Proposition**

*Under CP, ceteris paribus increases in $D$ result in decreases in the damage cost from emissions in Cases 2 and 4.*
Lower Level: Producers’ Problems and MCP

- Each optimisation problem is convex:
  \[ \max_{y_S \geq 0} \; p_{SYS} - C_{SYS} \]  \hspace{1cm} (21)
  \[ \max_{y_N \geq 0} \; p_{NYN} - C_{NYN} \]  \hspace{1cm} (22)

  \[ \max_t \; \left[ A_N - B_N y_N \right] t - \left[ A_S - B_S y_S \right] t - \frac{1}{2} (B_N + B_S) t^2 \]  \hspace{1cm} (23)

  s.t. \quad \text{Equations (2)-(4)}

- Problems (21)-(23) may be replaced by their KKT conditions
  \[ 0 \leq y_S \perp - \left[ A_S - B_S (y_S - t) \right] + C_S \geq 0 \]  \hspace{1cm} (24)
  \[ 0 \leq y_N \perp - \left[ A_N - B_N (y_N + t) \right] + C_N \geq 0 \]  \hspace{1cm} (25)

  \[ -A_N + B_N y_N + A_S - B_S y_S + B_N t + B_S t \]
  \[ + \lambda^+ - \lambda^- - \beta_N + \beta_S = 0 \text{ with } t \text{ u.r.s.} \]  \hspace{1cm} (26)

  \text{Equations (9)-(12)}
Lower Level: Producers’ Problems and MCP

- Each optimisation problem is convex:

\[
\max_{y_S \geq 0} \quad p_S y_S - C_S y_S
\]  
(21)

\[
\max_{y_N \geq 0} \quad p_N y_N - C_N y_N
\]  
(22)

\[
\max_t \quad [A_N - B_N y_N] t - [A_S - B_S y_S] t - \frac{1}{2} (B_N + B_S) t^2
\]  
(23)

s.t. Equations (2)–(4)

- Problems (21)–(23) may be replaced by their KKT conditions

\[
0 \leq y_S \perp - [A_S - B_S (y_S - t)] + C_S \geq 0
\]  
(24)

\[
0 \leq y_N \perp - [A_N - B_N (y_N + t)] + C_N \geq 0
\]  
(25)

\[
-A_N + B_N y_N + A_S - B_S y_S + B_N t + B_S t + \lambda^+ - \lambda^- - \beta_N + \beta_S = 0 \text{ with } t \text{ u.r.s.}
\]  
(26)

Equations (9)–(12)
Lower Level: Producers’ Problems and MCP

Each optimisation problem is convex:

\[
\max_{y_S \geq 0} \quad ps_S - C_S y_S \\
\max_{y_N \geq 0} \quad p_N y_N - C_N y_N
\]

\[
\max_t \quad [A_N - B_N y_N] t - [A_S - B_S y_S] t - \frac{1}{2} (B_N + B_S) t^2
\]

s.t. Equations (2)–(4)

Problems (21)–(23) may be replaced by their KKT conditions

\[
0 \leq y_S \perp - [A_S - B_S (y_S - t)] + C_S \geq 0
\]

\[
0 \leq y_N \perp - [A_N - B_N (y_N + t)] + C_N \geq 0
\]

\[
-A_N + B_N y_N + A_S - B_S y_S + B_N t + B_S t + \lambda^+ - \lambda^- - \beta_N + \beta_S = 0 \text{ with } t \text{ u.r.s.}
\]

Equations (9)–(12)
Lower Level: Producers’ Problems and MCP

- Each optimisation problem is convex:
  \[
  \max_{y_S \geq 0} \quad p_S y_S - C_S y_S \\
  \max_{y_N \geq 0} \quad p_N y_N - C_N y_N \\
  \max_t \quad [A_N - B_N y_N] t - [A_S - B_S y_S] t - \frac{1}{2} (B_N + B_S) t^2
  \]
  s.t. \quad Equations (2)–(4)

- Problems (21)–(23) may be replaced by their KKT conditions
  \[
  0 \leq y_S \perp - [A_S - B_S (y_S - t)] + C_S \geq 0 \\
  0 \leq y_N \perp - [A_N - B_N (y_N + t)] + C_N \geq 0 \\
  -A_N + B_N y_N + A_S - B_S y_S + B_N t + B_S t \\
  +\lambda^+ - \lambda^- - \beta_N + \beta_S = 0 \quad \text{with } t \text{ u.r.s.}
  \]
  Equations (9)–(12)
Lower Level: Producers’ Problems and MCP

- Each optimisation problem is convex:
  \[
  \max_{y_S \geq 0} \quad p_S y_S - C_S y_S \\
  \max_{y_N \geq 0} \quad p_N y_N - C_N y_N
  \]
  \[
  \max_t \quad [A_N - B_N y_N] t - [A_S - B_S y_S] t - \frac{1}{2} (B_N + B_S) t^2
  \]
  s.t.  Equations (2)–(4)

- Problems (21)–(23) may be replaced by their KKT conditions
  \[
  0 \leq y_S \perp - [A_S - B_S (y_S - t)] + C_S \geq 0
  \]
  \[
  0 \leq y_N \perp - [A_N - B_N (y_N + t)] + C_N \geq 0
  \]
  \[-A_N + B_N y_N + A_S - B_S y_S + B_N t + B_S t + \lambda^+ - \lambda^- - \beta_N + \beta_S = 0 \text{ with } t \text{ u.r.s.}
  \]
  Equations (9)–(12)
Lower Level: Producers’ Problems and MCP

- Each optimisation problem is convex:
  
  \[
  \text{max}_{y_S \geq 0} \quad p_S y_S - C_S y_S
  \]
  \[\text{(21)}\]
  
  \[
  \text{max}_{y_N \geq 0} \quad p_N y_N - C_N y_N
  \]
  \[\text{(22)}\]
  
  \[
  \text{max}_t \quad [A_N - B_N y_N] t - [A_S - B_S y_S] t - \frac{1}{2} (B_N + B_S) t^2
  \]
  \[\text{(23)}\]

  s.t. Equations (2)–(4)

- Problems (21)–(23) may be replaced by their KKT conditions
  
  \[
  0 \leq y_S \perp - [A_S - B_S (y_S - t)] + C_S \geq 0
  \]
  \[\text{(24)}\]
  
  \[
  0 \leq y_N \perp - [A_N - B_N (y_N + t)] + C_N \geq 0
  \]
  \[\text{(25)}\]

  \[
  -A_N + B_N y_N + A_S - B_S y_S + B_N t + B_S t + \lambda^+ - \lambda^- - \beta_N + \beta_S = 0 \text{ with } t \text{ u.r.s.}
  \]
  \[\text{(26)}\]

Equations (9)–(12)
Replacing the lower level via its MCP enables us to convert the bi-level problem into an MPEC:

\[
\begin{align*}
\max_{\{k \geq 0\} \cup \Omega \cup \Xi} & \quad \text{Equation (1)} \\
\text{s.t.} & \quad \text{Equations (9)–(12), (24)–(26)} \\
\Omega & \equiv \{y_N \geq 0, y_S \geq 0, t\}, \quad \Xi \equiv \{\lambda^+ \geq 0, \lambda^- \geq 0, \beta_N \geq 0, \beta_S \geq 0\}
\end{align*}
\]

Using the solutions \(\hat{y}_N(k), \hat{y}_S(k),\) and \(\hat{t}(k),\) we can next replace sets \(\Omega\) and \(\Xi\) in the MPEC, thereby turning it into an unconstrained non-linear programming (NLP) problem

\[
\begin{align*}
\max_{k \geq 0} & \quad A_S (\hat{y}_S(k) - \hat{t}(k)) - \frac{1}{2} B_S (\hat{y}_S(k) - \hat{t}(k))^2 - C_S \hat{y}_S(k) \\
& \quad + A_N (\hat{y}_N(k) + \hat{t}(k)) - \frac{1}{2} B_N (\hat{y}_N(k) + \hat{t}(k))^2 - C_N \hat{y}_N(k) \\
& \quad - C_T k - \frac{1}{2} D \hat{y}_S(k)^2
\end{align*}
\]
Upper Level: TSO’s MPEC and NLP

- Replacing the lower level via its MCP enables us to convert the bi-level problem into an MPEC:

\[
\max_{\{k \geq 0\} \cup \Omega \cup \Xi} \quad \text{Equation (1)}
\]

\[
\text{s.t.} \quad \text{Equations (9)–(12), (24)–(26)}
\]

\[\Omega \equiv \{y_N \geq 0, y_S \geq 0, t\}, \quad \Xi \equiv \{\lambda^+ \geq 0, \lambda^- \geq 0, \beta_N \geq 0, \beta_S \geq 0\}\]

- Using the solutions \(\hat{y}_N(k), \hat{y}_S(k),\) and \(\hat{t}(k),\) we can next replace sets \(\Omega\) and \(\Xi\) in the MPEC, thereby turning it into an unconstrained non-linear programming (NLP) problem

\[
\max_{k \geq 0} \quad A_S (\hat{y}_S(k) - \hat{t}(k)) - \frac{1}{2} B_S (\hat{y}_S(k) - \hat{t}(k))^2 - C_S \hat{y}_S(k)
\]

\[
+ A_N (\hat{y}_N(k) + \hat{t}(k)) - \frac{1}{2} B_N (\hat{y}_N(k) + \hat{t}(k))^2 - C_N \hat{y}_N(k)
\]

\[
- C_T k - \frac{1}{2} D \hat{y}_S(k)^2
\]

(28)
Solution under PC

- **Case 1:** $\hat{k} > 0$, $\hat{y}_N = 0$, $\hat{y}_S > 0$, $\hat{t} > 0$

  \[
  \hat{y}_S(\hat{k}) = \frac{A_S - C_S + B_S \hat{k}}{B_S} \tag{29}
  \]

  \[
  \hat{k} = \frac{D(C_S - A_S) + B_S(A_N - C_S - C_T)}{B_S(D + B_N)} \tag{30}
  \]

- **Case 2:** $\hat{k} > 0$, $\hat{y}_N > 0$, $\hat{y}_S > 0$, $\hat{t} > 0$

  \[
  \hat{y}_S(\hat{k}) = \frac{A_S - C_S + B_S \hat{k}}{B_S} \tag{31}
  \]

  \[
  \hat{y}_N(\hat{k}) = \frac{A_N - C_N - B_N \hat{k}}{B_N} \tag{32}
  \]

  \[
  \hat{k} = \frac{D(C_S - A_S) + B_S(C_N - C_S - C_T)}{B_SD} \tag{33}
  \]

- **Case 3:** $\hat{k} = 0$, $\hat{y}_N > 0$, $\hat{y}_S > 0$, $\hat{t} = 0$
Solution under PC

- **Case 1:** $\hat{k} > 0$, $\hat{y}_N = 0$, $\hat{y}_S > 0$, $\hat{t} > 0$

  \[ \hat{y}_S(\hat{k}) = \frac{A_S - C_S + B_S \hat{k}}{B_S} \]  
  \[ \hat{k} = \frac{D(C_S - A_S) + B_S(A_N - C_S - C_T)}{B_S(D + B_N)} \]  

- **Case 2:** $\hat{k} > 0$, $\hat{y}_N > 0$, $\hat{y}_S > 0$, $\hat{t} > 0$

  \[ \hat{y}_S(\hat{k}) = \frac{A_S - C_S + B_S \hat{k}}{B_S} \]  
  \[ \hat{y}_N(\hat{k}) = \frac{A_N - C_N - B_N \hat{k}}{B_N} \]  
  \[ \hat{k} = \frac{D(C_S - A_S) + B_S(C_N - C_S - C_T)}{B_S D} \]  

- **Case 3:** $\hat{k} = 0$, $\hat{y}_N > 0$, $\hat{y}_S > 0$, $\hat{t} = 0$
Properties of PC Solution

Proposition

Under a solution for PC, ceteris paribus increases in $D$ result in:

Case 1: $\frac{\partial \hat{k}}{\partial D} < 0$, $\frac{\partial \hat{y}_S}{\partial D} < 0$.

Case 2: $\frac{\partial \hat{k}}{\partial D} < 0$, $\frac{\partial \hat{y}_S}{\partial D} < 0$, $\frac{\partial \hat{y}_N}{\partial D} > 0$.

Case 3: $\frac{\partial \hat{y}_S}{\partial D} = 0$, $\frac{\partial \hat{y}_N}{\partial D} = 0$.

Proposition

Under a solution for PC, ceteris paribus increases in $D$ result in a decrease and an increase in the damage cost from emissions in Cases 2 and 3, respectively.
Lower Level: Producers’ Problems and MCP

- Each optimisation problem is convex:

\[
\max_{y_S \geq 0} [A_S - B_S (y_S - t)] y_S - C_S y_S
\]  
(34)

\[
\max_{y_N \geq 0} [A_N - B_N (y_N + t)] y_N - C_N y_N
\]  
(35)

- ISO’s problem is as before, and problems (34)–(35) may be replaced by their KKT conditions

\[
0 \leq y_S \perp - [A_S - B_S (2y_S - t)] + C_S \geq 0
\]  
(36)

\[
0 \leq y_N \perp - [A_N - B_N (2y_N + t)] + C_N \geq 0
\]  
(37)
Lower Level: Producers’ Problems and MCP

- Each optimisation problem is convex:

\[
\max_{y_S \geq 0} \ [A_S - B_S (y_S - t)] y_S - C_S y_S
\]

\[
(34)
\]

\[
\max_{y_N \geq 0} \ [A_N - B_N (y_N + t)] y_N - C_N y_N
\]

\[
(35)
\]

- ISO’s problem is as before, and problems \((34)-(35)\) may be replaced by their KKT conditions

\[
0 \leq y_S \perp - [A_S - B_S (2y_S - t)] + C_S \geq 0
\]

\[
(36)
\]

\[
0 \leq y_N \perp - [A_N - B_N (2y_N + t)] + C_N \geq 0
\]

\[
(37)
\]
Lower Level: Producers’ Problems and MCP

- Each optimisation problem is convex:

  \[\max_{y_S \geq 0} \ [A_S - B_S (y_S - t)] y_S - C_S y_S \]  
  \[\max_{y_N \geq 0} \ [A_N - B_N (y_N + t)] y_N - C_N y_N \]  

- ISO’s problem is as before, and problems (34)–(35) may be replaced by their KKT conditions

  \[0 \leq y_S \perp - [A_S - B_S (2y_S - t)] + C_S \geq 0 \]  
  \[0 \leq y_N \perp - [A_N - B_N (2y_N + t)] + C_N \geq 0 \]
Lower Level: Producers’ Problems and MCP

- Each optimisation problem is convex:

\[
\max_{y_S \geq 0} \ [A_S - B_S (y_S - t)] y_S - C_S y_S \tag{34}
\]

\[
\max_{y_N \geq 0} \ [A_N - B_N (y_N + t)] y_N - C_N y_N \tag{35}
\]

- ISO’s problem is as before, and problems (34)–(35) may be replaced by their KKT conditions

\[
0 \leq y_S \perp - [A_S - B_S (2y_S - t)] + C_S \geq 0 \tag{36}
\]

\[
0 \leq y_N \perp - [A_N - B_N (2y_N + t)] + C_N \geq 0 \tag{37}
\]
Upper Level: TSO’s MPEC and NLP

Replacing the lower level via its MCP enables us to convert the bi-level problem into an MPEC:

\[
\max_{\{k \geq 0\} \cup \Omega \cup \Xi} \quad \text{Equation (1)}
\]
\[
\text{s.t.} \quad \text{Equations (9)--(12), (26), (36)--(37)}
\]

Using the solutions \(y_N^*(k), y_S^*(k),\) and \(t^*(k),\) we can next replace sets \(\Omega\) and \(\Xi\) in the MPEC, thereby turning it into an unconstrained non-linear programming (NLP) problem

\[
\max_{k \geq 0} \quad A_S (y_S^*(k) - t^*(k)) - \frac{1}{2} B_S (y_S^*(k) - t^*(k))^2 - C_S y_S^*(k)
\]
\[
+ A_N (y_N^*(k) + t^*(k)) - \frac{1}{2} B_N (y_N^*(k) + t^*(k))^2 - C_N y_N^*(k)
\]
\[
- C_T k - \frac{1}{2} D y_S^*(k)^2
\]
Replacing the lower level via its MCP enables us to convert the bi-level problem into an MPEC:

\[
\max_{\{k \geq 0\} \cup \Omega \cup \Xi} \quad \text{Equation (1)} \\
\text{s.t.} \quad \text{Equations (9)-(12), (26), (36)-(37)}
\]  

Using the solutions \( y^*_N(k), y^*_S(k), \) and \( t^*(k) \), we can next replace sets \( \Omega \) and \( \Xi \) in the MPEC, thereby turning it into an unconstrained non-linear programming (NLP) problem

\[
\max_{k \geq 0} \quad A_S (y^*_S(k) - t^*(k)) - \frac{1}{2} B_S (y^*_S(k) - t^*(k))^2 - C_S y^*_S(k) \\
+ A_N (y^*_N(k) + t^*(k)) - \frac{1}{2} B_N (y^*_N(k) + t^*(k))^2 - C_N y^*_N(k) \\
- C_T k - \frac{1}{2} D y^*_S(k)^2
\]  

(39)
Solution under CO

- **Case 1:** \( k^* > 0, y_N^* = 0, y_S^* > 0, t^* > 0 \)

  \[ y_S^*(k^*) = \frac{A_S - C_S + B_S k^*}{2B_S} \quad (40) \]

  \[ k^* = \frac{4A_N - A_S - 3C_S - 4C_T - \frac{D}{B_S} (A_S - C_S)}{B_S + 4B_N + D} \quad (41) \]

- **Case 2:** \( k^* > 0, y_N^* > 0, y_S^* > 0, t^* > 0 \)

  \[ y_S^*(k^*) = \frac{A_S - C_S + B_S k^*}{2B_S} \quad (42) \]

  \[ y_N^*(k^*) = \frac{A_N - C_N - B_N k^*}{2B_N} \quad (43) \]

  \[ k^* = \frac{A_N + 3C_N - A_S - 3C_S - 4C_T - \frac{D}{B_S} (A_S - C_S)}{B_S + B_N + D} \quad (44) \]

- **Case 3:** \( k^* = 0, y_N^* > 0, y_S^* > 0, t^* = 0 \)

- **Case 4:** \( k^* > 0, y_N^* > 0, y_S^* > 0, t^* < 0 \)

  \[ y_S^*(k^*) = \frac{A_S - C_S - B_S k^*}{2B_S} \quad (45) \]

  \[ y_N^*(k^*) = \frac{A_N - C_N + B_N k^*}{2B_N} \quad (46) \]

  \[ k^* = \frac{A_S + 3C_S - A_N - 3C_N - 4C_T + \frac{D}{B_S} (A_S - C_S)}{B_S + B_N + D} \quad (47) \]
Solution under CO

- **Case 1:** $k^* > 0$, $y_N^* = 0$, $y_S^* > 0$, $t^* > 0$

  \[ y_S^*(k^*) = \frac{A_S - C_S + B_S k^*}{2B_S} \]  
  \[ k^* = \frac{4A_N - A_S - 3C_S - 4C_T - \frac{D}{B_S}(A_S - C_S)}{B_S + 4B_N + D} \]  

- **Case 2:** $k^* > 0$, $y_N^* > 0$, $y_S^* > 0$, $t^* > 0$

  \[ y_S^*(k^*) = \frac{A_S - C_S + B_S k^*}{2B_S} \]  
  \[ y_N^*(k^*) = \frac{A_N - C_N - B_N k^*}{2B_N} \]  
  \[ k^* = \frac{A_N + 3C_N - A_S - 3C_S - 4C_T - \frac{D}{B_S}(A_S - C_S)}{B_S + B_N + D} \]  

- **Case 3:** $k^* = 0$, $y_N^* > 0$, $y_S^* > 0$, $t^* = 0$

- **Case 4:** $k^* > 0$, $y_N^* > 0$, $y_S^* > 0$, $t^* < 0$

  \[ y_S^*(k^*) = \frac{A_S - C_S - B_S k^*}{2B_S} \]  
  \[ y_N^*(k^*) = \frac{A_N - C_N + B_N k^*}{2B_N} \]  
  \[ k^* = \frac{A_S + 3C_S - A_N - 3C_N - 4C_T + \frac{D}{B_S}(A_S - C_S)}{B_S + B_N + D} \]
Solution under CO

- **Case 1:** \( k^* > 0, \ y_N^* = 0, \ y_S^* > 0, \ t^* > 0 \)

\[
y_S^*(k^*) = \frac{A_S - C_S + B_S k^*}{2B_S} \quad (40)
\]

\[
k^* = \frac{4A_N - A_S - 3C_S - 4C_T - \frac{D}{B_S} (A_S - C_S)}{B_S + 4B_N + D} \quad (41)
\]

- **Case 2:** \( k^* > 0, \ y_N^* > 0, \ y_S^* > 0, \ t^* > 0 \)

\[
y_S^*(k^*) = \frac{A_S - C_S + B_S k^*}{2B_S} \quad (42)
\]

\[
y_N^*(k^*) = \frac{A_N - C_N - B_N k^*}{2B_N} \quad (43)
\]

\[
k^* = \frac{A_N + 3C_N - A_S - 3C_S - 4C_T - \frac{D}{B_S} (A_S - C_S)}{B_S + B_N + D} \quad (44)
\]

- **Case 3:** \( k^* = 0, \ y_N^* > 0, \ y_S^* > 0, \ t^* = 0 \)
- **Case 4:** \( k^* > 0, \ y_N^* > 0, \ y_S^* > 0, \ t^* < 0 \)

\[
y_S^*(k^*) = \frac{A_S - C_S - B_S k^*}{2B_S} \quad (45)
\]

\[
y_N^*(k^*) = \frac{A_N - C_N + B_N k^*}{2B_N} \quad (46)
\]

\[
k^* = \frac{A_S + 3C_S - A_N - 3C_N - 4C_T + \frac{D}{B_S} (A_S - C_S)}{B_S + B_N + D} \quad (47)
\]
Properties of CO Solution

Proposition

Under a solution for CO, ceteris paribus increases in $D$ result in:

Case 1: \( \frac{\partial k^*}{\partial D} < 0, \frac{\partial y^*_S}{\partial D} < 0. \)

Case 2: \( \frac{\partial k^*}{\partial D} < 0, \frac{\partial y^*_S}{\partial D} < 0, \frac{\partial y^*_N}{\partial D} > 0. \)

Case 3: \( \frac{\partial y^*_S}{\partial D} = 0, \frac{\partial y^*_N}{\partial D} = 0. \)

Case 4: \( \frac{\partial k^*}{\partial D} > 0, \frac{\partial y^*_S}{\partial D} < 0, \frac{\partial y^*_N}{\partial D} > 0. \)

Proposition

Under a solution for CO, ceteris paribus increases in $D$ result in an increase in the damage cost from emissions in Case 3.

Proposition

Under solutions for CP, PC, and CO, ceteris paribus increases in $D$ result in decreases in social welfare.
Carbon Tax: Perfect Competition

- The NRE sector’s problem under a carbon tax of rate \(0 \leq E \leq 1\):
  \[
  \max_{y_S \geq 0} \quad p_{sys} - C_{sys} - \frac{1}{2} EDy_S^2 \\
  \Rightarrow \quad 0 \leq y_S \perp - [A_S - B_S (y_S - t)] + C_S + EDy_S \geq 0
  \]

- The TSO’s MPEC is now:
  \[
  \max_{\{k \geq 0\} \cup \Omega \cup \Xi} \quad \text{Equation (1)} \\
  \text{s.t.} \quad \text{Equations (9)-(12), (25)-(26), (48)}
  \]

**Proposition**

*If a full carbon tax, i.e., \(E = 1\), is imposed under PC, then the TSO’s problem is equivalent to that of CP.*

**Proposition**

*Under a solution for PC with \(E \in (0, 1)\), a more stringent carbon tax leads to an increase (decrease) in the transmission capacity, i.e., \(\frac{\partial k}{\partial E} > 0\) (\(\frac{\partial k}{\partial E} < 0\)) for Cases 1 and 2 (Case 4).*
Carbon Tax: Perfect Competition

- The NRE sector’s problem under a carbon tax of rate $0 \leq E \leq 1$:
  $$\max_{y_S \geq 0} \quad p_{sys} - C_{sys} - \frac{1}{2} EDy_S^2$$
  $$\Rightarrow \quad 0 \leq y_S \perp - [A_S - B_S (y_S - t)] + C_S + EDy_S \geq 0 \quad (48)$$

- The TSO’s MPEC is now:
  $$\max_{\{k \geq 0\} \cup \Omega} \equiv$$
  s.t. \quad $$\text{Equation (1)} \quad (49)$$

Proposition

If a full carbon tax, i.e., $E = 1$, is imposed under PC, then the TSO’s problem is equivalent to that of CP.

Proposition

Under a solution for PC with $E \in (0, 1)$, a more stringent carbon tax leads to an increase (decrease) in the transmission capacity, i.e., $\frac{\partial k}{\partial E} > 0$ ($\frac{\partial k}{\partial E} < 0$) for Cases 1 and 2 (Case 4).
**Carbon Tax: Cournot Oligopoly**

- **The NRE sector’s problem under a carbon tax of rate**
  \[0 \leq E \leq 1:\]

  \[
  \max_{y_S \geq 0} \left[ A_S - B_S (y_S - t) \right] y_S - C_S y_S - \frac{1}{2} E D y_S^2
  \]

  \[
  \Rightarrow 0 \leq y_S \perp - \left[ A_S - B_S (2y_S - t) \right] + C_S + E D y_S \geq 0 \tag{50}
  \]

- **The TSO’s MPEC is now:**

  \[
  \max_{\{k \geq 0\} \cup \Omega \cup \Xi} \text{ Equation (1)}
  \]

  \[
  \text{s.t. } \quad \text{Equations (9)-(12), (26), (37), (50)}
  \]

**Proposition**

*Under a solution for CO with \( E \in (0, 1) \), if \( E > 1 - \frac{B_S}{D} \), then a more stringent carbon tax leads to a decrease (increase) in the transmission capacity, i.e., \( \frac{\partial k^*}{\partial E} < 0 \) (\( \frac{\partial k^*}{\partial E} > 0 \)) for Cases 1 and 2 (Case 4), and vice versa.*
Carbon Tax: Cournot Oligopoly

- **The NRE sector’s problem under a carbon tax of rate** $0 \leq E \leq 1$:

\[
\max_{y_S \geq 0} \left[ A_S - B_S (y_S - t) \right] y_S - C_S y_S - \frac{1}{2} E D y_S^2
\]
\[
\Rightarrow \quad 0 \leq y_S \perp - \left[ A_S - B_S (2y_S - t) \right] + C_S + E D y_S \geq 0 \quad (50)
\]

- **The TSO’s MPEC is now:**

\[
\max_{\{k \geq 0\} \cup \Omega} \quad \text{Equation (1)}
\]
\[
\text{s.t.} \quad \text{Equations (9)}-(12), (26), (37), (50)
\]

**Proposition**

*Under a solution for CO with $E \in (0, 1)$, if $E > 1 - \frac{B_S}{D}$, then a more stringent carbon tax leads to a decrease (increase) in the transmission capacity, i.e., $\frac{\partial k^*}{\partial E} < 0$ ($\frac{\partial k^*}{\partial E} > 0$) for Cases 1 and 2 (Case 4), and vice versa.*
Welfare Analysis of a Carbon Tax

Proposition

Under a solution for PC with $E \in (0, 1)$, a more stringent carbon tax leads to an increase in social welfare, i.e., $\frac{\partial sw}{\partial E} > 0$.

Proposition

Under a solution for CO with $E \in (0, 1)$, if $E > 1 - \frac{Bs}{D}$, then a more stringent carbon tax leads to a decrease in social welfare, i.e., $\frac{\partial sw^*}{\partial E} < 0$, and vice versa.
Numerical Examples
Optimal Net Import at Node $N$

$C_S = 20$, $C_T = 25$, $C_N = 80$, $A_S = 400$, $A_N = 200$, $B_S = 1$, $B_N = 1$, $D \in [0, 0.5]$
Fraction of Renewable Energy

Base Results

Fraction of Renewable Energy

Mathematical Formulation and Analytical Insights

Numerical Examples

Conclusions
Equilibrium Nodal Prices

\[ p_N \]

\[ p_S \]

\[ D \]

\[ CP \]

\[ PC \]

\[ CO \]
Maximised Social Welfare

![Graph showing maximised social welfare vs. D with three lines representing CP, PC, and CO.]

- **CP**
- **PC**
- **CO**
Effect of a Carbon Tax

Optimal Net Import at Node $N$ under PC

- $E = 0$
- $E = 0.25$
- $E = 0.50$
- $E = 0.75$
- $E = 1.00$
Maximised Social Welfare under PC

Effect of a Carbon Tax

Maximised Social Welfare under PC

Effect of a Carbon Tax
Power Sectors’ Incentives without a Carbon Tax under PC

\[ A_S(\hat{y}_S'(k) - 1) - B_S(\hat{y}_S(k) - k)(\hat{y}_S'(k) - 1) - C_S \hat{y}_S'(k) - C_N \hat{y}_N'(k) - C_T - D\hat{y}_S(k)\hat{y}_S'(k) = 0 \]

\[ \Rightarrow C_N = C_T + C_S + D\hat{y}_S(k) \]
Power Sectors’ Incentives with $E = 0.50$ Carbon Tax under PC

\[ A_S(\hat{y}_S'(k) - 1) - B_S(\hat{y}_S(k) - k)(\hat{y}_S'(k) - 1) - C_S\hat{y}_S'(k) - C_N\hat{y}_N'(k) - C_T - D\hat{y}_S(k)\hat{y}_S'(k) = 0 \]

\[ \Rightarrow C_N = C_T + C_S\hat{y}_S'(k) + D\hat{y}_S(k)\hat{y}_S'(k) - (\hat{y}_S'(k) - 1)\hat{p}_S \]
Power Sectors’ Incentives with $E = 0.50$ Carbon Tax under PC

\[ A_S(\hat{y}'_S(k) - 1) - B_S(\hat{y}_S(k) - k)(\hat{y}'_S(k) - 1) - C_S\hat{y}'_S(k) - C_N\hat{y}'_N(k) - C_T - D\hat{y}_S(k)\hat{y}'_S(k) = 0 \]

\[ \Rightarrow C_N = C_T + C_S\hat{y}'_S(k) + D\hat{y}_S(k)\hat{y}'_S(k) - (\hat{y}'_S(k) - 1)\hat{p}_S \]
Effect of a Carbon Tax

Optimal Net Import at Node $N$ under CO

Effect of $E$ on $N$ under CO

$D$ vs $t^*$

- $E = 0$
- $E = 0.25$
- $E = 0.50$
- $E = 0.75$
- $E = 1.00$
Maximised Social Welfare under CO

Effect of a Carbon Tax

Maximised Social Welfare under CO

\[ E = 0 \]
\[ E = 0.25 \]
\[ E = 0.50 \]
\[ E = 0.75 \]
\[ E = 1.00 \]
Effect of a Carbon Tax

Power Sectors’ Incentives without a Carbon Tax under $CO_2$

\begin{align*}
A_S(y_S'(k) + 1) - B_S(y_S^*(k) + k)(y_S'(k) + 1) - C_Sy_S'(k) + \\
A_N(y_N'(k) - 1) - B_N(y_N^*(k) - k)(y_N'(k) - 1) - C_Ny_N'(k) - \\
C_T - D_y^*(k)y_S'(k) &= 0 \\
\Rightarrow \frac{p_S^*}{2} + \frac{C_S}{2} + \frac{D_y^*(k)}{2} &= \\
C_T + \frac{p_N^*}{2} + \frac{C_N}{2}
\end{align*}
Power Sectors’ Incentives with $E = 0.50$ Carbon Tax under CO

\[
A_S(y_S'(k) + 1) - B_S(y_S^*(k) + k)(y_S'(k) + 1) - C_Sy_S^*(k) + \\
A_N(y_N^*(k) - 1) - B_N(y_N^*(k) - k)(y_N'(k) - 1) - C_Ny_N^*(k) - \\
C_T - Dy_S^*(k)y_S^*(k) = 0
\]

\[
\Rightarrow p_S^*(y_S'(k) + 1) - C_Sy_S^*(k) - \\
Dy_S^*(k)y_S^*(k) = C_T + \frac{p_N^*}{2} + \frac{C_N}{2}
\]
Effect of a Carbon Tax

Power Sectors’ Incentives with $E = 0.50$ Carbon Tax under CO

\[
A_S(y_S^*(k) + 1) - B_S(y_S^*(k) + k)(y_S^*(k) + 1) - C_S y_S^*(k) + A_N(y_N^*(k) - 1) - B_N(y_N^*(k) - k)(y_N^*(k) - 1) - C_N y_N^*(k) - C_T - D y_S^*(k) y_S^*(k) = 0
\]

\[
\Rightarrow p_S^*(y_S^*(k) + 1) - C_S y_S^*(k) - D y_S^*(k) y_S^*(k) = C_T + \frac{p_N^*}{2} + \frac{C_N}{2}
\]
Conclusions
Summary

- Complementarity approach to compare CP, PC, and CO settings in analysing sustainable transmission expansion
  - CP matches the most efficient resource with demand but may not curb emissions via RE until cost of damage is high
  - PC: TSO’s inability to induce consumption reduction leads to smaller line and relatively more RE generation initially
  - CO: power sectors’ market power is used by the TSO to induce a reversal in the prevalent flows
  - A full carbon tax results in perfect alignment of incentives under PC: increasing $E$ holding $k$ fixed reduces $\hat{y}_S(k)$, and reducing $\hat{y}_S(k)$ increases maximised SW by exactly $(1 - E)D\hat{y}_S(k)$
  - However, the same regulation worsens outcomes under CO: reducing $y^*_S(k)$ decreases maximised SW by exactly $B_Sy^*_S(k)$ while also increasing maximised SW by exactly $(1 - E)Dy^*_S(k)$

- Future work: stochastic model, endogenous carbon pricing, energy storage