

# Limited Cognitive Ability and Selective Information Processing<sup>1</sup>

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## **Abstract**

This paper studies the information processing behavior of a decision maker (DM) who has limited information processing ability. More specifically, the DM can process only a subset of all available information. Before taking an action, he chooses whether to process or ignore signals about the state of the world which he receives sequentially. I show that at the optimum, the DM processes only signals which are strong enough, but will process a weaker signal if it confirms his existing strong belief or if it supports a much more desirable state of the world. This explains some phenomena which have been well documented in the psychology literature, such as preference for strong signals, confirmation bias for individuals with strong prior and wishful thinking. Moreover, I analyze how the Internet, and in general changes in information structures, affects the processing behavior of the DM. The results shed light on different issues in the information era, including polarization and media strategy.

**Keywords:** bounded rationality, information overload, information avoidance, confirmation bias, wishful thinking, polarization

**JEL codes:** D83, D91

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# 1 Introduction

There is abundant evidence that people selectively process information in a systematically biased way. For instance, investors avoid looking at their financial portfolios when the market is down; individuals tend to ignore information that challenges their existing beliefs; people at risk for health conditions often eschew medical tests (see [Golman et al. \(2017\)](#) for a review of the literature of information avoidance). These behaviors of information avoidance could significantly worsen our well-being as we miss out information that improve our decision making.

As bad as information avoidance sounds, we are not able to process all available information, especially in this information era. Unavoidably, everyday we have to make many decisions on whether to process or ignore pieces of information. Processing a piece of information allows us to understand better its content and update our beliefs but consumes cognitive resources like time and attention which are limited. This limitation imposes a constraint on our processing capacity<sup>3</sup>, which gives rise to the possibility of information avoidance: it could be optimal for individuals to ignore a piece of information in order to save their cognitive resources for other pieces of information.

It is increasingly important nowadays to study individuals' processing behavior in this constrained environment as information overload becomes a prominent issue. There is no doubt that the advance in technology provides us with more information on different issues. However, we cannot process all available information and have to strategically use our scarce cognitive resources. On the other hand, there are some evidence that the Internet are associated with biased processing behavior ([Flaxman et al. \(2016\)](#)). In order to understand the impacts of the Internet, information policies, or in general changes in informational environment, it is important to understand how people process information when they have limited processing ability, and how their processing behavior changes with the policies.

To answer these questions, this paper proposes a simple model of sequen-

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<sup>3</sup>For instance, during an election, there are millions of articles on the Internet that could guide our voting decision. However, we are endowed with limited time and attention such that we could only process a subset of all the available articles.

tial information processing. Consider a decision maker (DM) who wants to match his action to an unknown state of the world, e.g., vote for a candidate if he/she is the best option, invest in a project if it is profitable, etc. Before taking the action, the DM gathers information as he receives sequentially  $T$  imperfect signals about the state of the world. However, he is endowed with limited cognitive resources such that he can only “process”  $\bar{T}$  ( $< T$ ) signals. After observing some preliminary (imperfect) information about the realization of the signal, e.g., the title of an article, he decides whether to “process” it, which means to learn perfectly its realization and update his belief. Otherwise, he “ignores” the signal without updating his belief. In other words, he makes his processing decision based on his (imperfect) knowledge about the signal realization<sup>4</sup>. Once he has consumed all his cognitive resources or has received all  $T$  pieces of information, he takes an action.

The model resembles a game of sequential search of information with a constraint on processing capacity. The DM updates his belief with only  $\bar{T}$  of the  $T$  available signals, knowing that he will take into account only a subset of all available information when he takes an action. On one hand, if the DM processes the current information, he loses one unit of capacity that could be used to process another piece of future information. The information he forgoes could be more informative and could lead to better decision making. On the other hand, if the DM ignores the current piece of information, he will not take it into account when he chooses which action he takes. It may lead to suboptimal decision, especially when the current information is very informative. This paper focuses on this dynamic trade-off of information processing, which is absent for instance in the literature of rational inattention<sup>5</sup>.

Facing such a trade-off, I show that the optimal processing strategy of the DM exhibits behavioral phenomena that are well-documented in the empirical and experimental literature. First, it exhibits a preference for strong

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<sup>4</sup>This is in particular different from models of information acquisition, where the DM decides whether to incur a fix cost to acquire a piece of information before knowing its realization. The DM’s inquisition decisions cannot, by definition, be different for belief-confirming and belief-challenging information.

<sup>5</sup>See the section of literature review for more detailed comparison.

signals (Itti and Baldi (2006)), *i.e.*, the DM tends to process strong signals and ignore weak signals. Second, the optimal processing strategy of confident individuals exhibits confirmation bias (Kahan et al. (2012)), *i.e.*, if the DM a priori believes strongly that one state is more probable than the other, he tends to process information which confirms his existing belief and ignores belief-challenging information. Third, the optimal processing strategy exhibits wishful thinking (Krizan and Windschitl (2007)), *i.e.*, if one state is much more desirable than the other as it is associated with a much higher maximum payoff, the DM pays more attention to information which supports the more desirable state. These results suggest that these “biases” are optimal and driven by the limitation in processing ability.

Furthermore, I analyze how changes in information structures, for example induced by the Internet or information policies, affect the processing behavior of individuals. I show that providing more or in average better information to the decision maker could strengthen his confirmation bias, *i.e.*, he has more incentive to process belief-confirming information and ignore belief-challenging information. This result suggests that the Internet could promote ideological polarization as individuals have more incentive to “cherry pick” the information that confirms their existing belief. Moreover, media has more incentive to selectively publish biased news stories as the demand increases. These results explain a number of empirical phenomena documented in the literature of ideological polarization.

The rest of the paper is organized as follows. In the next section, I present a review of the related literature. Section 3 shows a simple version of the model to illustrate the assumption of bounded rationality and the results. In section 4, I present the model setting. The analysis is shown in section 5 and 6. Section 7 presents a variation of the model while section 8 shows two simple applications. Lastly, I conclude.

## 2 Related Literature

This paper is related to a wide range of literature, spanning economics, political science and psychology. First of all, the core assumption of this paper,

*i.e.*, the DM has to use his limited cognitive resources in order to update his belief, is built on psychological theories and evidence. More specifically, it could arise from different channels as suggested by the literature, including the effort required to understand the information and to memorize the information<sup>6</sup>.

On one hand, cognitive psychological theories proposed by [Langdon and Coltheart \(2000\)](#), [Coltheart et al. \(2011\)](#) and [Connors and Halligan \(2015\)](#) suggest that beliefs are formed as explanations to information. Thus, it requires efforts to understand the information in order to integrate it with the individual's existing system of beliefs.

On the other hand, there is evidence from psychological studies showing that memory of information plays a big role in belief formation. One example is the seminal study of availability heuristic by [Tversky and Kahneman \(1973\)](#). They find that individuals evaluate the probability of an event by how much and how easily supportive evidence is retrieved from memory. Memory-based belief formation is also modeled in the theoretical economics literature to explain different behavioral phenomena, e.g., [Bénabou and Tirole \(2002\)](#) and [Baliga and Ely \(2011\)](#). Both mechanisms, *i.e.*, to understand information or to memorize information, imply that individuals have to spend cognitive resources to “process” the information in order to update their belief.

The idea of limited cognitive ability has been introduced in the fast-growing literature of rational inattention, e.g., [Sims \(2003\)](#), [Matejka and McKay \(2014\)](#) and many others. They model the limitation by a cost of information acquisition which is proportional to the reduction in Shannon entropy. Differently, the cost of processing in this paper is an opportunity cost<sup>7</sup>, which is arguably more realistic and easier to be interpreted compared

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<sup>6</sup>Memorizing a piece of information involves an encoding process, which can be strengthened by different factors, including time ([Goldstein et al. \(2011\)](#)), attention ([Shallice et al. \(1994\)](#),  [Craik, Govoni, et al. \(1996\)](#),  [Benjamin and Bjork \(2000\)](#) and  [Uncapher and Rugg \(2005\)](#)), and how “deep” the individual processes the information ( [Craik and Lockhart \(1972\)](#),  [Craik and Tulving \(1975\)](#),  [Wagner et al. \(1998\)](#) and  [Brewer et al. \(1998\)](#)). Strengthening the encoding process increases the probability of recalling the information, but consumes scarce cognitive resources.

<sup>7</sup>To process a signal, the DM uses one unit of his processing capacity which could be

to a cognitive cost with a particular functional form. Moreover, this paper focuses on the dynamics of information processing, while the literature of rational inattention studies a static problem of information acquisition<sup>8</sup>.

In terms of results, this paper contributes to the literature of information avoidance (see [Golman et al. \(2017\)](#) for an extensive review of the literature). For instance, [Eil and Rao \(2011\)](#) show that individuals update less when they receive negative information about their appearance or intelligence than when they receive positive information. [Karlsson et al. \(2009\)](#) and [Sicherman et al. \(2016\)](#) find that investors avoid checking their financial portfolios when the market is down. [Sullivan et al. \(2004\)](#) find that people who had been tested for HIV often failed to return to the test center to obtain the results. The papers mentioned above, alongside with many others, document different systematic biases in the processing behavior of individuals, including the well-known confirmation bias.

There are many economic theories that explain the confirmation bias in processing behavior. For instance, [Akerlof and Dickens \(1982\)](#), [Kőszegi \(2003\)](#) and [Brunnermeier and Parker \(2005\)](#) show that anticipatory utility or belief-dependent utility leads to the confirmation bias; [Carrillo and Mariotti \(2000\)](#) and [Bénabou and Tirole \(2002\)](#) show that confirmation bias can be used as a remedy for time inconsistent preferences; [Crémer \(1995\)](#) and [Aghion and Tirole \(1997\)](#) explain it with interpersonal strategic concerns; and the list goes on.

In contrast, this paper belongs to a relatively small, but growing, set of literature which suggests limitation in cognitive ability could explain a number of behavioral “biases”. [Compte and Postlewaite \(2012\)](#) and [Wilson \(2014\)](#) assume that the belief of the DM is constrained to a finite set of discrete memory states. Both papers show that the belief of the DM is non-responsive to weak information, while the latter also shows that the DM has

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used to process another signal.

<sup>8</sup>The literature of rational inattention gets around the mechanism of belief formation and assume the cost of reduction in uncertainty is proportional to the reduction in entropy. As a result, receiving several weak supportive evidence is equivalent to receiving one strong supportive evidence, which overlooks the dynamics of information processing and belief formation.

a tendency to update his belief with belief-confirming signals but not with belief-challenging signals. [Jehiel and Steiner \(2018\)](#) assume that the decision maker chooses which action he takes based on only one signal and can decide whether to redraw that signal. They provide a micro-foundation for theoretical models that individuals place linear attention weights on information. In contrast, this paper not only studies a different type of limitation in processing ability, but also derives a wider range of results including the comparative analysis on how changes in informational environment affect the processing behavior.

Lastly, the results in this paper shed light on different issues in the information era. It includes political polarization, which receives lots of attention in recent years (See [Prior \(2013\)](#) for a review.). On one hand, [Gentzkow and Shapiro \(2011\)](#) and [Flaxman et al. \(2016\)](#) show that online media expose individuals to belief-challenging information. On the other hand, there is evidence that the political ideology among US citizens is getting polarized ([Bartels \(2000\)](#), [Flaxman et al. \(2016\)](#)), especially among those who are more politically engaged and partisan ([Baldassarri and Gelman \(2008\)](#), [Abramowitz and Saunders \(2008\)](#), [Hetherington \(2009\)](#).). This paper shows that the limitation in processing ability could explain the wide range of phenomena documented in the literature.

### 3 An illustrative example

Consider a voter who must decide to vote for either a left wing or a right wing candidate. Only one of the candidates is “good”. Voting for the “good” candidate yields one util while voting for the “bad” candidate yields zero util.

Before he receives any information, the voter believes that there is a 70% probability that the left wing candidate is the good candidate. Before he must vote, he knows that he will receive two tweets from two journalists whom he trusts. Each tweet provides a signal about the quality of the candidates. More precisely, the tweets could be left leaning, right leaning or neutral. When the left wing candidate is the good candidate, the tweets are left leaning with probability  $1/2$ , right leaning with probability  $1/5$  and neu-

tral with probability 30%. Similarly, when the right wing candidate is the good candidate, the tweets are right leaning with probability  $1/2$ , left leaning with probability  $1/5$  and neutral with probability 30%. (Conditional on the identity of the good candidate, the tweets are independent of each other.) The information structure is represented on table 1.

	the tweet		
	left leaning	neutral	right leaning
left wing candidate is good	0.5	0.3	0.2
right wing candidate is good	0.2	0.3	0.5

Table 1: The probabilities of the tweets depending on the identity of the good candidate.

Now, the voter knows that he can “process” and use only one tweet for his voting decision. After he sees the first tweet, he knows whether it is left leaning, neutral or right leaning, but has to read the attached long article if he wants to update his belief and use the tweet for his voting decision. Indeed, “processing” the tweet takes time, so if the voter processes the first tweet, he will not be able to process the second tweet and will choose whom he votes for based on the first tweet. On the other hand, if he ignores the first tweet, he processes the second tweet and decide which candidate he will vote for based on it.

Given this limitation on processing ability, there are two possible assumptions that I could make about the way in which the voter updates his belief and makes his processing decision. I consider an “aware case” and an “unaware case”. In the “aware case”, after ignoring the first tweet and receiving the second tweet, the voter does not remember whether the first tweet is left leaning, neutral or right leaning. However, he knows there was one and can rationally infer information about it from the fact that he had chosen to ignore it. While in the first period, he takes the processing decision, knowing that if he ignores it, he will make rational inferences in the second period. He can compute the decision that he will take if he processes the first tweet

and also what he will do in different configurations of period 2 if he ignores it. Put differently, he is able to take the optimal processing decision given his knowledge of the first tweet. In this “aware” case, there is an incomplete information game between the voter’s first period self and his second period self.

On the other hand, in the “unaware case”, the voter is less sophisticated. When comes the second period, the voter does not remember at all that he received the first tweet. He updates his belief with the second tweet as if he lived in a world where there was only one journalist and as if he has only received the tweet from that journalist. Put differently, he does not infer information from the fact that he had chosen to ignore the first tweet. On the other hand, when the DM decides whether to process or to ignore the first tweet, he knows that there is a second tweet and is fully rational. He takes the optimal processing decision, knowing that if he ignores it, he will not infer information about the first tweet. This “unawareness” was documented in an experimental study by [Enke and Zimmermann \(forthcoming\)](#)<sup>9</sup>.

I will fully treat the example only in the unaware case. Nonetheless, as I will show in the section 7, the aware case gives similar results.

**Beliefs and Voting decisions** Before studying the processing decisions of the voter, I analyze his voting decision. Using the Bayesian formula<sup>10</sup>, the (unaware) voter’s belief given the tweet he had processed is presented on table 2.

After the voter processes a left leaning tweet, he becomes more confident that the left wing candidate is good; while if he processes a right leaning tweet, he believes that the right wing candidate is better than the left wing candidate in expectation. As a result, he will vote for the right wing candidate if he processes a right leaning tweet; in all other cases, in which he processes a neutral or left leaning tweet, he will vote for the left wing candidate. The

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<sup>9</sup>I will discuss the assumption of “awareness” and “unawareness” in more details in the section of model setting.

<sup>10</sup>Given a left leaning tweet, the voter’s posterior belief equals  $\frac{0.7 \times 0.5}{0.7 \times 0.5 + 0.3 \times 0.2} = \frac{35}{41}$ ; given a neutral tweet, his posterior belief equals  $\frac{0.7 \times 0.3}{0.7 \times 0.3 + 0.3 \times 0.3} = \frac{7}{10}$ ; given a right leaning tweet, his posterior belief equals  $\frac{0.7 \times 0.2}{0.7 \times 0.2 + 0.3 \times 0.5} = \frac{14}{29}$ .

	the tweet he had processed		
	left leaning	neutral	right leaning
the (unaware) voter's belief that the left wing candidate is good	$\frac{35}{41}$ $\approx 85\%$	$\frac{7}{10}$ $\approx 70\%$	$\frac{14}{29}$ $\approx 48\%$

Table 2: The (unaware) voter's belief given the tweet he has processed.

asymmetry in his voting decision stems from the fact that he believes a priori that the left wing candidate is better.

**Processing Decision** Now I analyze the voter's information processing decision. First consider the case where the first tweet is left leaning. He knows that if he processes the tweet, he will not be able to process the second tweet and will vote for the left wing candidate. The expected utility of processing the left leaning tweet is therefore equal to the conditional probability that the left wing candidate is good given the fact that the tweet is left leaning:

$$\frac{\Pr(\text{left wing candidate is good \& the tweet is left leaning})}{\Pr(\text{the tweet is left leaning})} = \frac{0.7 \times 0.5}{0.7 \times 0.5 + 0.3 \times 0.2} = \frac{35}{41}.$$

On the other hand, if the voter ignores the left leaning tweet, he processes and uses the second tweet for his voting decision. When he evaluates the expected utility of ignoring the tweet, he has to anticipate his voting decisions and expected utilities in different configurations of the second tweet. For instance, the voter knows that if he ignores the first tweet and the second tweet is right leaning, he will vote for the right wing candidate. From the view of his period 1 self before he makes the processing decision, the expected utility in that configuration is equal to the conditional probability that the right wing candidate is good, given the first tweet is left leaning and the

second tweet is right leaning:

$$\frac{0.3 \times 0.2 \times 0.5}{0.7 \times 0.5 \times 0.2 + 0.3 \times 0.2 \times 0.5} = \frac{3}{10}.$$

The probabilities, voting decisions and the expected utilities, associated to different configurations of the second tweet are shown in table 3.

second tweet	probability	voting decision	expected utility
left leaning	$187/410$	left	$175/187$
neutral	$3/10$	left	$35/41$
right leaning	$10/41$	right	$3/10$

Table 3: The probabilities of different configurations of the second tweet, the associated voting decisions and expected utilities, given that the voter ignores the first tweet which is left leaning.

The expected utility of ignoring the left leaning tweet is just equal to the weighted average of the expected utilities associated with different configurations of the second tweet:

$$\left(\frac{187}{410} \times \frac{175}{187}\right) + \left(\frac{3}{10} \times \frac{35}{41}\right) + \left(\frac{10}{41} \times \frac{3}{10}\right) = \frac{31}{41}.$$

It is smaller than  $35/41$ , *i.e.*, the expected utility of processing. Therefore, the voter (optimally) processes the left leaning tweet.

With similar computations, the expected utilities of processing and ignoring a neutral tweet are equal to  $7/10$  and  $71/100$  respectively; while the expected utilities of processing and ignoring a right leaning tweet are equal to  $15/29$  and  $187/290$  respectively. Therefore, the voter ignores the first tweet if it is neutral or right leaning. To summarize, *the voter processes the first tweet if and only if it is left leaning, i.e., it confirms his belief.*

The processing decisions of the voter exhibit a confirmation bias, which is well documented in the literature of information avoidance. Most of the theories in the literature explain the bias by belief-dependent utility, exogenous biases or interpersonal interaction. In contrast, the voter in this example is

rational, Bayesian and cares only about maximizing the probability of voting for the good candidate. In this example, this confirmation “bias” is optimal and solely driven by the limitation in processing ability.

Not only that the limitation in information processing ability drives a confirmation bias in processing behavior, it also drives a “bias” in voting decision. In this example, the voter votes for the right wing candidate if and only if both tweets are right leaning, or the first tweet is neutral and the second tweet is right leaning. Differently, if the voter can process both tweets, apart from the two scenarios just mentioned, he also votes for the right wing candidate if the first tweet is right leaning and the second tweet is neutral. *The voter votes for the right wing candidate with a lower probability if he can only process one tweet, than if he can process both tweets.*

Lastly, it is interesting to note that with the assumption of “unawareness”, the belief of the voter does not have the martingale property. The expectation of the voter’s posterior belief (after he processes a tweet), evaluated before he sees any tweets, is not equal to his prior belief. Furthermore, because of the confirmation bias, *the expectation of the voter’s posterior belief that the left wing candidate is “good”, evaluated before he sees any tweets, is bigger than 70%, i.e., it is biased towards the left wing candidate.*

**Intuition** First, it is very intuitive that the voter ignores the neutral tweet as it does not provide any information about which candidate is good. It is optimal for the voter to save his processing capacity for the second tweet, which could be informative. In the following, I present the intuition behind the asymmetry between the processing decisions regarding to the left leaning and right leaning tweet.

Consider the case where the first tweet is left leaning. To decide whether to process or to ignore the tweet is equivalent to decide whether to vote based on the first or the second tweet. On one hand, the voter knows that if he processes the first tweet, he will vote for the left wing candidate. On the other hand, if the voter votes based on the second tweet, he knows that he will also vote for the left wing candidate if the second tweet is neutral or left leaning. Therefore, in those two configurations of the second tweet,

processing or ignoring the first tweet yields the same action and associated payoff. The voter only has to compare the expected utility of processing and ignoring the first tweet under the configuration that the second tweet is right leaning and thus contradictory to the first tweet. The comparison is shown on the first row of table 4.

first tweet	second tweet	expected utility	
		process the first tweet	ignore the first tweet
left leaning	right leaning	$7/10$	$3/10$
right leaning	left leaning	$3/10$	$7/10$
right leaning	neutral	$15/29$	$14/29$

Table 4: The expected utility of processing and ignoring the first tweet, given the relevant configurations of the second tweet.

After the voter receives a left leaning tweet, he becomes more confident that the left wing candidate is the good candidate. Processing the first tweet and voting for the left wing candidate becomes a more attractive option. Moreover, the voter knows that if he ignores the first tweet, he will be underconfident about the left wing candidate and will take the suboptimal voting decision<sup>11</sup> should the second tweet be right leaning. This gives him incentive to process the left leaning tweet.

Now consider the case where the first tweet is right leaning. Only the configurations where the second tweet is left leaning or neutral are relevant as they induce a different voting decision compared to the first tweet. The comparisons are shown in the second and the third row of table 4. When the second tweet is left leaning, ignoring the first tweet and voting based on the second tweet gives a higher expected utility; while in the configuration that the second tweet is neutral, processing the first tweet and voting based on it gives a higher expected utility. Therefore, compared to processing the right

<sup>11</sup>When the first tweet is left leaning and the second tweet is right leaning, the optimal voting decision is to vote for the left wing candidate. However, the voter will vote for the right wing candidate if he ignores the first tweet. Note that this is true even if the right leaning tweet is marginally stronger than the left leaning tweet.

leaning tweet, ignoring it induces a loss when the second tweet is neutral but a gain when the second tweet is left leaning.

As the voter was confident (with prob. 70%) that the left wing candidate is good, he is not sure about voting for the right wing candidate even after receiving the right leaning tweet. The loss of ignoring the tweet is therefore small. On the other hand, as he is not sure about the right wing candidate, the benefit of waiting for the second tweet is large. The gain of ignoring the right leaning tweet outweighs the loss. This gives him incentive to ignore the right leaning tweet.

## 4 Model Setting

**Primitives** The decision maker faces a binary choice problem with two actions,  $a \in \{l, r\}$ . For example,  $l$  could represent voting for the left wing candidate and  $r$  could represent voting for the right wing candidate. There are two possible states of the world,  $\omega \in \{L, R\}$ . The DM wants to match his action to the state, *i.e.*, his utility function  $u(a | \omega)$ <sup>12</sup> is

$$\begin{aligned} u(l | L) &= u_L > 0; \\ u(r | R) &= u_R > 0; \\ u(l | R) &= u(r | L) = 0. \end{aligned}$$

The DM's prior belief is  $(p_L, p_R)$ , with  $p_L + p_R = 1$  and  $p_L, p_R > 0$ . Without loss of generality, I assume  $u_L p_L \geq u_R p_R$ . Action  $l$  is a priori more attractive than action  $r$ .

Before the DM takes the action, he looks for information about the two

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<sup>12</sup>The model is identical up to the following transformation in the utility function:

$$\begin{aligned} u(l | L) &= u_L + A; & u(l | R) &= B; \\ u(r | L) &= A; & u(r | R) &= u_R + B, \end{aligned}$$

for any constants  $A$  and  $B$ . What matters are the differences between the utility of the two actions given the state. It implies that the model can be applied to a setting where action  $l$  is risky, *i.e.*, it gives positive payoff  $u_L$  in state  $L$  and negative payoff  $-u_R$  in state  $R$ , while action  $r$  is safe and gives 0 payoff in both states.

candidates, for example on Google, Facebook or traditional media. The available information, for instance articles on the Google news page, is represented by a finite<sup>13</sup> sequence of signals, denoted  $(s_t)_{t=1}^T$ . The signal  $s_t$  is drawn from a set  $\mathcal{S}_t$  with the p.d.f. in states  $L$  and  $R$ , denoted  $f_{tL}$  and  $f_{tR}$  respectively. I assume

$$f_{t\omega}(s_t) > 0 \text{ for all } s_t \in \mathcal{S}_t \text{ and all } \omega \in \{L, R\}$$

which implies that no signal perfectly reveals the state. I allow the possibility for  $(f_{tL}, f_{tR})$  to vary with  $t$ , *i.e.*, different signals along the sequence may be drawn from different information structures. For example, the DM may expect that the quality of information decreases when he goes down the list on the Google news page.

By the Bayesian formula, the information conveyed by a signal  $s_t$  is represented by its likelihood ratio  $\frac{f_{tL}(s_t)}{f_{tR}(s_t)}$ . Therefore without loss of generality, I normalize signals as their likelihood ratio. For  $t = 1, \dots, T$ , I assume that  $\mathcal{S}_t$  is the set of strictly positive real numbers  $\mathcal{R}_{++}$  and

$$s_t = \frac{f_{tL}(s_t)}{f_{tR}(s_t)} \text{ for all } s_t \in \mathcal{R}_{++}.$$

For technical convenience, the c.d.f.  $F_{tL}$  and  $F_{tR}$  are assumed to be continuous but the results hold for more general distributions<sup>14</sup>. Furthermore, I denote the set of belief-confirming information and the set of belief-challenging information by  $\mathcal{S}_+$  and  $\mathcal{S}_-$  respectively:

$$\begin{aligned} \mathcal{S}_+ &= (1, \infty); \\ \mathcal{S}_- &= (0, 1). \end{aligned}$$

A signal  $s_t > 1$  supports state  $L$  and is belief-confirming; while a signal  $s_t < 1$  supports state  $R$  and is belief-challenging<sup>15</sup>. A signal  $s_t = 1$  is pure noise

<sup>13</sup>The results in this paper hold for arbitrarily big  $T$ .

<sup>14</sup>For examples, the results hold for discrete distributions, or for distributions with some mass points.

<sup>15</sup>Note that the definition of belief-confirming and belief-challenging information corresponds to whether the information supports the a priori optimal action. Therefore, it

which provides no information about the state of the world.

**Strength of Signals** The strength of a signal  $s_t$ , denoted by  $STR(s_t)$ , is defined as follows:

$$STR(s_t) = \max \{s_t, s_t^{-1}\} \in [1, \infty).$$

The larger  $s_t$ , the more convincingly<sup>16</sup> it supports state  $L$ ; similarly, the larger  $s_t^{-1}$ , the more convincingly it supports state  $R$ . Analogously, the strength of a set of signals  $E \subset (0, \infty)$  is defined as:

$$STR(s_t \in E) = \max \left\{ \frac{\int_E f_{tL}(s_t) ds_t}{\int_E f_{tR}(s_t) ds_t}, \frac{\int_E f_{tR}(s_t) ds_t}{\int_E f_{tL}(s_t) ds_t} \right\}.$$

Given that  $s_t \in E$ , the larger  $\int_E f_{tL}(s_t) ds_t$  compared to  $\int_E f_{tR}(s_t) ds_t$ , the more probable that the true state is  $L$ .

**Preliminary Information** The game is a sequential game with  $T + 1$  periods. At the last period, the DM takes action  $l$  or  $r$ . Before that, at each period  $t = 1, \dots, T$ , the DM observes some preliminary information about the signal  $s_t$  and decides whether to “process” or “ignore” it. In the context of Google news page, the preliminary information are the title of article, graphics, source, date of publication and preview<sup>17</sup>. Different from the illustrative example, the DM may not observe perfectly the realization of the signal when he makes his processing decision, *i.e.*, the preliminary information may not perfectly reveal the signal realization.

Formally, the preliminary information is represented by an experimentation of the signal  $s_t$ , which is denoted by  $\tilde{s}_t$ . I assume that  $\tilde{s}_t$  is drawn from a set  $\mathcal{E}$  and with a p.d.f.  $h(\cdot | s_t) : (0, \infty) \rightarrow \Delta_{\mathcal{E}}$ . For example, the DM could

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depends not only on the belief of the DM, but also on the utility matrix. In particular, the set of belief-confirming and belief-challenging information may differ for two individuals who have same the beliefs but different utility functions.

<sup>16</sup>That is, the signal is drawn with a higher probability in state  $L$  compared to that in state  $R$ .

<sup>17</sup>In general, the preliminary information could be interpreted as a brief understanding of the content of the article that consume negligible amount of cognitive resources.

observe whether the signal supports state  $L$  or state  $R$  but not its strength. In that case,  $\mathcal{E} = \{-, \pm, +\}$  with  $\tilde{s}_t = +$  if  $s_t > 1$ ,  $\pm$  if  $s_t = 1$ ,  $-$  if  $s_t < 1$ .

I denote by  $g(\cdot | \tilde{s}_t)$  the conditional probability distribution of  $s_t$  given the preliminary information  $\tilde{s}_t$ . In the rest of the paper, without loss of generality, I often use  $g$  instead of  $\tilde{s}_t$  to denote the preliminary information, as it represents the knowledge of the DM about the realization of  $s_t$ . Note that in the illustrative example,  $g$  is degenerate, *i.e.*, the DM observes perfectly the realization of  $s_t$  before taking the processing decision.

**Processing Decisions and Limitation** In this model, processing a piece of information has two effects. First, the DM learns perfectly the realization of  $s_t$ <sup>18</sup>, *i.e.*, he reads the article, understands better the content and learns whether it is left or right leaning and how convincing it is. Second, the DM updates his belief with  $s_t$ , *i.e.*, he must “process” the article to understand or memorize it in order to revise his belief<sup>19</sup>. I assume that he cannot return to a piece of information he has ignored<sup>20</sup>.

This mechanism of “processing” implies that the DM updates his belief only with a set of processed signals. Mathematically, let the set of processed signals at the beginning of period  $t$  as  $\mathcal{M}_t$ , which is also referred as the DM’s memory<sup>21</sup>. By construction,  $\mathcal{M}_1 = \emptyset$ . If the DM processes  $s_t$ ,  $\mathcal{M}_{t+1} = \mathcal{M}_t \cup \{s_t\}$ ; otherwise,  $\mathcal{M}_{t+1} = \mathcal{M}_t$ . With some abuse of notations, I write  $t' \in \mathcal{M}_t$  for some  $t' < t$  if and only if  $s_{t'} \in \mathcal{M}_t$ .  $t' \notin \mathcal{M}_t$  is defined analogously. The belief of the DM at period  $t$ , denoted  $(\Pr_L^t(\mathcal{M}_t), \Pr_R^t(\mathcal{M}_t))$ , is a function of his memory  $\mathcal{M}_t$  and his prior.

The time-line of the game is shown in figure 1. Note that once the DM has exhausted his processing capacity or has searched for all available pieces of information, the game jumps to period  $T + 1$  and he takes one of the two

<sup>18</sup>This effect is absent in the illustrative example, where  $g$  is degenerate.

<sup>19</sup>See section 2 for the related psychological theories and evidences.

<sup>20</sup>This assumption rules out the possibility that the DM keeps in mind the preliminary information of several pieces of news at the same time and decides which one(s) to process. It emphasize the limitation of the DM’s cognitive ability and the fact that the capacity of our working memory is small.

<sup>21</sup>This corresponds to one of the interpretation of processing, which is to memorize the information in order to update the one’s belief.

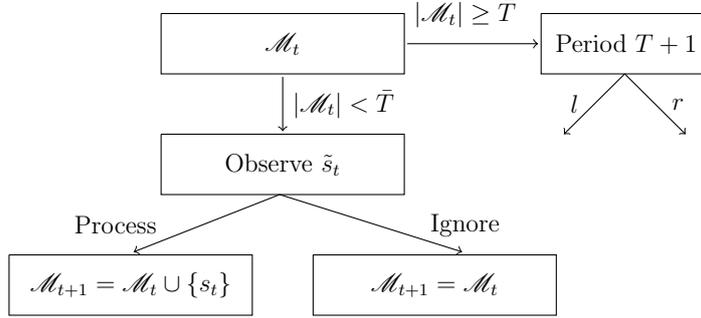


Figure 1: Time line of the game at any period  $t \leq T$ .

actions.

Apart from being subject to his processing constraint, the DM is rational and able to evaluate the expected utility of processing and ignoring a signal, using his memory  $\mathcal{M}_t$  and the preliminary information  $g$ . The processing strategy of the DM at period  $t$  is denoted by  $\sigma_t(g, \mathcal{M}_t) \in \{P, I\}$ , where  $P$  represents the decision to process the signal and  $I$  represents the decision to ignore it. The processing strategy from period 1 to period  $T$  is denoted by  $\sigma = (\sigma_t)_{t=1}^T$ . I assume that if the DM is indifferent between processing or ignoring a signal, he processes it when the signal is belief-confirming and ignores it otherwise.

**Posterior Belief and Equilibrium Concept** As in the illustrative example, I consider an “aware” and an “unaware” case on how the DM forms belief with his memory. The two cases correspond to two different levels of cognitive sophistication of the DM, and are defined as follows.

**Unaware DM** In the unaware case, when the DM forms his belief, he does not infer information from the fact that he had chosen to ignore some previous signals. Formally, the posterior belief of the unaware DM at period  $t$

satisfies:

$$\begin{aligned} \Pr_L^t(\mathcal{M}_t) &= \frac{p_L \prod_{s' \in \mathcal{M}_t} f_{s'L}(s')}{p_L \prod_{s' \in \mathcal{M}_t} f_{s'L}(s') + p_R \prod_{s' \in \mathcal{M}_t} f_{s'R}(s')} \\ &= \left( 1 + \frac{p_R}{p_L \prod_{s' \in \mathcal{M}_t} s'} \right)^{-1}; \\ \Pr_R^t(\mathcal{M}_t) &= 1 - \Pr_L(\mathcal{M}_t). \end{aligned} \tag{1}$$

This “unawareness” in belief formation has been documented in the literature, as shown in an experimental study conducted by [Enke and Zimmermann \(forthcoming\)](#). They show that a significant amount of their subjects are not aware of, and do not take into account the correlation between different pieces of information when they form beliefs. Analogously, the “unaware” DM is not aware of, and does not take into account his previous processing decisions when he forms belief. This assumption is also equivalent to the equilibrium concept introduced by [Jehiel \(2005\)](#) and [Eyster and Rabin \(2005\)](#), in which they argue that individuals may not take into account how other people’s action depend on these other people’s information<sup>22</sup>.

The game is a multi-selves dynamic game. With the “unawareness” assumption, the DM acts as if there is no incomplete information<sup>23</sup>. Therefore, the solution concept used is subgame perfect Nash equilibrium. At each period  $t = 1, \dots, T$ , he anticipates how his processing decision affects the decisions of all his future selves, using his belief of the state  $\Pr_L(\mathcal{M}_t)$  and the preliminary information  $g$  of the signal  $s_t$ . On the other hand, in period  $T+1$ , the DM takes actions  $l$  if  $\Pr_L(\mathcal{M}_{T+1}) > 1/2$  and  $r$  if  $\Pr_L(\mathcal{M}_{T+1}) < 1/2$ . In case of indifference, I assume that the DM takes action  $l$ .

In most parts (except section 7) of the main text, I study the DM’s processing behavior in only the unaware case. However, to help the reader to understand the differences between the two cases, I now briefly introduce the

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<sup>22</sup>In the current setting, when the unaware DM forms his belief, he does not take into account the fact that his previous selves’ processing decisions depend on their knowledge of the realization of the signals. As a result, he forms his belief as if the processing decisions of his previous selves did not correlate with the signals’ realization.

<sup>23</sup>The DM acts as if he had only received the signals in his memory. In other words, it is as if he “mistakenly thinks” that he is in a singleton information set.

definition of the aware case (the formal definition is presented in section 7). The results in the two cases are similar.

**Aware DM** In the aware case, the DM is more sophisticated. When he updates his belief with his memory, he rationally infers information about the signals in periods  $\{t' \mid t' \notin \mathcal{M}_t\}$  from the fact that he had chosen to ignore them. The game is a multi-selves dynamic game with incomplete information and the corresponding solution concept is perfect Bayesian Nash equilibrium. When the DM updates his belief with  $\mathcal{M}_t$ , he does not know his processing strategy in periods  $t' < t$ , but forms a conjecture denoted by  $(\tilde{\sigma}_{t'})_{t'=1}^{t-1}$ .

In equilibrium, for all  $t = 1, \dots, T+1$ , the conjecture  $(\tilde{\sigma}_{t'})_{t'=1}^{t-1}$  of the DM's period  $t$  self has to be consistent with the equilibrium processing strategies of his previous selves. On the other hand, when the DM decides whether to process or ignore a signal, he is fully aware of the fact that his future self will make rational inference. He evaluates the expected utility of processing and ignoring a signal and takes the optimal processing decision, using his knowledge of the state and the corresponding signal realization.

**Interpretations** Before I present the analysis, I first present a few interpretations of the belief formation mechanism assumed in this paper.

*1. Short-term-Long-term-Memory Conversion*

The first interpretation of the model is a memory-based belief formation mechanism<sup>24</sup>. The DM forms belief only with his (long-term) memory of the information. Upon receiving a piece of information, it first enters as a short-term memory. The DM then decides whether or not to consume his cognitive resources to convert it into long-term memory, which is used to form belief.

*2. Title (or a brief look) and the Main text of Articles*

The second interpretation is related to the way individuals process news articles. The DM first reads the title of the article and roughly gets an impression of its main message. He then decides whether or not to read the main text based on this rough impression, which allows him to update his belief accordingly. If he chooses not to read the main text, his rough impression of

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<sup>24</sup>See section 2 for related literature.

the article has negligible effect on his belief.

### 3. *Optimal Selective Processing*

Taking a step back, there is abundant evidence that individuals update differently with belief-confirming and belief-challenging information. As the DM decides whether or not to update his belief after he learns (imperfectly) the signal realization, this model ensures the optimality of the selective processing behavior given the DM's limited processing ability.

## 5 Processing Strategy

I now characterize the best response processing strategy of the DM's period  $t$  self, taking the strategies of his future selves as given. In period  $t$ , the DM learns from the preliminary information that  $s_t \sim g$ . I first analyze the case where  $g$  is degenerate, *i.e.*, the preliminary information  $g$  perfectly reveals the realization of the signal  $s_t$ , and then proceed to discuss the differences with non-degenerate  $g$ . A degenerate distribution  $g$  is simply denoted by  $s_t$ . Also denote by  $(p_L^t, p_R^t)$  the belief of the DM at the beginning of period  $t$ :

$$\begin{aligned} p_L^t &= \Pr_L^t(\mathcal{M}_t); \\ p_R^t &= \Pr_R^t(\mathcal{M}_t). \end{aligned}$$

Similarly, I assume  $p_L^t u_L \geq p_R^t u_R$ . To evaluate the expected utility of processing and ignoring a signal  $s_t$ , the DM anticipates how that will affect his future processing decisions, which in turn will affect his final action. To simplify expressions, denote by  $\Pr_{\mathcal{M}_{t+1}}^{t+1}(a \mid \omega)$  as the probability that the DM takes action  $a$  in state  $\omega$ , evaluated at the beginning of period  $t + 1$  given his memory  $\mathcal{M}_{t+1}$ . The function  $\Pr_{\mathcal{M}_{t+1}}^{t+1}(a \mid \omega)$  takes into account the DM's processing strategies<sup>25</sup> and information structures from period  $t + 1$  to period  $T$ . Note that  $\Pr_{\mathcal{M}_{t+1}}^{t+1}(l \mid L)$  and  $\Pr_{\mathcal{M}_{t+1}}^{t+1}(r \mid R)$  measures the quality of decision making in state  $L$  and  $R$  respectively, *i.e.*, they equal the probability

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<sup>25</sup>For instance,  $\Pr_{\mathcal{M}_{t+1}}^{t+1}(r \mid \omega)$  equals the probability that the DM receives and processes strong enough evidence that support state  $R$  from period  $t + 1$  to period  $T$ , such that he takes action  $r$  in the final period.

that the DM chooses the optimal action in the two states.

If the DM processes the signal  $s_t$ , he incorporates it into his memory  $\mathcal{M}_{t+1}$  and that affects his choice of the final action. Denote by  $\Delta_{s_t, \mathcal{M}_t}^t(a | \omega)$  the corresponding change in probability of choosing action  $a$  in state  $\omega$ , *i.e.*,

$$\Delta_{s_t, \mathcal{M}_t}^t(a | \omega) = \Pr_{\mathcal{M}_t \cup \{s_t\}}^{t+1}(a | \omega) - \Pr_{\mathcal{M}_t}^{t+1}(a | \omega).$$

The DM's expected utility of processing  $s_t$ , denoted by  $U_P(s_t, \mathcal{M}_t)$ , is equal to a weighted sum of the expected utilities in state  $L$  and state  $R$ , *i.e.*,

$$U_P(s_t, \mathcal{M}_t) = u_L \frac{p_L^t s_t}{p_L^t s_t + p_R^t} \Pr_{\mathcal{M}_t \cup \{s_t\}}^{t+1}(l | L) + u_R \frac{p_R^t}{p_L^t s_t + p_R^t} \Pr_{\mathcal{M}_t \cup \{s_t\}}^{t+1}(r | R),$$

where  $\frac{p_L^t s_t}{p_L^t s_t + p_R^t}$  and  $\frac{p_R^t}{p_L^t s_t + p_R^t}$  are the conditional probability of state  $L$  and state  $R$  given the signal  $s_t$ . Similarly, his expected utility after ignoring the signal, denoted by  $U_I(s_t, \mathcal{M}_t)$ , follows:

$$U_I(s_t, \mathcal{M}_t) = u_L \frac{p_L^t s_t}{p_L^t s_t + p_R^t} \Pr_{\mathcal{M}_t}^{t+1}(l | L) + u_R \frac{p_R^t}{p_R^t + p_L^t s_t} \Pr_{\mathcal{M}_t}^{t+1}(r | R).$$

Therefore,  $U_P(s_t, \mathcal{M}_t) \geq U_I(s_t, \mathcal{M}_t)$  if and only if:

$$\begin{aligned} u_L \frac{p_L^t s_t}{p_L^t s_t + p_R^t} \Delta_{s_t, \mathcal{M}_t}^t(l | L) &\geq -u_R \frac{p_R^t}{p_R^t + p_L^t s_t} \Delta_{s_t, \mathcal{M}_t}^t(r | R) \\ \iff u_L p_L^t s_t \Delta_{s_t, \mathcal{M}_t}^t(l | L) &\geq -u_R p_R^t \Delta_{s_t, \mathcal{M}_t}^t(r | R). \end{aligned} \quad (2)$$

Equation (2) shows how the DM's processing decision depends on the change in the quality of decision making in the two states. For instance, if both  $\Delta_{s_t, \mathcal{M}_t}^t(l | L)$  and  $\Delta_{s_t, \mathcal{M}_t}^t(r | R)$  are positive, processing the signal improves the quality of decision making in both states, *i.e.*, there is a higher probability that the DM chooses the optimal action in both states. Unsurprisingly, by equation (2), the DM processes the signal. The opposite holds true when both

$\Delta_{s_t, \mathcal{M}_t}^t(l | L)$  and  $\Delta_{s_t, \mathcal{M}_t}^t(r | R)$  are negative<sup>26</sup>. The trade-off kicks in when the two functions have opposite signs, *i.e.*, processing the signal improves the quality of decision making in one state but worsens it in another state.

**Lemma 1.** *Assume  $\Delta_{s_t, \mathcal{M}_t}^t(l | L)$  and  $\Delta_{s_t, \mathcal{M}_t}^t(r | R)$  are of opposite signs with neither equal to 0, and suppose the DM is not indifferent between processing and ignoring. At period  $t$ , the DM's best response processing strategy, taking  $\Delta_{s_t, \mathcal{M}_t}^t(l | L)$  and  $\Delta_{s_t, \mathcal{M}_t}^t(r | R)$  as given, is as follows:*

1. *if  $\Delta_{s_t, \mathcal{M}_t}^t(l | L) > 0$ , processing the signal  $s_t$  increases his probability of taking action  $l$  in both states. The DM chooses to process it if and only if it is a strong enough evidence for state  $L$ , *i.e.*,*

$$s_t > \frac{u_R p_R^t}{u_L p_L^t} \times \frac{\Delta_{s_t, \mathcal{M}_t}^t(r | R)}{\Delta_{s_t, \mathcal{M}_t}^t(r | L)}; \quad (3)$$

2. *if  $\Delta_{s_t, \mathcal{M}_t}^t(r | R) > 0$ , processing the signal  $s_t$  increases his probability of taking action  $r$  in both states. The DM chooses to process it if and only if it is a strong enough evidence for state  $R$ , *i.e.*,*

$$s_t^{-1} > \frac{u_L p_L^t}{u_R p_R^t} \times \frac{\Delta_{s_t, \mathcal{M}_t}^t(l | L)}{\Delta_{s_t, \mathcal{M}_t}^t(l | R)}. \quad (4)$$

*Proof.* The results are implied by inequality (2) combined with

$$\begin{aligned} \Delta_{s_t, \mathcal{M}_t}^t(r | L) &= -\Delta_{s_t, \mathcal{M}_t}^t(l | L); \\ \Delta_{s_t, \mathcal{M}_t}^t(l | R) &= -\Delta_{s_t, \mathcal{M}_t}^t(r | R). \end{aligned} \quad \square$$

To understand the intuition of the result, let us look into case 1 where  $\Delta_{s_t, \mathcal{M}_t}^t(l | L) > 0 > \Delta_{s_t, \mathcal{M}_t}^t(r | R)$ . If the DM processes the signal, he knows that he will be more inclined to take action  $l$  in both state  $L$  and  $R$ . This

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<sup>26</sup>Note that with limited attention, processing a signal does not necessarily increase the probability of choosing the optimal action. It is because processing lowers the number of signals the DM can process in the future. Take an extreme example where  $s_t = 1$ , processing the signal does not help the DM to know better about the state of the world, but consumes one unit of processing capacity that could be used on an informative future signal.

induces a gain in utility in state  $L$  but a loss in state  $R$ . If the signal is a strong evidence that state  $L$  is true, the gain in state  $L$  outweighs the loss in state  $R$ . This gives the DM incentive to process the signal.

On the other hand, the difference between inequality (3) and (4) shows that the DM's equilibrium processing strategy is "biased", *i.e.*, his best response processing strategy for signals that encourage the choice of action  $l$  and action  $r$  are generically different. In the following, I illustrate the "asymmetry" of the equilibrium processing strategy by comparing the DM's processing decisions for two "symmetric" signals, which will be defined in below.

**Definition 1 (Symmetric Signals).** *Suppose processing a signal  $s_t$  increases the probability of taking action  $l$  in both states, *i.e.*,  $\Delta_{s_t, \mathcal{M}_t}(l | L) > 0 > \Delta_{s_t, \mathcal{M}_t}(r | R)$ . Another signal  $s'_t$  is symmetric to  $s_t$  if and only if processing  $s'_t$  increases the probability of taking action  $r$  in both states, *i.e.*,  $\Delta_{s'_t, \mathcal{M}_t}(r | R) > 0 > \Delta_{s'_t, \mathcal{M}_t}(l | L)$  and*

$$\begin{aligned} s'_t &= s_t^{-1}; \\ \Delta_{s_t, \mathcal{M}_t}^t(l | L) &= \Delta_{s'_t, \mathcal{M}_t}^t(r | R); \\ \Delta_{s_t, \mathcal{M}_t}^t(r | R) &= \Delta_{s'_t, \mathcal{M}_t}^t(l | L). \end{aligned}$$

That is, for two symmetric signals  $s_t$  and  $s'_t$ , the signal  $s'_t$  supports state  $R$  as strongly as  $s_t$  supports state  $L$ ; and processing  $s'_t$  encourages the choice of action  $r$  to the same extent as processing  $s_t$  encourages the choice of action  $l$ .

**Corollary 1.** *Consider a signal  $s_t$  which encourages the choice of action  $l$ , and a signal  $s'_t$  which is symmetric to  $s_t$ . The DM pays more attention to  $s_t$  than to  $s'_t$ , *i.e.*, if the DM processes  $s'_t$ , he also processes  $s_t$  while the opposite is not necessarily true.*

*Proof.* The results are implied directly by lemma 1 as  $\frac{u_L p_L}{u_R p_R} \geq 1 \geq \frac{u_R p_R}{u_L p_L}$ .  $\square$

When action  $l$  is a priori more attractive, it implies that either the DM believes with a high probability that state  $L$  is true ( $p_L$  is big) or the payoff associated with state  $L$  is much higher than that with state  $R$  ( $u_L \gg u_R$ ). In either of two cases, the DM cares more about choosing the optimal action

in state  $L$  compared to that in state  $R$ . Therefore, he is more inclined to process signals which encourage the choice of action  $l$ , in comparison to symmetric signals which encourage the choice of action  $r$ . In other words, the “asymmetry” in the a priori attractiveness of actions drives the “asymmetry” in the equilibrium processing strategy. As I will show in the section 6, this “asymmetry” explains a number of behavioral phenomena documented in the literature.

**Non-degenerate  $g$**  Now I analyze the case where  $g$  is non-degenerate, *i.e.*, the preliminary information  $g$  does not perfectly reveal the realization of the signal  $s_t$ . In this case, processing the signal has two effects: it allows the DM to learn the realization of  $s_t$  and update his belief. The expected utilities of processing and ignoring a signal with preliminary information  $g$  are:

$$U_P(g, \mathcal{M}_t) = \int U_P(s_t, \mathcal{M}_t)g(s_t | \tilde{s}_t) ds_t;$$

$$U_I(g, \mathcal{M}_t) = \int U_I(s_t, \mathcal{M}_t)g(s_t | \tilde{s}_t) ds_t,$$

which gives the following result.

**Corollary 2.** *Suppose when  $g$  is degenerate, the DM’s best response processing strategy is to process  $s_t$  if and only if its realization is in some set  $\bar{\mathcal{S}}$ , *i.e.*,  $\sigma_t(s_t, \mathcal{M}_t) = P$  if and only if  $s_t \in \bar{\mathcal{S}}$ .*

*Now suppose the DM observes that  $s_t \sim g$  for some non-degenerate  $g$ . If there is a large enough probability that the signal realization is in the set  $\bar{\mathcal{S}}$ , his best response processing strategy is also to process the signal, *i.e.*,  $\sigma_t(g, \mathcal{M}_t) = P$  if  $\int_{s \in \bar{\mathcal{S}}} g(s | \tilde{s}_t) ds$  is big enough.*

The intuition of corollary 2 is best understood as follows: when the DM observes perfectly the realization of  $s_t$ , he knows that processing and ignoring the signal induce two different probability distribution of his final action. Choosing whether to process or ignore the signal is equivalent to choosing between two lotteries, let say lottery  $P_{s_t}$  and lottery  $I_{s_t}$ . In contrast, when the DM observes that  $s_t \sim g$ , it is as if he does not know exactly which two lotteries he is choosing between. Instead, he only knows how the pair of

lotteries is distributed. It is then straightforward to see that if there is a big enough probability that lottery  $P_{s_t}$  yields higher utility than lottery  $I_{s_t}$ , the DM will choose the former, which corresponds to the decision of processing the signal.

Importantly, corollary 2 implies that one could focus on the case where  $g$  is degenerate, without losing much qualitative insights. For instance, if the DM processes belief-confirming signals but ignores belief-challenging signals, he also processes a signal if he observes that the signal is likely to be belief-confirming. In the rest of the paper (unless when specified), I analyze only the case where  $g$  is degenerate.

## 6 Behavioral Implications

In this section, I present the full equilibrium analysis of this multi-selfs dynamic game and its behavioral implications. For simplicity as well as other reasons that will be discussed below, I assume  $\bar{T} = 1$ , *i.e.*, the DM can only process one signal.

Furthermore, I will only look into the case where  $T = 2$ . In appendix B, I show that it is without loss of generality<sup>27</sup> to do so when  $\bar{T} = 1$ . The processing decision boils down to whether the DM processes  $s_1$  or ignores  $s_1$  in order to save his cognitive resources for  $s_2$ . By lemma 1, the equilibrium processing strategy of the DM is characterized in the following proposition.

**Proposition 1.** *In period 1, the DM processes a signal  $s_1$  if and only if it is strong enough, *i.e.*,*

- *he ignores pure noise  $s_1 = 1$ ;*
- *he processes a belief-confirming signal  $s_1 > 1$  if and only if*

$$STR(s_1) = s_1 \geq \Phi^+; \tag{5}$$

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<sup>27</sup>Loosely speaking, I show that the equilibrium processing strategy of the DM in any period  $t < T$  is equivalent to the equilibrium processing strategy of the DM in period 1 in some simplified game where  $T = 2$ .

- he processes a belief-challenging signal  $s_1 < 1$  if and only if

$$STR(s_1) = s_1^{-1} > \Phi^-, \quad (6)$$

where  $\Phi^+$  and  $\Phi^-$  are defined as follows:

$$\begin{aligned} \Phi^+ &= \frac{u_R p_R}{u_L p_L} \times STR \left( s_2 \in \left( 0, \frac{u_R p_R}{u_L p_L} \right) \right); \\ \Phi^- &= \frac{u_L p_L}{u_R p_R} \times STR \left( s_2 \in \left[ \frac{u_R p_R}{u_L p_L}, \infty \right) \right) > \frac{u_L p_L}{u_R p_R}. \end{aligned} \quad (7)$$

The equilibrium processing strategy of the DM is a threshold-strategy, *i.e.*, he processes the signal if and only if the signal strength is larger than some threshold. As shown by equation (7), the two thresholds,  $\Phi^+$  and  $\Phi^-$  which characterize the processing strategy for belief-confirming and belief-challenging information respectively, are generically different. Moreover, they depend on the strength of a subset of all configurations of the future signal  $s_2$ , which corresponds to the realizations of  $s_2$  that induce a different action compared to  $s_1$  (see table 5). Intuitively, when an individual decides whether not to read a piece of left-wing news, he is concerned about losing time and/or attention for a right-leaning article as it provides contradictory information. This feature has important implications on the comparative analysis, which will be discussed later in this section.

## 6.1 Behavioral Phenomena

The equilibrium processing strategy has behavioral implications which explain some of the “biased” processing behaviors documented in the behavioral economics and psychological literature. The first implication is that there is a preference for strong signals, *i.e.*, the DM processes only strong enough signals in equilibrium. This implies that individuals react more to striking or convincing information but ignore coarse or ambiguous information. Analogously, in an experiment which studies visual attention, [Itti and Baldi \(2006\)](#) shows that individuals selectively allocate visual attention to details that induce a large difference between prior and posterior belief.

$s_1$	$s_2$	Final action	
		process $s_1$	ignore $s_1$
$> 1$	$\geq \frac{u_R p_R}{u_L p_L}$	$l$	$l$
	$< \frac{u_R p_R}{u_L p_L}$	$l$	$r$
$< \frac{u_R p_R}{u_L p_L}$	$\geq \frac{u_R p_R}{u_L p_L}$	$r$	$l$
	$< \frac{u_R p_R}{u_L p_L}$	$r$	$r$

Table 5: The table shows the final action given  $(s_1, s_2)$  and the processing decision. The colored boxes highlight the configurations of  $s_2$  that induce different actions compared to  $s_1$ , which corresponds to the subset of realizations of  $s_2$  shown in equation (7).

Another implication relates to the confirmation bias, which is defined as follows:

**Definition 2 (Confirmation Bias).** *The equilibrium processing strategy exhibits confirmation bias if  $\Phi^- > \Phi^+$ . That is, the DM processes a larger set of belief-confirming information, compared to belief-challenging information.*

When  $\Phi^- > \Phi^+$ , the DM processes a belief-confirming signal  $s$  but ignores an equally strong belief-challenging signal  $s^{-1}$  for all  $s \in [\Phi^+, \Phi^-]$ . On the other hand, there does not exist any  $s \in (1, \infty)$  such that the DM ignores a belief-confirming signal  $s$  but processes an equally strong belief-challenging signal  $s^{-1}$ .

The following proposition presents two behavioral implications of the equilibrium processing strategy.

**Proposition 2.** *The equilibrium processing strategy of the DM explains the following behavioral phenomena:*

1. **(preference for strong signals)**

*The DM processes a signal if and only if it is strong enough, as shown in equation (5) and (6).*

2. **(confirmation bias for confident individuals)**

*If the DM is a priori confident enough that state  $L$  is true, his processing strategy exhibits confirmation bias, i.e.,  $\Phi^- > \Phi^+$  if  $p_L$  is big enough.*

The intuition of proposition 2 can be understood as a trade-off between current and future information. First, when the current information  $s_1$  is strong, the DM becomes much more confident about which state is true. Therefore, he knows that processing the current signal and choosing which action he takes based on it yields high utility. On the other hand, as he becomes more confident about which state is true, he has less incentive to look for future information. He knows that if he ignores the current signal and does not update his belief with it, he will be highly under-confident in period 2 which could lead to suboptimal decision. All of these induce the preference for strong signals.

On the other hand, when the DM is a priori confident enough that state  $L$  is true, even if he receives a weak belief-confirming signal, he becomes almost sure that action  $l$  is the optimal action. He has (almost) no incentive to look for future information and will process the weak belief-confirming signal. In contrary, even when the DM has processed a strong belief-challenging signal, he is not sure about the optimality of action  $r$ . Processing the belief-challenging signal and choosing action  $r$  yields low expected utility, which is close to 0.5. As a result, the DM has incentive to ignore the belief-challenging signal and look for future information as the benefit outweighs the loss. This asymmetry in the incentive to look for future information drives the confirmation bias.

The result of confirmation bias also holds when  $g$  is non-degenerate. To illustrate that, consider two pieces of preliminary information,  $g^+$  and  $g^-$ , where

$$\begin{aligned} g^+(s) &= 0 \text{ for all } s \leq 1; \\ g^-(s) &= 0 \text{ for all } s \geq 1. \end{aligned}$$

That is, the DM observes that the signal  $s_1$  is belief-confirming if  $s_1 \sim g^+$  and is belief-challenging if  $s_1 \sim g^-$ . Moreover, in both cases, he is not sure about the signal strength.

**Corollary 3.** *Suppose a priori the DM believes strongly that state  $L$  is true, i.e.,  $p_L$  is big enough. In period 1, he processes the signal if he observes*

that  $s_1 \sim g^+$  but ignores the signal if he observes that  $s_1 \sim g^-$ .

Both proposition 2 and corollary 3 explain the phenomenon of information avoidance, in particular the avoidance of belief-challenging information (Kahan et al. (2012)). Although this behavior of information avoidance looks like a systematic bias against belief-challenging information, the results aforementioned suggest that when one takes into account the limitation in cognitive ability, “confirmation bias” actually arises as an optimal strategy under a setting with rational and Bayesian individuals. This is in particular different from the literature which explains the processing behavior with belief-dependent utility, exogenous biases such as time-inconsistent preference or interpersonal interaction<sup>28</sup>. Moreover, the results also help us to understand how the “confirmation bias” differs across different subjects in experiments.

It is interesting to note the limitation in processing ability drives not only a confirmation bias, but also a “bias” in action. More specifically, if the DM is a priori confident enough, he chooses action  $l$  with a higher probability in the current setting, than in a setting where there is no limitation in processing ability.

**Corollary 4 (Bias in action).** *Suppose the DM a priori believes strongly that state  $L$  is true, i.e.,  $p_L$  is big enough. He chooses action  $l$  with a higher probability when he can only process one of the two signals  $s_1$  and  $s_2$ , than when he can process both.*

Now I present another behavioral implication, which relates to wishful thinking. It refers to individuals’ tendency to form optimistic belief about some desirable outcomes. In the psychological theory proposed by Krizan and Windschitl (2007), one of the mechanisms behind wishful thinking is that individuals tend to process and encode information that suggests some desirable outcomes. In the current setting,  $u_L$  and  $u_R$  measures the how desirable the two states are. When  $u_L$  is larger than  $u_R$ , state  $L$  is more desirable than state  $R$  as it is associated with a higher achievable payoff. By analogy to confirmation bias, wishful thinking is defined as follows:

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<sup>28</sup>See section 2 for the references.

**Definition 3 (Wishful Thinking).** *Suppose state  $L$  is more desirable than state  $R$ , i.e.,  $u_L \geq u_R$ . The equilibrium processing strategy of the DM exhibits wishful thinking if  $\Phi^- > \Phi^+$ . That is, he processes a larger set of information that supports state  $L$ , compared to that supports state  $R$ .*

It is clear from equation (5) and (6) that an increase in  $u_L/u_R$  has the same effect as an increase in  $p_L/p_R$ . Therefore, the second point of proposition 2 implies the following result.

**Corollary 5 (Wishful thinking).** *When state  $L$  is much more desirable than state  $R$ , the equilibrium processing strategy of the DM exhibits wishful thinking, i.e.,  $\Phi^- > \Phi^+$  when  $u_L/u_R$  is big enough<sup>29</sup>.*

## 6.2 Comparative Analysis

I now analyze how a change in the information structures  $(f_{2L}, f_{2R})$ , which could be induced by the Internet or information policies, affects the information processing behavior of the DM. In particular, I study whether the change strengthens the confirmation bias. Consider two environments A and B, which are associated with two different equilibrium processing strategies characterized by two sets of thresholds  $(\Phi_i^+, \Phi_i^-)$ ,  $i = A, B$ . The confirmation biases under the two environments are compared as follows:

**Definition 4 (Comparison of confirmation bias).** *The confirmation bias is stronger under environment A than under environment B if  $\Phi_A^+ \leq \Phi_B^+$  and  $\Phi_A^- \geq \Phi_B^-$ , where at least one of the two inequalities is strict. That is, under environment A, the DM processes only a subset of belief-challenging information that he would process under environment B; while the reverse is true for belief-confirming information.*

In the sequel, I analyze two specific types of change in the information structure, in order to illustrate how the Internet could strengthen the confirmation bias of the DM. The first type of change captures the idea that

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<sup>29</sup>Note that the model also explains a reverse phenomenon of wishful thinking, namely that individuals tend to form pessimistic belief about some undesirable outcomes when the associated loss increases (Dunning and Balcells (2013)). The formal result is presented as corollary 11 in the appendix

the Internet facilitates a better access to information, thanks to the decrease in information transmission cost and the advance of search technology. Formally, I assume that under environment  $j$ , where  $j = A, B$ , the signal  $s_2$  is drawn from the following distribution<sup>30</sup>:

$$s_2 = \begin{cases} 1 & \text{with probability } 1 - \lambda_j; \\ s & \text{otherwise, where } s \sim (f_{2L}, f_{2R}). \end{cases}$$

The bigger  $\lambda_j$  is, the better the access to information is. When  $\lambda_A > \lambda_B$ , there is a higher probability to receive informative signals  $s_2 \neq 1$  under environment  $A$  than that under environment  $B$ .

**Proposition 3.** *Suppose environment  $A$  facilitates a better access to information compared to environment  $B$ , i.e.,  $\lambda_A > \lambda_B$ . The confirmation bias is stronger under environment  $A$  than under environment  $B$ , i.e.,*

$$\Phi_A^+ = \Phi_B^- \quad \text{and} \quad \Phi_A^- > \Phi_B^+.$$

Before I present the intuition, let me first present the analysis on the second type of change. It corresponds to the idea that the Internet lowers the barrier to information production. As a result, it facilitates the development of new media, and therefore provides individuals with more information sources. The quality of the new information sources could be different from the old ones. Intuitively, the skewness of the information structure of the new sources have a great impact on the processing strategy of the DM. For instance, if the new information sources produce strong belief-challenging information but weak belief-confirming information, the DM has more incentive to process the former and ignore the latter. To avoid assuming whether the new information sources are biased towards one state or the other, I analyze the case where the information structure of the new sources are in aggregate symmetric.

Formally, I assume that under environment  $B$ , the signal  $s_2$  is drawn from

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<sup>30</sup>Note that here  $F_{2L}, F_{2R}$  are not continuous. Nonetheless, the results in previous sections still hold.

the following distribution:

$$s_2 = \begin{cases} 1 & \text{with probability } 1 - \lambda \\ s_B & \text{otherwise, where } s_B \sim (f_{2L}^B, f_{2R}^B), \end{cases} \quad (8)$$

while under environment  $A$ , the signal  $s_2$  is drawn from the following distribution:

$$s_2 = \begin{cases} 1 & \text{with probability } 1 - \lambda - \delta \\ s_B & \text{with probability } \lambda, \text{ where } s_B \sim (f_{2L}^B, f_{2R}^B); \\ s_A & \text{with probability } \delta, \text{ where } s_A \sim (f_{2L}^A, f_{2R}^A). \end{cases} \quad (9)$$

for some symmetric distribution  $(f_{2L}^A, f_{2R}^A)$ , *i.e.*,  $f_{2L}^A(s) = f_{2R}^A(s^{-1})$ . Put differently, environment  $A$  transfers some mass  $\delta$  of pure noise to the new information sources, which is characterized by a symmetric distribution  $(f_{2L}^A, f_{2R}^A)$ .

**Proposition 4.** *Consider two environments,  $A$  and  $B$ , defined in equation (9) and (8). The confirmation bias is stronger under environment  $A$  compared to environment  $B$  if and only if the strong belief-challenging information drawn from the new sources is weaker than the same information drawn from the old information sources, *i.e.*,*

$$STR\left(s_A \in \left(-\infty, \frac{u_R p_R}{u_L p_L}\right)\right) \leq STR\left(s_B \in \left(-\infty, \frac{u_R p_R}{u_L p_L}\right)\right).$$

Note that both types of change are defined in a way such that the information structure under environment  $A$  is in average more informative<sup>31</sup> than that under environment  $B$ . When the future information becomes more informative, classical intuition suggests that the DM should have more incentive to ignore the current information and process the future information. However, in this model, the processing decision depends not on the average informativeness of all realizations of  $s_2$ , but only on the set of relevant configurations of  $s_2$ . For instance, when the DM decides whether to process

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<sup>31</sup>The average strength of signal  $s_2$  in environment  $A$  is bigger than that in environment  $B$ , *i.e.*,  $E_A(STR(s_2)) > E_B(STR(s_2))$ .

an left leaning article, he does not worry about losing attention for another left leaning article but worry more about losing attention for a right leaning article. While environment  $A$  increases the average strength of  $s_2$  compared to environment  $B$ , it does not necessarily increase the average strength of the strong belief-challenging information  $s_2 < \frac{u_R p_R}{u_L p_L}$ . As a result, providing in average “better” information to the DM could strength his confirmation bias.

The Internet undoubtedly facilitates better access to information and lowers the cost of information production. As the Internet lowers the entry cost of information production, the new information sources are in general of lower quality. Proposition 3 and 4 contribute theoretical explanations and insights to the literature of political polarization, as it suggests that the Internet promotes biased processing behaviors or so called “cherry-picking” on information, in the presence of information overload.

## 7 Aware DM

In this section, I show how the results in previous sections hold in the “aware” case. Throughout the section, I assume that the preliminary information perfectly reveals the realization of the signal.

First, let me formally define the “aware” case. When the DM forms his belief with his memory  $\mathcal{M}_t$ , he rationally infers information about the signals in periods  $\{t' \mid t' \notin \mathcal{M}_t\}$  from the fact that he had chosen to ignore them. More specifically, he forms a conjecture  $\tilde{\sigma}_{t'}$  about his processing strategies in those periods  $t'$  and his posterior is given by the following equations:

$$\begin{aligned} \Pr_L^t(\mathcal{M}_t, (\tilde{\sigma}_{t'})_{t'=1}^{t-1}) &= \left( 1 + \frac{p_R}{p_L \prod_{t' \in \mathcal{M}_t} s_{t'} \prod_{t' \notin \mathcal{M}_t} \mathcal{R}_{t'}(\tilde{\sigma}_{t'})} \right)^{-1}; \\ \Pr_R^t(\mathcal{M}_t, (\tilde{\sigma}_{t'})_{t'=1}^{t-1}) &= 1 - \Pr_L^t(\mathcal{M}_t, (\tilde{\sigma}_{t'})_{t'=1}^{t-1}), \end{aligned}$$

where  $\mathcal{R}_{t'}(\tilde{\sigma}_{t'})$  is the ratio of the conjecture probability of ignoring a period  $t'$

signal in state  $L$  over state  $R$ :

$$\mathcal{R}_{t'}(\tilde{\sigma}_{t'}) = \frac{\int (\mathbb{1}_{\tilde{\sigma}_{t'}(s_{t'}, \mathcal{M}_{t'})=I}) f_{t'L}(s_{t'}) ds_{t'}}{\int (\mathbb{1}_{\tilde{\sigma}_{t'}(s_{t'}, \mathcal{M}_{t'})=I}) f_{t'R}(s_{t'}) ds_{t'}}.$$

On the other hand, when he decides whether to process or ignore a signal, he knows that if he ignores it, his future selfs will rationally infer information about the signal. The game is a multi-self dynamic game with incomplete information. The solution concept used is therefore perfect Bayesian Nash equilibrium, which requires the optimality of the processing strategies as well as the consistency of the DM's conjecture, i.e.,  $\tilde{\sigma}_{t'}$  has to coincide with the equilibrium processing strategy for all  $t' = 1, \dots, T$ .

First, I analyze the case where  $\bar{T} > 1$ . As in section 5, I analyze the best response processing strategy at period  $t$ , taking  $\Pr_{\mathcal{M}_t}^t(a | \omega)$  and  $\Delta_{s_t, \mathcal{M}_t}^t(a | \omega)$  as given. Note that two functions do not specify how the DM forms his posterior belief, and therefore encompass both the aware and unaware case. In other words, all the results in section 5 hold in the aware case. In particular, the equilibrium processing strategy is “asymmetric, *i.e.*, it is generically different for symmetric signals that encourages the choice of action  $l$  and action  $r$ .”

**Corollary 6 (Analogue of Corollary 1).** *Consider a signal  $s_t$  which encourages the choice of action  $l$ , and a signal  $s'_t$  which is symmetric to  $s_t$ . The DM pays more attention to  $s_t$  than to  $s'_t$ , *i.e.*, if the DM processes  $s'_t$ , he also processes  $s_t$  while the opposite is not necessarily true.*

Now I assume  $\bar{T} = 1$ . As in the unaware case, it is without loss of generality to further simply the model to  $T = 2$ . The equivalent result is presented in appendix B. Different from the unaware case, there is no guarantee that there exists a unique equilibrium in the aware case. The following proposition proves the existence of a perfect Bayesian Nash equilibrium.

**Proposition 5.** *There exists a perfect Bayesian Nash equilibrium in the aware case.*

The proof of the proposition is shown in the appendix. It follows classical fixed point arguments. Note that there may exist multiple equilibria

because of the self-fulfilling nature of the incomplete information game. If the future selfs of the DM conjecture that his period  $t$  self ignores only belief-challenging information, his period  $t$  self will have more incentive to ignore belief-challenging information as he knows that his future selfs will rationally infer information from it. On the other hand, he also has more incentive to process belief-confirming information because otherwise, his future selfs will mistakenly conjecture that he has received a belief-challenging information in period  $t$ .

In the following, I show that the behavioral implications presented in section 6 hold qualitatively for all equilibria in the aware case. First, I characterize the equilibrium processing strategy, as in proposition 1.

**Proposition 6.** *The equilibrium processing strategy of the aware DM is as follows:*

- *he processes a belief confirming signal  $s_1 > 1$  if and only if*

$$STR(s_1) = s_1 \geq \Phi^+;$$

- *he processes a pure noise or a weak belief-challenging signal  $s_1 \in [\frac{u_{RPR}}{u_{LPL}}, 1]$  if and only if*

$$s_1 > \Phi^+;$$

- *he processes a strong belief-challenging signal  $s_1 < \frac{u_{RPR}}{u_{LPL}}$  if and only if*

$$STR(s_1) = s_1^{-1} > \Phi^-,$$

where  $\Phi^+$  and  $\Phi^-$  are the fixed point of the following system of equa-

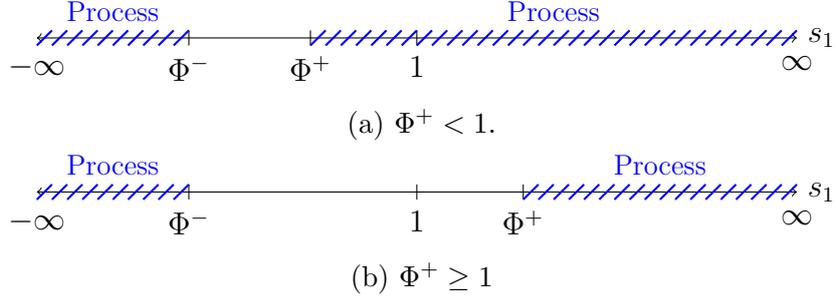


Figure 2: Equilibrium Information Processing Strategy: the DM processes  $s_1$  if and only if its realization falls on the shaded area.

tions:

$$\begin{aligned} \Phi^+ &= \frac{u_R p_R}{u_L p_L} STR \left( s_2 \in \left( 0, \frac{u_R p_R}{u_L p_L \frac{F_{1L}(\Phi^+) - F_{1L}(1/\Phi^-)}{F_{1R}(\Phi^+) - F_{1R}(1/\Phi^-)}} \right) \right) \\ \Phi^- &= \frac{u_L p_L}{u_R p_R} STR \left( s_2 \in \left[ \frac{u_R p_R}{u_L p_L \frac{F_{1L}(\Phi^+) - F_{1L}(1/\Phi^-)}{F_{1R}(\Phi^+) - F_{1R}(1/\Phi^-)}}, \infty \right) \right) \end{aligned} \quad (10)$$

The proof is very similar to that of proposition 1 and is therefore skipped. Different from the unaware case, the DM may process weak belief-challenging information (when  $\Phi^+ < 1$ , as shown in figure 2). This difference is driven by the self-fulfilling nature of the game: suppose the DM's period 2 self conjectures that his period 1 self ignores strong belief-challenging signals but processes weak belief-challenging signals. In the view of his period 1 self, he knows that if he ignores the weak belief-challenging signals, his period 2 self will mistakenly infer that  $s_1$  strongly supports state  $R$ . It in turn gives the DM's period 1 self incentive to process weak belief-challenging signals and take action  $l$ , as otherwise he will be under-confident about state  $L$  and switch to action  $r$  too easily in period 2.

Despite the slight difference, the behavior implications hold qualitatively. First, the DM processes strong enough signals, but may ignore weak signals, *i.e.*, it resembles a preference for strong signals (although it is weaker than

the version in the unaware case). Moreover, by equation (10), the limits of  $\Phi^+$  and  $\Phi^-$  when  $u_L p_L / u_R p_R \rightarrow +\infty$  are the same as in the unaware case<sup>32</sup>. Hence, when  $u_L p_L / u_R p_R$  is large enough,  $\Phi^- < \Phi^+$ . If the DM processes a signal  $s^{-1}$  that supports state  $R$ , he also processes a equally strong signal  $s$  that supports state  $L$ . The reverse is not necessarily true.

**Proposition 7.** *The equilibrium processing strategy of the DM explains the following behavioral phenomena:*

1. ***(Preference for strong signals)***

*The DM processes a signal  $s_1$  if it is strong enough.*

2. ***(Confirmation bias for confident individuals)***

*If the DM a priori believes strongly that state  $L$  is true, his processing strategy exhibits confirmation bias, i.e.,  $\Phi^- > \Phi^+$  if  $p_L$  is large enough.*

3. ***Wishful thinking if one state is much more desirable than the others***

*If state  $L$  is much more desirable than state  $R$ , the DM's processing strategy exhibits confirmation bias, i.e.,  $\Phi^- > \Phi^+$  if  $u_L / u_R$  is large enough.*

Note that the results hold in all equilibria in the aware case, i.e., all equilibrium processing strategies of the DM have (qualitatively) the same behavioral implications as in the unaware case.

Lastly, because there is an issue of multiple equilibria, the result of the comparative analysis would not be as clean as in the unaware case. However, the insights still hold true. As shown in equation (10), the thresholds which characterize the equilibrium processing strategy depend only on a subset of all realizations of  $s_2$ . As in the unaware case, providing in average more informative signal to the DM does not necessarily increases the average strength

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<sup>32</sup>That is, as shown in the proof of proposition 2,

$$\begin{aligned} \lim_{u_L p_L / u_R p_R \rightarrow +\infty} \Phi^+ &= 1 \\ \lim_{u_L p_L / u_R p_R \rightarrow +\infty} \Phi^- &= +\infty \end{aligned}$$

of a subset of all realizations of  $s_2$ . Therefore, it could strengthen the confirmation bias of the DM.

## 8 Applications

In this section, I provide two simple examples, which relates to polarization and media competition in the presence of information overload. For simplicity, I only look at the case where  $u_L = u_R = 1$ ,  $T = 2$  and  $\bar{T} = 1$ . Moreover, as in the illustrative example, I assume  $s_1$  and  $s_2$  follow a symmetric information structure with three possible realizations, denoted as  $1/q, 1, q$  where  $q > 1$ . The information structure is represented in the following table:

	$f_{t\omega}(1/q)$	$f_{t\omega}(1)$	$f_{t\omega}(q)$
$\omega = L$	$\lambda(1+q)^{-1}$	$1-\lambda$	$\lambda q(1+q)^{-1}$
$\omega = R$	$\lambda q(1+q)^{-1}$	$1-\lambda$	$\lambda(1+q)^{-1}$

Table 6: The distribution of  $s_t$ ,  $t = 1, 2$ , given the state of the world.

An increase in  $\lambda$  represents a better access to valuable information while  $q$  represents the strength/quality of the informative signals. In the illustrative example,  $\lambda = 0.3$  and  $q = 2.5$ . Throughout the section, I focus on the unaware case and assume  $g$  is degenerate.

### 8.1 Polarization

Many empirical studies have documented the phenomenon of political polarization or stronger partisanship in the US in recent years. [Bartels \(1998\)](#) and [Bartels \(2000\)](#) show that party identification has become a better predictor of vote decisions and document a decline in volatility of election outcomes. Moreover, the polarization is stronger among citizens who are more politically engaged and partisan. (See [Evans \(2003\)](#), [Baldassarri and Gelman \(2008\)](#), [Abramowitz and Saunders \(2008\)](#) and [Hetherington \(2009\)](#)). In contrast,

Gentzkow and Shapiro (2011) and Flaxman et al. (2016) show that online media exposes individuals to belief-challenging information, although somewhat counter-intuitively, Flaxman et al. (2016) also find that online media are associated to an increase in ideological distance between individuals.

In the following, I show how the model of limited cognitive ability presented in this paper sheds light on the wide range of phenomena documented in the literature. Assume that Alice and Bob have opposite prior beliefs about the state of the world with obvious notations  $p_L^a = p_R^b = p > 1/2$ . The informative signals are strong enough, *i.e.*,  $q > \frac{p}{1-p}$ , which rules out the trivial case where the two individuals always take their a priori optimal action. Without loss of generality, I assume that  $L$  is the true state.

I analyze three indicators of polarization:

1. the probability that both individuals take the same action, which is denoted as  $P_{consensus}$ . It is used to assess the intuition where more/better information results in a higher probability of achieving consensus;
2. the probability that the individual takes his/her a priori optimal action, which is denoted as  $P_{default}^j$ ,  $j = A, B$ . It measures how well the prior belief, or analogously party identification, predicts voting decision, which is studied in the empirical literature of political science;
3. the change in the distance between the beliefs of the two individuals after receiving information. It corresponds to belief polarization and is widely analyzed in the theoretical economics literature<sup>33</sup>.

Proposition 1 allows me to characterize the processing strategy of the two individuals as follows:

**Corollary 7.** *In period 1, both Alice and Bob process  $s_1$  if it is belief-confirming and ignore it if it is pure noise. There exists some thresholds  $q^-$ ,  $\lambda^-$  and  $p^-$  such that they process belief-challenging signal if and only if*

- *their prior is weak enough, *i.e.*,  $p < p^-$ , or;*

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<sup>33</sup>For example, see Acemoglu et al. (2007), Baldassarri and Gelman (2008).

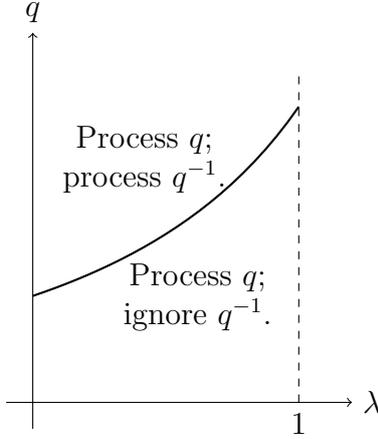


Figure 3: Alice's processing strategy in period 1 as a function of  $\lambda$  and  $q$ , fixing  $p_L^a = p = 0.7$ .

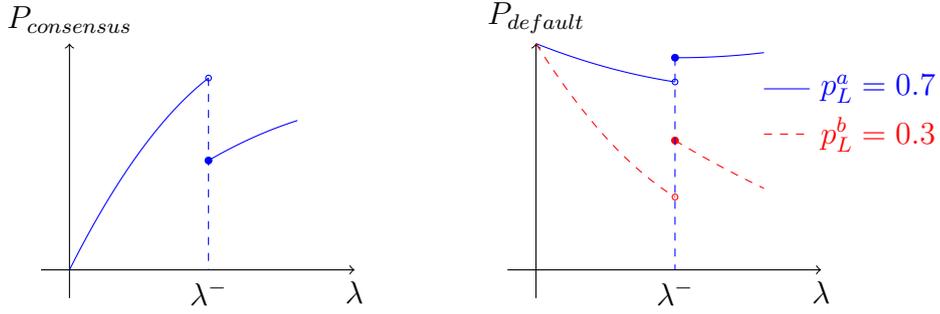
- the informative signals are strong enough, i.e.,  $q > q^-$ , or;
- the access to information is poor enough, i.e.,  $\lambda < \lambda^-$ .

The proof is shown in the appendix and the result is illustrated in figure 3. The following proposition, illustrated in figure 4, presents how a change in the access to information  $\lambda$  affects the first two indicators of polarization,  $P_{consensus}$  and  $P_{default}^j$ .

**Proposition 8.** *The probability that Alice and Bob take the same action,  $P_{consensus}$ , is non-monotonic in  $\lambda$ , i.e., it is increasing in the range  $[0, \lambda^-)$  and  $[\lambda^-, 1]$ , but exhibits a downward jump at  $\lambda = \lambda^-$ .*

*Similarly, the probability that Alice/Bob takes her/his a priori optimal action,  $P_{default}^a$  or  $P_{default}^b$ , is also non-monotonic in  $\lambda$ , i.e., both are decreasing in  $\lambda$  in the range  $[0, \lambda^-)$ , but exhibit a upward jump at  $\lambda = \lambda^-$ .*

When there is better access to information, fixing the processing strategies of the two individuals, the probability that they takes the optimal action  $l$  increases. However, as shown in proposition 3 and corollary 7, a better access to information promotes biased processing behavior, i.e., when  $\lambda$  is big enough, the two individuals ignore belief-challenging information. As a result, it could reduce the probability of achieving consensus and increase the



(a)  $P_{consensus}$  with  $p = 0.7$  and  $\Phi = 0.7$ .

(b)  $P_{default}$  with  $\Phi = 0.7$

Figure 4: The effect of  $\lambda$  on the first two indicators of polarization.

probability that the individuals take their a priori optimal action, despite the availability of more valuable public information. The limitation in processing ability hinders the benefits of information technology because individuals strategically allocate their cognitive resources in the presence of information overload.

Moreover, as individuals with different prior beliefs adopt different processing strategies, their beliefs can be polarized even if they receive the same sequence of information.

**Corollary 8.** *When  $\lambda \geq \lambda^-$ ,  $q \leq q^-$  or  $p \geq p^-$ , the distance between the beliefs of Alice and Bob increases after receiving information if the signals in the two periods support different states. More specifically, if  $s_1 > 1 > s_2$  or  $s_2 > 1 > s_1$ , Alice becomes more confident that state  $L$  is true while Bob becomes more confident that state  $R$  is true.*

*Moreover, the probability that  $s_1 > 1 > s_2$  or  $s_2 > 1 > s_1$  increases in  $\lambda$  and decreases in  $q$ .*

*Proof.* Without loss of generality, I analyze the case where  $s_1 = q$  and  $s_2 = 1/q$ . When  $\lambda \geq \lambda^-$ ,  $q \leq q^-$  or  $p \geq p^-$ , Alice and Bob process belief-confirming information but ignore belief-challenging information in period 1. Therefore, Alice processes  $s_1$  while Bob ignores  $s_1$  and processes  $s_2$ . Their beliefs in

period 3 are:

$$\begin{aligned}\Pr_L^a(\mathcal{M}_3^a) &= \Pr_L^a(q) = \left(1 + \frac{1-p}{pq}\right)^{-1} > p_L^a & ; \\ \Pr_L^b(\mathcal{M}_3^b) &= \Pr_L^b(q^{-1}) = \left(1 + \frac{p}{(1-p)q^{-1}}\right)^{-1} < p_L^b.\end{aligned}$$

On the other hand, the probability that  $s_1 > 1 > s_2$  or  $s_2 > 1 > s_1$  equals

$$2\lambda^2 q(1+q)^{-2},$$

which increases in  $\lambda$  and decreases in  $q$ . □

Corollary 8 shows that even when the same sequence of information is available for both individuals, the difference in their prior beliefs induces different processing strategies and could polarize their beliefs. Moreover, it happens only when there are sufficiently good access to information, or when the prior beliefs of the two individuals are strong enough. In other words, when the two individuals are partisan enough, a better access to information gives rise to the possibility of polarization even under a setting with public information. This result sheds light on the empirical evidence that polarization is much stronger among individuals who are more partisan.

On the other hand, belief polarization happens when the information available in the two periods are contradictory, *i.e.*,  $s_1 > 1 > s_2$  or  $s_2 > 1 > s_1$ . Its probability increases when there is better access to information and when the quality of information decreases. Arguably, the Internet contributes to both. While it provides us with enormous amount of information, it also facilitates the spread of rumors, fake news and low-quality information. It is much easier to find contradictory information on the Internet. It gives more incentive for individuals to selectively attend to belief-confirming information, which increases the probability of belief polarization.

## 8.2 Media Competition

In this second application, I study media strategy in the information era. In order to introduce the role of the media, I present a variation of the main model.

Formally, there is a continuum of media, indexed by  $i \in \mathcal{I}$ . In period  $t = 1, 2$ , each media  $i$  collects a signal  $s_{it}$  about the state of the world and publishes a piece of news  $m_{it}$ . The signal  $s_{it}$  collected by different media are independent and follow the distribution defined in the beginning of the section, *i.e.*, table 6. I assume that the media cannot post fake news and therefore either publishes the signal it receives or publishes nothing, *i.e.*,  $m_{it} \in \{\emptyset, s_{it}\}$ . To simplify the analysis, I assume that there are three types of non-strategic (biased) media,  $\{T_L, T_R, T_N\}$ , which publish according to the following fixed rules:

- media  $\{i \mid i \in T_L\}$  is biased towards state  $L$ : he publishes  $s_{it}$  if it supports state  $L$  ( $s_{it} = q$ ), but publishes  $\emptyset$  if he receives  $s_{it} \in \{1/q, 1\}$ ;
- media  $\{i \mid i \in T_R\}$  has an opposite bias: he publishes a signal if it supports state  $R$  ( $s_{it} = 1/q$ ) but publishes  $\emptyset$  if he receives  $s_i \in \{1, q\}$ ;
- media  $\{i \mid i \in T_N\}$  has no bias and publishes any informative signal he receives, *i.e.*,  $m_{it} = s_i$  if and only if  $s_i \neq 1$ .

I assume that each media belongs to one and only one of the three types, and the type of each media is a public information. The time line of the game is as follows:

**Period 1** The DM chooses which media he visits and processes the piece of news  $m_{i1}$  posted by the media.

**Period 2** If the media visited by the DM in period 1 published nothing, he chooses again a media outlet to visit. Otherwise, he visits no media as processing the information in period 1 takes time.

**Period 3** The DM forms his belief with his memory of information and takes an action  $l$  or  $r$ .

Note that this variation differs from the main model only in terms of interpretation. Here the DM control his “diet” of information by choosing

which (biased or unbiased) media to visit, instead of choosing whether to process or ignore the signals he receives. For example, if the DM visits media  $i \in T_L$  in period 1, it is as if he chooses to process the period 1 signal if and only if it supports state  $L$ . By corollary 7, the DM visits the biased media in period 1 if his prior belief is strong enough.

**Corollary 9.** *Suppose the DM has prior belief  $(p_L, 1 - p_L)$  where  $p_L \geq 1/2$ . In period 1, there exists a threshold  $p^- \in (1/2, 1)$  such that if  $p_L \geq p^-$ , he visits a media  $i$  where  $i \in T_L$ ; otherwise, he visits a media  $i$  where  $i \in T_N$ .*

*If the media visited by the DM in period 1 published nothing, he visits a media  $i$  where  $i \in T_N$  in period 2.*

Now I turn to analyze the viewership of the three types of media. In the society, individuals' prior belief  $p_L$  are distributed according to  $g(p)$  where  $g(p) > 0$  for all  $p \in (0, 1)$ . Its c.d.f. is denoted by  $G$ . Define the viewership of media  $\{i \mid i \in T_j\}$  as the mass of individuals which visit media  $\{i \mid i \in T_j\}$  across the two periods. Denote it by  $V_j$  for  $j = L, R, N$ . By corollary 9,

$$\begin{aligned} V_L &= 1 - G(p^-); \\ V_R &= G(1 - p^-); \\ V_N &= 2 - V_L - V_R - \lambda. \end{aligned}$$

That is, biased media attracts views from individuals with strong beliefs, while unbiased media serves the others.

**Corollary 10.** *When there are better access to information ( $\lambda$  increases), the viewership of the biased media increases while the viewership of unbiased media decreases, i.e.,  $V_L$  and  $V_R$  increase with  $\lambda$  while  $V_N$  decreases in  $\lambda$ .*

*On the other hand, when the quality of information increases ( $q$  increases), the viewership of the biased media decreases while the viewership of unbiased media increases, i.e.  $V_L$  and  $V_R$  decrease with  $q$  while  $V_N$  increases in  $q$ .*

*Proof.* The result follows from corollary 7. □

As shown in previous sections, a better access to information or an decrease in quality of information strengthens the confirmation bias of individuals, which in turn increases the viewership of the biased media. The increase in viewership increases the profitability of biased media and thus incentivize media to adopt a biased strategy. This result sheds light on the emergence of partisan media in recent years as the Internet promotes biased processing behavior.

## 9 Conclusion

In conclusion, this paper investigates the information processing behavior of a decision maker who can process only a subset of all available signals. I show that this limitation in processing ability drives a number of well-documented behavioral “biases”, including preference of strong signals, confirmation bias for confident individuals and wishful thinking.

These “biases” has been attracting lots of attention in the behavioral economics literature, in which many have analyzed how these “biases” affects different market outcomes by introducing directly the “biases” in traditional economics models. In contrast, instead of taking the “biases” as they are, this paper aims to improve our understanding by analyzing their cause. In particular, I show that these “biases” are features of optimal strategies if we take into account our limited cognitive ability as a human being.

This approach allows us to analyze the “biases” as an outcome of an optimization problem. It brings two advantages. First, not only that it explains the existence of the “biases”, but using standard techniques of comparative statistics, it also explains how the “biases” change among individuals with different personal characteristics or abilities, and how they change in different situations faced by the individuals. Second, it allows us to study how regulatory policies could play a role in changing these “biases” and associated market outcomes.

Thus, looking forward and building from the insights of this paper, there are two different ways to further develop the literature. First, in policy analysis with behavioral settings, modeling the “biases” as optimal strategies

allows us to take into account the indirect effects of policy interventions on behaviors of individuals. It contributes to a more complete analysis than if we take the “biases” as they are. For example (loosely speaking), if providing more information to consumers strengthens their confirming bias, it might back fire as it could weaken competition and increase prices. Second, more experimental or empirical work has to be done to understand how “biases” are formed and vary across different individuals or settings. It will give us a better understanding on whether the “biases” do indeed relate to the limitation in ability. And as an (early) answer to that question, an ongoing experimental study conducted by myself and co-authors, [Goette et al. \(2018\)](#), does find evidences that a larger cognitive load strengthens confirmation bias in belief formation.

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## A Optimality of the processing strategy

Before I present the omitted proofs in the main text, I now discuss the optimality of the equilibrium processing strategy. In the main text, the DM decides whether to process or ignore the signal given the preliminary information about its realization. Therefore, the equilibrium processing strategy  $\sigma_t(g, \mathcal{M}_t)$  is optimal for any preliminary information  $g$ . Moreover, as it maximizes the DM's expected utility for all possible preliminary information in each period, it also maximizes his expected ex-ante utility in every period  $t$ , which gives the following result.

**Proposition A.1.** *The processing strategy characterized by lemma 1 maximizes the expected utility of the DM's period  $t$  self.*

In the maintext, I show that the optimal processing strategy explains some well-documented behavioral phenomena in the presence of limitation in processing ability. The optimality is achieved as the DM evaluates the expected utility of processing and ignoring a signal given his knowledge of the signal realization. In contrast, proposition A.1 suggests that the results also hold if we consider strategies that maximize the DM's expected utility at the beginning of each period. This result is also useful for the proof for the equivalence result, as shown in the next section.

## B Equivalence Results

In this section, I present two equivalence results which allow me to simplify the model with  $T > 2$  and  $\bar{T} = 1$  to a model with  $T = 2$  and  $\bar{T} = 1$ . The idea is to show that the equilibrium processing strategy at any period  $t < T$  in a model with  $T > 2$  is equivalent to the equilibrium processing strategy at period 1 in the simplified model with  $T = 2$ . I first present the result in the unaware case.

## B.1 Unaware Case

**Proposition B.1.** *There exists some p.d.f.  $(\tilde{f}_{2L}, \tilde{f}_{2R})$  such that the following two equilibrium strategies are equivalent:*

1. *the equilibrium processing strategy of the DM with belief  $(p_L^t, p_R^t)$  at period  $t < T$ , under a setting with  $T > 2$ ,  $\bar{T} = 1$  and information structure  $\{(f_{L'}^t, f_{R'}^t)\}_{t'=t+1}^T$ ;*
2. *the equilibrium processing strategy of the DM with prior belief  $(p_L, p_R) = (p_L^t, p_R^t)$  at period 1, under a setting with  $T = 2$ ,  $\bar{T} = 1$  and information structure  $(\tilde{f}_{2L}, \tilde{f}_{2R})$ .*

*Proof.* I prove the proposition under the assumption the preliminary information perfectly reveals the signal realization. The proof is similar in the general setting where  $g$  could be degenerate<sup>34</sup>. First consider the first point where  $T > 2$  and without loss of generality assume  $u_L p_L^t \geq u_R p_R^t$ . By lemma 1, the decision maker processes signal  $s_t \geq \frac{u_R p_R^t}{u_L p_L^t}$  if and only if

$$\begin{aligned}
 s_t &\geq \frac{u_R p_R^t \Delta_{s_t, \mathcal{M}_t}^t(r | R)}{u_L p_L^t \Delta_{s_t, \mathcal{M}_t}^t(r | L)} \\
 &= \frac{u_R p_R^t \sum_{k=t+1}^T \left[ \mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t}} \mathbb{1}_{\sigma_k(s)=P} f_{kR}(s) ds}{u_L p_L^t \sum_{k=t+1}^T \left[ \mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t}} \mathbb{1}_{\sigma_k(s)=P} f_{kL}(s) ds}.
 \end{aligned} \tag{B.1}$$

On the other hand, he processes signal  $s_t < \frac{u_R p_R^t}{u_L p_L^t}$  if and only if

$$\begin{aligned}
 s_t^{-1} &> \frac{u_L p_L^t \Delta_{s_t, \mathcal{M}_t}^t(l | L)}{u_R p_R^t \Delta_{s_t, \mathcal{M}_t}^t(l | R)} \\
 &= \frac{u_L p_L^t \left[ 1 - \sum_{k=t+1}^T \left[ \mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h^*(s')=I}) f_{hL}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t}} \mathbb{1}_{\sigma_k^*(s)=P} f_{kL}(s) ds \right]}{u_R p_R^t \left[ 1 - \sum_{k=t+1}^T \left[ \mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h^*(s')=I}) f_{hR}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t}} \mathbb{1}_{\sigma_k^*(s)=P} f_{kR}(s) ds \right]}.
 \end{aligned} \tag{B.2}$$

<sup>34</sup>The proof is identical if I replace  $\mathbb{1}_{\sigma_h(s)=P}$  by  $\int \mathbb{1}_{\sigma_h(s)=P} g(s) dg$  and  $\mathbb{1}_{\sigma_h(s)=I}$  by  $\int \mathbb{1}_{\sigma_h(s)=I} g(s) dg$ .

Now consider point 2 where  $T = 2$  with information structure  $(\tilde{f}_{2L}, \tilde{f}_{2R})$  and prior belief  $p_L = p_L^t$ . By lemma 1, the decision maker processes signal  $s_1 \geq \frac{u_R p_R}{u_L p_L}$  if

$$s_t \geq \frac{u_R p_R}{u_L p_L} \frac{\tilde{F}_{2R} \left( \frac{u_R p_R}{u_L p_L} \right)}{\tilde{F}_{2L} \left( \frac{u_R p_R}{u_L p_L} \right)}, \quad (\text{B.3})$$

and he processes signal  $s_1 < \frac{u_R p_R}{u_L p_L}$  if and only if

$$s_t^{-1} > \frac{u_L p_L}{u_R p_R} \frac{1 - \tilde{F}_{2L} \left( \frac{u_R p_R}{u_L p_L} \right)}{1 - \tilde{F}_{2R} \left( \frac{u_R p_R}{u_L p_L} \right)}. \quad (\text{B.4})$$

By comparing equation (B.1) to equation (B.3), and equation (B.2) to equation (B.4), the equilibrium processing strategy in the two cases are equivalent if and only if

$$\begin{aligned} \tilde{F}_{2L} \left( \frac{u_R p_R}{u_L p_L} \right) &= \sum_{k=t+1}^T \left[ \mathbf{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbf{1}_{\sigma_h^*(s')=I}) f_{hL} ds' + \mathbf{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t}} \mathbf{1}_{\sigma_k^*(s)=P} f_{kL}(s) ds; \\ \tilde{F}_{2R} \left( \frac{u_R p_R}{u_L p_L} \right) &= \sum_{k=t+1}^T \left[ \mathbf{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbf{1}_{\sigma_h^*(s')=I}) f_{hR} ds' + \mathbf{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t}} \mathbf{1}_{\sigma_k^*(s)=P} f_{kR}(s) ds. \end{aligned} \quad (\text{B.5})$$

To prove that there exists p.d.f.  $\tilde{f}_{2L}$  and  $\tilde{f}_{2R}$  that generates the c.d.f.s evaluated at  $\left( \frac{u_R p_R}{u_L p_L} \right)$  with values defined in equation (B.5), it remains to prove that

$$\frac{\tilde{F}_{2R} \left( \frac{u_R p_R}{u_L p_L} \right)}{\tilde{F}_{2L} \left( \frac{u_R p_R}{u_L p_L} \right)} > \frac{u_L p_L}{u_R p_R}.$$

Note that by definition, equation (B.5) implies that

$$\frac{\tilde{F}_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)}{\tilde{F}_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)} = \frac{\Pr_{\mathcal{M}_{t+1}}^{t+1}(r | R)}{\Pr_{\mathcal{M}_{t+1}}^{t+1}(r | L)}.$$

On the other hand, by proposition A.1, the expected utility of the DM at the beginning of period  $t + 1$  is weakly greater than that if he chooses to process all  $s_t$ :

$$\begin{aligned} & u_L p_L (1 - \Pr_{\mathcal{M}_{t+1}}^{t+1}(r | L)) + u_R p_R \Pr_{\mathcal{M}_{t+1}}^{t+1}(r | R) \\ & \geq u_L p_L \left(1 - F_{(t+1)L}\left(\frac{u_R p_R}{u_L p_L}\right)\right) + u_R p_R F_{(t+1)R}\left(\frac{u_R p_R}{u_L p_L}\right) \\ & > u_L p_L, \end{aligned}$$

where the last inequality is implied by the fact receiving one signal always improves expected utility, in comparison to receiving no signal. Rearranging gives:

$$\begin{aligned} u_R p_R \Pr_{\mathcal{M}_{t+1}}^{t+1}(r | R) & > u_L p_L \Pr_{\mathcal{M}_{t+1}}^{t+1}(r | L); \\ \frac{\Pr_{\mathcal{M}_{t+1}}^{t+1}(r | R)}{\Pr_{\mathcal{M}_{t+1}}^{t+1}(r | L)} & > \frac{u_L p_L}{u_R p_R}. \end{aligned}$$

The results follow. □

## B.2 Aware Case

**Proposition B.2.** *There exists some p.d.f.  $(\tilde{f}_{2L}, \tilde{f}_{2R})$  such that the following two sets of equilibrium strategies are equivalent:*

1. *the set of equilibrium processing strategies of the DM with belief  $(p_L^t, p_R^t)$  at period  $t < T$ , under a setting with  $T > 2$ ,  $\bar{T} = 1$  and information structure  $\{(f_{t'L}, f_{t'R})\}_{t'=t+1}^T$  (assuming that it exists);*
2. *the set of equilibrium processing strategies of the DM with prior belief  $(p_L, p_R) = (p_L^1, p_R^1)$  at period 1, under a setting with  $T = 2$ ,  $\bar{T} = 1$*

and information structure  $(\tilde{f}_{2L}, \tilde{f}_{2R})$ .

*Proof.* First consider point 1 where  $T > 2$  and without loss of generality assume  $u_L p_L^t \geq u_R p_R^t$ . For simplicity and with a bit of abuse in notations, define  $\mathcal{R}_{t'}^* = \prod_{t=1}^{t'-1} \mathcal{R}_t(\tilde{\sigma}_t)$ , which is the conjectured ratio of ignoring all the previous signals in state  $L$  over than in state  $R$ . Note that at the beginning of period  $t' > t$ , the expected utility of action  $l$  over that of action  $r$  equals  $\frac{u_L p_L^t \mathcal{R}_{t'}^*}{u_R p_R^t}$ . By lemma 1, the decision maker processes signal  $s_t \geq \frac{u_R p_R^t}{u_L p_L^t}$  if and only if

$$\begin{aligned} s_t &\geq \frac{u_R p_R^t \Delta_{s_t, \mathcal{M}_t}^t(r | R)}{u_L p_L^t \Delta_{s_t, \mathcal{M}_t}^t(r | L)} \\ &= \frac{u_R p_R^t \sum_{k=t+1}^T \left[ \mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(s)=P} f_{kR}(s) ds}{u_L p_L^t \sum_{k=t+1}^T \left[ \mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(s)=P} f_{kL}(s) ds}. \end{aligned} \quad (\text{B.6})$$

On the other hand, he processes signal  $s_t < \frac{u_R p_R^t}{u_L p_L^t}$  if and only if

$$\begin{aligned} s_t^{-1} &> \frac{u_L p_L^t \Delta_{s_t, \mathcal{M}_t}^t(l | L)}{u_R p_R^t \Delta_{s_t, \mathcal{M}_t}^t(l | R)} \\ &= \frac{u_L p_L^t \left[ 1 - \sum_{k=t+1}^T \left[ \mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(s)=P} f_{kL}(s) ds \right]}{u_R p_R^t \left[ 1 - \sum_{k=t+1}^T \left[ \mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(s)=P} f_{kR}(s) ds \right]}. \end{aligned} \quad (\text{B.7})$$

The equilibrium strategy is given by the equation (B.6) and (B.7). Now

define  $\tilde{s} = s\mathcal{R}_k^*$ , note that

$$\begin{aligned}
& \frac{\sum_{k=t+1}^T \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \int_0^{\frac{u_{RP}^t}{u_{LP}^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(s)=P} f_{kR}(s) ds}{\sum_{k=t+1}^T \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' \right] \int_0^{\frac{u_{RP}^t}{u_{LP}^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(s)=P} f_{kL}(s) ds} \\
&= \frac{\sum_{k=t+1}^T \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \int_0^{\frac{u_{RP}^t}{u_{LP}^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*) d\tilde{s}}{\sum_{k=t+1}^T \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' \right] \int_0^{\frac{u_{RP}^t}{u_{LP}^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kL}(\tilde{s}/\mathcal{R}_k^*) d\tilde{s}} \\
&= \frac{\int_0^{\frac{u_{RP}^t}{u_{LP}^t \mathcal{R}_k^*}} \sum_{k=t+1}^T \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*) d\tilde{s}}{\int_0^{\frac{u_{RP}^t}{u_{LP}^t \mathcal{R}_k^*}} \sum_{k=t+1}^T \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kL}(\tilde{s}/\mathcal{R}_k^*) d\tilde{s}}.
\end{aligned}$$

Now define  $(\tilde{f}_{2L}, \tilde{f}_{2R})$  as follows and verify whether it is a probability distribution function and whether the equilibrium processing strategy of the DM in period  $t = 1$  where  $T = 2$  is equivalent to that characterized in equation (B.6) and (B.7):

$$\begin{aligned}
\tilde{f}_{2L}(\tilde{s}) &= \sum_{k=t+1}^T \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kL}(\tilde{s}/\mathcal{R}_k^*); \\
\tilde{f}_{2R}(\tilde{s}) &= \sum_{k=t+1}^T \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*).
\end{aligned}$$

First, it is clear that  $\int \tilde{f}_{2L}(\tilde{s}) d\tilde{s} = \int \tilde{f}_{2R}(\tilde{s}) d\tilde{s} = 1$  as the DM always processes

one signal in equilibrium. On the other hand,

$$\begin{aligned}
\frac{\tilde{f}_{2L}(\tilde{s})}{\tilde{f}_{2R}(\tilde{s})} &= \frac{\sum_{k=t+1}^T \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbf{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' \right] \mathbf{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kL}(\tilde{s}/\mathcal{R}_k^*)}{\sum_{k=t+1}^T \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbf{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbf{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*)} \\
&= \frac{\sum_{k=t+1}^T \mathcal{R}_k^* \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbf{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbf{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kL}(\tilde{s}/\mathcal{R}_k^*)}{\sum_{k=t+1}^T \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbf{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbf{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*)} \\
&= \frac{\sum_{k=t+1}^T \tilde{s} \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbf{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbf{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*)}{\sum_{k=t+1}^T \left[ \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbf{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbf{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*)} \\
&= \tilde{s}.
\end{aligned}$$

Denote  $(\tilde{F}_{2L}, \tilde{F}_{2R})$  as the c.d.f. associated with  $(\tilde{f}_{2L}, \tilde{f}_{2R})$ . In point 2 where  $T = 2$ , the DM processes signal  $s_1 \geq \frac{u_R p_R}{u_L p_L}$  if and only if

$$s_1 \geq \frac{u_R p_R}{u_L p_L} \frac{\tilde{F}_{2R} \left( \frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)}{\tilde{F}_{2L} \left( \frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)},$$

and processes signal  $s_t < \frac{u_R p_R}{u_L p_L}$  if and only if

$$s_1^{-1} \geq \frac{u_L p_L}{u_R p_R} \frac{1 - \tilde{F}_{2L} \left( \frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)}{1 - \tilde{F}_{2R} \left( \frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)}.$$

The result follows. □

## C Omitted Results and Proofs

### C.1 Proof of Proposition 1

*Proof.* By lemma 1, the decision maker processes  $s_1 \geq \frac{u_R p_R}{u_L p_L}$  if and only if

$$s_1 \geq \frac{u_R p_R}{u_L p_L} \frac{F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)}{F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)} = \frac{u_R p_R}{u_L p_L} \times STR\left(s_2 \in \left(0, \frac{u_R p_R}{u_L p_L}\right)\right) > 1. \quad (\text{C.1})$$

The last inequality follows from the fact that the strength of the set of signals  $s_2 \in (0, \frac{u_R p_R}{u_L p_L})$  is higher than the strength of  $s_2 = \frac{u_R p_R}{u_L p_L}$ , *i.e.*,

$$\frac{F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)}{F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)} > \frac{f_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)}{f_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)} = \frac{u_L p_L}{u_R p_R}.$$

The last inequality of equation (C.1) also implies that the DM ignores all weak belief-challenging signals  $s_1 \in [\frac{u_R p_R}{u_L p_L}, 1]$ .

On the other hand, the DM processes  $s_1 < \frac{u_R p_R}{u_L p_L}$  if and only if

$$s_1^{-1} > \frac{u_L p_L}{u_R p_R} \frac{1 - F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)}{1 - F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)} = \frac{u_L p_L}{u_R p_R} \times STR\left(s_2 \in \left[\frac{u_R p_R}{u_L p_L}, \infty\right)\right) > \frac{u_L p_L}{u_R p_R}. \quad (\text{C.2})$$

The last inequality is implied by

$$\begin{aligned} F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right) &> F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right); \\ 1 - F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right) &> 1 - F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right); \\ STR\left(s_2 \in \left[\frac{u_R p_R}{u_L p_L}, \infty\right)\right) &> 1. \end{aligned}$$

Combining inequalities (C.1) and (C.2) proves the results.  $\square$

## C.2 Proof of Proposition 2

*Proof.* Point 1 of the proposition is directly implied by the proposition 1.

For point 2, by proposition 1,  $\Phi^- > \Phi^+$  if and only if:

$$\frac{u_L^2 p_L^2}{u_R^2 p_R^2} \times \frac{\left(1 - F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)\right) F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)}{\left(1 - F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)\right) F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)} > 1.$$

First, note that

$$\begin{aligned} \lim_{u_L p_L / u_R p_R \rightarrow \infty} \frac{F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)}{F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)} &= \lim_{u_L p_L / u_R p_R \rightarrow \infty} \frac{f_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)}{f_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)} \\ &= \lim_{u_L p_L / u_R p_R \rightarrow \infty} \frac{u_R p_R}{u_L p_L} \end{aligned}$$

which implies

$$\begin{aligned} \lim_{u_L p_L / u_R p_R \rightarrow \infty} \frac{u_L p_L F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)}{u_R p_R F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)} &= 1 > 0; \\ \lim_{u_L p_L / u_R p_R \rightarrow \infty} \frac{u_L^2 p_L^2}{u_R^2 p_R^2} \times \frac{\left(1 - F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)\right) F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)}{\left(1 - F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)\right) F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)} &= +\infty > 0. \end{aligned}$$

The result follows given the continuity of  $\Phi^-$  and  $\Phi^+$  in  $p_L$ . □

### C.3 Proof of Corollary 3

*Proof.* I prove the corollary at the limit where  $p_L \rightarrow 1$ . Follow from the proof of proposition 2,

$$\begin{aligned}\lim_{u_L p_L / u_R p_R \rightarrow \infty} \Phi^+ &= 1; \\ \lim_{u_L p_L / u_R p_R \rightarrow \infty} \Phi^- &= +\infty.\end{aligned}$$

It is then obvious that as  $p_L \rightarrow 1$ , for all  $s > 1$ ,

$$\begin{aligned}U_P(s, \mathcal{M}_1) &> U_I(s, \mathcal{M}_1) \\ U_P(s^{-1}, \mathcal{M}_1) &< U_I(s^{-1}, \mathcal{M}_1).\end{aligned}$$

The result follows from corollary 2. □

### C.4 Proof of Corollary 4

*Proof.* I prove the corollary at the limit where  $p_L \rightarrow 1$ . Follow from the proof of proposition 2,

$$\begin{aligned}\lim_{u_L p_L / u_R p_R \rightarrow \infty} \Phi^+ &= 1; \\ \lim_{u_L p_L / u_R p_R \rightarrow \infty} \Phi^- &= +\infty.\end{aligned}$$

Now suppose  $\Phi^+ \rightarrow 1$  and  $\Phi^- \rightarrow +\infty$ . The probability that the DM chooses action  $r$  converges to

$$p_L F_{1L}(1) F_{2L} \left( \frac{u_R p_R}{u_L p_L} \right) + p_R F_{1R}(1) F_{2R} \left( \frac{u_R p_R}{u_L p_L} \right). \quad (\text{C.3})$$

On the other hand, if the DM can process both signals, the probability that he chooses action  $r$  equals

$$p_L \int_0^\infty F_{2L} \left( \frac{u_R p_R}{u_L p_L s} \right) f_{1L}(s) ds + p_R \int_0^\infty F_{2R} \left( \frac{u_R p_R}{u_L p_L s} \right) f_{1R}(s) ds. \quad (\text{C.4})$$

Equation (C.3) is strictly smaller than equation (C.4) as for  $\omega = L$  and  $R$ ,

$$\begin{aligned} \int_0^\infty F_{2\omega} \left( \frac{u_R p_R}{u_L p_L s} \right) f_{1\omega}(s) ds &> \int_0^1 F_{2\omega} \left( \frac{u_R p_R}{u_L p_L s} \right) f_{1\omega}(s) ds \\ &> \int_0^1 F_{2\omega} \left( \frac{u_R p_R}{u_L p_L} \right) f_{1\omega}(s) ds \\ &= F_{2\omega} \left( \frac{u_R p_R}{u_L p_L} \right) F_{1\omega}(1). \end{aligned}$$

The result follows by the continuity of the probability of choosing action  $r$  in  $\Phi^+$  and  $\Phi^-$ .  $\square$

## C.5 Reverse Wishful Thinking

To illustrate reverse wishful thinking, I normalize the utility function as follows:

$$\begin{aligned} u(l | L) &= -u_L < 0; \\ u(l | R) &= u_R > 0; \\ u(r | R) &= u(r | L) = 0. \end{aligned}$$

State  $L$  is the undesirable outcome which yields weakly negative utility while state  $R$  yields weakly positive utility. By analogy to the definition of wishful thinking, the processing strategy of the DM exhibits reverse wishful thinking when  $\Phi^- < \Phi^+$ . That is, he processes a larger set of information that supports the undesirable state, compare to that supports the desirable state.

**Corollary 11 (Reverse Wishful thinking).** *When state  $L$  is very undesirable, the equilibrium processing strategy of the DM exhibits reverse wishful thinking, i.e.,  $\Phi^- > \Phi^+$  when  $|-u_L/u_R|$  is big enough.*

## C.6 Proof of Proposition 3

*Proof.* By proposition 1,

$$\begin{aligned}\Phi_A^+ &= \frac{u_R p_R}{u_L p_L} \times \frac{F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)}{F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)} \\ &= \Phi_B^+\end{aligned}$$

On the other hand,

$$\begin{aligned}\Phi_A^- &= \frac{u_L p_L}{u_R p_R} \times \frac{1 - \lambda_A + \lambda_A \left(1 - F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)\right)}{1 - \lambda_A + \lambda_A \left(1 - F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)\right)} \\ &> \frac{u_L p_L}{u_R p_R} \times \frac{1 - \lambda_B + \lambda_B \left(1 - F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)\right)}{1 - \lambda_B + \lambda_B \left(1 - F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)\right)} \\ &= \Phi_B^-\end{aligned} \tag{C.5}$$

where the second inequality of equation (C.5) follows from the fact that  $1 - F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right) > 1 - F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)$ .  $\square$

## C.7 Proof of Proposition 4

*Proof.* First, by proposition 1,

$$\begin{aligned}
\Phi_A^- &= \frac{u_L p_L}{u_R p_R} \frac{1 - \lambda - \delta + \lambda \left(1 - F_{2L}^B\left(\frac{u_R p_R}{u_L p_L}\right)\right) + \delta \left(1 - F_{2L}^A\left(\frac{u_R p_R}{u_L p_L}\right)\right)}{1 - \lambda - \delta + \lambda \left(1 - F_{2R}^B\left(\frac{u_R p_R}{u_L p_L}\right)\right) + \delta \left(1 - F_{2R}^A\left(\frac{u_R p_R}{u_L p_L}\right)\right)} \\
&> \frac{u_L p_L}{u_R p_R} \frac{1 - \lambda + \lambda \left(1 - F_{2L}^B\left(\frac{u_R p_R}{u_L p_L}\right)\right)}{1 - \lambda + \lambda \left(1 - F_{2R}^B\left(\frac{u_R p_R}{u_L p_L}\right)\right)} \\
&= \Phi_B^-
\end{aligned} \tag{C.6}$$

where the second inequality of equation (C.6) follows from the fact that  $1 - F_{2L}^A\left(\frac{u_R p_R}{u_L p_L}\right) > 1 - F_{2R}^A\left(\frac{u_R p_R}{u_L p_L}\right)$ .

Now I compare  $\Phi_A^+$  and  $\Phi_B^+$ . By proposition 1,

$$\begin{aligned}
\Phi_A^+ &= \frac{u_R p_R}{u_L p_L} \frac{\lambda F_{2R}^B\left(\frac{u_R p_R}{u_L p_L}\right) + \delta F_{2R}^A\left(\frac{u_R p_R}{u_L p_L}\right)}{\lambda F_{2L}^B\left(\frac{u_R p_R}{u_L p_L}\right) + \delta F_{2L}^A\left(\frac{u_R p_R}{u_L p_L}\right)}, \\
\Phi_B^+ &= \frac{u_R p_R}{u_L p_L} \frac{\lambda F_{2R}^B\left(\frac{u_R p_R}{u_L p_L}\right)}{\lambda F_{2L}^B\left(\frac{u_R p_R}{u_L p_L}\right)}.
\end{aligned}$$

Therefore,  $\Phi_A^+ \leq \Phi_B^+$  if and only if

$$\frac{F_{2R}^A\left(\frac{u_R p_R}{u_L p_L}\right)}{F_{2L}^A\left(\frac{u_R p_R}{u_L p_L}\right)} \leq \frac{F_{2R}^B\left(\frac{u_R p_R}{u_L p_L}\right)}{F_{2L}^B\left(\frac{u_R p_R}{u_L p_L}\right)}.$$

□

## C.8 Proof of Proposition 5

*Proof.* By proposition B.2, it is sufficient to prove that there exists a perfect Bayesian Nash equilibrium for  $T = 2$ . First note that the equilibrium strategy is a threshold strategy, with thresholds  $(\Phi^+, \Phi^-)$ . Denote  $\mathcal{R}_1$  as

$$\mathcal{R}_1 = \frac{F_{1L}(\Phi^+) - F_{1L}(1/\Phi^-)}{F_{1R}(\Phi^+) - F_{1R}(1/\Phi^-)}.$$

As  $\Phi^+ \geq \frac{u_R p_R}{u_L p_L}$  and  $\Phi^- > \frac{u_L p_L}{u_R p_R}$ ,  $\mathcal{R}_1 \in \left( \frac{F_{1L}(1)}{F_{1R}(1)}, \frac{1-F_{1L}(1)}{1-F_{1R}(1)} \right)$ . Given the optimality of the processing strategy, the perfect Bayesian Nash equilibrium is a fixed point of the following equation:

$$\mathcal{R}_1 = \frac{F_{1L} \left( \frac{u_R p_R}{u_L p_L} \frac{F_{2R} \left( \frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)}{F_{2L} \left( \frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)} \right) - F_{1L} \left( \frac{u_R p_R}{u_L p_L} \frac{1 - F_{2R} \left( \frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)}{1 - F_{2L} \left( \frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)} \right)}{F_{1R} \left( \frac{u_R p_R}{u_L p_L} \frac{F_{2R} \left( \frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)}{F_{2L} \left( \frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)} \right) - F_{1R} \left( \frac{u_R p_R}{u_L p_L} \frac{1 - F_{2R} \left( \frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)}{1 - F_{2L} \left( \frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)} \right)}$$

Denote the equation as  $\mathcal{R}_1 = \psi(\mathcal{R}_1)$ . The mapping  $\psi$  is continuous mapping from  $\left[ \frac{F_{1L}(1)}{F_{1R}(1)}, \frac{1-F_{1L}(1)}{1-F_{1R}(1)} \right]$  to itself, which is a convex and compact set. By the Brouwer's fixed point theorem, there exists a fixed point of the equation which belongs to the set  $\left[ \frac{F_{1L}(1)}{F_{1R}(1)}, \frac{1-F_{1L}(1)}{1-F_{1R}(1)} \right]$ . As the two end points  $\frac{F_{1L}(1)}{F_{1R}(1)}$  and  $\frac{1-F_{1L}(1)}{1-F_{1R}(1)}$  are not a fix point of the equation, there exists a fixed point  $\mathcal{R}_1^*$  such that

$$\mathcal{R}_1^* \in \left( \frac{F_{1L}(1)}{F_{1R}(1)}, \frac{1 - F_{1L}(1)}{1 - F_{1R}(1)} \right);$$

$$\mathcal{R}_1^* = \psi(\mathcal{R}_1^*).$$

□

## C.9 Proof of Corollary 7

*Proof.* Without loss of generality, I analyze only the processing strategy of Alice. First by corollary 1, Alice processes signal  $q$  because

$$q > \frac{1-p}{p} \times q,$$

while she processes signal  $q^{-1}$  if and only if

$$q > \frac{p}{1-p} \times \frac{1-\lambda + \lambda q(1+q)^{-1}}{1-\lambda + \lambda(1+q)^{-1}} = \frac{p}{1-p} \times \frac{1-\lambda + q}{1+q(1-\lambda)}.$$

Define  $\mathcal{F}(p, q, \lambda) = q - \frac{p}{1-p} \times \frac{1-\lambda + q}{1+q(1-\lambda)}$ . Its first derivatives w.r.t.  $p$  and  $\lambda$  are

$$\begin{aligned} \frac{\partial \mathcal{F}}{\partial p} &= -\frac{1}{(1-p)^2} \frac{1-\lambda + q}{1+q(1-\lambda)} < 0; \\ \frac{\partial \mathcal{F}}{\partial \lambda} &= -\frac{p}{1-p} \frac{q^2 - 1}{(1+q(1-\lambda))^2} < 0. \end{aligned}$$

Moreover,

$$\begin{aligned} \mathcal{F}(0, q, \lambda) = q > 0 &> \lim_{(p/1-p) \rightarrow q} \mathcal{F}(p, q, \lambda) = q \frac{\lambda(1-q)}{1+q(1-\lambda)}; \\ \mathcal{F}(p, q, 0) = q - \frac{p}{1-p} &> 0 > \mathcal{F}(p, q, 1) = q(1 - \frac{p}{1-p}). \end{aligned}$$

Therefore, there exists  $p^-$  and  $\lambda^-$  such that  $\mathcal{F} > 0$  if and only if  $p < p^-$  and if and only if  $\lambda < \lambda^-$ . It remains to prove point 2 of the corollary. First,  $\mathcal{F}$  is strictly convex in  $q$

$$\frac{\partial^2 \mathcal{F}}{\partial q^2} = \frac{p}{1-p} \frac{2\lambda(1-\lambda)(2-\lambda)}{(1+q(1-\lambda))^3} > 0,$$

and the two roots of  $\mathcal{F}(q) = 0$  are

$$\frac{-(1 - \frac{p}{1-p}) \pm \sqrt{(1 - \frac{p}{1-p})^2 + 4(1-\lambda)^2 \frac{p}{1-p}}}{2(1-\lambda)}. \quad (\text{C.7})$$

One of the two roots is negative, which contradicts the fact that  $q > 1$ . Hence, there exists a  $q^-$ , which is the positive root defined in equation (C.7), such that  $\mathcal{F} > 0$  if and only if  $q > q^-$ .  $\square$

## C.10 Proof of Proposition 8

*Proof.* When  $\lambda < \lambda^-$ , Alice and Bob processes both belief-confirming and belief-challenging information. They take the same action if and only if at least one of  $s_1$  and  $s_2$  is informative.

$$P_{concensus} = 1 - (1 - \lambda)^2 = \lambda(2 - \lambda). \quad (\text{C.8})$$

On the other hand, when  $\lambda \geq \lambda^-$ , they ignore belief-challenging signals in period 1. Therefore, Alice and Bob take the same action if and only if the signals in both periods support the same state, or if  $s_1$  is pure noise while  $s_2$  is not.

$$P_{concensus} = \lambda^2(q^2(1+q)^{-2} + (1+q)^{-2}) + \lambda(1 - \lambda) \quad (\text{C.9})$$

Similarly, the probability that Alice/Bob takes her/his a priori optimal action follows:

$$\begin{aligned} P_{default}^a &= \begin{cases} 1 - \lambda(1+q)^{-1} - \lambda(1-\lambda)(1+q)^{-1} & \text{if } \lambda < \lambda^- \\ 1 - \lambda^2(1+q)^{-2} - \lambda(1-\lambda)(1+q)^{-1} & \text{if } \lambda \geq \lambda^- \end{cases} \\ P_{default}^b &= \begin{cases} 1 - \lambda q(1+q)^{-1} - \lambda(1-\lambda)q(1+q)^{-1} & \text{if } \lambda < \lambda^- \\ 1 - \lambda^2 q^2(1+q)^{-2} - \lambda(1-\lambda)q(1+q)^{-1} & \text{if } \lambda \geq \lambda^- \end{cases} \end{aligned} \quad (\text{C.10})$$

First, I prove the first part of the proposition. First, notice that equation (C.8) is increasing and convex in  $\lambda$ . On the other hand, the first and second derivative of equation (C.9) w.r.t.  $\lambda$  are given by:

$$\begin{aligned} \frac{\partial P_{concensus}}{\partial \lambda} &= 1 - 2\lambda(1 - (q^2(1+q)^{-2} + (1+q)^{-2})) > 0 \\ \frac{\partial^2 P_{concensus}}{\partial^2 \lambda} &= -2(1 - (q^2(1+q)^{-2} + (1+q)^{-2})) < 0 \end{aligned}$$

respectively. The two inequalities are implied by the fact that  $q > 1$  and  $\lambda \in [0, 1]$ .<sup>35</sup> Secondly, equation (C.9) is clearly smaller than equation (C.8) for any given  $\lambda$ , which implies that there is a downward jump at  $\lambda = \tilde{\lambda}$ .

Now I move on to the proof for  $P_{default}^j$ ,  $j = a, b$ . First, part 1 is implied by the fact that both functions

$$\begin{aligned} P_{default}^a &= 1 - \lambda(1+q)^{-1} - \lambda(1-\lambda)(1+q)^{-1} \\ P_{default}^b &= 1 - \lambda q(1+q)^{-1} - \lambda(1-\lambda)q(1+q)^{-1} \end{aligned}$$

decrease in  $\lambda$  as  $\lambda(2-\lambda)$  increases in  $\lambda$  for  $\lambda \in [0, 1]$ . On the other hand, from equation (C.10), it is obvious that there is an upward jump at  $\lambda = \tilde{\lambda}$ .  $\square$

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<sup>35</sup>First,  $[q^2(1+q)^{-2} + (1+q)^{-2}] \in (1/2, 1)$  as it is increasing in  $q$ . Therefore,  $1 - (q^2(1+q)^{-2} + (1+q)^{-2}) \in (0, 1/2)$ . Combined with the fact that  $\lambda \in [0, 1]$ , it implies the first inequality.