

Reciprocity in Dynamic Employment Relationships*

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This paper explores how the norm of reciprocity shapes relational contracts. Developing a theoretical model of a long-term employment relationship, I show that reciprocal preferences are more important when an employee is close to retirement. At earlier stages, direct incentives provided by the relational contract are more effective because more future rents increase the employer's commitment. Generally, a combination of direct and reciprocity-based incentives is optimal in a dynamic incentive scheme. I also show that more competition can magnify the use of reciprocity-based incentives. Moreover, I demonstrate that, with asymmetric information on the employee's reciprocal preferences, an early separation of types is mostly optimal. Then, the employer might actually benefit from not knowing the employee's type.

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1 Motivation

Contracts allow to realize gains from cooperation in case preferences are not aligned. They are enforced by courts or, in the absence of third-party enforcement, might as well be honored by the involved parties who do not want to put their relationship at risk. More recently, contracts have been acknowledged to also establish reference points which cause feelings of entitlement and consequently ex-post inefficiencies (Hart and Moore, 2008), and – in particular if only informal “handshake agreements” are feasible – to even generate an inherent enforcement mechanism by establishing norms parties feel obliged to honor (Kessler and Leider, 2012).

In this paper, I explore how dynamic employment relationships, which are characterized by a large degree of incompleteness and therefore commonly governed by relational contracts, are affected by norms. Following Macneil, 1980, I start with the presumption that a relational contract can establish a norm to reciprocate. This norm is enforced by social preferences and makes an employee willing to exert higher effort after receiving a generous wage. In addition to direct incentives where a bonus is promised in exchange for effort, the norm of reciprocity thus generates an enforcement mechanism, which has crucial implications for the optimal incentive system in a dynamic employment relationship.

I show that the relative importance of direct and reciprocity-based incentives in a relational contract depends on the career phase of an employee: At early stages, direct incentives are more important because higher future rents allow the employer to credibly promise a higher bonus. At later stages, incentives using the norm of reciprocity become more important for implementing high effort, and gradually replace direct incentives. Moreover, I find that more intense competition for labor might magnify the importance of reciprocity-based incentives in the relational contract. I also analyze the consequences of asymmetric information on an employee’s reciprocal preferences and claim that an early separation of types is mostly optimal. This implies that pooling equilibria where “reciprocal” imitate “selfish” types might be less important than often assumed.

More precisely, I develop a repeated principal-agent model with a finite time horizon. The risk-neutral agent can exert costly effort, which benefits the risk-neutral principal and is observable but not verifiable. Hence formal, court-enforceable, contracts cannot be used to motivate the agent. Instead, both parties can form a self-enforcing relational contract, which determines (wage and bonus) payments the principal is supposed to make as a reward for the agent’s effort. In addition, the relational contract specifies a *norm of reciprocity*, implying that a generous wage payment by the principal is supposed to be reciprocated by the agent via higher effort. This incorporates insights from the law literature, in particular by Macneil, 1980 and Macneil, 1983, who has developed a norms-based approach to contracting. He claims that a relational contract is a manifestation of norms supposed to govern the behavior of the involved parties.¹ In my setting, the norm is enforced by preferences for reciprocity on the agent’s side. These preferences are activated by wage components that are not paid as a reward for past effort and induce the agent to reciprocate with higher effort. Therefore, the principal has two means to provide incentives. First, directly by promising a bonus to be made *af-*

¹Kessler and Leider, 2012 deliver evidence for norms indeed providing incentives to cooperate. In a lab experiment, they find that individuals take significantly more prosocial actions after having made handshake agreements as part of the contracting process.

ter the agent has exerted effort. Then, the principal uses “relational incentives”, where the bonus is enforced by repeated-game incentives. Second, indirectly via the norm of reciprocity and paying a generous wage *before* the agent is exerting effort. Then, the principal uses “reciprocity-based incentives”, which are enforced by the agent’s preferences for reciprocity.

Moreover, the following norm function lets the agent’s preferences for reciprocity respond to the history of the game: In case the principal reneges on promised bonus, not only the relational contract breaks down (as is standard in the literature), but also the agent’s preferences for reciprocity towards the principal disappear. This norm function is inspired by Cox et al., 2007 and Cox et al., 2008, who develop an approach to modelling reciprocity that is grounded in neoclassical preference theory. Preferences for reciprocity are “emotional state-dependent” and stronger if actions upset the status quo. In my setting, the status quo corresponds to the equilibrium prescribed by the relational contract, and (downward) deviations by the principal not only constitute a violation of the relational contract, but also affect the agent’s reciprocal preferences.

Importantly, this norm function allows the use of relational incentives even *despite the existence of a predetermined last period*: Because the agent’s preferences for reciprocity disappear once the principal reneges on a promised bonus, and because the principal’s profits in the last period of the game are higher with reciprocal preferences than without, her behavior in the penultimate period affects her profits in the last period. This interaction between relational and reciprocity-based incentives carries over to earlier periods. The principal’s credibility when using relational incentives in a given period is determined by a so-called dynamic enforcement constraint, which states that a promised bonus must not exceed the difference between future discounted profits on and off the equilibrium path. Since future on-path profits increase in the extent of the agent’s reciprocal preferences, the principal can also provide stronger relational incentives today if the agent is more reciprocal. This source of complementarity between relational incentives and the agent’s reciprocal preferences is amended by an additional channel. The principal’s dynamic enforcement constraint is relaxed and more effort can consequently be implemented if she pays a generous wage, implying that reciprocity-based preferences are particularly valuable in periods where the constraint is tight. Therefore, relational and reciprocity-based incentives are complements because more effort can be implemented with a combination of the two, a result that has received empirical support from Boosey and Goerg, 2018. Both are dynamic substitutes, though, in the sense that, as time proceeds, relational incentives are gradually replaced by reciprocity-based incentives. The reason is that the dynamic enforcement constraint is tighter in later periods – less remaining periods reduce the principal’s future profits – which amplifies the benefits of reciprocity-based incentives as time moves on.

This implies that the profit-maximizing incentive scheme has implemented effort at its highest level in early stages of the employment relationship, where it remains until the dynamic enforcement constraint starts to bind. Then, the principal’s reduced credibility effectively constrains her ability to pay a sufficiently high bonus. This decreases implementable effort, which in turn lets the principal respond with an increase of the fixed wage in order to (partially) mitigate the necessary effort reduction. Once the dynamic enforcement constraint starts binding, effort thus gradually decreases and reaches its lowest level in the last period of the game. The effort reduction goes hand in hand with a gradual increase of the wage. In line with this result, there is indeed evidence that the productivity of workers declines once they approach retirement (Halti-

wanger et al., 1999, Skirbekk 2004, or Lallemand and Rycx, 2009).²

Moreover, effort is generally higher if the agent has more pronounced preferences for reciprocity. This result has received empirical support from Dohmen et al. (2009), who use data from the German Socio-Economic Panel (SOEP), which also contains information on the reciprocal inclinations of individuals. The positive effect of reciprocal preferences on implemented effort is stronger if reciprocity-based incentives are more important, that is, in later periods when the principal's dynamic enforcement constraint binds. This outcome is in line with evidence provided by Fahn et al., 2017. Using the same data and approach as Dohmen et al. (2009), they show that the positive interaction between reciprocal inclinations and effort is substantially stronger for older workers who are close to retirement.

In a next step, I explore how labor market competition affects the optimal dynamic incentive scheme. There, I follow Schmidt (2011) and assume that a more intense competition for workers decreases the principal's outside option and increases the wage the agent must at least be offered. I also assume that this "minimum wage" serves as a reference wage for the norm of reciprocity, in the sense that the agent only perceives higher wages as generous. Better outside opportunities generally reduce the relationship surplus and consequently the potential strength of relational incentives. Now, a more intense labor market competition has opposing effects on the principal's outside option and the agent's reference wage. If the effect on the agent's reference wage dominates, a more intense labor market competition increases the importance of reciprocity-based incentives, in order to make up for the reduced effectiveness of relational incentives. Otherwise, more intense labor market competition allows for stronger relational incentives, hence reciprocity-based incentives are used less extensively.

In a number of extensions, I explore the robustness of my results and derive additional insights, all within a two-period setting. First, in Section 5.1, I let the agent's preferences for reciprocity not merely be triggered by non-discretionary, but by all realized payments, hence also by wages that are paid as a response for past effort. Then, also relational incentives are provided with fixed wages paid at the beginning of a period, and the back-loading of wages is more pronounced than in my main model.

In Section 5.2, I allow for asymmetric information on the agent's reciprocal preferences. There, I assume that the agent might either be reciprocal (as in the previous analysis), or be selfish without any reciprocal preferences. If the likelihood of facing a reciprocal agent is high, a "separating contract" is optimal for the principal. This incorporates high effort in the first period, which however will only be exerted by the reciprocal type, whereas the selfish type shirks and is subsequently fired. If the likelihood of facing a selfish agent is high, it might be optimal for the principal to offer a "pooling contract". This incorporates low effort in the first period, which is exerted by both types. In the second period, the selfish type collects the wage and subsequently shirks. The pooling contract resembles outcomes derived in the reputation literature (see Mailath and Samuelson, 2006 for an overview), where the presence of even a small fraction of "commitment types" can motivate selfish agents to cooperate in a finitely repeated game – because it allows them to maintain a reputation for (potentially) being cooperative. It is a common perception that, in lab experiments with repeated interaction, selfish types who imitate cooperative (or "fair") types are responsible for observing high cooperation in early periods (Fehr et al., 2009a).

²Upward sloping wage curves are supported by a vast amount of evidence as well (see Waldman, 2012 for a summary), but can also be explained by many theoretical approaches.

However the existence of the pooling contract in my setting relies on a Perfect Bayesian Equilibrium being played where *any* deviation from equilibrium effort lets the principal assign probability 1 to facing the selfish type. But, even if preferred by the principal, such a pooling contract may not satisfy the Intuitive Criterion (Cho and Kreps, 1987): A deviation to a higher effort than the one specified by the pooling contract would only be incentive compatible for the reciprocal, but not for the selfish type. Such an upward deviation would thus reveal the agent to be reciprocal, and allow for an adjustment of the second-period wage that makes principal and reciprocal agent better off. Then, only a separating contract can be sustained, which can have implications for the interpretation of many experimental results: High effort in early and low effort in later periods can also be due to an early separation of types, followed by a relational contract between remaining matches. Indeed, the experimental exercise conducted by (Brown et al., 2004) exactly generates this outcome. Their theoretical explanation (some players have fairness preferences; those without imitate the fair players early on) can only account for the observed effort dynamics, but not for the high amount of separations in initial periods. Therefore, I provide a complementary interpretation for the standard notion that “selfish-imitates-fair-types” explains higher cooperation with repeated interaction in lab experiments.

Moreover, I show that the principal might actually benefit from asymmetric information on the agent’s reciprocal inclinations, because then a reciprocal type has larger incentives to exert effort than with symmetric information. The reason is that the principal will never fire an agent she knows to be reciprocal. But with a separating contract under asymmetric information, a deviation from equilibrium effort will lead to a termination of the relationship. Therefore, the reciprocal type loses his future rent upon a deviation, which provides additional incentives to exert effort. If the ex-ante probability of facing a reciprocal type is sufficiently high, the principal can even benefit from the existence of selfish types. Therefore, not only the existence of reciprocal agents might induce selfish agents to exert more effort, but it can also be other way round – that the willingness to separate themselves from selfish types makes reciprocal types work harder.

In Section 5.3, I explore the implications of negative reciprocity, in the sense that the agent wants to retaliate in case the principal has reneged on a promised payment. To do so, I apply ideas developed by Hart and Moore (2008) to my setting and assume that the relational contract provides a reference point for the agent. A deviation by the principal lets the agent subsequently shade on performance, where the extent of shading is determined by the agent’s preferences. To avoid the consequences of shading, though, the principal might fire the agent when planning to renege on the bonus. She will do so if the costs of shading imposed on the principal are higher than her future profits, which holds if the agent’s preferences for negative reciprocity are sufficiently strong. Then, the principal’s trade-off amounts to paying the bonus versus sacrificing future profits, which is equivalent to the trade-off captured by the dynamic enforcement constraint in the main model of this paper. In addition, this outcome does not need the agent’s preferences for positive reciprocity to disappear after a deviation by the principal. Therefore, negative reciprocity does not affect my results, as long as the agent’s preferences for negative reciprocity are sufficiently strong.

In Section 5.4, I endogenize the reference wage that has to be paid at least in order to let the agent respond to the norm of reciprocity. There, I assume that the first-period wage determines the reference wage in the second period. This reduces the importance

of reciprocity-based incentives early on because a positive wage paid in the first period – albeit still inducing higher effort via the norm of reciprocity – increases the reference wage later on and makes it more expensive for the principal to use reciprocity-based incentives.

Finally, in Section 5.5, I assume that the agent's preferences for reciprocity are triggered by the material rent he is bound to receive in a period, which equals the difference between on-path payments and effort costs. Then, the principal is less inclined to pay a positive wage early on, because also the bonus activates the norm of reciprocity.

Related Literature

It is one of the most robust, thoroughly researched, outcomes in behavioral economics that many individuals not only maximize their own material payoffs, but also take others' well-being into account when making decisions (DellaVigna, 2009). Many individuals thus possess social preferences, where an important component is captured by preferences for intrinsic reciprocity. Based on Akerlof's conceptual idea of gift exchange, which states that employees exert voluntary effort if they feel treated well by firms (Akerlof, 1982), and firms thus might find it optimal to pay wages above the market-clearing level, a plethora of research has found experimental support for the existence of reciprocal preferences (starting with Fehr et al. (1993, 1998); see (see, e.g., Camerer and Weber, 2013, for an overview of existing experimental research)). Concerning real-world evidence, Dohmen et al. (2009) use data on individual-level survey measures for reciprocity from the German Socio-Economic Panel (SOEP), and show that reciprocal inclinations are linked to high effort, high wages, and general life success. . (Huang and Cappelli, 2010; Englmaier et al., 2015) explore outcomes such as monitoring, teamwork, wage levels, and firm productivity and find at least suggestive evidence for the importance of reciprocity in employment relationships. Moreover, Bellemare and Shearer (2009, 2011) show that monetary gifts increase effort in a real-world working environment, however only temporarily.

Although these findings have been very robust, the actual consequences for real-world employment relationships, which are inherently dynamic, are uncertain. In one-shot gift-exchange experiments, for example, higher wages trigger higher effort, however realized effort levels on average are far from efficient. As soon as subjects are allowed to interact repeatedly, though, positive reactions to generous wages are way more pronounced (see, e.g., Falk et al., 1999 Brown et al. (2004), or Fehr et al., 2009a). This is mostly attributed to asymmetric information on an individual's reciprocal inclinations and reputational concerns regarding one's type. Individuals are more willing to cooperate in order to establish or maintain a reputation for being reciprocal, indicating that intrinsic reciprocity and repeated-game incentives indeed complement each other (for an overview of the theoretical reputation literature see Mailath and Samuelson, 2006, and Andreoni and Miller (1993) or Gächter and Falk (2002) for support of this hypothesis).

Still, uncertainty on one's reciprocal inclinations will eventually be revealed, and it is crucial to understand how repeated interaction affects optimal incentives for individuals who are known to be reciprocal. Then, the importance of reciprocal preferences in the workplace relies on gift-exchange considerations not being marginalized by repeated-game incentives. Some experimental studies have approached this question and disentangled the two motives for cooperation. Reuben and Suetens (2012) use

an infinitely-repeated prisoner's dilemma to assess the relative importance of strategic motives (i.e., driven by repeated interaction) and intrinsic reciprocity and find that cooperation is mostly driven by strategic concerns. Similarly, Dreber et al. (2014) find that strategic motives seem to be more important than social preferences in an infinitely repeated prisoner's dilemma. Cabral et al. (2014) conduct an infinitely repeated veto game to distinguish between different explanations for generous behavior. They find strategic motives to be the predominant motivation, but also present evidence for the importance of intrinsic reciprocity.

Hence, experimental evidence suggests that repeated-game incentives are an important mode to support cooperation even for individuals with reciprocal preferences. However, a sound understanding of how firms optimally design incentive schemes if they have both means, gift-exchange as well as "direct" incentives, at hand, is still lacking. The present paper addresses this gap by providing a theoretical framework to explore the optimal provision of dynamic incentives if agents have reciprocal preferences and formal contracts are not feasible.

The theoretical literature on intrinsic reciprocity can be arranged along the lines whether reciprocal behavior is triggered by intentions or by outcomes. This partially includes the question whether a person's preferences for reciprocity can be used strategically by others. The classic gift-exchange approach developed by Akerlof (1982) allows firms to strategically raise wages in order to induce their employees to work harder. Applying this idea to a moral hazard framework, Englmaier and Leider (2012a) show that generous compensation can be a substitute for performance-based pay and consequently increase profits. On the other hand, Rabin (1993) argues that the perceived kindness of an action should be the driving force to induce reciprocal behavior. He develops the techniques for incorporating intentions into game theory. Dufwenberg and Kirchsteiger (2004) apply this psychological game theory to extensive games and explicitly account for the sequential structure of the respective games. Netzer and Schmutzler (2014) show that if only intentions matter, a self-interested firm cannot benefit from its employees' reciprocal preferences.

Whereas these two approaches assume that either only outcomes or only intentions are relevant, Falk and Fischbacher (2006) develop a theory incorporating both aspects. They assume that an action is perceived as kind if the opponent has the option to treat someone less kind. Their exercise incorporates evidence that, while individuals respond to outcomes, those responses are considerably stronger if choices are at the counterpart's discretion (cf. Falk et al., 2006; Fehr et al., 2009a; Camerer and Weber, 2013). Cox et al. (2007) and Cox et al. (2008) develop a theoretical framework that can generate such results without having to resort to psychological game theory. Their approach is based on neoclassical preference theory, and individuals merely respond to observable events and opportunities, instead of beliefs about others' intentions or types. I build upon these ideas and apply them to a dynamic setting. Reciprocity is triggered by generous wages, i.e., wages that exceed the one specified by a direct incentive system. Moreover, the agent's inclination to reciprocate disappears once the principal has broken any implicit promise.

I also contribute to the literature on relational contracts. Bull (1987) and MacLeod and Malcomson (1989) derive relational contracts with observable effort, whereas Levin (2003) shows that those also take a rather simple form in the presence of asymmetric information with respect to effort and the agent's characteristics. Malcomson (2013) delivers an extensive overview of relational contracts. Within this broader area,

few papers have started to investigate how relational contracts and social preferences interact: Dur and Tichem (2015) incorporate social preferences into a model of relational contracts. They show that altruism undermines the credibility of termination threats which may reduce productivity and payoffs. Contreras and Zanarone (2017) assume that employees suffer when their formal wage is below that of their colleagues. They show that these “social comparison costs” can be managed by having a homogeneous formal governance structure, while achieving necessary customizations through relational contracts. To the best of my best knowledge, this is the first paper to incorporate intrinsic preferences for reciprocity into a relational contracting framework. There, my approach allows relational contracts to also work with a predefined last period.

2 Model Setup

2.1 Environment and Technology

There is one risk-neutral principal (“she”) and one risk-neutral agent (“he”). At the beginning of every period $t \in \{1, \dots, T\}$, with $1 < T < \infty$, the principal makes an employment offer to the agent which specifies a fixed wage $w_t \geq 0$ and the promise to pay a discretionary bonus $b_t \geq 0$.³ The agent’s acceptance/rejection decision is described by $d_t \in \{0, 1\}$. Upon acceptance ($d_t = 1$), the agent chooses an effort level $e_t \geq 0$, which is associated with effort costs $c(e) = e^3/3$.⁴ Effort generates a deterministic output $e_t\theta$ which is subsequently consumed by the principal. If the agent rejects the principal’s offer ($d_t = 0$), both consume their outside option utilities, which (for now) are set to zero.

2.2 Relational Contracts, Preferences, and the Norm of Reciprocity

Neither effort nor output are verifiable, however can be observed by both parties. Therefore, no formal but only a relational contract can be used to motivate the agent. The relational contract is a self-enforcing agreement determined by principal and agent, and constitutes an equilibrium of the game. There, in addition to the standard components of a game – players, information, action space, preferences and equilibrium concept – I incorporate a norm function that maps the game’s history into the agent’s preferences. Thereby, I integrate a perception first developed in the law literature: Macneil, 1980 or Macneil, 1983 state that a relational contract specifies norms – among which one of them constitutes the norm of reciprocity – and these norms determine the supposed behavior of individuals. Economic models to incorporate reciprocal behavior have mostly applied social preferences, assuming that individuals are endowed with intrinsic preferences to reciprocate. I follow up on this literature and adapt the approach introduced by Cox et al. (2008) to my environment. The modelling of reciprocal

³Non-negativity constraints on payments do not affect any results, however simplify the definition of reciprocity below in a sense that we do not have to differentiate between positive and negative payments.

⁴I assume this specific functional form for analytical tractability. Other (convex) cost functions would deliver similar results as long as the third derivative is positive. A positive third derivative is necessary to guarantee an interior solution in Section 3.1.

preferences in Cox et al. (2008) is grounded in neoclassical preference theory, hence no psychological game theory and no assessment of the other player's intentions are needed. They assume that an action by one player is perceived as more (less) generous – and consequently causes a stronger reciprocal reaction – if it allows the other player to obtain a higher (lower) monetary payoff (Cox et al. (2008), Definitions 1 and 2; Axiom R). Moreover, (positive or negative) reciprocal reactions are stronger whenever an action upsets the status quo, compared to this same action that only upholds the status quo (Cox et al. (2008), Axiom S, Part 1).

I capture the first aspect by assuming that *realized* payments made in a given period trigger reciprocal behavior by the agent in the respective period. Concerning, the second aspect, I take the “standard” role of a relational contract into account, in the sense that it establishes a direct incentive system where payments are promised in return for effort. This direct incentive system accounts for the status quo in Cox et al. (2008), hence wages and bonuses that are paid as a reward for past effort do *not* trigger reciprocal behavior. Hence, the norm of reciprocity only refers to non-discretionary wages w_t that do not depend on the agent's past effort choices.

To formally describe my approach, I denote the events in a period t by $h_t = (w_t, d_t, e_t, b_t)$. h_t is public information. A *history* of length $t - 1$, h^{t-1} (for $t \geq 2$) collects the public events up to, and including, time $t - 1$, i.e. $h^{t-1} := (h_\tau)_{\tau=1}^{t-1}$. The set of histories of length $t - 1$ is denoted by \mathcal{H}^{t-1} (and $\mathcal{H}^0 = \{\emptyset\}$). The relational contract prescribes actions as a function of the history, which then constitute an equilibrium of the game (defined below). For the agent, it determines an acceptance function $d(h^{t-1}, w_t)$ that specifies whether the agent is supposed to accept a certain offer, as well as an effort function $e(h^{t-1}, w_t, d_t)$. For the principal, the relational contract determines a wage function $w(h^{t-1})$ and a bonus function $b(h^{t-1}, w_t, d_t, e_t)$. The total wage $w(h^{t-1})$ is split into a discretionary component $w^d(h^{t-1})$, which includes wages that are paid as a reward for the agent's past effort, and a non-discretionary component. The latter is independent of the agent's past choices, hence is defined as $w^{nd}(h^{t-1} \setminus \{e^{t-1}, d^{t-1}\})$, where $e^{t-1} := (e_\tau)_{\tau=1}^{t-1}$ and $d^{t-1} := (d_\tau)_{\tau=1}^{t-1}$.

Whereas bonus and discretionary wage constitute the “direct” incentive system that grants payments as a reward for previously exerted effort, the non-discretionary wage w_t^{nd} stipulates subsequent effort by the agent in order to adhere to the norm of reciprocity. The agent's responsiveness to this norm is given by his utility function, which – in a period t – equals

$$u_t = d_t (b_t + w_t - c(e_t^*) + \eta(h^{t-1})w_t^{nd}e_t\theta).$$

The term $\eta(h^{t-1}) \in [0, \infty)$ captures the agent's inherent preferences for *positive reciprocity* (negative reciprocity is considered in Section 5.3) and lets the principal's output enter the agent's utility. Its value in a given period depends on the history via a *norm function*, which takes the following form: $\eta(h^{t-1})$ remains at a constant, individual-specific, level η in case the principal has not deviated in the past. In case of at least one (downward) deviation from the actions specified by the relational contract, η_t drops to zero in all subsequent periods. Therefore, $\eta(\emptyset) = \eta$; in all periods $t \geq 2$,

$$\eta(h^{t-1}) = \begin{cases} \eta & \text{if } b_\tau \geq b(h^{\tau-1}, w_\tau, d_\tau, e_\tau) \text{ and } w_\tau \geq w^d(h^{\tau-1}), \text{ all } \tau \leq t \\ 0 & \text{otherwise.} \end{cases}$$

This implies that η_t drops to zero once the agent exerts equilibrium effort but is not

rewarded accordingly. It does *not* drop to zero after a deviation by the agent and if no bonus is paid in response, or if w_t^{nd} is smaller than expected.

This specific approach allows to separate “standard” direct incentives (which in the following are denoted “relational incentives) from those that make use of the norm of reciprocity (denoted “reciprocity-based incentives”). Later, I also explore the consequences of letting all realized payments, hence even the ones made as a reward for past effort (Section 5.1), as well as the on-path rent the agent is bound to receive in a period (Section 5.5) trigger reciprocal behavior. Moreover, in Section 5.3, I incorporate negative reciprocity and allow for “shading” by the agent in case the principal has deviated (as in Hart and Moore, 2008).

The principal has no preferences for reciprocity; her per-period profits are

$$\pi_t = d_t (e_t \theta - b_t - w_t).$$

Finally, principal and agent share the discount factor $\delta \in (0, 1]$.

2.3 Equilibrium and Objective

The relational contract has to constitute an equilibrium of the finitely repeated game. There, I focus on pure strategies. For the agent, a pure strategy specifies what wage offers to accept in each period as a function of the previous history, and what level of effort to exert if he accepts employment as a function of the previous history and current-period wages. Formally, it is a sequence of mappings $\{\sigma_t^A\}_{t=1}^\infty$, where, for each $t \leq T$, $\sigma_t^A = (d_t, e_t)$, and $d_t : \mathcal{H}^{t-1} \times \mathbb{R}_+ \rightarrow \{0, 1\}$, $(h^{t-1}, w_t) \mapsto d_t(h^{t-1}, w_t)$ and $e_t : \mathcal{H}^{t-1} \times \mathbb{R}_+ \times \{0, 1\} \rightarrow \mathbb{R}_+$, $(h^{t-1}, w_t, d_t) \mapsto e_t(h^{t-1}, w_t, d_t)$.

In each period, a pure strategy for the principal specifies her wage offer as a function of the previous history, as well as the bonus payment as a function of the previous history, current-period wages and effort. Formally, it is a sequence of mappings $\{\sigma_t^P\}_{t=1}^\infty$, where, for each $t \leq T$, $\sigma_t^P = (w_t, b_t)$, and $w_t : h^{t-1} \rightarrow \mathbb{R}_+$, $h^{t-1} \mapsto w_t(h^{t-1})$, $b_t : h^{t-1} \times \mathbb{R}_+ \times \{0, 1\} \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $(h^{t-1}, w_t, d_t, e_t) \mapsto b_t(h^{t-1}, w_t, d_t, e_t)$. Because of the absence of private information, the equilibrium concept is Subgame Perfect Equilibrium (SPE).

In a SPE where $d_t = 1$ in all periods of the game, the following recursive formulations describe players’ discounted payoff streams on the equilibrium path:

$$\begin{aligned} \Pi_t &= e_t^* \theta - b_t - w_t + \delta \Pi_{t+1} \\ U_t &= b_t + w_t - c(e_t^*) + \eta w_t^{nd} e_t^* \theta + \delta U_{t+1}. \end{aligned}$$

In what follows, the objective is to characterize a SPE that maximizes the principal’s profits at the beginning of the game, Π_1 . Before doing so, I discuss the assumptions regarding the agent’s preferences for reciprocity, as well as regarding the finite time horizon of the game.

2.4 Discussion of Assumptions

Reciprocity I assume that the principal can strategically use the norm of reciprocity and hence payment of w_t^{nd} . The agent understands the purpose of a gift received by the profit-maximizing principal but still reciprocates. This presumption is supported by experimental evidence from Malmendier and Schmidt (2017), who show that subjects

reciprocate to gifts even though they understand that the giver is selfish and expects something in return.

Moreover, I assume that η drops to zero after a deviation, hence one player's preferences are affected by another player's actions. This approach is inspired by Cox et al. (2007) or Cox et al. (2008), however not needed for my results if negative reciprocity is explicitly considered. In Section 5.3, I assume that the agent "shades" on the principal if the latter has deviated from the reference point provided by the relational contract (adapting an approach developed by Hart and Moore, 2008 to my setting). Such an assumption has received empirical support from Malmendier and Schmidt (2017), who show that individuals exert negative reciprocity upon a potential gift giver if they expected a gift but did not receive one. Now, the agent's shading is costly for the principal, who can avoid these costs by firing the agent after a deviation. Then, if the agent's preferences for negative reciprocity are sufficiently severe, outcomes are equivalent to my main setting, even if η remains constant throughout.

I also assume that reciprocity only enters the agent's stage-game payoffs. This notion is consistent with evidence delivered Bellemare and Shearer (2009), who show that a gift causes a positive effort response – but that this effect is only temporary. Moreover, in Section 5.4, I analyze a situation where a positive wage today increases the reference wage (above which the agent's reciprocity is triggered) tomorrow.

Furthermore, reciprocal behavior is triggered by (non-discretionary) payments and not by the agent's actual or perceived rent. Indeed, there is evidence (in particular from the lab) that generous wages cause reciprocal behavior even in the absence of performance-based incentives (cf. Fehr et al., 2009b; Charness and Kuhn, 2011). In Section 5.5, I also show that my results are robust to letting the agent's reciprocal preferences respond to any rent he expects to receive in a given period.

The reciprocity term in the agent's utility function contains θ , and hence the extent to which the principal benefits from the agent's effort. This follows evidence pointing out that an important factor for reciprocity is the agent's assessment of the value generated for the principal (Hennig-Schmidt et al., 2010; Englmaier and Leider, 2012b).

Finally, I assume that the principal knows η . In Section 5.2, I explore the consequences of asymmetric information concerning the agent's preferences for reciprocity.

Finite Time Horizon I analyze a game of T periods, and most results on the dynamics of the employment relationship rely on the time horizon being finite. Whereas many real-life employment relationships either have a pre-defined ending date or an increasing probability of termination (which could be captured by a decreasing discount factor and generate the same dynamics, because those rely on the gradual decrease of discounted future profits), most people work in multi-worker firms that continue to exist when workers retire. In my setting, this would imply that the principal generally also has the option to hire other agents for the job under consideration – after period T or potentially even before. Taking this into account, my results survive as long as multilateral punishments are not feasible, for example because deviations in one relationship cannot be observed by other (prospective) employees. With multilateral punishments, the principal's commitment in the employment relationship would not necessarily be smaller in later periods (a result which drives most of the dynamics). However, although deviations have to be private information of one match to render multilateral relational contracts (as in Levin, 2002) unfeasible, it would be perfectly fine for outsiders to observe whether the agent is employed or gets fired. Then, only a pre-

mature termination could be punished by any “new” agent. This would make it costly for the principal to replace the agent early on, leaving my results valid.

Finally, if I completely ruled out that a premature termination was punished by prospective new agents, the opportunity to employ other agents would manifest in a positive outside option for the principal, which I explore in Section 4. There, I assume that this outside option is sufficiently small for the principal to never have an incentive to terminate an employment relationship on the equilibrium path. This could be due to replacement costs when hiring a new agent, like labor market frictions (such as search costs) or direct replacement costs. Moreover, a sufficiently small outside option of the principal also rules out the use of efficiency wages, because any firing threat would not be credible.

3 Results

3.1 Reciprocity Spot Contract

I first derive a profit-maximizing spot contract and hence omit time subscripts. Besides serving as a benchmark, such a contract will also be offered in the final period T . In a spot contract, $b = 0$ because the principal has no incentives to make a payment to the agent after the latter has exerted effort. Therefore, the only means to incentivize the agent is a positive non-discretionary wage. Since $w = w^{nd}$, I will subsequently omit the “nd”-superscript. Given w , and presuming he decides to work for the principal, the agent chooses effort in order to maximize his per-period utility $u = w - e^3/3 + \eta w e \theta$.

The conditions for using the first order approach hold, hence the agent’s incentive compatibility (IC) constraint yields

$$e^* = \sqrt{\eta w \theta}. \quad (\text{IC})$$

The principal sets w to maximize her expected per-period profits $\pi = e^* \theta - w$. There, she has to take into account that accepting the contract must be optimal for the agent. This is captured by the agent’s individual rationality (IR) constraint,

$$e^* b + w - \frac{(e^*)^3}{3} + \eta w e^* \theta \geq 0. \quad (\text{IR})$$

Concluding, the principal’s problem is to

$$\max_w e^* \theta - w,$$

subject to (IR) and (IC) and non-negativity constraints.

Lemma 1 *The profit-maximizing reciprocity spot contract contract has $w = \eta \theta^3/4$ and $e^* = \eta \theta^2/2$. Therefore, $\pi = \eta \theta^3/4$ and $u = \eta \theta^3/4 + \eta^3 \theta^6/12$.*

The proof, as well as all other omitted proofs, can be found in Appendix 6.

Intuitively, a positive wage lets the agent partially internalize the principal’s payoff, which is why he reciprocates and selects a positive effort level.

3.2 Relational Contracts

Now, I analyze how a relational contract can and will be used to motivate the agent. There, two aspects are of particular interest, namely the enforceability of the relational contract, and how the norm of reciprocity affects outcomes. These aspects will be explored in the next subsections, where I furthermore derive the properties of a profit-maximizing relational contract.

3.2.1 Preliminaries

Note that it is without loss of generality for the principal to only use bonus payments for the provision of relational incentives. Therefore, in the following I assume $w_t = w_t^{nd}$, hence all fixed wages are non-discretionary. This allows to easily separate relational incentives (provided by b_t) from reciprocity-based incentives (provided by w_t). If either bonus or discretionary wages also triggered direct reciprocal responses by the agent, the respective payments would merely assume a larger relative weight in the optimal incentive scheme (see Sections 5.1 and 5.5 below).

The relational contract specifies a bonus function $b(h^{t-1}, w_t, d_t, e_t)$. Since effort is public information, it is without loss to have $b(h^{t-1}, w_t, d_t, e_t^*) \equiv b_t \geq 0$ (where e_t^* is equilibrium effort) if play has so far been on the equilibrium path, and $b(h^{t-1}, w_t, d_t, \hat{e}_t) = 0$ for any $\hat{e}_t \neq e_t^*$. The promise to pay b_t must be credible, which is captured by a dynamic enforcement (DE) constraint for every period t ,

$$-b_t + \delta \Pi_{t+1} \geq \delta \tilde{\Pi}_{t+1}. \quad (\text{DE})$$

There, Π_{t+1} describes the principal's on-path and $\tilde{\Pi}_{t+1}$ her off-path continuation profits. The (DE) constraint states that future on-path profits must be sufficiently large compared to future off-path profits so that they offset today's costs of paying the bonus. It indicates that a bonus payment is only feasible if $\Pi_{t+1} > \tilde{\Pi}_{t+1}$, i.e., if future equilibrium play can be made contingent on the principal's current behavior.

Generally, relational contracts require a (potentially) infinite time horizon because of a standard unraveling argument that can be applied once a predetermined last period exists: If the equilibrium outcome in the last period is unique, the same holds for all preceding periods. In my case, however, the situation is different because the norm function lets $\eta(h^{t-1})$ subsequently drop to zero in case the principal refuses to pay a specified bonus. Moreover, the "standard" grim-trigger punishment is imposed afterwards and relational contracts are not feasible anymore (adapting Abreu, 1988 to my setting, in the sense that any observable deviation from agreed-upon behavior should be punished by a reversion to a player's minmax-payoff).

Hence, the principal's continuation profits are $\tilde{\Pi}_{t+1} = 0$ in case she does not pay $b(h^{t-1}, w_t, d_t, e_t)$, and her behavior in any period $t < T$ indeed affects her future profits. All this indicates that not only the relational contract determines whether a given payment "activates" the agent's reciprocal preferences, but the latter are a prerequisite for the relational contract to work in the present setting with a finite time horizon.

Finally, I assume that subsequent equilibrium play is unaffected in case the principal does not pay the equilibrium level of w_t^{nd} . This assumption has no impact on my results, though, because the agent's period- t effort is independent of any w_τ^{nd} , $\tau > t$ (see below).

3.2.2 Incentive Compatibility

In this section, I explore the agent's incentives to exert equilibrium effort e_t^* . Those are (potentially) determined by a combination of reciprocity-based incentives (via a positive w_t) and relational incentives (via b_t). Recall that my specification of the norm function implies that, after a deviation by the agent, the reciprocity parameter is not reduced but remains at η . This indicates that the agent does not necessarily deviate to an effort level of zero. In addition, continuation play in subsequent periods is not affected by the agent's behavior, who only forgoes the period's bonus after a deviation. This is due to the following two reasons. First, instead of letting continuation play be unaffected by a deviation of the agent, the principal could fire the agent. This would increase the agent's incentives to exert effort, but would not be subgame perfect since a spot reciprocity contract (which could always be offered instead) yields positive profits. Second, only the relational contract might end after a deviation by the agent, being replaced by a reciprocity spot contract in each subsequent period. This would also not be optimal, because a spot contract always yields a higher per-period utility for the agent than the profit-maximizing equilibrium with a relational contract (see Section 3.2.4). Thus, an arrangement where a deviation by the agent causes a permanent reversion to reciprocity spot contracts would actually *reduce* the agent's incentives to exert effort.

Therefore, the agent's period- t incentive compatibility (IC) constraint for any off-path effort level \tilde{e}_t equals

$$b_t + w_t - \frac{(e_t^*)^3}{3} + \eta w_t e_t^* \theta \geq w_t - \frac{(\tilde{e}_t)^3}{3} + \eta w_t \tilde{e}_t \theta.$$

If the agent deviates, he will select an effort level $\tilde{e}_t = \operatorname{argmax}(-e^3/3 + \eta w_t e \theta)$, i.e., $\tilde{e}_t = \sqrt{\eta w_t \theta}$, and the (IC) constraint becomes

$$b_t - \frac{(e_t^*)^3}{3} + \eta w_t e_t^* \theta \geq 2/3 \left(\sqrt{\eta w_t \theta} \right)^3. \quad (\text{IC})$$

This implies that an (IR) constraint for the agent is automatically satisfied because his per-period rent, $b_t + w_t - (e_t^*)^3/3 + \eta w_t e_t^* \theta$, is non-negative given the (IC) constraint. Also note that $e_t^* \geq \tilde{e}_t$ (because $b_t \geq 0$).

3.2.3 The Complementarity of Relational and Reciprocity-Based Incentives

In this section, I derive some first results and show that reciprocity-based incentives can improve the power of relational incentives for a given value of η , and vice versa. To simplify the principal's problem, note that the (IC) constraint must bind in any profit-maximizing equilibrium. If it did not bind, the bonus b_t could be slightly reduced, which would increase profits and relax the (DE) constraint without violating the (IC) constraint. This allows to plug $b_t = (e_t^*)^3/3 - \eta w_t e_t^* \theta + 2/3 \left(\sqrt{\eta w_t \theta} \right)^3$ into the (DE) constraint, which becomes

$$\frac{(e_t^*)^3}{3} - \eta w_t \theta e_t^* \leq \delta \Pi_{t+1} - \frac{2}{3} \left(\sqrt{\eta w_t \theta} \right)^3. \quad (\text{DE})$$

The enforceability of relational contracts is generally determined by a comparison of today's effort costs with discounted future (net) payoffs generated in the relationship.

Only if the latter are large enough, they are sufficient to cover today's costs of exerting effort. Here, two additional terms enter if the (non-discretionary) wage is positive; first, the agent's preferences for reciprocity reduce the necessary bonus payment to achieve a certain effort level e_t^* ; second, if the agent deviates, he still selects a positive effort level.

Concluding, the principal's problem is to maximize

$$\Pi_1 = \sum_{t=1}^T \delta^{t-1} \pi_t,$$

subject to a (DE) constraint for every period t , and subject to $w_t \geq 0 \forall t$.⁵

The equilibrium is sequentially efficient, hence the problem is equivalent to maximizing

$\pi_t = e_t \theta - b_t - w_t = e_t \theta - \left((e_t^*)^3 / 3 - \eta w_t e_t^* \theta + 2/3 (\sqrt{\eta w_t \theta})^3 \right) - w_t$ in every period t , subject to the relevant constraints.

Now, I will explore the relationship between relational and reciprocity-based incentives. To do so, I first abstract from issues of enforceability. Put differently, I assume that the (DE) constraint does not bind, i.e., is satisfied for the principal's preferred effort level and derive respective effort and wage levels. Note that this situation is equivalent to one where formal contracts based on effort would be feasible.

Lemma 2 *Assume the (DE) constraint does not bind in a period $t < T$. Then, there is a $\bar{\eta} > 0$ such that setting a strictly positive wage is optimal for $\eta > \bar{\eta}$, whereas the optimal wage equals zero for $\eta \leq \bar{\eta}$.*

Lemma 2 implies that even if the principal is not restricted in setting her preferred effort-based bonus b_t , i.e., if her discounted future on-path profits are sufficiently large, she might still decide to grant the agent a rent. This is because the agent's concern for the norm of reciprocity reduces his effective effort costs, but only in combination with a strictly positive wage w_t . The principal thus faces a trade-off between the higher costs when paying a positive wage, and the higher effort the agent is willing to exert in response. If the agent's preferences for reciprocity are sufficiently large (more precisely, if $\eta > \sqrt{1/\theta^3}$), the latter effect dominates. Then, the optimal wage $w_t = (\eta^2 \theta^3 - 1)^2 / 4\eta^3 \theta^3$ yields effort $e_t^* = (1 + \eta^2 \theta^3) / 2\eta \theta$. For $\eta \leq \bar{\eta}$, setting $w_t = 0$ is optimal, together with an effort level $e_t^* = \sqrt{\theta}$. In the following, I will refer to the implemented effort and wage levels for a non-binding (DE) constraint as *first-best* levels. At these first-best levels, the costs for the principal to implement one additional unit of effort are the same when using relational as when using reciprocity-based incentives for $\eta > \bar{\eta}$, and those costs are equal to the principal's marginal benefits. For $\eta \leq \bar{\eta}$, it is more effective for the principal to only rely on relational incentives.

In a next step, I assess how the agent's preferences for reciprocity affect outcomes if the commitment problem has bite, i.e., if her future profits are not sufficiently large to credibly commit to her preferred effort-based bonus.

Lemma 3 *Assume the (DE) constraint binds in a period $t < T$. Then, implemented effort is smaller than with a non-binding (DE) constraint. Moreover, if paying a fixed wage is*

⁵Note that in period T , the (DE) constraint equals $\frac{(e_T^*)^3}{3} - \eta w_T \theta e_T^* \leq -\frac{2}{3} (\sqrt{\eta w_T \theta})^3$, which for $e_T^* = \sqrt{\eta w \theta}$ (the agent's effort in a spot reciprocity contract) is trivially satisfied.

optimal in the situation with a non-binding (DE) constraint (i.e., if $\eta > \bar{\eta}$), the fixed wage now is larger. Otherwise, (i.e., if $\eta \leq \bar{\eta}$), there exists a $\tilde{\eta}_t < \bar{\eta}$ (with $\tilde{\eta}_t$ increasing in δ) such that setting a strictly positive wage is optimal for $\eta > \tilde{\eta}_t$, whereas the optimal wage equals zero for $\eta \leq \tilde{\eta}_t$.

Besides reducing effective effort costs, a fixed wage also relaxes the principal's (DE) constraint by decreasing the bonus that must be paid in order to implement a given effort level. Therefore, if the (DE) constraint binds (meaning it does not hold for first-best values), the fixed wage is larger than when it does not bind for $\eta > \bar{\eta}$. Moreover, even if $\eta \leq \bar{\eta}$ and consequently $w_t = 0$ with a non-binding (DE) constraint, paying a positive wage can be optimal – in particular if discounted future profits are small and the (DE) constraint is tight. Then, the effect of a fixed wage relaxing the (DE) constraint is more valuable.

All this implies that relational and reciprocity-based incentives are complements at a given point in time – a combination yields more efficient and profitable outcomes than the use of only one of them – a result that has received empirical support from Boosey and Goerg, 2018. They conduct a laboratory experiment where a manager and a worker interact for two periods. The worker can spend time completing a series of real effort tasks and is paid a fixed wage in every period. In addition, the principal potentially has the opportunity to pay a fixed bonus between the two periods, after first-period output has been observed. Boosey and Goerg, 2018 find that average output is considerably larger with this option, compared to the treatments where the principal either is not able to pay a bonus, or where the bonus can be paid at the beginning or end of the game. This supports my result that a relational contract can boost productivity also with agents known to be reciprocal, and that a relational contract can even be sustained with a finite time horizon if the agent is reciprocal.

3.2.4 Relational and Reciprocity-Based Incentives as Dynamic Substitutes

In this section, I characterize how the interaction between relational and reciprocity-based incentives evolves over the course of the employment relationship for a given value of η .

First, note that the (DE) constraint might or might not bind in any period $t < T$, depending on discount factor δ , reciprocity parameter η and productivity θ . Furthermore, the (DE) constraint becomes tighter in later periods.

Lemma 4 *The principal's dynamic enforcement constraint might or might not bind in period $T - 1$. More precisely, for any discount factor δ , the (DE) constraint holds for first-best effort and wage levels if η is sufficiently large. For any values η and θ , the (DE) constraint does not hold for first-best effort if the discount factor is sufficiently small.*

Furthermore, $\Pi_{t-1} > \Pi_t$ for all $t \leq T$.

The principal's commitment in a relational contract is determined by what she has to lose given she deviates. If the discount factor is small, she cares less about a potential reduction of future profits and is therefore less willing to pay a bonus to compensate the agent for his effort. In addition, a larger reciprocity parameter η increases future profits on the equilibrium path, and furthermore reduces today's effective effort costs. The second part of Lemma 4 states that the difference between on- and off-path continuation profits goes down over time. As time elapses, the remaining time horizon

and therefore the periods in which profits can be generated is reduced. Moreover, this triggers a re-enforcing effect because implementable effort in a period is increasing in the difference between on- and off-path continuation profits. Since $\Pi_T > 0$, the (DE) constraint allows to implement a larger effort level in period $T - 1$ than in period T . Then, per-period profits in period $T - 1$ are larger than in period T , and implementable effort in period $T - 2$ is even larger than in period $T - 1$, and so on. Hence, the (DE) constraint in earlier periods is less tight than later on.

All this implies that, if the (DE) constraint binds in a given period \tilde{t} , it will thus also bind in all *subsequent* periods $t > \tilde{t}$. If it is slack in a given period \hat{t} , it will also be slack in all *previous* periods $t < \hat{t}$. This yields the following effort and (non-discretionary) wage dynamics.

Proposition 1 *Equilibrium effort is weakly decreasing over time and equilibrium wage weakly increasing, i.e., $e_t^* \leq e_{t-1}^*$ and $w_t \geq w_{t-1}$.*

Furthermore, $e_t^ < e_{t-1}^*$ and $w_t > w_{t-1}$ imply $e_{t+1}^* < e_t^*$ and $w_{t+1} > w_t$, whereas $e_{t+1}^* = e_t^*$ and $w_{t+1} = w_t$ imply $e_t^* = e_{t-1}^*$ and $w_t = w_{t-1}$.*

Proposition 1 states that effort and wage are time-invariant in early stages of the employment relationship, as long as the future is sufficiently valuable for the (DE) constraint to hold for first-best values. Once the end of the employment relationship is sufficiently close and the (DE) constraint binds, the effort profile becomes downward sloping and the wage profile upward sloping. This is because the principal cannot credibly promise her preferred bonus payment anymore. On the one hand, this reduces equilibrium effort. On the other hand, the principal might respond with a wage increase which increases equilibrium effort due to the agent's preferences for reciprocity. However, the effort increase caused by a higher wage does not fully compensate for the effort reduction caused by the binding (DE) constraint because the costs of implementing an additional unit of effort are now larger with reciprocity-based than with relational incentives. As time proceeds, the (DE) constraint becomes tighter and tighter (Lemma 4). Hence, towards the end of an employment relationship relational incentives are gradually substituted by reciprocity-based incentives (fixed wage \uparrow), with the substitution however being incomplete (effort \downarrow).

An upward sloping wage curve is in line with a substantial amount of evidence on individual career paths (see Waldman, 2012 for a summary). Many explanations have been developed to explain this pattern, for example a downward-stickiness of wages because of an optimal risk-sharing arrangement between a risk-neutral firm and a risk-averse worker (Harris and Holmstrom (1982)), the back-loading of incentives in order to prevent shirking in earlier periods (Lazear (1979)), or symmetric learning on a worker's ability combined with human capital acquisition (Gibbons and Waldman (1999)). I complement these explanations and show that an increasing wage profile is also part of a profit-maximizing dynamic incentive system for reciprocal workers.

Moreover, and in line with the effort dynamics generated by my model, there is evidence that a worker's productivity is decreasing once he approaches retirement. Using US data, Haltiwanger et al. (1999) find that a firm's productivity is higher if it has a lower fraction of workers older than 55. Skirbekk (2004) report that older workers generally have lower productivities, and are – in particular – overpaid relative to their productivity. Using Belgian data Lallemand and Rycx (2009) show that having a high share of workers above 49 is harmful for a firm's productivity. Theoretical explanations for productivity reductions in the last periods before retirement are scarce, in particular

compared to the abundance of explanations for upward-sloping wage profiles. Whereas a career concerns model (such as Holmström, 1999) would predict a monotonically decreasing productivity (that is, not concentrated in the last years before retirement)⁶, Lazear (1979) has employees shirking mostly in the very last period of an employment relationship⁷.

Reduced effort in the last periods of an employment relationship has also been observed in many lab experiments (for example, see Brown et al. (2004), or Fehr et al. (2009a)). These results have mainly been explained by selfish individuals imitating those with social preferences early on, in order to collect later rents. I further explore this aspect in Section 5.2 and show that my model can deliver an alternative explanation for many results delivered by Brown et al. (2004).

Further Results

Finally, I present results on the dynamics of total wage payments and payoffs.

Lemma 5 *The bonus b_t is weakly decreasing over time, i.e., $b_t \leq b_{t-1}$. Moreover, $b_t < b_{t-1}$ implies $b_{t+1} < b_t$, whereas $b_{t+1} = b_t$ implies $b_t = b_{t-1}$. The agent's total compensation, $w_t + b_t$, might increase or decrease over time.*

Over time, the substitution of relational with reciprocity-based incentives also reduces bonus payments. There, the direct effect (smaller future profits tighten the (DE) constraint) dominates the indirect effect (a higher wage relaxes the (DE) constraint). The dynamics of the agent's total compensation, $w_t + b_t$, are not necessarily monotone, and depend on the relative importance of relational versus reciprocity-based incentives. Relating these results to real-world phenomena, though, one ought to be careful because an employee's compensation often consists of plenty other components than just monetary payments (in particular if supposed to assume a reward for non-verifiable aspects of effort such as the bonus in my setting). For instance, Gibbons and Henderson (2012) conceive an individual's payoffs to include "everything that might affect an individual's experience of his or her job, including factors such as job assignment, degree of autonomy, status with the firm or work group, and other intangibles such as feelings of belonging or that one is making a difference" (Gibbons and Henderson, 2012, p. 1353).

Finally, payoff dynamics are as follows. Whereas the principal's per-period profits decrease over time (once (DE) binds), the opposite is true for the agent's per-period utilities. This result is also driven by the gradual replacement of relational with reciprocity-based incentives; because of a binding (IC) constraint, the agent only receives a rent for the latter.

Lemma 6 *The principal's per-period profits π_t are weakly decreasing over time, i.e., $\pi_t \leq \pi_{t-1}$. Moreover, $\pi_t < \pi_{t-1}$ implies $\pi_{t+1} < \pi_t$, whereas $\pi_{t+1} = \pi_t$ implies $\pi_t = \pi_{t-1}$.*

The agent's per-period utility u_t is weakly increasing over time, i.e., $u_t \geq u_{t-1}$. Moreover, $u_t > u_{t-1}$ implies $u_{t+1} > u_t$, whereas $u_{t+1} = u_t$ implies $u_t = u_{t-1}$.

⁶If combined with contracts (Gibbons and Murphy, 1992), the productivity dynamics in a career concerns model are ambiguous.

⁷Some might also shirk in earlier periods, depending on the private benefits of shirking which vary stochastically (Lazear, 1981).

3.3 Reciprocity

In the previous sections, I have derived the properties of a profit-maximizing relational contract that also specifies a norm for reciprocity. Now, I explore how the agent's preferences for reciprocity affect implemented effort over the course of his career.

Proposition 2 *In every period t , equilibrium profits Π_t are increasing in η . Moreover, equilibrium effort e_t^* is (weakly) increasing in η . This positive effect is stronger if the (DE) constraint binds, i.e., in the later stages of the employment relationship.*

First, a higher η directly raises e_t for a given $w_t > 0$ due to the reduction of effective effort costs, and consequently profits. Second, there is an indirect effect. Because profits Π_{t+1} increase in η , the (DE) constraint in period t is relaxed. This further leads to higher effort and profits.

Proposition 2 also indicates that the positive effect of η on effort is stronger if the principal's (DE) constraint binds, i.e., at later stages of the agent's career. Then, the incentive system puts more weight on reciprocal incentives and the role of η is intensified. Therefore, the reduction of incentive costs caused by a higher η is more pronounced and equilibrium effort reacts more strongly.

Evidence on the generally positive relationship between reciprocity and effort has been provided by Dohmen et al. (2009). They use data from the German Socio-Economic Panel (SOEP). The SOEP is an annual panel survey that is representative of the German population and contains a wide range of questions on the personal and socioeconomic situation as well as labor market status and income of respondents. In a number of years (2005, 2010 and 2015) it also contained questions designed to capture individual reciprocal inclinations. As a measure for (non-verifiable) effort, Dohmen et al. (2009) use overtime work, and show that individuals with stronger reciprocal inclinations are more likely to work overtime.

Evidence on the dynamics of the effect of reciprocity and effort is provided by Fahn et al., 2017. They confirm the results of Dohmen et al. (2009), and in addition show that the positive link between reciprocity on overtime is significantly more pronounced for older workers close to retirement.

4 Competition

An important question in behavioral economics deals with the effect of competition on the relevance of social preferences. A number of theoretical and empirical contributions indicate that social preferences are driven out by competition if contracts are complete (Fehr and Schmidt, 1999; Dufwenberg et al., 2011). With incomplete contracts (such as in the present setting), though, the situation is different (Fehr and Fischbacher, 2002; Schmidt, 2011). Schmidt (2011) uses a static model to analyze how labor market competition might affect the utilization of fairness preferences by firms. He shows that induced effort levels are the same for all degrees of competition, only rents are shifted between firms and workers.

In this section, I discuss how competition shapes the optimal use of reciprocal preferences in an optimal *dynamic* incentive scheme. I show that, in a more competitive labor market, the principal might actually make *more* use of reciprocity-based incentives – if more intense labor-market competition reduces the principal's future profits and consequently her credibility in the relational contract.

My approach to model labor market competition follows Schmidt (2011), where the degree of competition determines the outside options of principal and agent. This reduced-form approach substantially simplifies the analysis and still allows to generate a number of insights.

The principal's outside option equals $\bar{\Pi} \geq 0$ and is the same in every period t . As discussed in Section 2.4, this view can be supported by the presumption that T reflects only the agent's time horizon, whereas the firm's is potentially infinite. Then, $\bar{\Pi}$ would include profits from hiring new agents once the current employment relationship is terminated (with multilateral punishments not being feasible). Moreover, $\bar{\Pi}$ is smaller than profits in period T (because of sufficiently high costs of replacing the agent with a new one), hence a premature on-path termination, as well as the use of efficiency wages, are not optimal. I assume that a larger degree of competition for workers decreases $\bar{\Pi}$, for example because more intense competition increases the costs and time to find a new agent.

Following Schmidt (2011), I capture the agent's outside opportunities by the wage $\bar{w} \geq 0$ he could secure when working for a different employer. Naturally, more intense competition for workers yields a higher outside wage \bar{w} . Moreover, I assume that the agent only reacts reciprocally to any wage paid *above* the new reference wage \bar{w} . Therefore, the agent's per-period payoff in a period t amounts to

$$u_t = w_t + b_t + \eta(w_t - \bar{w})\theta e_t - \frac{e_t^3}{3},$$

also taking into account that a wage below \bar{w} would not be accepted by the agent.

First, I characterize effort and wage in a spot reciprocity contract.

Lemma 7 *Effort in the profit-maximizing spot reciprocity contract is independent of \bar{w} and $\bar{\Pi}$. Moreover, $\partial w / \partial \bar{w} = 1$, and $\partial w / \partial \bar{\Pi} = \partial e / \partial \bar{w} = \partial e / \partial \bar{\Pi} = 0$.*

The principal responds to a higher \bar{w} with an increase of w_t in order to keep incentives constant. Therefore, labor market competition does not affect the importance of the agent's reciprocity for the optimal provision of incentives in a *static setting*. This replicates the results Schmidt (2011) has derived for the case of fairness preferences. \bar{w} only causes a redistribution of rents, whereas outcomes are entirely independent of $\bar{\Pi}$ (as long as $\pi = \frac{\eta\theta^3}{4} - \bar{w} \geq \bar{\Pi}$, i.e., $\frac{\eta\theta^3}{4} \geq \bar{w} + \bar{\Pi}$, which I implicitly presume in this section).

To derive a profit-maximizing relational contract with positive outside options, I first characterize the agent's (IC) constraint for a general $\bar{w} \geq 0$,

$$b_t - \frac{(e_t)^3}{3} + \eta(w_t - \bar{w})e_t\theta \geq \frac{2}{3} \left(\sqrt{\eta(w_t - \bar{w})\theta} \right)^3.$$

The outside wage \bar{w} enters the agent's (IC) constraint only via the associated increase of the reference wage above which reciprocity is triggered. This is different from a "standard" efficiency wage effect, where a better outside option of an employee also reduces his incentives to work hard. Here, any firing threat (which is an important component of the efficiency wage mechanism) would not be credible because keeping the agent gives the principal a higher payoff than $\bar{\Pi}$.

The principal's (DE) constraint amounts to

$$-b_t + \delta\Pi_{t+1} \geq \delta\bar{\Pi}.$$

(DE) is tightened by a larger $\bar{\Pi}$ and consequently relaxed by a more competitive labor market. Therefore, the principal can *ceteris paribus* commit to a *larger* bonus if facing a tighter competition for labor, because her relative benefits of maintaining a cooperative relationship go up.

As before, the (IC) constraint will bind in a profit-maximizing equilibrium. Moreover, the general structure of a profit-maximizing relational contract will be as in my main model, with constant wage and effort levels as long as (DE) is slack, and upward-sloping wage and downward-sloping effort profiles once (DE) becomes binding. Still, $\bar{\Pi}$ and \bar{w} crucially affect the importance of reciprocity-based incentives, as described in Proposition 3.

Proposition 3 *Larger values of \bar{w} and/or $\bar{\Pi}$ tighten the (DE) constraint.*

If (DE) does not bind in a period t , $\partial w_t / \partial \bar{w} = 1$. Moreover, $\partial w_t / \partial \bar{\Pi} = \partial e_t / \partial \bar{w} = \partial e_t / \partial \bar{\Pi} = 0$.

If (DE) binds in a period t , $\partial w_t / \partial \bar{w} > 1$ and $\partial w_t / \partial \bar{\Pi} > 0$. Moreover, $\partial e_t / \partial \bar{w} < 0$ and $\partial e_t / \partial \bar{\Pi} < 0$.

Finally, for given values of \bar{w} and $\bar{\Pi}$, effort and wage dynamics are as in Proposition 1

Larger values of \bar{w} and $\bar{\Pi}$ have no direct effect on the optimal provision of incentives, hence the principal implements the same effort level for all values of \bar{w} and $\bar{\Pi}$ in case (DE) does not bind (i.e., in earlier periods of the employment relationship). Then, as in a reciprocity spot contract, a higher \bar{w} causes a mere redistribution of rents from principal to agent (and $\partial w_t / \partial \bar{w} = 1$). Even though implementing a certain effort level becomes more expensive for the principal, the marginal costs of doing so remain constant.

Still, there is an indirect effect, because higher values of \bar{w} and $\bar{\Pi}$ reduce the principal's future profits. This tightens the (DE) constraint and, once the constraint binds, restricts the principal's possibility to use relational incentives. As in the main analysis (see Lemma 1 and Proposition 1), she mitigates the necessary effort reduction by an expansion of reciprocity-based incentives and raises w_t beyond the increase induced by a larger \bar{w} . Hence, $\partial w_t / \partial \bar{w} > 1$ and $\partial w_t / \partial \bar{\Pi} > 0$ if (DE) binds.

Now, a more competitive labor increases \bar{w} but reduces $\bar{\Pi}$, both of which have opposite effects. If the effect of a lower $\bar{\Pi}$ dominates, a more competitive labor market lets the effort reduction induced by a binding (DE) materialize at a later point in time. Moreover, effort is generally higher and fixed wages are lower, hence reciprocity-based incentives become *less* important. The opposite happens if the effect of a higher \bar{w} dominates. Then, the (DE) constraint binds earlier if the labor market is more competitive, effort is lower and wages are higher. All this is driven by the reduced commitment in the relational contract, letting reciprocity-based incentives become more important in a profit-maximizing dynamic incentive scheme.

Discussion The results with respect to the effects of a higher \bar{w} could also be applied to analyze the consequences of a minimum wage. There, I would expect that, if effort is not contractible, firms pay reciprocal agents more than the minimum wage, and that any minimum wage increase yields a more-than-proportional wage hike.

Finally, I briefly discuss how the relational contract might be influenced by varying degrees of product market competition. In my model, the latter could be captured by different values of θ , with a lower θ representing more intense competition on the

product market. Different from labor market competition, changes in θ already affect the optimal spot reciprocity contract, where a lower θ reduces effort as well as wages (since $w_T = \eta\theta^3/4$ and $e_T^* = \eta\theta^2/2$). The same holds for the relational contract with a non-binding (DE) constraint, i.e., in early periods of the employment relationship. In these cases, a more intense product market competition would reduce the use of reciprocity-based incentives. However, the reduction of future profits generally also limits the enforceability of the relational contract, which can increase the importance of reciprocity-based incentives. If both effects are active (i.e., if (DE) binds), either of them might dominate.

5 Extensions and Robustness

In the following, I explore how my results are affected by changes in some of the assumptions made so far. Doing so, I focus on the case of two periods, hence $T = 2$.

5.1 Reciprocity Triggered by all Current Payments

First, I sketch the implications of the agent's preferences for reciprocity being triggered by *all* realized current payments. Then, wages paid as a reward for effort in previous periods (and not only w_t^{nd}) also induce the agent to reciprocate. This does not hold for the bonus, though, because it is paid after effort has been exerted (in Section 5.5, I also allow equilibrium bonus payments to affect the agent's reciprocity). Therefore, only wages are used to provide incentives – because those can assume the role of the bonus and additionally induce reciprocal behavior.

To formally underpin this claim, note that the agent's second-period effort still maximizes $u_2 = w_2 - c(e_2) + \eta w_2 \theta e_2$, hence

$$e_2^* = \sqrt{\eta w_2 \theta}.$$

Different from before, though, w_2 is not chosen to maximize π_2 because it can also be contingent on e_1^* and is therefore set to maximize the principal's total discounted profit stream, Π_1 (subject to constraints). In the first period, the agent's effort e_1^* must satisfy his (IC) constraint. There, I assume that once the agent deviates, $b_1 = 0$, and w_2 is set such that π_2 is myopically maximized (in which case $w_2^* = \eta\theta^3/4$, $e_2^* = \eta\theta^2/2$, and $u_2 = \eta\theta^3/4 + \eta^3\theta^6/12$).⁸ Therefore, if the agent deviates, he chooses \tilde{e}_1 to maximize $\tilde{u}_1 = w_1 - e_1^3/3 + \eta w_1 \theta e_1$, hence $\tilde{e}_1 = \sqrt{\eta w_1 \theta}$.

All this implies that the agent's (IC) constraint equals

$$\begin{aligned} b_1 - \frac{e_1^3}{3} + \eta w_1 \theta e_1^* + \delta \left[w_2 + \frac{2(\sqrt{\eta w_2 \theta})^3}{3} \right] \\ \geq \frac{2(\sqrt{\eta w_1 \theta})^3}{3} + \delta \left(\frac{\eta\theta^3}{4} + \frac{\eta^3\theta^6}{12} \right). \end{aligned} \quad (\text{IC})$$

The principal is only willing to make equilibrium payments if her (DE) constraint holds,

$$-b_1 + \delta(e_2\theta - w_2) \geq 0. \quad (\text{DE})$$

⁸As before, a firing threat – which would maximize the power of incentives – is not credible.

For consistency, η now also drops to zero if w_2 differs from the amount promised at the beginning of period 1.

Then, the principal sets w_1 , w_2 and b_1 to maximize $\Pi_1 = e_1^*\theta - w_1 - b_1 + \delta(e_2^*\theta - w_2)$, subject to (IC) and (DE), and taking into account that $e_2^* = \sqrt{\eta w_2 \theta}$.

The structure of the optimal arrangement is similar to the one in the main part, with two exceptions. First, it is optimal to set $b_1 = 0$: To the contrary, assume there is a profit-maximizing equilibrium that has $b_1 > 0$. Then, a reduction of b_1 by a small $\varepsilon > 0$, together with an increase of w_2 by ε/δ does not affect (DE) and Π_1 , but relaxes (IC). Therefore, w_2 is set above the level maximizing π_2 and bounded by the condition that second-period profits must be non-negative. That implies that the back-loading of wages is more pronounced than before.

Second, the principal's profits will be larger than in the main model. This is because payments used to provide relational incentives also trigger reciprocal behavior, an aspect not present before.⁹

5.2 Asymmetric Information

Previously, I have assumed that the principal is aware of the agent's η , for example because of personality tests used in the hiring process. In this section, I explore potential implications of asymmetric information on the agent's reciprocal inclinations. I assume that the agent can either be a "reciprocal" type with $\eta > 0$ (with probability $p \in (0, 1)$) or a "selfish" type with no reciprocal preferences (with probability $1 - p$). Moreover, the agent's type is his private information. Assuming that the principal can design the incentive scheme and will do so in a profit-maximizing way, she will choose one of the following two options. Either, the principal asks for a first-period effort level that only the reciprocal, but not the selfish agent is willing to exert. Then, the selfish agent collects the first-period wage, but is subsequently detected and fired (because he would exert no effort in the second period). I call this a "separation contract". Or, the effort request is sufficiently low that it satisfies the selfish type's (IC) constraint. In this case, the agent's effort choice cannot be used to screen agents, and both types are also employed in the second period. Only then, the selfish agent – after collecting w_2 – shirks by exerting zero-effort. I call this arrangement a "pooling contract".

Before going on, note that I stick to the setting of the previous Section 5.1, where the agent is also motivated by wages that are paid as a reward for past effort. I do so because, in a separation contract, the agent takes into account that he will only remain employed if he exerts equilibrium effort in the first period. Therefore, his incentives to exert effort in the first period should be affected by his second-period utility from remaining employed, which includes the utility generated by his reciprocal inclinations. Taking this into account, no bonus but only future wages are used to motivate the agent (Section 5.1). Different from Section 5.1, a deviation from equilibrium effort might result in a termination of the employment relationship and henceforth zero off-path continuation utilities, namely if the deviation lets the principal assign probability 1 to facing the selfish type.

Now, I derive a Perfect Bayesian Equilibrium where *any* deviation by the agent lets the principal assign probability 1 to facing the selfish type. Then, a separation and pooling contract both are feasible. The (IC) constraints, one for the selfish type (ICS),

⁹One can show that, with equilibrium values from the main part and where w_2 is adjusted to include the first-period bonus, the (IC) constraint now would be slack.

and one for the reciprocal type (ICR), already taking into account that $e_2 = \sqrt{\eta w_2 \theta}$, amount to

$$-\frac{e_1^3}{3} + \delta w_2 \geq 0 \quad (\text{ICS})$$

$$-\frac{e_1^3}{3} + \eta w_1 \theta e_1 + \delta \left[w_2 + \frac{2(\sqrt{\eta w_2 \theta})^3}{3} \right] \geq -\frac{\tilde{e}_1^3}{3} + \eta w_1 \theta \tilde{e}_1, \quad (\text{ICR})$$

with $\tilde{e}_1 = \sqrt{\eta w_1 \theta}$.

For any effort level $e_1 \geq \tilde{e}_1$ (ICS) is tighter than (ICR) (this is shown in the proof to Proposition 4). Therefore, if the principal offered the profit-maximizing contract for a reciprocal type (which involves a binding (ICR) constraint), this would automatically result in a separation of types. Moreover, effort in a pooling contract will be determined by a binding (ICS) constraint.

Proposition 4 *In a profit-maximizing Perfect Bayesian Equilibrium where any deviation from equilibrium effort induces the principal to assign probability 1 to facing a selfish type, a pooling contract is optimal if p is sufficiently small. If p is sufficiently large, a separating contract is optimal.*

Proposition 4 indicates that, if the probability of facing a reciprocal type is sufficiently close to zero, a pooling contract will be optimal – for reasons similar to the “classic” reputation literature (see Mailath and Samuelson 2006). This is because the principal faces the following trade-off: Either, with a pooling contract, first-period effort is rather low (determined by a binding (ICS) constraint), however exerted by both types. Moreover, only the reciprocal type exerts effort in the second period, whereas both are paid w_2 . In this case, the principal’s expected profits are $\Pi_1^P = e_1 \theta - w_1 + \delta [p(\sqrt{w_2 \eta \theta} - w_2) - (1-p)w_2]$. Or, with a separating contract, first-period effort is higher (determined by a binding (ICR) constraint), however only exerted by the reciprocal type. Moreover, both types are paid w_1 , whereas the selfish type is fired and only the reciprocal type remains employed in the second period, then exerting effort accordingly. In this case, the principal’s expected profits are $\Pi_1^S = p [e_1 \theta + \delta (\sqrt{w_2 \eta \theta} - w_2)] - w_1$. If p is sufficiently small, the principal prefers the former case. This pooling contract, though, relies on the assumption that the reciprocal type cannot reveal himself by choosing a higher effort level. But this restriction generally does not survive the Intuitive Criterion as a refinement of Perfect Bayesian Equilibrium (Cho and Kreps, 1987): Assume that, in a pooling contract, an agent chooses an effort level that is slightly higher than equilibrium effort. Since the selfish type’s (IC) constraint binds, whereas the reciprocal type’s is slack, a deviation to a higher effort level should indicate that the principal in fact faces the reciprocal type. But if the principal responds to this revelation by offering the profit-maximizing second-period wage for the reciprocal type, and if this gives the latter a higher utility than equilibrium play, an upward deviation by the reciprocal type indeed increases his utility.

To support the relevance of this argument, note that, in the proof to Proposition 4, I show that, for low p and consequently a pooling contract,¹⁰ $e_1^* = \sqrt[3]{3\delta p^2 \eta \theta^3}$ and

¹⁰More precisely, for $p^2 \leq \left(\sqrt{\frac{2}{\theta}}\right)^3 / 3\delta\eta$.

$w_2 = e_1^3/3\delta = p^2\eta\theta^3$. If the reciprocal type deviates and chooses an effort level $e_1^* + \varepsilon$, the principal will take this as a signal that she faces the reciprocal type, and might instead offer $w_2 = \eta\theta^3/4$ (the second-period wage that maximizes her profits with a reciprocal type; see the proof to Lemma 1). It turns out that this wage also increases the reciprocal type's utility if $p < 1/2$, in which case the pooling equilibrium would not survive an application of the Intuitive Criterion.

Although a more general characterization of an optimal arrangement under asymmetric information is beyond the scope of this paper, note the following: There is a large amount of evidence that, in gift-exchange experiments, cooperation is larger in repeated than in one-shot interactions, even with a pre-defined last period. This is usually attributed to selfish types imitating those with social preferences, in order to collect future rents (see Fehr et al., 2009a). I aim at providing support for an alternative story: If the uninformed party is able to determine the incentive scheme, and in particular ask for a certain effort level, pooling equilibria where a selfish type imitates a reciprocal type are much harder to maintain. Then, an early separation of types can be achieved by requiring an effort level that just satisfies the reciprocal type's (IC) constraint, with the remaining matches thereafter having a relational contract that produces outcomes resembling my main results (high effort in early periods, declining effort once the last period approaches). Such results have actually been observed in the lab experiments conducted by Brown et al. (2004). They compare different settings, in particular one where players (among whom one side assumes the role of firms and the other side represents workers) either have the option to form long-term relationships, or where they are randomly matched in each of 15 rounds. Firms pay wages in every period and ask for effort from "their" workers, who subsequently choose their effort levels. Brown et al. (2004) find that effort is significantly larger in the treatment where long-term relationships are feasible, and that effort only goes down in the last two periods. They present the theoretical explanation that some players have fairness preferences, and that those without imitate the fair players early on by exerting high effort, in order to collect rents at the end of the game. What this explanation does not address, though, but my explanation can explain, is that many separations occur early on (70 percent in period 1, 65 percent in period 2), whereas way less matches separate in later periods. Moreover, a further result not fully consistent with their theoretical explanation, but in line with my prediction of an early separation of types, is that, in the treatment where long-term relationships are feasible, effort *in the last period* is considerably higher and the positive effect of wages on effort larger than in the treatment without long-term relationships.

Naturally, the setting in Brown et al. (2004) differs from my theoretical model along several dimensions. For example, also students who assume the role of firms might have social preferences, whereas in my setting only agents are reciprocal. Nevertheless, I think that the presented theoretical analysis, together with a careful analysis of experimental results such as in Brown et al. (2004), justifies the notion that not only the "selfish-types-mimick-fair-types" story might contribute to explaining experimental results. In particular if players do not face a rather inflexible environment such as a standard prisoner's dilemma, but have more flexibility in determining transfers and effort levels, the possibility to separate types early on and subsequently have a relational contract might also contribute to the high cooperation observed in repeated, but finite, gift-exchange experiments.

Finally, note that this section can deliver an additional theoretical result: The poten-

tial existence of a purely selfish agent induces the reciprocal agent to exert *more* effort in the first period (with a separating contract) than if the principal knew his type for sure. This is because, in the former case, the principal fires an agent who does not exert equilibrium. Conversely, if the principal assigns probability 1 to facing a reciprocal agent, it is not subgame perfect to fire the agent after a deviation. In this case, she would rather offer the profit-maximizing contract in the second period, which the agent would accept because of the positive rent involved with it.

Indeed, the reciprocal type's (IC) constraint with symmetric information equals

$$\begin{aligned} & -\frac{e_1^3}{3} + \eta w_1 \theta e_1 + \delta \left[w_2 + \frac{2(\sqrt{\eta w_2 \theta})^3}{3} \right] \\ & \geq -\frac{\tilde{e}_1^3}{3} + \eta w_1 \theta \tilde{e}_1 + \delta \left(\frac{\eta \theta^3}{4} + \frac{\eta^3 \theta^6}{12} \right), \end{aligned}$$

whereas the (IC) constraint in a separating equilibrium with asymmetric information (and $p < 1$) equals

$$\begin{aligned} & -\frac{e_1^3}{3} + \eta w_1 \theta e_1 + \delta \left[w_2 + \frac{2(\sqrt{\eta w_2 \theta})^3}{3} \right] \\ & \geq -\frac{\tilde{e}_1^3}{3} + \eta w_1 \theta \tilde{e}_1. \end{aligned}$$

This constraint is tighter in the first case, where the agent has less to lose in case he deviates. Hence, for given wages, more effort can be implemented with asymmetric information, and not only the existence of reciprocal agents can induce selfish agents to work harder (as in the reputation literature), but it might also be the other way round.

If p is sufficiently close to, but still below, 1, the principal's profits will actually be larger than if $p = 1$. This is a result that has, to the best of my knowledge, not been identified before. It might, for example, have implications for a firm's hiring process, which then should focus on employing workers with reciprocal preferences, but still potentially allow for some workers who are entirely selfish.

5.3 Negative Reciprocity

So far, I have focused on the positive effects of reciprocity. I have abstracted from any potential "dark side" of reciprocal preferences, in the sense that if an agent is granted a lower payment than expected, he wants to actively harm the principal. The potential consequences of negative reciprocity have been widely explored, for example by Dufwenberg and Kirchsteiger (2004), Dohmen et al. (2009), or Netzer and Schmutzler (2014). In this section, I introduce negative reciprocity and show that it leads to the same results as in the main part of this paper, even if η does not drop to zero after a deviation by the principal. This section therefore also serves as a robustness device to show that my results can also be generated if the agent's preferences are unaffected by the principal's behavior.

I use the approach introduced by Hart and Moore (2008), who have derived a tractable model to analyze negative reciprocity. They assume that the terms of a contract provide reference points, which determine a party's ex post performance. If someone gets less than what he feels entitled to, he shades on performance, thereby causing

a deadweight loss that has to be borne by the other party. I adapt the setting of Hart and Moore (2008) to my environment, and assume that the relational contract determines the agent's reference point.

Therefore, the agent feels entitled to the equilibrium bonus b_1^* . If he receives a lower bonus, his period-1 utility is decreased by $\underline{\eta}(b_1^* - b_1)$, where $\underline{\eta} \geq 0$ and b_1 the bonus actually paid by the principal. Moreover, the agent can reduce this utility loss via “shading” (for example by sabotaging the principal), by an amount σ that is at the agent's discretion. I assume that the agent still has to be employed by the principal in order to shade, and the principal can fire the agent before making the choice whether to pay the bonus. Thereby, she is able to escape the shading costs σ , but would then also sacrifice potential future profits

All this implies that the utility stream of the agent, conditional on not being fired, amounts to

$$U_1 = b_1 + w_1 - c(e_1^*) + \eta w_1 \theta e_1^* - \max\{(\underline{\eta}(b_1^* - b_1) - \sigma), 0\} + \delta [w_2 - c(e_2^*) + \eta w_2 \theta e_2^*].$$

The principal's payoff stream, in case she does not fire the agent before paying the bonus, amounts to

$$\Pi_1 = e_1 \theta - w_1 - b_1 - \sigma + \delta (e_2 \theta - w_2).$$

Since shading is not costly for the agent, it is optimal to set $\sigma = \underline{\eta}(b_1^* - b_1)$ (for $b_1 \leq b_1^*$). Furthermore, second period effort and wage equal $w_2 = \eta \theta^3 / 4$ and $e_2^* = \eta \theta^2 / 2$, respectively, hence second period profits are $\pi_2 = \eta \theta^3 / 4$ (see the proof to Lemma 1).

The principal faces two decisions. First, which bonus $b_1 \in [0, b_1^*]$ to pay, and second whether to fire the agent. Concerning the first decision, note that if the principal decides to pay a bonus $b_1 \leq b_1^*$ (and not fire the agent), her profits amount to

$$\Pi_1 = e_1 \theta - w_1 + (\underline{\eta} - 1) b_1 - \underline{\eta} b_1^* + \delta \frac{\eta \theta^3}{4}.$$

This immediately reveals that $b_1 = 0$ is optimal for $\underline{\eta} < 1$, whereas $b_1 = b_1^*$ for $\underline{\eta} \geq 1$. Since $b_1 = b_1^*$ on the equilibrium path, $\underline{\eta} < 1$ also implies $b_1^* = b_1 = 0$, and only reciprocity spot contracts are feasible in this case.

Now, assume $\underline{\eta} \geq 1$. Then, the principal sets $b_1 = b_1^*$ in case she does not fire the agent. She will terminate the relationship, though, if the bonus is larger than period-2 profits, i.e., if $b_1^* > \delta \pi_2$.

The principal's optimization problem becomes to maximize $\pi_1 = e_1^* \theta - b_1^* - w_1$, subject to the agent's binding (IC) constraint, which yields $b_1^* = (e_1^*)^3 / 3 - \eta w_1 e_1^* \theta + 2/3 (\sqrt{\eta w_1 \theta})^3$, as well as subject to $b_1^* \leq \delta \pi_2$. The last condition is equivalent to the (DE) constraint, hence the problem in this section is the same as the optimization problem in our main part.

This immediately yields Lemma 8:

Lemma 8 *The profit-maximizing equilibrium with negative reciprocity, and η being unaffected by the principal's behavior, has the following characteristics:*

- If $\underline{\eta} < 1$, $b_1^* = 0$. Moreover, $e_1^* = e_2^* = \eta\theta^2/2$ and $w_1 = w_2 = \eta\theta^3/4$.
- If $\eta \geq 1$, $b_1^* > 0$, and outcomes are as characterized in Section 3.2, with $w_1 < w_2 = \eta\theta^3/4$ and $e_1^* > e_2^* = \eta\theta^2/2$, as well as $de_2^*/d\eta > de_1^*/d\eta$.

This implies that, if $\underline{\eta} \geq 1$, all my previous results do *not* rely on the presumption that preferences change as a result of a deviation from equilibrium.

5.4 Adjustment of Reference Wage

Some evidence points towards a declining effect of gifts in long-term interactions. Individuals respond to higher wages by an increase in effort, but effort eventually goes down again. Gneezy and List (2006) conduct a real-world experiment, where they permanently increase the wages of recruited workers. Although workers respond with an immediate effort increase, this is only temporary, and effort falls to an amount that is only slightly above the initial level. Jayaraman et al. (2016) explore the effects of a mandated 30% wage increase for tea pluckers in India. They find that productivity substantially increased immediately after the wage raise. However, it started falling again around the second month after the change, and returned to its initial levels after four months.

This evidence suggests that individuals adapt to wage increases and update reference wages above which they are willing to reciprocate with higher effort. Further support in favor of an adaptation of reference wages is provided by Sliwka and Werner (2017), who examine how reciprocal effort is affected by the timing of wage increases. They document that a permanent wage increase only temporarily increases effort, and that the only way to permanently benefit from an individual's reciprocal behavior is to constantly raise wages.

In the following, I incorporate this evidence and assume that the reference wage above which the agent is willing to reciprocate increases in past wages. More precisely, the agent starts with a reference wage of zero. In the second period, the first-period wage w_1 determines the second-period reference wage. Hence, the agent's utilities are

$$\begin{aligned} u_1 &= b_1 + w_1 - c(e_1^*) + \eta w_1 e_1^* \theta \\ u_2 &= b_2 + w_2 - c(e_2^*) + \max\{0, \eta(w_2 - w_1) e_2^* \theta\}. \end{aligned}$$

Spot Reciprocity Contract First, I compute the profit-maximizing spot reciprocity contract in the last period, $t = 2$. Then, no bonus is paid, and (taking into account that setting $w_2 \geq w_1$ will be optimal) effort maximizes $-\frac{e_2^3}{3} + \eta(w_2 - w_1) e_2 \theta$. As shown in Lemma 7 in Section 4, effort will be unaffected by the higher reference wage, hence $e_2^* = \eta\theta^2/2$ and $w_2^* = \eta\theta^3/4 + w_1$.

Before deriving first-period outcomes of an optimal relational contract, I derive a benchmark where the principal only uses spot reciprocity contracts. There, in period 1, the principal takes into account how w_1 affects period-2 profits via the adjustment of the reference wage. Hence, he would choose w_1 to maximize

$$\Pi_1^{SC} = e_1 \theta - w_1 + \delta(e_2 \theta - w_2) = \sqrt{\eta w_1 \theta} - w_1 + \delta \left(\frac{\eta \theta^3}{4} - w_1 \right),$$

which also incorporates the agent's first-period (IC). The optimal period-1 spot reciprocity contract would then have $w_1^* = \eta\theta^3 / [4(1 + \delta)^2]$ and $e_1^* = \eta\theta^2 [2(1 + \delta)]$. Therefore, $w_2^* > w_1^*$ and $e_2^* > e_1^*$, and wages and effort would increase over time.

Relational Contract Outcomes for an optimal relational contract are given in Lemma 5.4.

Lemma 9 *Assume the second-period reference wage is equal to w_1 . Then, $w_1 < w_2$. Moreover, the (DE) constraint might or might not bind.*

- *If it does not bind, $de_1^*/d\eta < de_2^*/d\eta$. Furthermore, there exists a $\bar{\eta} > 0$ such that the optimal wage equals zero for $\eta \leq \bar{\eta}$. In this case, $e_1^* > e_2^*$. For $\eta > \bar{\eta}$, setting a strictly positive wage is optimal, and e_1^* can be smaller or larger than e_2^* .*
- *If it binds, there exists a $\tilde{\eta} > 0$ such that the optimal wage equals zero for $\eta \leq \tilde{\eta}$, whereas it is strictly positive for $\eta \geq \tilde{\eta}$. In both cases, e_1^* can be smaller or larger than e_2^* .*

$\bar{\eta}$ can be smaller or larger than $\tilde{\eta}$, and both are larger than if the second-period reference wage equals zero.

The principal is reluctant to trigger the agent's reciprocal preferences already in the first period. In particular if δ is large, she rather wants to maintain this opportunity until later on – when relational contracts are not feasible anymore. Therefore, the threshold for η above which a positive first-period wage is paid is larger than in the main part. Different from before, a higher w_1 also does not necessarily relax the (DE) constraint (which implies that $\tilde{\eta}$ does not have to be smaller than $\bar{\eta}$). This is because a positive first-period wage has two different effects on the tightness of the (DE) constraint. On the one hand, the necessary bonus to implement a certain effort level is reduced, which relaxes the constraint (as in the main part). On the other hand, future profits are reduced via the adjustment of the reference wage, which tightens the constraint. Moreover, e_1^* is not necessarily larger than e_2^* – unless (DE) is slack and $w_1 = 0$ – because the reluctance to pay a positive w_1 also reduces the agent's willingness to exert effort in the first period.

5.5 Reciprocity Triggered by Rent

Finally, I explore the implications of reciprocity being triggered by the agent's material rent, in contrast to only by monetary payments. More precisely, the agent's per-period utilities are

$$\begin{aligned} u_1 &= (b_1 + w_1 - c(e_1)) (1 + \eta e_1 \theta) \\ u_2 &= (w_2 - c(e_2)) (1 + \eta e_2 \theta). \end{aligned}$$

Importantly, when choosing his effort level, the agent also reciprocates on the equilibrium bonus of this period, *before* it is paid. Still, our main results remain valid, in particular that effort is higher in the first than in the second period. Moreover, the principal generally is less likely to pay a positive fixed wage already in the first period. The reason is that also the bonus triggers reciprocal behavior. Only if a sufficiently tight (DE) constraint allows for only a relatively low bonus, w_1 will be positive. However, the (DE) constraint is *ceteris paribus* tighter than in the main model, because reciprocity in the second period is triggered by the agent's rent $w_2 - c(e_2)$ instead of merely the wage.

Formally, effort in the second period is given by the agent's first order condition,

$$-e_2^2 - \frac{4}{3}e_2^3\eta\theta + w_2\eta\theta = 0.$$

This is taken into account by the principal who sets w_2 in order to maximize $\pi_2 = e_2\theta - w_2$.

In the first period, the principal's (DE) constraint still equals $-b_1 + \delta\pi_2 \geq 0$, whereas the agent's (IC) constraint becomes

$$\left(b_1 + w_1 - \frac{(e_1^*)^3}{3}\right) (1 + \eta e_1^*\theta) \geq \left(w_1 - \frac{(\tilde{e}_1)^3}{3}\right) (1 + \eta \tilde{e}_1\theta). \quad (\text{IC})$$

There, \tilde{e}_1 is characterized by $-\tilde{e}_1^2 - \frac{4}{3}\tilde{e}_1^3\eta\theta + w_1\eta\theta = 0$, and $e_1^* > \tilde{e}_1$ if $b_1 > 0$.

Lemma 10 *Assume that the agent's preferences for reciprocity are triggered by his material rent. Then, the (DE) constraint binds given $T = 2$ and $\delta \leq 1$. Moreover, there exists a $\tilde{\eta} > 0$ such that the optimal wage equals zero for $\eta \leq \tilde{\eta}$, whereas it is strictly positive for $\eta \geq \tilde{\eta}$.*

In any case, $e_1^ > e_2^*$ and $w_1 < w_2$.*

With $T = 2$, second-period profits cannot be sufficiently large for a non-binding (DE) constraint given $\delta \leq 1$. However, in a more general setting with more than two periods, (DE) might indeed be slack. In this case, the proof to Lemma 10 reveals that paying a positive wage could not be optimal. The reason is that the purpose of a positive wage – triggering the agent's reciprocal inclinations – can equivalently be achieved by a bonus, which additionally allows for higher effort via the relational contract. With a binding (DE) constraint, the principal might pay a fixed wage in the first period, but only if η is large enough.

6 Discussion & Conclusion

I have shown that relational incentives and preferences for positive reciprocity can interact in intricate ways. The two are dynamic substitutes, but complements once a specific point in time is considered.

Proofs

Proof of Lemma 1 I maximize profits $\pi = e\theta - w$, taking into account that effort equals $e = \sqrt{\eta w\theta}$, and that the agent's (IR) constraint, $u = w - e^3/3 + \eta w e\theta = w + (2/3)\sqrt{\eta w\theta^3} \geq 0$, must be satisfied. Naturally, the latter holds for any $w \geq 0$.

In a next step, I solve for the profit-maximizing wage level. There I first omit the non-negative condition $w \geq 0$ and later show that is satisfied. The first-order condition equals

$$\frac{d\pi}{dw} = \frac{de}{dw}\theta - 1 = 0,$$

which yields

$$w = \frac{\eta\theta^3}{4}.$$

Hence,

$$e^* = \frac{\eta\theta^2}{2},$$

and $\pi = \frac{\eta\theta^3}{4}$, $u = \frac{\eta\theta^3}{4} + \frac{\eta^3\theta^6}{12} > 0$. ■

Proof of Lemma 2 If the (DE) constraint does not bind in a period t , the principal maximizes profits $\pi_t = e_t\theta - \left((e_t)^3/3 - \eta w_t e_t\theta + 2/3 (\sqrt{\eta w_t\theta})^3 \right) - w_t$, subject to $w_t \geq 0$.

The Lagrange function equals $L_t = e_t\theta - (e_t)^3/3 + \eta w_t e_t\theta - 2/3 (\sqrt{\eta w_t\theta})^3 - w_t + \lambda_{w_t} w_t$, where $\lambda_{w_t} \geq 0$ represents the Lagrange parameter for the agent's limited liability constraint, giving first order conditions

$$\begin{aligned} \frac{\partial L_t}{\partial e_t^*} &= \theta - (e_t)^2 + \eta w_t\theta = 0 \\ \frac{\partial L_t}{\partial w_t} &= \eta\theta \left(e_t - \sqrt{\eta w_t\theta} \right) - 1 + \lambda_{w_t} = 0 \end{aligned}$$

First, assume that $\lambda_{w_t} = 0$. Then, the first order conditions yield $w_t = \frac{(\eta^2\theta^3 - 1)^2}{4\eta^3\theta^3}$ and $e_t^* = \frac{1 + \eta^2\theta^3}{2\eta\theta}$. Second, assume that $\lambda_{w_t} > 0$ and hence $w_t = 0$. Then, $e_t^* = \sqrt{\theta}$ and $\pi_t = \frac{2}{3} \left(\sqrt{\theta} \right)^3$. To establish the existence of $\bar{\eta}$, note that $\frac{d\pi_t}{dw_t} |_{w_t=0} = \sqrt{\eta^2\theta^3} - 1$. This is positive for $\eta > \sqrt{1/\theta^3}$, hence a strictly positive wage is optimal in this case and not otherwise. ■

Proof of Lemma 3 Including the respective (DE) constraints, the Lagrange function of the principal's maximization problem in a period t becomes

$$\begin{aligned} L_t &= e_t\theta - e_t^3/3 + \eta w_t e_t\theta - 2/3 \left(\sqrt{\eta w_t\theta} \right)^3 - w_t \\ &+ \lambda_{DEt} \left[\delta \Pi_{t+1} - \frac{2}{3} \left(\sqrt{\eta w_t\theta} \right)^3 - e_t^3/3 + \eta w_t\theta e_t \right] + w_t \lambda_{w_t}, \end{aligned}$$

where $\lambda_{w_t} \geq 0$ represents the Lagrange parameter for the agent's limited liability constraint and $\lambda_{DE_t} \geq 0$ represents the Lagrange parameter for the principal's dynamic enforcement constraint.

First-order conditions are

$$\begin{aligned}\frac{\partial L}{\partial e_t^*} &= \theta - e_t^2 + \eta w_t \theta + \lambda_{DE_t} [-e_t^2 + \eta w_t \theta] = 0 \\ \frac{\partial L}{\partial w_t} &= \eta \theta e_t - \eta \theta \sqrt{\eta w_t \theta} - 1 + \lambda_{DE_t} [-\eta \theta \sqrt{\eta w_t \theta} + \eta \theta e_t] + \lambda_{w_t} = 0.\end{aligned}$$

First, assume that $\lambda_{w_t} = 0$. Then, the first order conditions yield $w_t = \frac{(\eta^2 \theta^3 (1 + \lambda_{DE_t}) - 1)^2}{4\eta^3 \theta^3 (1 + \lambda_{DE_t})^2}$ and $e_t^* = \frac{1 + \eta^2 \theta^3 (1 + \lambda_{DE_t})}{2\eta \theta (1 + \lambda_{DE_t})}$. It follows that, given $\lambda_{DE_t} > 0$ and $\eta > \bar{\eta}$ (i.e., $\eta^2 \theta^3 - 1$, implying that $w_t > 0$ if (DE) does not bind), $\frac{(\eta^2 \theta^3 (1 + \lambda_{DE_t}) - 1)^2}{4\eta^3 \theta^3 (1 + \lambda_{DE_t})^2} > \frac{(\eta^2 \theta^3 - 1)^2}{4\eta^3 \theta^3}$ and $\frac{1 + \eta^2 \theta^3 (1 + \lambda_{DE_t})}{2\eta \theta (1 + \lambda_{DE_t})} < \frac{1 + \eta^2 \theta^3}{2\eta \theta}$. Second, assume that $\lambda_{w_t} > 0$ and hence $w_t = 0$. Then, $e_t^* = \sqrt{\frac{\theta}{1 + \lambda_{DE_t}}}$. To establish the existence of $\tilde{\eta}$, note that $\frac{\partial L}{\partial w_t} |_{w_t=0} = \sqrt{\eta^2 \theta^3 (1 + \lambda_{DE_t})} - 1 = 0$. This is positive for $\eta > \sqrt{1/\theta^3 (1 + \lambda_{DE_t})}$, hence a strictly positive wage is optimal in this case and not otherwise. Finally, for $\lambda_{DE_t} > 0$, $\tilde{\eta} = \sqrt{1/\theta^3 (1 + \lambda_{DE_t})} < \bar{\eta} = \sqrt{1/\theta^3}$. Moreover, $\tilde{\eta}$ increases in δ because λ_{DE_t} decreases in δ (see the proof to Lemma 4). ■

Proof of Lemma 4 The (DE) constraint in period $T - 1$ (where on-path continuation profits are $\Pi_T = \eta \theta^3 / 4$) equals $(e_t^*)^3 / 3 - \eta w_t \theta e_t^* \leq \delta \frac{\eta \theta^3}{4} - \frac{2}{3} (\sqrt{\eta w_t \theta})^3$. First, note that for $\eta \leq \bar{\eta}$ and consequently first-best effort equals $\sqrt{\theta}$, whereas the first-best wage is zero, the (DE) constraint equals $(\sqrt{\theta})^3 / 3 \leq \delta \frac{\eta \theta^3}{4}$. This cannot hold if $\eta \leq \bar{\eta} = \sqrt{1/\theta^3}$, even for $\delta \rightarrow 1$.

Therefore, assume $\eta > \bar{\eta}$ for the remainder of this proof. Then, first-best effort and wage levels are $e = \frac{1 + \eta^2 \theta^3}{2\eta \theta}$ and $w = \frac{(\eta^2 \theta^3 - 1)^2}{4\eta^3 \theta^3}$, and the (DE) constraint in period $T - 1$ becomes

$$\frac{3\eta^2 \theta^3 - 1}{6\eta^3 \theta^3} \leq \delta \frac{\eta \theta^3}{4}. \quad (1)$$

Because $\eta > \bar{\eta}$, the left-hand-side is strictly positive. Therefore, the constraint is violated for first-best effort and wage levels if $\delta \rightarrow 0$.

To show that first-best effort can be implemented in period $T - 1$ if η is sufficiently large, I compute the derivative of the left-hand-side of 1 and obtain

$(1 - \eta^2 \theta^3) / 2\eta^4 \theta^3$, which is negative for $\eta > \bar{\eta}$. Moreover, $\lim_{\eta \rightarrow \infty} \frac{3\eta^2 \theta^3 - 1}{6\eta^3 \theta^3} = 0$, whereas the right-hand side of 1 is strictly positive and increasing in η . Therefore, 1 is satisfied if η is sufficiently large.

Concerning the second part of the Lemma, recall that the equilibrium is sequentially efficient, hence the principal's maximization problem is equivalent to maximizing $\pi_t = e_t^* \theta - b_t^* - w_t$ in every period t , subject to the (DE) constraint $(e_t^*)^3 / 3 - \eta w_t \theta e_t^* \leq \delta \Pi_{t+1} - \frac{2}{3} (\sqrt{\eta w_t \theta})^3$. It follows that, for a given w_t , implementable effort in period t is ceteris paribus strictly increasing in Π_{t+1} , whereas per-period profits π_t are consequently weakly increasing in Π_{t+1} . Furthermore, per-period profits in periods $t < T$ can be expressed as functions of Π_{t+1} , i.e. $\pi_t(\Pi_{t+1})$, with $\pi_t' \geq 0$.

The profit-maximizing spot reciprocity contract is the principal's optimal choice in the last period T , hence $\pi_T = \Pi_T = \eta\theta^3/4$. In all previous periods, the principal still has the option to implement the spot reciprocity contract (by setting $b_t^* = 0$ and $w_t = \eta\theta^3/4$), therefore $\pi_t \geq \pi_T \forall t$.

Now, I apply proof by induction to verify that $\Pi_{t-1} > \Pi_t$. First, $\Pi_{T-1} > \Pi_T$ because

$$\Pi_{T-1} = \pi_{T-1} + \delta\Pi_T \geq \pi_T + \delta\Pi_T = \Pi_T(1 + \delta) > \Pi_T.$$

Now, assume that $\Pi_t > \Pi_{t+1}$. Since $\pi_t'(\Pi_{t+1}) \geq 0$, $\pi_{t-1} \geq \pi_t$. Therefore, $\Pi_{t-1} = \pi_{t-1} + \delta\Pi_t \geq \pi_t + \delta\Pi_t > \pi_t + \delta\Pi_{t+1} = \Pi_t$, which completes the proof. \blacksquare

Proof of Proposition 1 First, assume $\eta > \bar{\eta} = \sqrt{1/\theta^3}$, hence $w_t > 0 \forall t$. Furthermore, in Lemmas 2 and 3, I have established that $w_t = \frac{(\eta^2\theta^3(1+\lambda_{DE_t})-1)^2}{4\eta^3\theta^3(1+\lambda_{DE_t})^2}$ and $e_t^* = \frac{1+\eta^2\theta^3(1+\lambda_{DE_t})}{2\eta\theta(1+\lambda_{DE_t})}$, where λ_{DE_t} is the Lagrange parameter associated with the (DE) constraint in period t . Hence, $w_t = w_{t-1}$ and $e_t^* = e_{t-1}^*$ if $\lambda_{DE_t} = \lambda_{DE_{t-1}} = 0$. By Lemma 3, if $\lambda_{DE_{t-1}} = 0$ but $\lambda_{DE_t} > 0$, then $w_t > w_{t-1}$ and $e_t^* < e_{t-1}^*$. Finally, assume that $\lambda_{DE_{t-1}} > 0$. First, I show that in this case also $\lambda_{DE_t} > 0$: Plugging $w_{t-1} = \frac{(\eta^2\theta^3(1+\lambda_{DE_{t-1}})-1)^2}{4\eta^3\theta^3(1+\lambda_{DE_{t-1}})^2}$ and $e_{t-1}^* = \frac{1+\eta^2\theta^3(1+\lambda_{DE_{t-1}})}{2\eta\theta(1+\lambda_{DE_{t-1}})}$ into the binding (DE) constraint for period $t-1$ yields

$$\frac{3\eta^2\theta^3(1+\lambda_{DE_{t-1}})-1}{6\eta^3\theta^3(1+\lambda_{DE_{t-1}})^3} = \delta\Pi_t.$$

By the implicit function theorem, $\frac{d\lambda_{DE_{t-1}}}{d\Pi_t} = \frac{2\delta\eta^3\theta^3(1+\lambda_{DE_{t-1}})^4}{1-2\eta^2\theta^3(1+\lambda_{DE_{t-1}})} < 0$ (since $\eta > \bar{\eta}$ implies $\eta^2\theta^3 > 1$). Hence, Lemma 4 yields $\lambda_{DE_{t-1}} < \lambda_{DE_t}$, which implies $\lambda_{DE_{t-1}} > 0 \Rightarrow \lambda_{DE_t} > 0$. Furthermore, if $\lambda_{DE_t} = 0$ in a period t , this also holds for all previous periods.

The wage schedule is increasing in periods $t < T$ since $\frac{\partial w_t}{\partial \lambda_{DE_t}} = \frac{(\eta^2\theta^3(1+\lambda_{DE_t})-1)}{2\eta^3\theta^3(1+\lambda_{DE_t})^3} > 0$, whereas the effort path is decreasing because of $\frac{\partial e_t^*}{\partial \lambda_{DE_t}} = \frac{-1}{2\eta\theta(1+\lambda_{DE_t})^2} < 0$. Finally, wage and effort in period T are $e_T^* = \frac{\eta\theta^2}{2}$ and $w_T = \frac{\eta\theta^3}{4}$, respectively. $e_T^* < e_t^*$ for all $t < T$ follows from $\frac{\eta\theta^2}{2} < \frac{1+\eta^2\theta^3(1+\lambda_{DE_t})}{2\eta\theta(1+\lambda_{DE_t})}$ ($\Leftrightarrow \eta^2\theta^3(1+\lambda_{DE_t}) < 1 + \eta^2\theta^3(1+\lambda_{DE_t})$). $w_T > w_t$ for all $t < T$ follows from $\frac{\eta\theta^3}{4} > \frac{(\eta^2\theta^3(1+\lambda_{DE_t})-1)^2}{4\eta^3\theta^3(1+\lambda_{DE_t})^2}$ ($\Leftrightarrow 2\eta^2\theta^3(1+\lambda_{DE_t}) > 1$).

For the remainder of the proof, assume $\eta \leq \bar{\eta}$, hence $w_t = 0$ and $e_t^* = \sqrt{\theta}$ if $\lambda_{DE_t} = 0$. As before, $\lambda_{DE_t} = 0$ implies $\lambda_{DE_{t-1}} = 0$, and $\lambda_{DE_t} > 0$ implies $\lambda_{DE_{t+1}} > \lambda_{DE_t}$.

The following cases still have to be explored:

- $\lambda_{DE_t} > 0$ and $w_t > 0$. Then, $w_t = \frac{(\eta^2\theta^3(1+\lambda_{DE_t})-1)^2}{4\eta^3\theta^3(1+\lambda_{DE_t})^2}$ and $e_t^* = \frac{1+\eta^2\theta^3(1+\lambda_{DE_t})}{2\eta\theta(1+\lambda_{DE_t})}$, and the previous analysis regarding wages w_τ and effort levels e_τ , for $\tau > t$, can be applied. The previous analysis can also be applied if $\lambda_{DE_{t-1}} > 0$ and $w_{t-1} > 0$.

Now, assume $\lambda_{DE_{t-1}} > 0$ and $w_{t-1} = 0$. Then, $e_{t-1} = \sqrt{\frac{\theta}{(1+\lambda_{DE_{t-1}})}}$ (see the proof

to Lemma 3), and I have to show that

$$\sqrt{\frac{\theta}{(1 + \lambda_{DE_{t-1}})}} > \frac{1 + \eta^2\theta^3(1 + \lambda_{DE_t})}{2\eta\theta(1 + \lambda_{DE_t})}.$$

In the proof to Lemma 3, I have proven that $w_{t-1} = 0$ implies $\eta \leq \sqrt{1/\theta^3(1 + \lambda_{DE_{t-1}})}$, which can be re-written to $\sqrt{\theta/(1 + \lambda_{DE_{t-1}})} \geq \eta\theta^2$. Therefore, it is sufficient to show that $\eta\theta^2 > [1 + \eta^2\theta^3(1 + \lambda_{DE_t})]/[2\eta\theta(1 + \lambda_{DE_t})]$, which becomes $\eta > \sqrt{1/\theta^3(1 + \lambda_{DE_t})}$. This, however, is implied by $w_t > 0$ (see the proof to Lemma 3).

- $\lambda_{DE_t} = 0$ and $\lambda_{DE_{t+1}} > 0$, with $w_{t+1} = 0$. Now, $e_{t+1}^* < e_t^*$ follows from $e_t^* = \sqrt{\theta/(1 + \lambda_{DE_t})}$, $e_{t+1}^* = \sqrt{\theta/(1 + \lambda_{DE_{t+1}})}$ and $\lambda_{DE_{t+1}} > \lambda_{DE_t}$.
- $\lambda_{DE_t} = 0$ and $\lambda_{DE_{t+1}} > 0$, with $w_{t+1} > 0$. Now, $e_t^* = \sqrt{\theta}$ and $e_{t+1}^* = \frac{1 + \eta^2\theta^3(1 + \lambda_{DE_{t+1}})}{2\eta\theta(1 + \lambda_{DE_{t+1}})}$, and I have to show that

$$\begin{aligned} \sqrt{\theta} &> \frac{1 + \eta^2\theta^3(1 + \lambda_{DE_{t+1}})}{2\eta\theta(1 + \lambda_{DE_{t+1}})} \\ \Leftrightarrow (1 + \lambda_{DE_{t+1}}) (2\sqrt{\eta^2\theta^3} - \eta^2\theta^3) &> 1 \end{aligned}$$

Again, $w_{t+1} > 0$ implies $(1 + \lambda_{DE_{t+1}}) > 1/\eta^2\theta^3$ (see the proof to Lemma 3), hence it is sufficient to prove that (taking into account that $\eta \leq \bar{\eta}$ implies $2\sqrt{\eta^2\theta^3} - \eta^2\theta^3 > 0$)

$$\begin{aligned} \frac{(2\sqrt{\eta^2\theta^3} - \eta^2\theta^3)}{\eta^2\theta^3} &\geq 1 \\ \Leftrightarrow 2\sqrt{\eta^2\theta^3} (1 - \sqrt{\eta^2\theta^3}) &\geq 0, \end{aligned}$$

which holds because of $\eta \leq \bar{\eta}$. ■

Proof of Lemma 5. The binding (IC) constraint delivers $b_t = (e_t^*)^3/3 - \eta w_t e_t^* \theta + 2/3 (\sqrt{\eta w_t \theta})^3$. It follows that, if $w_t = \frac{(\eta^2\theta^3(1 + \lambda_{DE_t}) - 1)^2}{4\eta^3\theta^3(1 + \lambda_{DE_t})^2} > 0$,

$$b_t = \frac{3\eta^2\theta^3(1 + \lambda_{DE_t}) - 1}{6\eta^3\theta^3(1 + \lambda_{DE_t})^3},$$

with $\frac{db_t}{d(1 + \lambda_{DE_t})} = \frac{-2\eta^2\theta^3(1 + \lambda_{DE_t}) + 1}{2\eta^3\theta^3(1 + \lambda_{DE_t})^4} < 0$.

Moreover, if $w_t = 0$, then

$$b_t = \frac{\left(\sqrt{\frac{\theta}{(1 + \lambda_{DE_t})}}\right)^3}{3},$$

with $db_t/d(1 + \lambda_{DE_t}) < 0$. The remainder of the first part of the proof then proceeds analogously to the proof of Proposition 1.

Concerning the second part, I focus on the case $\eta^2\theta^3 > 1$, hence $w_t = \frac{(\eta^2\theta^3(1+\lambda_{DE_t})-1)^2}{4\eta^3\theta^3(1+\lambda_{DE_t})^2} >$

0. Then,

$$\frac{d(w_t + b_t)}{d(1 + \lambda_{DE_t})} = \frac{-2\eta^2\theta^3(1 + \lambda_{DE_t}) + 1 + \eta^2\theta^3(1 + \lambda_{DE_t})^2 - (1 + \lambda_{DE_t})}{4\eta^3\theta^3(1 + \lambda_{DE_t})^4},$$

which is negative for $\lambda_{DE_t} \rightarrow 0$. To show that this expression can also be positive, note that a binding (DE) constraint delivers

$$\frac{3\eta^2\theta^3(1 + \lambda_{DE_t}) - 1}{6\eta^3\theta^3(1 + \lambda_{DE_t})^3} = \delta\Pi_{t+1},$$

hence $3\eta^2\theta^3(1 + \lambda_{DE_t}) - 1 \geq 0$. At $3\eta^2\theta^3(1 + \lambda_{DE_t}) - 1 = 0$, the numerator of $d(w_t + b_t)/d(1 + \lambda_{DE_t})$ becomes $(3\eta^2\theta^3 - 2)/9\eta^2\theta^3 > 0$. ■

Proof of Lemma 6. First, I consider the case $w_t > 0$, hence $(\eta^2\theta^3(1 + \lambda_{DE_t}) - 1)$.

Then,

$$\begin{aligned} u_t &= w_t + b_t - \frac{e_t^3}{3} + \eta w_t e_t^* \theta \\ &= w_t + 2/3 \left(\sqrt{\eta w_t \theta} \right)^3 \\ &= \frac{(\eta^2\theta^3(1 + \lambda_{DE_t}) - 1)^2}{4\eta^3\theta^3(1 + \lambda_{DE_t})^2} \left[1 + \frac{(\eta^2\theta^3(1 + \lambda_{DE_t}) - 1)}{3(1 + \lambda_{DE_t})} \right] \text{ and} \\ \frac{\partial u_t}{\partial(1 + \lambda_{DE_t})} &= \frac{(\eta^2\theta^3(1 + \lambda_{DE_t}) - 1)}{2\eta^3\theta^3(1 + \lambda_{DE_t})^3} \left[1 + \frac{2(\eta^2\theta^3(1 + \lambda_{DE_t}) - 1)}{6(1 + \lambda_{DE_t})} \right] \\ &\quad + \frac{(\eta^2\theta^3(1 + \lambda_{DE_t}) - 1)^2}{12\eta^3\theta^3(1 + \lambda_{DE_t})^3} > 0 \end{aligned}$$

Moreover,

$$\begin{aligned} \pi_t &= e\theta - w - b \text{ and} \\ \frac{\partial \pi_t}{\partial(1 + \lambda_{DE_t})} &= \frac{\partial e}{\partial(1 + \lambda_{DE_t})} \theta - \frac{\partial w}{\partial(1 + \lambda_{DE_t})} - \frac{\partial b}{\partial(1 + \lambda_{DE_t})} \\ &= -\frac{\eta^2\theta^3(1 + \lambda_{DE_t})\lambda_{DE_t} + (\eta^2\theta^3(1 + \lambda_{DE_t}) - 1)}{2\eta^3\theta^3(1 + \lambda_{DE_t})^4} < 0 \end{aligned}$$

Furthermore,

$$\begin{aligned} \lim_{\lambda_{DE_t} \rightarrow \infty} u_t &= \frac{\left(\eta^4\theta^6 - \frac{2\eta^2\theta^3}{(1+\lambda_{DE_t})} + \frac{1}{(1+\lambda_{DE_t})^2} \right)}{4\eta^3\theta^3} \left[1 + \frac{\left(\eta^2\theta^3 - \frac{1}{(1+\lambda_{DE_t})} \right)}{3} \right] \\ &= \frac{\eta\theta^3}{4} \left[1 + \frac{\eta^2\theta^3}{3} \right] = u_T \end{aligned}$$

and

$$\begin{aligned} \lim_{\lambda_{DE_t} \rightarrow \infty} \pi_t &= \frac{\frac{1}{(1+\lambda_{DE_t})} + \eta^2 \theta^3}{2\eta\theta} \theta - \frac{\left(\eta^4 \theta^6 - \frac{2\eta^2 \theta^3}{(1+\lambda_{DE_t})} + \frac{1}{(1+\lambda_{DE_t})^2} \right)}{4\eta^3 \theta^3} \\ &= \frac{3\eta^2 \theta^3 - \frac{1}{(1+\lambda_{DE_t})}}{6\eta^3 \theta^3 (1+\lambda_{DE_t})^2} = \frac{\eta \theta^3}{4} = \pi_T \end{aligned}$$

Second, I consider the case $w_t = 0$.

Then

$$\begin{aligned} u_t &= 0, \\ \pi_t &= \sqrt{\frac{\theta}{(1+\lambda_{DE_t})}} \left(\theta - \frac{\theta}{3(1+\lambda_{DE_t})} \right) \text{ and} \\ \frac{\partial \pi_t}{\partial (1+\lambda_{DE_t})} &= -\sqrt{\frac{\theta^3}{(1+\lambda_{DE_t})^3}} \frac{\lambda_{DE_t}}{2(1+\lambda_{DE_t})} < 0 \end{aligned}$$

Finally, note that $w_t > 0$ for λ_{DE_t} sufficiently large, hence the first case applies for $\lambda_{DE_t} \rightarrow \infty$. ■

Proof of Proposition 2. First, note that $e_T^* = \eta\theta^2/2$, which is obviously increasing in η . Second, assume that a positive wage is optimal in any period $t < T$ (i.e., if $\eta > \bar{\eta}$ with a non-binding (DE) constraint and $\eta > \tilde{\eta}$ with a binding (DE) constraint). Then, $e_t^* = \frac{1+\eta^2\theta^3(1+\lambda_{DE_t})}{2\eta\theta(1+\lambda_{DE_t})}$, with $\lambda_{DE_t} \geq 0$, and

$$\frac{\partial e_t^*}{\partial \eta} = \frac{\eta^2 \theta^3 (1 + \lambda_{DE_t}) - 1}{2\eta^2 \theta (1 + \lambda_{DE_t})} - \frac{1}{2\eta\theta (1 + \lambda_{DE_t})^2} \frac{\partial \lambda_{DE_t}}{\partial \eta} > 0.$$

There, $\partial \lambda_{DE_t} / \partial \eta \leq 0$ because λ_{DE_t} is decreasing in Π_{t+1} (see the proof to Proposition 1), and because profits in all periods increase in η : This is obviously true for $\pi_T = \eta\theta^3/4$. Therefore, (DE) constraints in all periods $\tau < T$ are relaxed. Moreover, the agent's (IC) constraints in all periods $\tau < T$ are relaxed by a higher η if $w_\tau > 0$ and stay unaffected if $w_\tau = 0$.

Now, assume that $w_t = 0$ is optimal in any period $t < T$. Then, $e_t^* = \sqrt{\theta/(1+\lambda_{DE_t})}$, with $\lambda_{DE_t} \geq 0$, and

$$\frac{\partial e_t^*}{\partial \eta} = -\frac{1}{2} \sqrt{\frac{\theta}{(1+\lambda_{DE_t})^3}} \frac{\partial \lambda_{DE_t}}{\partial \eta} \geq 0.$$

The second part ($\partial e_t^* / \partial \eta$ is larger if $\lambda_{DE_t} > 0$) immediately follows. ■

Proof of Lemma 7. For a given $w \geq \bar{w}$, the agent chooses an effort level that maximizes $u = w + \eta(w - \bar{w})\theta e - \frac{e^3}{3}$, hence $e^* = \sqrt{\eta(w - \bar{w})\theta}$. Taking this into account, the principal maximizes profits $\pi = e^*\theta - w = \sqrt{\eta(w - \bar{w})\theta}\theta - w$, subject to $w \geq \bar{w}$. First ignoring the latter constraint, the principal's first-order condition equals

$$\frac{\eta\theta^2}{2\sqrt{\eta(w - \bar{w})\theta}} - 1 = 0.$$

This yields

$$w = \frac{\eta\theta^3}{4} + \bar{w},$$

which is larger than \bar{w} .

Hence,

$$e^* = \frac{\eta\theta^2}{2},$$

$$\text{and } \pi = \frac{\eta\theta^3}{4} - \bar{w}, u = \frac{\eta\theta^3}{4} + \frac{\eta^3\theta^6}{12} + \bar{w}. \quad \blacksquare$$

Proof of Proposition 3. In any period t , the principal maximizes

$\pi_t = e_t\theta - \left((e_t)^3/3 - \eta(w_t - \bar{w})e_t\theta + 2/3 \left(\sqrt{\eta(w_t - \bar{w})\theta} \right)^3 \right) - w_t$, subject to (DE) and $w_t \geq \bar{w}$. First, I assume that (DE) does not bind (which is possible if η and/or δ are sufficiently large – see the proof to Lemma 2). Then, the Lagrange function equals

$$L_t = e_t\theta - (e_t)^3/3 + \eta(w_t - \bar{w})e_t\theta - 2/3 \left(\sqrt{\eta(w_t - \bar{w})\theta} \right)^3 - w_t + \lambda_{w_t}(w_t - \bar{w}),$$

with first order conditions

$$\begin{aligned} \frac{\partial L_t}{\partial e_t^*} &= \theta - (e_t)^2 + \eta(w_t - \bar{w})\theta = 0 \\ \frac{\partial L_t}{\partial w_t} &= \eta\theta \left(e_t - \sqrt{\eta(w_t - \bar{w})\theta} \right) - 1 + \lambda_{w_t} = 0 \end{aligned}$$

I start with $\lambda_{w_t} = 0$. Then, the first order conditions yield $w_t = \frac{(\eta^2\theta^3 - 1)^2}{4\eta^3\theta^3} + \bar{w}$ and $e_t^* = \frac{1 + \eta^2\theta^3}{2\eta\theta}$. Now, assume that $\lambda_{w_t} > 0$ and hence $w_t = \bar{w}$. Then, $e_t^* = \sqrt{\theta}$. Moreover, note that $\frac{dw_t}{d\eta} \Big|_{w_t=\bar{w}} = \sqrt{\eta^2\theta^3} - 1$. This is positive for $\eta > \sqrt{1/\theta^3}$, hence a strictly positive wage is optimal in this case and not otherwise. Therefore, effort levels in both cases ($w_t > 0$ and $w_t = 0$) are not affected by \bar{w} , as well as the threshold $\bar{\eta}$ above which $w_t > 0$ is optimal. Therefore, equilibrium effort is independent of \bar{w} . It follows that e_t^* and w_t are independent of $\bar{\Pi}$.

Now, I include the respective (DE) constraints, which yields the Lagrange function of the principal's maximization problem in a period t

$$\begin{aligned} L_t &= e_t\theta - e_t^3/3 + \eta(w_t - \bar{w})e_t\theta - 2/3 \left(\sqrt{\eta(w_t - \bar{w})\theta} \right)^3 - w_t \\ &+ \lambda_{DE_t} \left[\delta (\Pi_{t+1} - \bar{\Pi}) - \frac{2}{3} \left(\sqrt{\eta(w_t - \bar{w})\theta} \right)^3 - e_t^3/3 + \eta(w_t - \bar{w})\theta e_t \right] \\ &+ \lambda_{w_t}(w_t - \bar{w}), \end{aligned}$$

where $\lambda_{w_t} \geq 0$ represents the Lagrange parameter for the agent's limited liability constraint and $\lambda_{DE_t} \geq 0$ represents the Lagrange parameter for the principal's dynamic enforcement constraint.

First-order conditions are

$$\begin{aligned}\frac{\partial L}{\partial e_t^*} &= \theta - e_t^2 + \eta(w_t - \bar{w})\theta + \lambda_{DE_t} [-e_t^2 + \eta(w_t - \bar{w})\theta] = 0 \\ \frac{\partial L}{\partial w_t} &= \eta\theta e_t - \eta\theta\sqrt{\eta(w_t - \bar{w})\theta} - 1 \\ &\quad + \lambda_{DE_t} [-\eta\theta\sqrt{\eta(w_t - \bar{w})\theta} + \eta\theta e_t] + \lambda_{w_t} = 0.\end{aligned}$$

I start with $\lambda_{w_t} = 0$. Then, the first order conditions yield

$$w_t = \frac{((1+\lambda_{DE_t})\eta^2\theta^3 - 1)^2}{(1+\lambda_{DE_t})^2 4\eta^3\theta^3} + \bar{w} \text{ and } e_t = \frac{1+(1+\lambda_{DE_t})\eta^2\theta^3}{(1+\lambda_{DE_t})2\eta\theta}.$$

It follows that, given $\lambda_{DE_t} > 0$ and $\eta^2\theta^3 - 1 \geq 0$, $\frac{(\eta^2\theta^3(1+\lambda_{DE_t})-1)^2}{4\eta^3\theta^3(1+\lambda_{DE_t})^2} > \frac{(\eta^2\theta^3-1)^2}{4\eta^3\theta^3}$ and $\frac{1+\eta^2\theta^3(1+\lambda_{DE_t})}{2\eta\theta(1+\lambda_{DE_t})} < \frac{1+\eta^2\theta^3}{2\eta\theta}$.

Now, assume that $\lambda_{w_t} > 0$ and hence $w_t = \bar{w}$. Then, $e_t^* = \sqrt{\frac{\theta}{(1+\lambda_{DE_t})}}$. To show that both cases, $w_t = \bar{w}$ and $w_t > \bar{w}$, are feasible note that $\frac{\partial L}{\partial w_t} |_{w_t=\bar{w}} = \sqrt{\eta^2\theta^3(1+\lambda_{DE_t})} - 1 = 0$. This is positive for $\eta > \sqrt{1/\theta^3(1+\lambda_{DE_t})}$, hence a strictly positive wage is optimal in this case and not otherwise.

Now, I show that Π_t is decreasing in \bar{w} . This implies that (DE) is more likely to bind for a higher \bar{w} , and thus – once (DE) binds – λ_{DE_t} increases with \bar{w} (see the proof to Lemma 4). First, I have already shown (in the proof to Lemma 7) that $\Pi_T = \pi_T$ is decreasing in \bar{w} . Therefore, (DE) in period $T-1$ is tightened, and consequently profits π_{T-1} and Π_{T-1} are reduced for larger values of \bar{w} . This tightens the (DE) constraint in period $T-2$ and reduces profits π_{T-2} and Π_{T-2} , and so on. Therefore, Π_t is decreasing in \bar{w} for all t , and λ_{DE_t} , if positive, is increasing.

Therefore,

$$\begin{aligned}\frac{\partial e_t^*}{\partial \bar{w}} &= -\frac{1}{2\eta\theta(1+\lambda_{DE_t})^2} \frac{\partial \lambda_{DE_t}}{\partial \bar{w}} < 0 \\ \frac{\partial w_t}{\partial \bar{w}} &= \frac{((1+\lambda_{DE_t})\eta^2\theta^3 - 1)}{2\eta^3\theta^3(1+\lambda_{DE_t})^3} \frac{\partial \lambda_{DE_t}}{\partial \bar{w}} + 1 > 1.\end{aligned}$$

if $w_t > \bar{w}$. If $w_t = \bar{w}$,

$$\begin{aligned}\frac{\partial e_t^*}{\partial \bar{w}} &= -\frac{1}{2} \sqrt{\frac{\theta}{(1+\lambda_{DE_t})^3}} \frac{\partial \lambda_{DE_t}}{\partial \bar{w}} < 0 \\ \frac{\partial w_t}{\partial \bar{w}} &= 1.\end{aligned}$$

Finally, λ_{DE_t} increases in $\bar{\Pi}$ because a larger $\bar{\Pi}$ tightens (DE) (see the proof to Lemma 4).

Therefore,

$$\begin{aligned}\frac{\partial e_t^*}{\partial \bar{\Pi}} &= -\frac{1}{2\eta\theta(1+\lambda_{DE_t})^2} \frac{\partial \lambda_{DE_t}}{\partial \bar{\Pi}} < 0 \\ \frac{\partial w_t}{\partial \bar{\Pi}} &= \frac{((1+\lambda_{DE_t})\eta^2\theta^3 - 1)}{2\eta^3\theta^3(1+\lambda_{DE_t})^3} \frac{\partial \lambda_{DE_t}}{\partial \bar{\Pi}} + 1 > 1.\end{aligned}$$

■

Proof of Proposition 4. First, I show that, for $p \rightarrow 1$, a separating contract dominates a pooling contract. There, note that, in any profit-maximizing equilibrium, (ICS), the selfish type's (IC) constraint, is tighter than (ICR), the reciprocal type's (IC) constraint:

$$-\frac{e_1^3}{3} + \delta w_2 \geq 0 \quad (\text{ICS})$$

$$\begin{aligned} & -\frac{e_1^3}{3} + \eta w_1 \theta e_1 + \delta \left[w_2 + \frac{2(\sqrt{\eta w_2 \theta})^3}{3} \right] \\ & \geq \frac{2}{3} \left(\sqrt{\eta w_1 \theta} \right)^3. \end{aligned} \quad (\text{ICR})$$

With $w_1 = 0$, (ICS) is tighter than (ICR) for any second-period wage w_2 because second-period utilities are larger for the reciprocal type. A strictly positive w_1 can only possibly be optimal for the principal if it further relaxes (ICR) ((ICS) is unaffected by w_1), which confirms that (ICS) is tighter than (ICR) in any profit-maximizing equilibrium. This implies that a strictly higher effort level can be implemented with a separating contract (then however only exerted by the reciprocal type) than with a pooling contract (then exerted by both). For $p \rightarrow 1$, profits under both regimes approach $e_1 \theta - w_1 + \delta (\sqrt{w_2 \eta \theta \theta} - w_2)$, which is larger with a separating contract because of the higher effort implemented in this case.

To show that a pooling contract dominates a separating contract for $p \rightarrow 0$, I first assume that the principal offers a pooling contract and explore its properties. Then, I do the same with a separating contract, and finally compare both alternatives.

Pooling contract In any profit-maximizing equilibrium, (ICS) is tighter than (ICR). Therefore, (ICS) determines feasible effort in a pooling contract. This also implies that $w_1 = 0$, because a positive w_1 would (potentially) only relax (ICR).

Now, the principal maximizes Π_1 , subject to her own (DE) constraint, $p e_2 \theta - w_2 \geq 0$, as well as the selfish agent's (IC) constraint, $-\frac{e_1^3}{3} + \delta w_2 \geq 0$. This will bind because, otherwise, the principal could ask for a higher first-period effort level without violating any constraint. Moreover, the reciprocal type exerts an effort level $e_2 = \sqrt{w_2 \eta \theta}$ in the second period, whereas the selfish type's second period effort amounts to zero, hence $\Pi_1 = e_1 \theta + \delta (p \sqrt{w_2 \eta \theta \theta} - w_2)$.

Taking all this into account, the Lagrange function becomes

$$L = e_1 \theta + \delta \left[p \sqrt{\frac{e_1^3}{3\delta} \eta \theta \theta} - \frac{e_1^3}{3\delta} \right] + \lambda_{DE} \left[p \sqrt{\frac{e_1^3}{3\delta} \eta \theta \theta} - \frac{e_1^3}{3\delta} \right],$$

and the first-order condition

$$\frac{\partial L}{\partial e_1} = \theta + \left[\frac{p \eta \theta^2}{2 \sqrt{\frac{e_1^3}{3\delta} \eta \theta}} - 1 \right] \frac{e_1^2}{\delta} (\delta + \lambda_{DE}) = 0.$$

First, assume $\lambda_{DE} = 0$. Then, e_1^* is characterized by

$$2 \sqrt{\frac{\eta \theta}{3\delta}} (\theta - (e_1^*)^2) + p \eta \theta^2 \sqrt{e_1^*} = 0. \quad (2)$$

Second, assume $\lambda_{DE} > 0$. Then, e_1^* is determined by the binding (DE) constraint,

$$e_1^* = \sqrt[3]{3\delta p^2 \eta \theta^3}.$$

To compute the condition for when (DE) actually binds, I plug $e_1^* = \sqrt[3]{3\delta p^2 \eta \theta^3}$ into the (FOC),

$$\begin{aligned} \theta + \left[\frac{p\eta\theta^2}{2\sqrt{\frac{e_1^3}{3\delta}\eta\theta}} - 1 \right] \frac{e_1^2}{\delta} (\delta + \lambda_{DE}) \\ = \theta - \frac{1}{2} \frac{e_1^2}{\delta} (\delta + \lambda_{DE}) \\ = \theta - \frac{1}{2} \frac{\left(\sqrt[3]{3\delta p^2 \eta \theta^3} \right)^2}{\delta} (\delta + \lambda_{DE}) = 0. \end{aligned}$$

Therefore, (DE) binds if $\theta - \frac{1}{2}\theta^2 \left(\sqrt[3]{3\delta p^2 \eta} \right)^2 \geq 0$, or

$$p^2 \leq \frac{\left(\sqrt{\frac{2}{\theta}} \right)^3}{3\delta\eta}.$$

In this case, which is the relevant case for $p \rightarrow 0$, the principal's profits with a pooling equilibrium are

$$\Pi_1^P = e_1^* \theta = \sqrt[3]{3\delta p^2 \eta \theta^2}.$$

Otherwise, $\Pi_1^P = e_1 \theta + \delta \left[p \sqrt{\frac{(e_1^*)^3}{3\delta} \eta \theta} - \frac{(e_1^*)^3}{3\delta} \right] = e_1^* \left[\theta + \frac{(e_1^*)^2 - 2\theta}{3} \right]$, where e_1^* is characterized by (2).

Separating contract In case she offers a separating contract, the principal maximizes $\Pi_1 = p[e_1\theta + \delta(e_2\theta - w_2)] - w_1$, where $e_2 = \sqrt{w_2\eta\theta}$, subject to her own (DE) constraint, $e_2\theta - w_2 \geq 0$ (which is relevant in case the agent turns out to be reciprocal), the limited liability constraint, $w_1 \geq 0$, as well as the reciprocal agent's binding (IC) constraint,

$$\begin{aligned} -\frac{(e_1^*)^3}{3} + \eta w_1 \theta e_1^* + \delta \left[w_2 + \frac{2(\sqrt{\eta w_2 \theta})^3}{3} \right] \\ = \frac{2(\sqrt{\eta w_1 \theta})^3}{3}. \end{aligned} \tag{IC}$$

There, note that

$$\begin{aligned} \frac{de_1^*}{dw_1} &= \eta\theta \frac{e_1^* - \sqrt{\eta w_1 \theta}}{(e_1^*)^2 - \eta w_1 \theta} = \frac{\eta\theta}{e_1^* + \sqrt{\eta w_1 \theta}} \\ \frac{de_1^*}{dw_2} &= \frac{\delta [1 + \sqrt{\eta w_2 \theta} \eta\theta]}{(e_1^*)^2 - \eta w_1 \theta}. \end{aligned}$$

Therefore, the Lagrange function becomes $L = p [e_1\theta + \delta (\sqrt{w_2\eta\theta} - w_2) + \lambda_{DE} (\sqrt{w_2\eta\theta} - w_2)] - w_1 + \lambda_{w_1} w_1$, with first-order conditions

$$\frac{\partial L}{\partial w_1} = p \frac{\eta\theta}{e_1 + \sqrt{\eta w_1 \theta}} - 1 + \lambda_{w_1} = 0$$

$$\frac{\partial L}{\partial w_2} = p \left[\frac{\delta [1 + \sqrt{\eta w_2 \theta \eta \theta}]}{e_1^2 - \eta w_1 \theta} \theta + \delta \left(\frac{\eta\theta^2}{2\sqrt{w_2\eta\theta}} - 1 \right) + \lambda_{DE} \left(\frac{\eta\theta^2}{2\sqrt{w_2\eta\theta}} - 1 \right) \right] = 0$$

For later use, note that the first condition implies that $w_1 = 0$ for $p \rightarrow 0$ (because e_1^* is bounded away from zero for any strictly positive δ).

First, assume $\lambda_{DE} = 0$, hence

$$\frac{[1 + \sqrt{\eta w_2 \theta \eta \theta}]}{e_1^2 - \eta w_1 \theta} \theta + \left(\frac{\eta\theta^2}{2\sqrt{w_2\eta\theta}} - 1 \right) = 0.$$

This, together with the reciprocal agent's (IC) constraint, determines outcomes if $w_1 = 0$. If $w_1 > 0$, outcomes are additionally given by

$$p \frac{\eta\theta^2}{e_1^* + \sqrt{\eta w_1 \theta}} - 1 = 0,$$

and an explicit characterization of the results is not feasible.

Now, assume $\lambda_{DE} > 0$. Then, a binding (DE) constraint implies $w_2 = \eta\theta^3$.

If $w_1 = 0$, (IC) yields

$$e_1^* = \sqrt[3]{3\delta \left[\eta\theta^3 + \frac{2\eta^3\theta^6}{3} \right]}$$

To compute the condition for when $w_1 = 0$ (if (DE) binds), I plug these values into the first (FOC), $p \frac{\eta\theta^2}{\sqrt[3]{3\delta \left[\eta\theta^3 + \frac{2\eta^3\theta^6}{3} \right]}} - 1 + \lambda_{w_1} = 0$. Therefore, $w_1 = 0$ if $p \frac{\eta\theta^2}{\sqrt[3]{3\delta \left[\eta\theta^3 + \frac{2\eta^3\theta^6}{3} \right]}} - 1 \leq 0$, or

$$p^3 \leq \frac{3\delta \left[\eta\theta^3 + \frac{2\eta^3\theta^6}{3} \right]}{\eta^3\theta^6}.$$

To compute the condition for when (DE) binds (if $w_1 = 0$), I plug these values into the second (FOC), $p \left[\frac{\delta [1 + \eta\theta^2\eta\theta]}{\left(\sqrt[3]{3\delta \left[\eta\theta^3 + \frac{2\eta^3\theta^6}{3} \right]} \right)^2} \theta - \frac{1}{2}\delta - \frac{1}{2}\delta\lambda_{DE} \right] = 0$. Therefore, (DE) binds for

$$\frac{\delta [1 + \eta\theta^2\eta\theta]}{\left(\sqrt[3]{3\delta \left[\eta\theta^3 + \frac{2\eta^3\theta^6}{3} \right]} \right)^2} \theta - \frac{1}{2}\delta \geq 0, \text{ or}$$

$$\delta^2 \leq \frac{8\theta^3 [1 + \eta^2\theta^3]^3}{9 \left[\eta\theta^3 + \frac{2\eta^3\theta^6}{3} \right]^2}.$$

The right hand side of this condition is larger than 1, hence (DE) *always* binds if $w_1 = 0$. Therefore, (DE) always binds if $p \rightarrow 0$ because then, $w_1 = 0$ (see above). On a general note, though, I want to emphasize that this might change in a more general setup with a longer time horizon.

All this implies that, for $p \rightarrow 0$, profits with a separating contract are

$$\Pi_1^S = p e_1^* \theta = p \theta \sqrt[3]{3\delta \left[\eta\theta^3 + \frac{2\eta^3\theta^6}{3} \right]}$$

Comparison For $p \rightarrow 0$, profits with a pooling contract are $\Pi_1^P = \sqrt[3]{3\delta p^2 \eta \theta^2}$, and $\Pi_1^S = p\theta \sqrt[3]{3\delta \left[\eta \theta^3 + \frac{2\eta^3 \theta^6}{3} \right]}$ for a separating contract. Therefore,

$$\begin{aligned} \Pi_1^P &> \Pi_1^S \\ \Leftrightarrow \sqrt[3]{3\delta p^2 \eta \theta^3} &\geq p\theta \sqrt[3]{3\delta \left[\eta \theta^3 + \frac{2\eta^3 \theta^6}{3} \right]} \\ \Leftrightarrow 1 &\geq p \left(1 + \frac{2\eta^2 \theta^3}{3} \right), \end{aligned}$$

which holds for $p \rightarrow 0$. ■

Proof of Lemma 9. The principal maximizes

$$\Pi_1 = e_1 \theta - (e_1^*)^3/3 + \eta w_1 e_1^* \theta - 2/3 \left(\sqrt{\eta w_1 \theta} \right)^3 - w_1 + \delta \left(\frac{\eta \theta^3}{4} - w_1 \right),$$

subject to $w_1 \geq 0$ and

$$\frac{(e_1^*)^3}{3} - \eta w_1 \theta e_1^* + \frac{2}{3} \left(\sqrt{\eta w_1 \theta} \right)^3 \leq \delta \left(\frac{\eta \theta^3}{4} - w_1 \right). \quad (\text{DE})$$

This yields the Lagrange function

$$\begin{aligned} L = & e_1 \theta - (e_1)^3/3 + \eta w_1 e_1 \theta - 2/3 \left(\sqrt{\eta w_1 \theta} \right)^3 - w_1 + \delta \left(\frac{\eta \theta^3}{4} - w_1 \right) \\ & + \lambda_{w_1} w_1 + \lambda_{DE} \left[\eta w_1 \theta e_1 + \delta \left(\frac{\eta \theta^3}{4} - w_1 \right) - \frac{2}{3} \left(\sqrt{\eta w_1 \theta} \right)^3 - \frac{(e_1)^3}{3} \right], \end{aligned}$$

where $\lambda_{w_1} \geq 0$ represents the Lagrange parameter for the agent's limited liability constraint, and $\lambda_{DE} \geq 0$ the Lagrange parameter for the principal's dynamic enforcement constraint.

First-order conditions are

$$\begin{aligned} \frac{\partial L}{\partial e_1} &= \theta - e_1^2 + \eta w_1 \theta + \lambda_{DE} [\eta w_1 \theta - e_1^2] = 0 \\ \frac{\partial L}{\partial w_1} &= \eta e_1 \theta - \eta \theta \sqrt{\eta w_1 \theta} - 1 - \delta + \lambda_{w_1} \\ &+ \lambda_{DE} [\eta \theta e_1^* - \delta - \eta \theta \sqrt{\eta w_1 \theta}] = 0. \end{aligned}$$

First, assume $\lambda_{DE} = 0$. Then, I have to consider the two cases $w_1 = 0$ and $w_1 > 0$.

If $w_1 = 0$, $e_1^* = \sqrt{\theta}$ and $\Pi_1 = \frac{2}{3} \left(\sqrt{\theta} \right)^3 + \delta \frac{\eta \theta^3}{4}$. Moreover, $\frac{d\Pi_1}{dw_1} |_{w_1=0} = \sqrt{\eta^2 \theta^3} - 1 - \delta$, therefore $w_1 = 0$ for $\eta^2 \theta^3 \leq (1 + \delta)^2$, whereas $w_1 > 0$ for $\eta^2 \theta^3 > (1 + \delta)^2$. Recall that the condition for a positive wage in case (DE) is non-binding in the main part (i.e., without an adjustment of the reference wage), has been $\eta^2 \theta^3 > 1$.

Furthermore, $e_1^* > e_2^* \Leftrightarrow \eta^2\theta^3 < 4$, which holds because $\eta^2\theta^3 < (1 + \delta)^2$. Moreover, $0 = w_1 < w_2 = \frac{\eta\theta^3}{4}$, and $\frac{de_1^*}{d\eta} = 0 < \frac{de_2^*}{d\eta}$.

To check the feasibility of the case with $\lambda_{DE} = 0$ and $w_1 = 0$, I plug the respective values into the (DE) constraint, and obtain

$$\frac{16}{9\delta^2} \leq \eta^2\theta^3.$$

This is consistent with $\eta^2\theta^3 \leq (1 + \delta)^2$ if $\frac{3}{4}\delta(1 + \delta) \geq 1$, hence if δ is sufficiently large.

Now, assume $\eta^2\theta^3 > (1 + \delta)^2$ and $\lambda_{DE} = 0$. Hence, $\lambda_{w_1} = 0$, and the first order conditions yield $e_1 = \frac{(1+\delta)^2 + \eta^2\theta^3}{2\eta\theta(1+\delta)}$ and $w_1 = \frac{[\eta^2\theta^3 - (1+\delta)^2]^2}{4(1+\delta)^2\eta^3\theta^3}$. Moreover, $e_1^* > e_2^* \Leftrightarrow \delta\theta^3\eta^2 < (1 + \delta)^2$, which only is consistent with $\eta^2\theta^3 > (1 + \delta)^2$ if δ is sufficiently small. In any case, $w_1 < w_2$ and $\frac{de_1^*}{d\eta} < \frac{de_2^*}{d\eta}$, where the latter condition is equivalent to $\delta\eta^2\theta^3 > -(1 + \delta)^2$.

To check the feasibility of the case with $\lambda_{DE} = 0$ and $w_1 > 0$, I plug the respective values into the (DE) constraint, and obtain

$$2 \leq \delta \left(\frac{(1 + \delta)^2 \eta^2\theta^3 - 1}{(1 + \delta)^2} \right) + (1 + \delta)^2 \frac{1}{3} \frac{(2 - \delta)}{\eta^2\theta^3}.$$

The right hand side is increasing in η^2 if δ is large enough. Since $\eta^2\theta^3 > (1 + \delta)^2$, I plug $\eta^2\theta^3 = (1 + \delta)^2$ into the condition, which becomes

$$\frac{4}{3} \leq \delta^2(2 + \delta) + \frac{2}{3}\delta - \frac{\delta}{(1 + \delta)^2}$$

The right hand side is increasing in δ and, for $\delta \rightarrow 1$, approaches $3 + \frac{5}{12} > \frac{4}{3}$. Hence, this case is feasible if η and/or δ are large enough.

Now, assume that the (DE) constraint binds, hence $\lambda_{DE} > 0$.

First, I assume that $\lambda_{w_t} > 0$, hence $w_t = 0$ and $e_t^* = \sqrt{\frac{\theta}{(1 + \lambda_{DE_t})}}$. To establish the existence of $\tilde{\eta}$, note that $\frac{\partial L}{\partial w_t} |_{w_t=0} = \left(\eta\theta \sqrt{\frac{\theta}{(1 + \lambda_{DE_t})}} - \delta \right) (1 + \lambda_{DE}) - 1$, which is positive for $\eta^2\theta^3 > \frac{(1 + \delta(1 + \lambda_{DE}))^2}{(1 + \lambda_{DE})}$. This threshold is larger than with a non-binding (DE) if $\lambda_{DE} > \frac{(1 - \delta^2)}{\delta^2}$, which might or might not hold. Moreover, provided $\eta^2\theta^3 \leq \frac{(1 + \delta(1 + \lambda_{DE}))^2}{(1 + \lambda_{DE})}$ $e_1^* > e_2^* \Leftrightarrow \eta^2\theta^3(1 + \lambda_{DE_t}) < 4$, which might or might not hold.

Second, I assume $\eta^2\theta^3 > \frac{(1 + \delta(1 + \lambda_{DE}))^2}{(1 + \lambda_{DE})}$, hence $\lambda_{w_1} = 0$. Then, the first order conditions yield

$$e_1 = \frac{\eta^2\theta^3(1 + \lambda_{DE}) + (1 + \delta(1 + \lambda_{DE}))^2}{2\eta\theta(1 + \lambda_{DE})(1 + \delta(1 + \lambda_{DE}))}$$

$$w_1 = \frac{[\eta^2\theta^3(1 + \lambda_{DE}) - (1 + \delta(1 + \lambda_{DE}))]^2}{4\eta^3\theta^3(1 + \lambda_{DE})^2(1 + \delta(1 + \lambda_{DE}))^2}.$$

Now, $e_1^* > e_2^* \Leftrightarrow \frac{(1 + \delta(1 + \lambda_{DE}))^2}{\delta(1 + \lambda_{DE})} > \eta^2\theta^3$, which might or might not be consistent with $\eta^2\theta^3 > \frac{(1 + \delta(1 + \lambda_{DE}))^2}{(1 + \lambda_{DE})}$. ■

Proof of Lemma 10. First, note that the principal maximizes $\pi_2 = e_2\theta - w_2$ in the second period, where e_2 is given by

$$-e_2^2 - \frac{4}{3}e_2^3\eta\theta + w_2\eta\theta = 0.$$

This yields

$$\begin{aligned} e_2^* &= \frac{\sqrt{1 + 4\eta^2\theta^3} - 1}{4\eta\theta} \\ w_2 &= \frac{e_2^2 + \frac{4}{3}e_2^3\eta\theta}{\eta\theta} \\ \pi_2 &= e_2^* \left(\frac{8\eta^2\theta^3 + 1 - \sqrt{1 + 4\eta^2\theta^3}}{12\eta^2\theta^2} \right) \\ u_2 &= \frac{(e_2^*)^2 (1 + \eta e_2^*\theta)^2}{\eta\theta}. \end{aligned}$$

Note that last-period profits in the main setup are $\eta\theta^3/4$, which is larger than the amount obtained here.

In the first period, note that, at e_1^* , u_1 is decreasing in e_1 . If it were increasing, the agent would further raise his effort level. This implies that (IC) is binding in a profit-maximizing equilibrium. If it were not binding, the principal could ask for a higher effort level without paying more.

Plugging the binding (IC) constraint,

$$b_1 = \frac{(e_1^*)^3}{3} - w_1 + \left(w_1 - \frac{(\tilde{e}_1)^3}{3} \right) \frac{(1 + \eta\tilde{e}_1\theta)}{(1 + \eta e_1^*\theta)},$$

into profits and the (DE) constraint yields the Lagrange function

$$\begin{aligned} L &= e_1\theta - \frac{(e_1^*)^3}{3} - \left(w_1 - \frac{(\tilde{e}_1)^3}{3} \right) \frac{(1 + \eta\tilde{e}_1\theta)}{(1 + \eta e_1^*\theta)} \\ &\quad + \lambda_{w_1} w_1 + \lambda_{DE} \left[-\frac{(e_1^*)^3}{3} + w_1 - \left(w_1 - \frac{(\tilde{e}_1)^3}{3} \right) \frac{(1 + \eta\tilde{e}_1\theta)}{(1 + \eta e_1^*\theta)} + \delta\pi_2 \right], \end{aligned}$$

where $\lambda_{w_1} \geq 0$ represents the Lagrange parameter for the agent's limited liability constraint, and $\lambda_{DE} \geq 0$ the Lagrange parameter for the principal's dynamic enforcement constraint.

First-order conditions are

$$\begin{aligned} \frac{\partial L}{\partial e_1} &= \theta - (e_1^*)^2 + \left(w_1 - \frac{(\tilde{e}_1)^3}{3} \right) \frac{\eta\theta (1 + \eta\tilde{e}_1\theta)}{(1 + \eta e_1^*\theta)^2} \\ &\quad + \lambda_{DE} \left[-(e_1^*)^2 + \left(w_1 - \frac{(\tilde{e}_1)^3}{3} \right) \frac{\eta\theta (1 + \eta\tilde{e}_1\theta)}{(1 + \eta e_1^*\theta)^2} \right] = 0 \\ \frac{\partial L}{\partial w_1} &= - \left(1 - (\tilde{e})^2 \frac{d\tilde{e}}{dw_1} \right) \frac{(1 + \eta\tilde{e}\theta)}{(1 + \eta e_1^*\theta)} - \left(w_1 - \frac{(\tilde{e})^3}{3} \right) \frac{\eta\theta}{(1 + \eta e_1^*\theta)} \frac{d\tilde{e}}{dw_1} \\ &\quad + \lambda_{DE} \left[1 - \left(1 - (\tilde{e})^2 \frac{d\tilde{e}}{dw_1} \right) \frac{(1 + \eta\tilde{e}\theta)}{(1 + \eta e_1^*\theta)} - \left(w_1 - \frac{(\tilde{e})^3}{3} \right) \frac{\eta\theta}{(1 + \eta e_1^*\theta)} \frac{d\tilde{e}}{dw_1} \right] \\ &\quad + \lambda_{w_1} = 0 \end{aligned}$$

Using $-\tilde{e}_1^2 - \frac{4}{3}\tilde{e}_1^3\eta\theta + w_1\eta\theta = 0$, which implies $w_1 = \frac{\tilde{e}_1^2}{\eta\theta} + \frac{4}{3}\tilde{e}_1^3$, those conditions become

$$\begin{aligned}\frac{\partial L}{\partial e_1} : \theta - \left((e_1^*)^2 - \tilde{e}_1^2 \frac{(1 + \eta\tilde{e}_1\theta)^2}{(1 + \eta e_1^*\theta)^2} \right) (1 + \lambda_{DE}) &= 0 \\ \frac{\partial L}{\partial w_1} : - \frac{(1 + \eta\tilde{e}\theta)}{(1 + \eta e_1^*\theta)} (1 + \lambda_{DE}) + \lambda_{w_1} + \lambda_{DE} &= 0\end{aligned}$$

First, assume $\lambda_{DE} = 0$. Then, I have to consider the two cases $w_1 = 0$ and $w_1 > 0$. However, $w_1 > 0$ and consequently $\lambda_{w_1} = 0$ cannot be optimal, since in this case, the second condition would imply $-\frac{(1+\eta\tilde{e}\theta)}{(1+\eta e_1^*\theta)} = 0$.

Therefore, $\lambda_{DE} = 0$ implies $w_1 = 0$, hence $\tilde{e} = 0$ and

$$e_1^* = \sqrt{\theta}.$$

Moreover $e_1^* = \sqrt{\theta} > \frac{\sqrt{1+4\eta^2\theta^3}-1}{4\eta\theta} = e_2^*$ and $w_1 = 0 < \frac{e_2^2 + \frac{4}{3}e_2^3\eta\theta}{\eta\theta} = w_2$.

However, note that in case of two periods and $\delta \leq 1$, $\lambda_{DE} = 0$ is not feasible: For $w_1 = 0$, $b_1 = (\sqrt{\theta})^3/3$ and $e_1^* = \sqrt{\theta}$, the (DE) constraint becomes

$$-\frac{(\sqrt{\theta})^3}{3} + \delta \frac{(1 + 4\eta^2\theta^3) \sqrt{1 + 4\eta^2\theta^3} - 6\eta^2\theta^3 - 1}{24\eta^3\theta^3} \geq 0.$$

There, the second term increases in η and approaches $\delta 2\sqrt{\theta^3}/9$ for $\eta \rightarrow \infty$. Therefore, the constraint does not hold for any θ and η if $\delta \leq 1$.

Now, assume that (DE) binds. Again, I start with $w_1 = 0$. Then, $e_1^* = \sqrt{\theta/(1 + \lambda_{DE})}$, and

$$\begin{aligned}\lim_{w_1 \rightarrow 0} \frac{\partial L}{\partial w_1} &= - \frac{(1 + \lambda_{DE})}{(1 + \eta e_1^*\theta)} + \lambda_{DE} \\ &= - \frac{(1 + \lambda_{DE})}{\left(1 + \sqrt{\frac{\eta^2\theta^3}{(1 + \lambda_{DE})}}\right)} + \lambda_{DE},\end{aligned}$$

which is positive for

$$\eta^2\theta^3 > \frac{1 + \lambda_{DE}}{\lambda_{DE}^2}.$$

Put differently,

$$e_1^* = \sqrt{\frac{\theta}{(1 + \lambda_{DE})}},$$

if $\eta^2\theta^3\lambda_{DE}^2 - \lambda_{DE} - 1 \leq 0$, hence if $\lambda_{DE} \leq \frac{1 + \sqrt{1 + 4\eta^2\theta^3}}{2\eta^2\theta^3}$. In this case,

$$\begin{aligned}e_1^* &\geq \sqrt{\frac{\theta}{\left(1 + \frac{1 + \sqrt{1 + 4\eta^2\theta^3}}{2\eta^2\theta^3}\right)}} \\ &= \sqrt{\frac{2\eta^2\theta^4}{2\eta^2\theta^3 + 1 + \sqrt{1 + 4\eta^2\theta^3}}}\end{aligned}$$

This is larger than $e_2^* = \frac{\sqrt{1+4\eta^2\theta^3}-1}{4\eta\theta}$, if

$$12\eta^4\theta^6 > 0.$$

Therefore, $e_1^* > e_2^*$ and $w_1 < w_2$ in this case.

Now, assume that $\lambda_{DE} > \frac{1+\sqrt{1+4\eta^2\theta^3}}{2\eta^2\theta^3}$, hence $w_1 > 0$. Then, solving the first first-order condition for λ_{DE} and plugging it into the second yields

$$\eta\theta^2 - \frac{[1 + \eta\theta(e_1^* + \tilde{e}_1)] [e_1^*(1 + e_1^*\eta\theta) + \tilde{e}_1(1 + \eta\tilde{e}_1\theta)]}{(1 + \eta e_1^*\theta)} = 0,$$

which, together with the binding (DE) constraint, which – using $-\tilde{e}_1^2 - \frac{4}{3}\tilde{e}_1^3\eta\theta + w_1\eta\theta = 0 \Rightarrow w_1 = \frac{\tilde{e}_1^2}{\eta\theta} + \frac{4}{3}\tilde{e}_1^3$ – becomes

$$-\frac{((e_1^*)^3 - \tilde{e}_1^3)}{3} + \tilde{e}_1^2(e_1^* - \tilde{e}_1) \frac{(1 + \tilde{e}_1\eta\theta)}{(1 + \eta e_1^*\theta)} + \delta\pi_2 = 0,$$

determines e_1^* as well as \tilde{e}_1 (and consequently w_1).

In order to prove $e_1^* > e_2^*$ and $w_1 < w_2$, I first show that e_1^* is increasing and \tilde{e}_1 is decreasing in $\delta\pi_2$:

$$\begin{aligned} \frac{de_1^*}{d(\delta\pi_2)} &= \frac{\begin{vmatrix} 0 & -\frac{\eta\theta\{e_1^*e_1^*\eta\theta + \eta\tilde{e}_1\tilde{e}_1\theta + 2(e_1^* + \tilde{e}_1)(1 + \eta\tilde{e}_1\theta)\} + (1 + 2\eta\tilde{e}_1\theta)}{(1 + \eta e_1^*\theta)} \\ -1 & -\frac{((e_1^*)^2 + e_1^*\tilde{e}_1 + \tilde{e}_1^2) + (e_1^* - \tilde{e}_1)(e_1^* + 2\tilde{e}_1)}{3} + \frac{2\tilde{e}_1e_1^* + 3e_1^*\tilde{e}_1^2\eta\theta - 3\tilde{e}_1^2 - 4\tilde{e}_1^3\eta\theta}{(1 + \eta e_1^*\theta)} \end{vmatrix}}{\begin{vmatrix} \frac{\tilde{e}_1\eta\theta(1 + \eta\tilde{e}_1\theta)^2 - [(1 + \eta\theta\tilde{e}_1)(1 + e_1^*\eta\theta) + \eta\theta e_1^*(1 + 2e_1^*\eta\theta)](1 + \eta e_1^*\theta)}{(1 + \eta e_1^*\theta)^2} & -\frac{\eta\theta\{e_1^*e_1^*\eta\theta + \tilde{e}_1\tilde{e}_1\eta\theta + 2(e_1^* + \tilde{e}_1)(1 + \eta\tilde{e}_1\theta)\} + (1 + 2\eta\tilde{e}_1\theta)}{(1 + \eta e_1^*\theta)} \\ -(e_1^*)^2 + \tilde{e}_1^2 \frac{(1 + \tilde{e}_1\eta\theta)^2}{(1 + \eta e_1^*\theta)^2} & \tilde{e}_1^2 + \frac{2\tilde{e}_1e_1^* + 3e_1^*\tilde{e}_1^2\eta\theta - 3\tilde{e}_1^2 - 4\tilde{e}_1^3\eta\theta}{(1 + \eta e_1^*\theta)} \end{vmatrix}} \\ &= \frac{\begin{vmatrix} 0 & -\frac{\eta\theta\{e_1^*e_1^*\eta\theta + \eta\tilde{e}_1\tilde{e}_1\theta + 2(e_1^* + \tilde{e}_1)(1 + \eta\tilde{e}_1\theta)\} + (1 + 2\eta\tilde{e}_1\theta)}{(1 + \eta e_1^*\theta)} \\ 1 & -\frac{((e_1^*)^2 + e_1^*\tilde{e}_1 + \tilde{e}_1^2) + (e_1^* - \tilde{e}_1)(e_1^* + 2\tilde{e}_1)}{3} + \frac{2\tilde{e}_1e_1^* + 3e_1^*\tilde{e}_1^2\eta\theta - 3\tilde{e}_1^2 - 4\tilde{e}_1^3\eta\theta}{(1 + \eta e_1^*\theta)} \end{vmatrix}}{\begin{vmatrix} \frac{\tilde{e}_1\eta\theta(1 + \eta\tilde{e}_1\theta)^2 - [(1 + \eta\theta\tilde{e}_1)(1 + e_1^*\eta\theta) + \eta\theta e_1^*(1 + 2e_1^*\eta\theta)](1 + \eta e_1^*\theta)}{(1 + \eta e_1^*\theta)^2} & -\frac{\eta\theta\{e_1^*e_1^*\eta\theta + \tilde{e}_1\tilde{e}_1\eta\theta + 2(e_1^* + \tilde{e}_1)(1 + \eta\tilde{e}_1\theta)\} + (1 + 2\eta\tilde{e}_1\theta)}{(1 + \eta e_1^*\theta)} \\ -(e_1^*)^2 + \tilde{e}_1^2 \frac{(1 + \tilde{e}_1\eta\theta)^2}{(1 + \eta e_1^*\theta)^2} & \frac{2\tilde{e}_1(e_1^* - \tilde{e}_1)(1 + 2\tilde{e}_1)}{(1 + \eta e_1^*\theta)} \end{vmatrix}} \end{aligned}$$

There, the numerator equals

$$-\frac{\eta\theta\{e_1^*e_1^*\eta\theta + \eta\tilde{e}_1\tilde{e}_1\theta + 2(e_1^* + \tilde{e}_1)(1 + \eta\tilde{e}_1\theta)\} + (1 + 2\eta\tilde{e}_1\theta)}{(1 + \eta e_1^*\theta)} < 0,$$

and the denominator

$$\begin{aligned} &\frac{\tilde{e}_1\eta\theta(1 + \eta\tilde{e}_1\theta)^2 - \eta\theta\tilde{e}_1(1 + e_1^*\eta\theta)^2 - (1 + e_1^*\eta\theta)^2 - \eta\theta e_1^*(1 + 2e_1^*\eta\theta)(1 + \eta e_1^*\theta)}{(1 + \eta e_1^*\theta)^2} \left[2(e_1^* - \tilde{e}_1)\tilde{e}_1 \frac{1 + 2\tilde{e}_1\eta\theta}{(1 + \eta e_1^*\theta)} \right] \\ &+ \left(-(e_1^*)^2 + \tilde{e}_1^2 \frac{(1 + \tilde{e}_1\eta\theta)^2}{(1 + \eta e_1^*\theta)^2} \right) \frac{\eta\theta\{e_1^*e_1^*\eta\theta + \tilde{e}_1\tilde{e}_1\eta\theta + 2(e_1^* + \tilde{e}_1)(1 + \eta\tilde{e}_1\theta)\} + (1 + 2\eta\tilde{e}_1\theta)}{(1 + \eta e_1^*\theta)}, \end{aligned}$$

which is negative because of $e_1^* > \tilde{e}_1$. Therefore,

$$\frac{de_1^*}{d(\delta\pi_2)} > 0.$$

If $\delta\pi_1 = 0$, $b_1 = 0$, and π_1 is maximized by setting $w_1 = w_2$, implying $e_1^* = e_2^*$. Therefore, $e_1^* > e_2^*$ given $\delta\pi_1 > 0$.

Moreover,

$$\frac{d\tilde{e}_1}{d(\delta\pi_2)} = \frac{\left| \begin{array}{cc} \frac{\tilde{e}_1\eta\theta(1+\eta\tilde{e}_1\theta)^2 - [(1+\eta\theta\tilde{e}_1)(1+e_1^*\eta\theta) + \eta\theta e_1^*(1+2e_1^*\eta\theta)](1+\eta e_1^*\theta)}{(1+\eta e_1^*\theta)^2} & 0 \\ -(e_1^*)^2 + \tilde{e}_1^2 \frac{(1+\tilde{e}_1\eta\theta)^2}{(1+\eta e_1^*\theta)^2} & -1 \end{array} \right|}{\left| \begin{array}{cc} \frac{\tilde{e}_1\eta\theta(1+\eta\tilde{e}_1\theta)^2 - [(1+\eta\theta\tilde{e}_1)(1+e_1^*\eta\theta) + \eta\theta e_1^*(1+2e_1^*\eta\theta)](1+\eta e_1^*\theta)}{(1+\eta e_1^*\theta)^2} & -\frac{\eta\theta\{e_1^*e_1^*\eta\theta + \tilde{e}_1\tilde{e}_1\eta\theta + 2(e_1^* + \tilde{e}_1)(1+\eta\tilde{e}_1\theta)\} + (1+2\eta\tilde{e}_1\theta)}{(1+\eta e_1^*\theta)} \\ -(e_1^*)^2 + \tilde{e}_1^2 \frac{(1+\tilde{e}_1\eta\theta)^2}{(1+\eta e_1^*\theta)^2} & \frac{2\tilde{e}_1(e_1^* - \tilde{e}_1)(1+2\tilde{e}_1)}{(1+\eta e_1^*\theta)} \end{array} \right|}$$

This is negative, since the denominator is negative, and the numerator, which becomes $\frac{[(1+\eta\theta\tilde{e}_1)(1+e_1^*\eta\theta) + \eta\theta e_1^*(1+2e_1^*\eta\theta)](1+\eta e_1^*\theta) - \tilde{e}_1\eta\theta(1+\eta\tilde{e}_1\theta)^2}{(1+\eta e_1^*\theta)^2}$, is positive.

Therefore,

$$\frac{dw_1}{d(\delta\pi_2)} < 0.$$

If $\delta\pi_1 = 0$, $b_1 = 0$, and π_1 is maximized by setting $w_1 = w_2$. Therefore, $w_1 < w_2$ given $\delta\pi_1 > 0$. ■

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