Communication & Cooperation

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Introduction

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- One model--pre-play comm. in one-shot games
  - Comm $\longrightarrow$ coordination and correlation
- Our model--in-play comm. in repeated games
  - Comm $\longrightarrow$ improves monitoring

Antitrust Law

- Tacit vs. explicit collusion
  - "Tacit collusion … describes a process, not in itself unlawful, by which …firms in a concentrated market might in effect share monopoly power, setting their prices at a profit-maximizing supracompetitive level …" (US Supreme Court, 1993)
- Theoretical basis for the distinction?

Cartel meetings

Industry	Frequency of meetings
Choline chloride	
Citric acid	
Copper tubes	
Lysine	?
Plasterboard	
Vitamins A and E	
Zinc phosphate	

Cartel meetings

Industry	Frequency of meetings
Choline chloride	2-3 weeks
Citric acid	monthly
Copper tubes	1-2 months
Lysine	monthly
Plasterboard	quarterly
Vitamins A and E	weekly/quarterly
Zinc phosphate	monthly

Example 1

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

Claim: With **perfect** monitoring, $\forall \delta \geq 1/3$, there is an efficient equilibrium.

Imperfect private monitoring

	C	D
C	2, 2	-1, 3
D	3 -1	0, 0

	h	l
h	½	0
l	0	½

	h	l
h	¼	¼
l	¼	¼

Players only see private signals h or l after every period.

Imperfect private monitoring

	C	*D*
C	2, 2	-1, 3
D	3 -1	0, 0

	h	*l*
h	½	0
l	0	½

	h	*l*
h	¼	¼
l	¼	¼

Players only see private signals *h* or *l* after every period.

Private monitoring precludes cooperation

π

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

	h	l
h	½	0
l	0	½

	h	l
h	¼	¼
l	¼	¼

Claim: $\forall \delta$, DD is the only eqm path.

Permanent deviation to D leaves marginal distribution on other's signals unchanged.

Communication restores cooperation

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

	h	l
h	½	0
l	0	½

	h	l
h	¼	¼
l	¼	¼

CHEAP TALK

Claim: $\forall \delta \geq$ ½, CC is an eqm path w/comm.

Play C and report truthfully; if reports match, play C; otherwise, play D forever.

Example 1

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

	h	l
h	½	0
l	0	½

	h	l
h	¼	¼
l	¼	¼

Example 2

	C	*D*
C	2, 2	-1, 3
D	3, -1	0, 0

	h	*l*
h	¼	¼
l	¼	¼

	h	*l*
h	½	0
l	0	½

Example 2

	C	*D*
C	2, 2	-1, 3
D	3, -1	0, 0

	h	*l*
h	¼	¼
l	¼	¼

	h	*l*
h	½	0
l	0	½

π

Communication $\not\Rightarrow$ cooperation

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

	h	l
h	¼	¼
l	¼	¼

	h	l
h	½	0
l	0	½

Claim: $\forall \delta \geq ½$, only DD is played w/ or w/o comm.

Communication $\not\Rightarrow$ cooperation

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

	h	l
h	¼	¼
l	¼	¼

	h	l
h	½	0
l	0	½

Play D and lie ("h" and "l" with prob. ½). Joint dist. over P1's reports and P2's signals is unchanged.

Example 3

	R	P	S
R	10, 10	0, 11	11, 0
P	11, 0	10, 10	0, 11
S	0, 11	11, 0	10,10

$\frac{1}{3}$	0	0
0	$\frac{1}{3}$	0
0	0	$\frac{1}{3}$

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Communication is not necessary for cooperation

	R	P	S
R	10, 10	0, 11	11, 0
P	11, 0	10, 10	0, 11
S	0, 11	11, 0	10,10

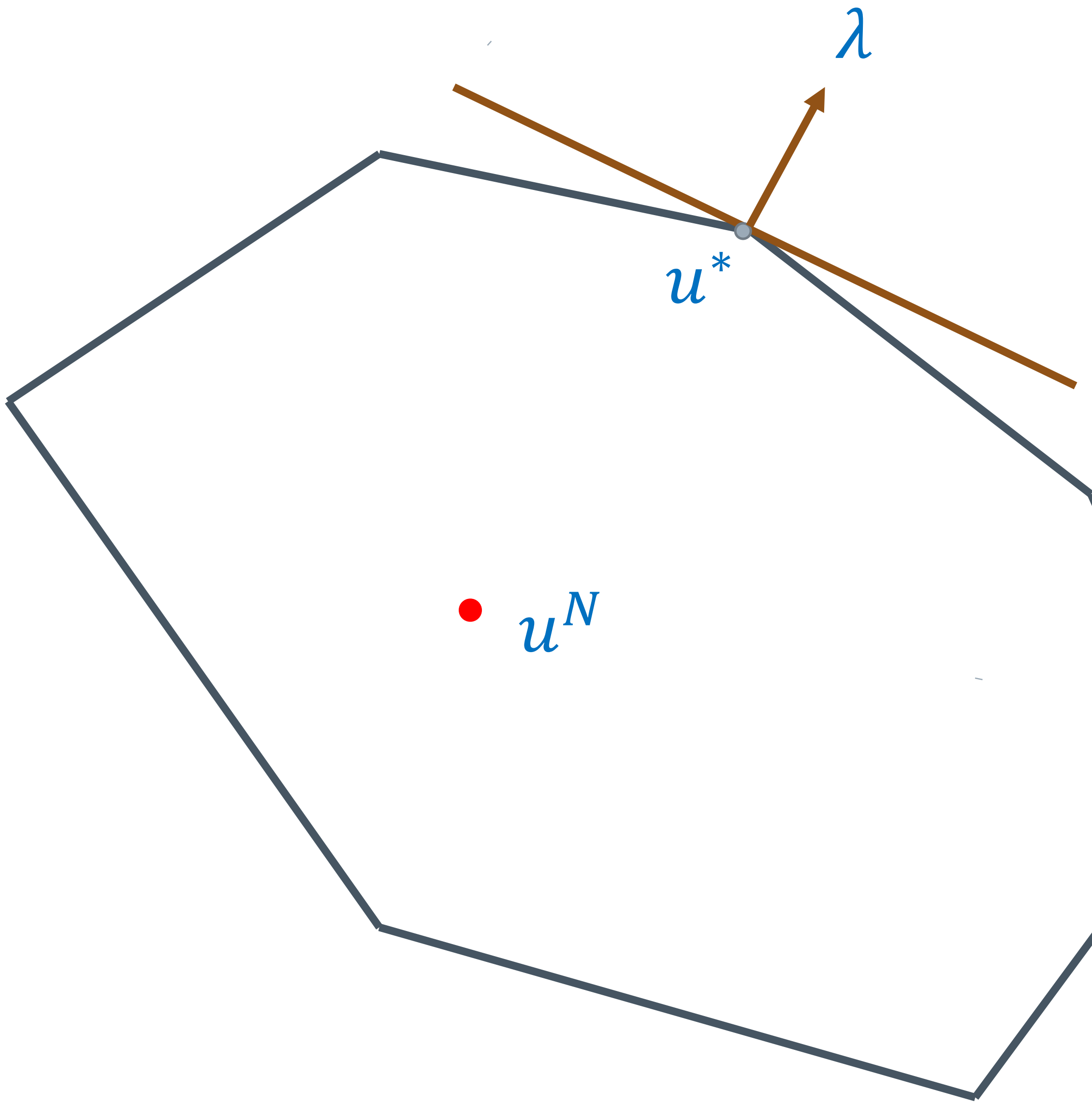
$\frac{1}{3}$	0	0
0	$\frac{1}{3}$	0
0	0	$\frac{1}{3}$

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Claim: $\forall \delta \geq \frac{3}{11}$, there is an efficient eqm w/o comm.

One-shot game

- n players
- Finite actions A_i
- Payoffs $u_i: A \to \Re$
- Nash equilibrium a^N
- Efficient action $a^*: \max_a \lambda \cdot u(a)$
- a^* not a Nash equilibrium and $u(a^*) \gg u(a^N)$



λ
u^*
u^N

Repeated game

- Actions a_i
- Signals $y_i \in Y_i$
- Actions generate signals as $q(\cdot\,|a) \in \Delta(Y)$
- Observe private signal y_i only

- Strategy maps own past actions and own past signals to current actions

Repeated game with communication

- Actions a_i
- Signals $y_i \in Y_i$
- Actions generate signals as $q(\cdot\,|a) \in \Delta(Y)$
- Observe private signal y_i only
- Exchange cheap talk messages
- Strategy maps own past actions, own past signals and all past messages to current actions

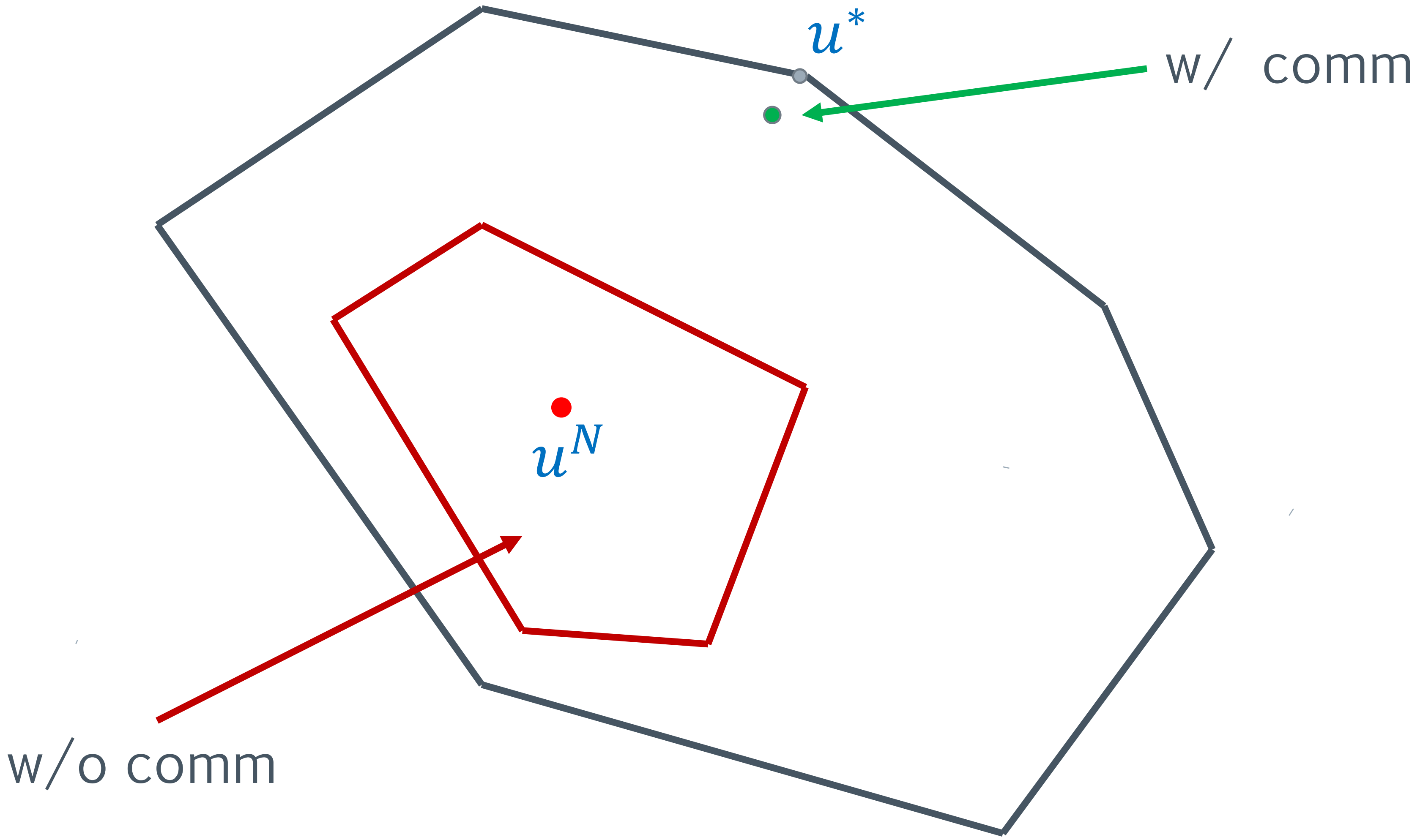
Communication is necessary for cooperation

Main result: Fix any large δ. Under certain conditions on the monitoring, there is a nearly efficient eqm w/ comm whereas all eqa w/o comm are bounded away from eff.

Step 1: bound on payoffs w/o comm

Step 2: construct a nearly efficient eqm w/comm

Not a folk-theorem!


u^*
w/ comm
u^N
w/o comm

Background

Folk Theorem with communication:

Compte (*Ecta*, 1998), Kandori & Matsushima (*Ecta*, 1998)

Folk Theorem without communication:

Sugaya (2013)

Necessity of communication:

Awaya & Krishna (*AER*, 2016)

Step 1: Bound without communication

Bound depends on

- Payoffs: u
- Monitoring quality:

$$\eta = \max_i \max_{a,b_i} \left\| q_{-i}\left(\cdot \mid b_i, a_{-i}\right) - q_{-i}\left(\cdot \mid a\right) \right\|_{TV}$$

- Discount factor: δ

Step 1: Bound without communication

Proposition:

$$NE\left(G_{\delta,q}\right) \subseteq \varepsilon\text{-}CCE(G)$$

where $\varepsilon = 2\frac{\delta^2}{1-\delta}\eta \times \|u\|_\infty$.

Step 1: Bound without communication

Proposition:

One-shot game

$$NE(G_{\delta,q}) \subseteq \varepsilon\text{-}CCE(G)$$

Repeated game

where $\varepsilon = 2\frac{\delta^2}{1-\delta}\eta \times \|u\|_\infty$

Step 1: Bound without communication

Proposition:

$$NE\left(G_{\delta,q}\right) \subseteq \varepsilon\text{-}CCE(G)$$

Coarse correlated equilibria

where $\varepsilon = 2\frac{\delta^2}{1-\delta}\eta \times \|u\|_\infty$.

Monitoring quality

Coarse Correlated Equilibrium

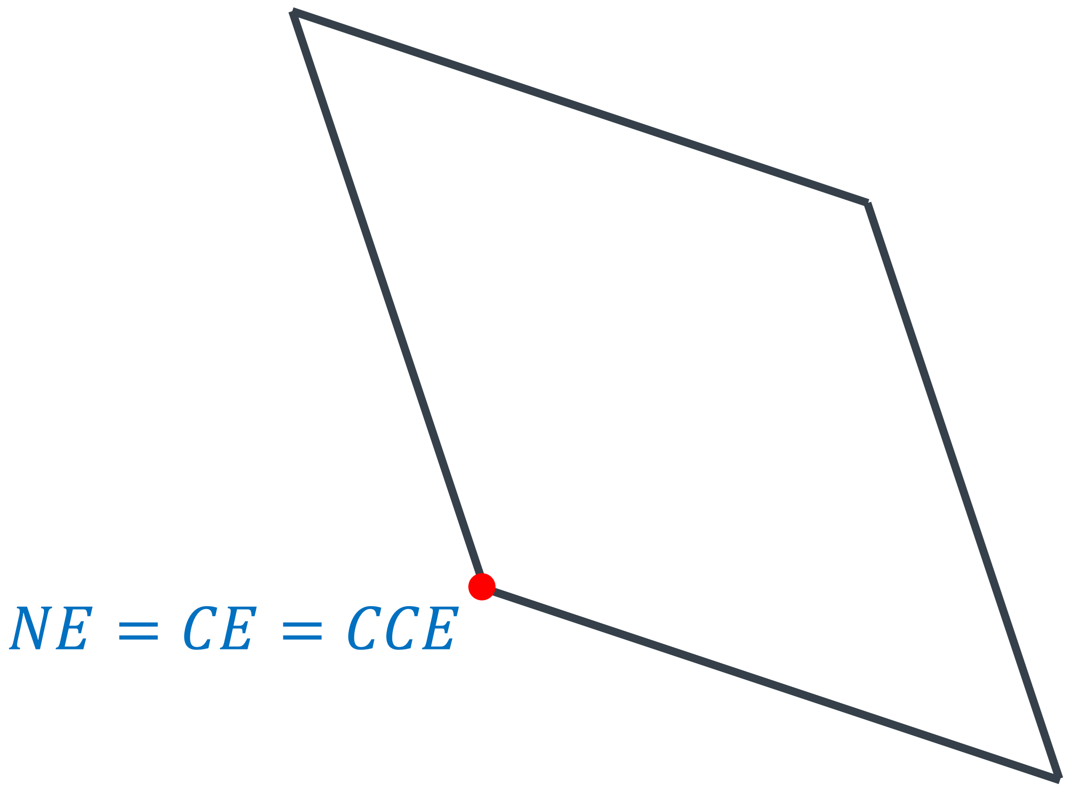
$\beta \in \Delta(\times_{j \in N} A_j)$ is a coarse correlated equilibrium if for all i and $a_i \in A_i$,

$$u_i(\beta) \geq u_i(a_i, \beta_{-i})$$

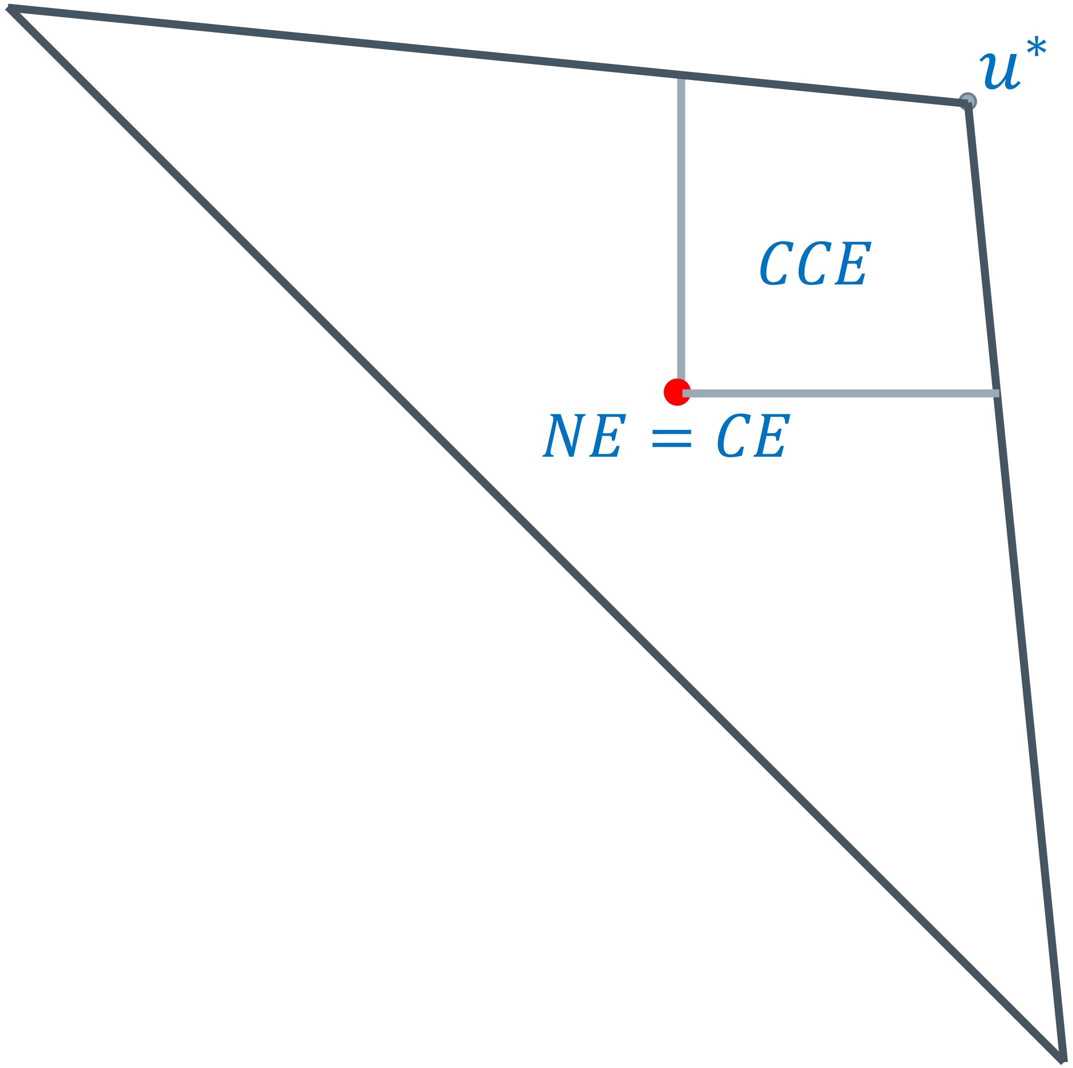
Moulin & Vial (1979); Roughgarden (2016)

$$NE \subset CE \subset CCE$$

Prisoners' Dilemma



R-P-S

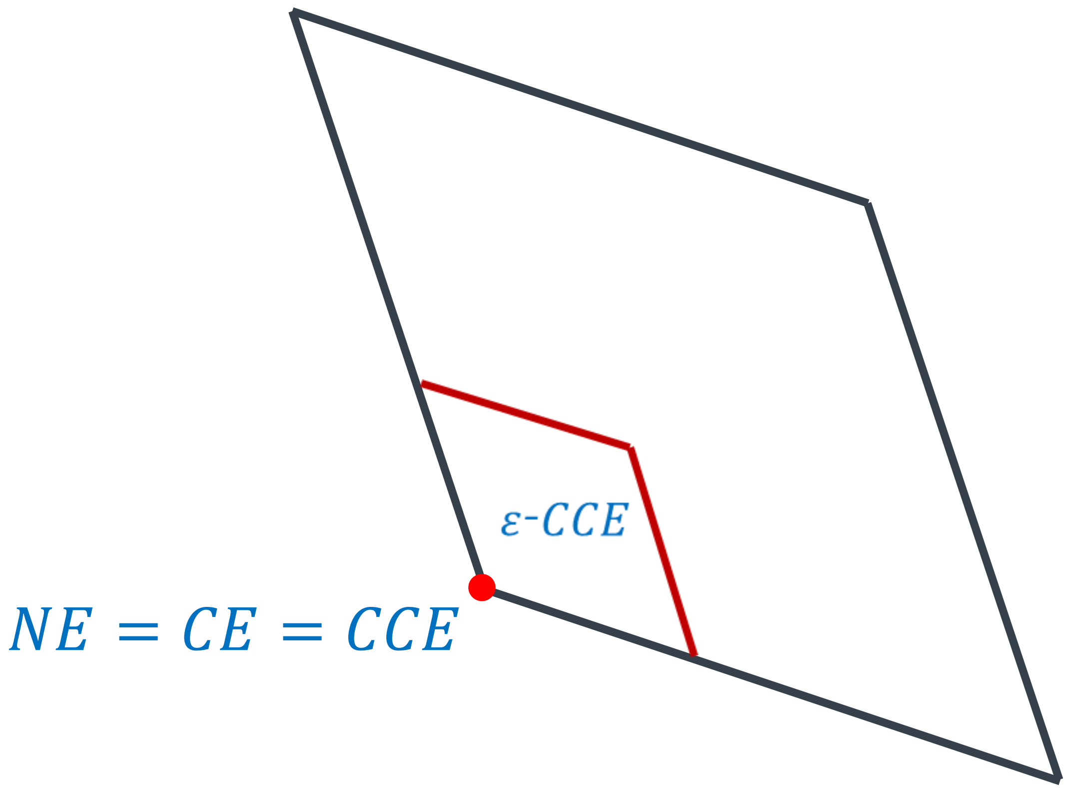


ε-Coarse Correlated Equilibrium

$\beta \in \Delta(\times_{j \in N} A_j)$ is an ε-coarse correlated equilibrium if for all i and $a_i \in A_i$,

$$u_i(\beta) \geq u_i(a_i, \beta_{-i}) - \varepsilon$$

Prisoners' Dilemma



Static incentives

For $v \in F$ define:

$$\Phi(v) = \min_{\beta \in \Delta(A)} \max_i \max_{a_i} u_i(a_i, \beta_{-i}) - u_i(\beta)$$
$$\text{s.t.} \quad u(\beta) = v$$

- Φ is convex
- $\Phi(v) \leq \varepsilon \Leftrightarrow \varepsilon\text{-}CCE(G)$

Dynamic incentives

Lemma 1:

$$\Phi(v) = \min_{\alpha \in \Delta(A)^\infty} \max_i \max_{\bar{\sigma}_i} v_i(\bar{\sigma}_i, \alpha_{-i}) - v_i(\alpha)$$

$$\text{s.t.} \quad v(\alpha) = v$$

Proof:

Dynamic smoothing of incentives (Φ is convex).

Step 1: Bound without communication

- Let σ be a strategy with payoffs $v(\sigma)$ such that

$$v \equiv v(\sigma) \notin \varepsilon\text{–}CCE(G)$$

- Then $\Phi(v) > \varepsilon$
- $\exists i$ and $\overline{\sigma}_i$ such that $v_i(\overline{\sigma}_i, \alpha_{-i}) - v_i(\sigma) > \varepsilon$

Step 1: Bound without communication

$$v_i(\overline{\sigma}_i, \sigma_{-i}) - v_i(\sigma) =$$

$$v_i(\overline{\sigma}_i,\ \sigma_{-i}) - v_i(\overline{\sigma}_i,\ \alpha_{-i}) + v_i(\overline{\sigma}_i,\ \alpha_{-i}) - v_i(\sigma)$$

↑ Loss when punished

↑ Gain when unpunished

Step 1: Bound without communication

Lemma 2:

$$|v_i(\bar{\sigma}_i,\ \sigma_{-i}) - v_i(\bar{\sigma}_i,\ \alpha_{-i})| < 2\frac{\delta^2}{1-\delta}\eta \times \|u\|_\infty$$

η is monitoring quality

$\bar{u}$ is largest payoff

Step 1: Bound without communication

$$v_i(\overline{\sigma}_i, \sigma_{-i}) - v_i(\sigma) =$$

$$v_i(\overline{\sigma}_i, \sigma_{-i}) - v_i(\overline{\sigma}_i, \alpha_{-i}) + v_i(\overline{\sigma}_i, \alpha_{-i}) - v_i(\sigma)$$

$$\vee \qquad\qquad\qquad\qquad \vee$$

$$-2\frac{\delta^2}{1-\delta}\eta \times \|u\|_\infty \qquad\qquad\qquad\qquad \varepsilon$$

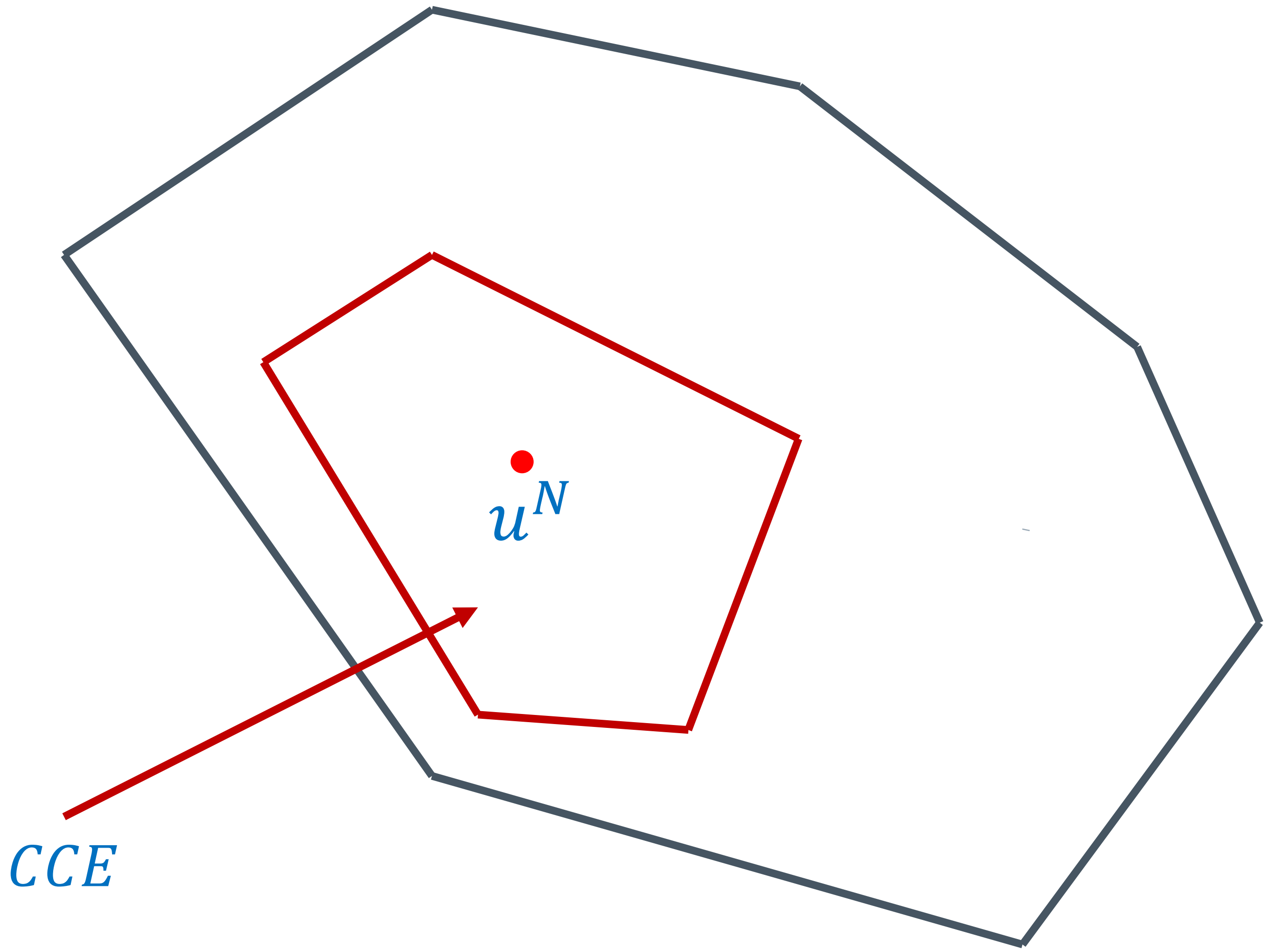
(Lemma 2) (Lemma 1)

Step 1: Bound without communication

Proposition:

$$NE\left(G_{\delta,q}\right) \subseteq \varepsilon\text{-}CCE(G)$$

where $\varepsilon = 2\frac{\delta^2}{1-\delta}\eta \times \|u\|_\infty$.


u^N
CCE

Effective bound without communication

Condition 1: $\nexists$ an efficient coarse correlated eqm.

Ensures bound is effective when $\eta = 0$.

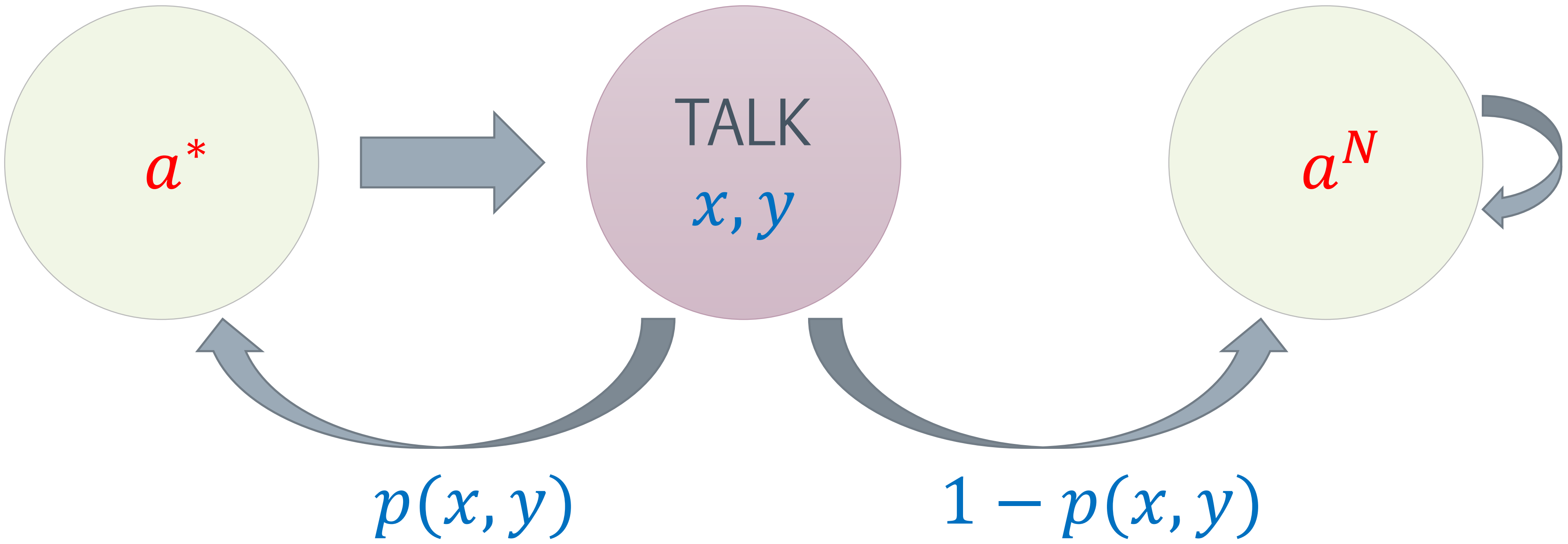
Fails in Example 3 (RPS)

Step 2: Equilibrium with communication

Strategy:

- In state a^* play a_i^*; otherwise, play a_i^N
- If played a_i^*, report signal truthfully; otherwise optimize
- State transitions depend on signal reports x and y

Equilibrium with communication



Truth-telling

Proposition: If types are strictly affiliated, there exists a strictly IC mechanism in which every player has the same **ex post** payoffs.

Truth-telling

Proposition: With strictly affiliated types, there exists a strictly IC mechanism in which every player has the same **ex post** payoffs.

Truth-telling

Suppose q^* is strictly affiliated over $[0,1]^2$. Define

$$P(x,y) = xy - \int_0^x E[Y|X = s]ds - \int_0^y E[X|Y = t]dt$$

Truth-telling

Suppose q^* is strictly affiliated over $[0,1]^2$. Define

$$P(x,y) = xy - \int_0^x E[Y|X=s]ds - \int_0^y E[X|Y=t]dt$$

Claim: q^* is a **strict** correlated eqm of (P,P).

Proof: Report z when signal is x

$$E[P(z,Y)\,|X=x] = zE[Y|X=x] - \int_0^z E[Y|X=s]ds - C(x)$$

First-order condition

$$0 = E[Y|X=x] - E[Y|X=z]$$

Correlation

- Normalize P to $p \in [0,1]$.
- Let $\gamma = \|q(\cdot|a^*) - q^0\|_{TV}$ where q^0 is closest perfectly correlated distribution.
- As $\gamma \to 0$, $p(x,y) \to p^0(x,y) = \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$
- p^0 leads to full efficiency
- Find $\underline{\delta}$ so that eqm with p^0

Main result

Theorem: Fix $\delta > \underline{\delta}$.

There exist $(\bar{\eta}, \bar{\gamma})$ such that for all q with $(\eta, \gamma) \ll (\bar{\eta}, \bar{\gamma})$, there is an eqm w/ comm whose welfare exceeds that from any eqm w/o comm.

η = monitoring quality

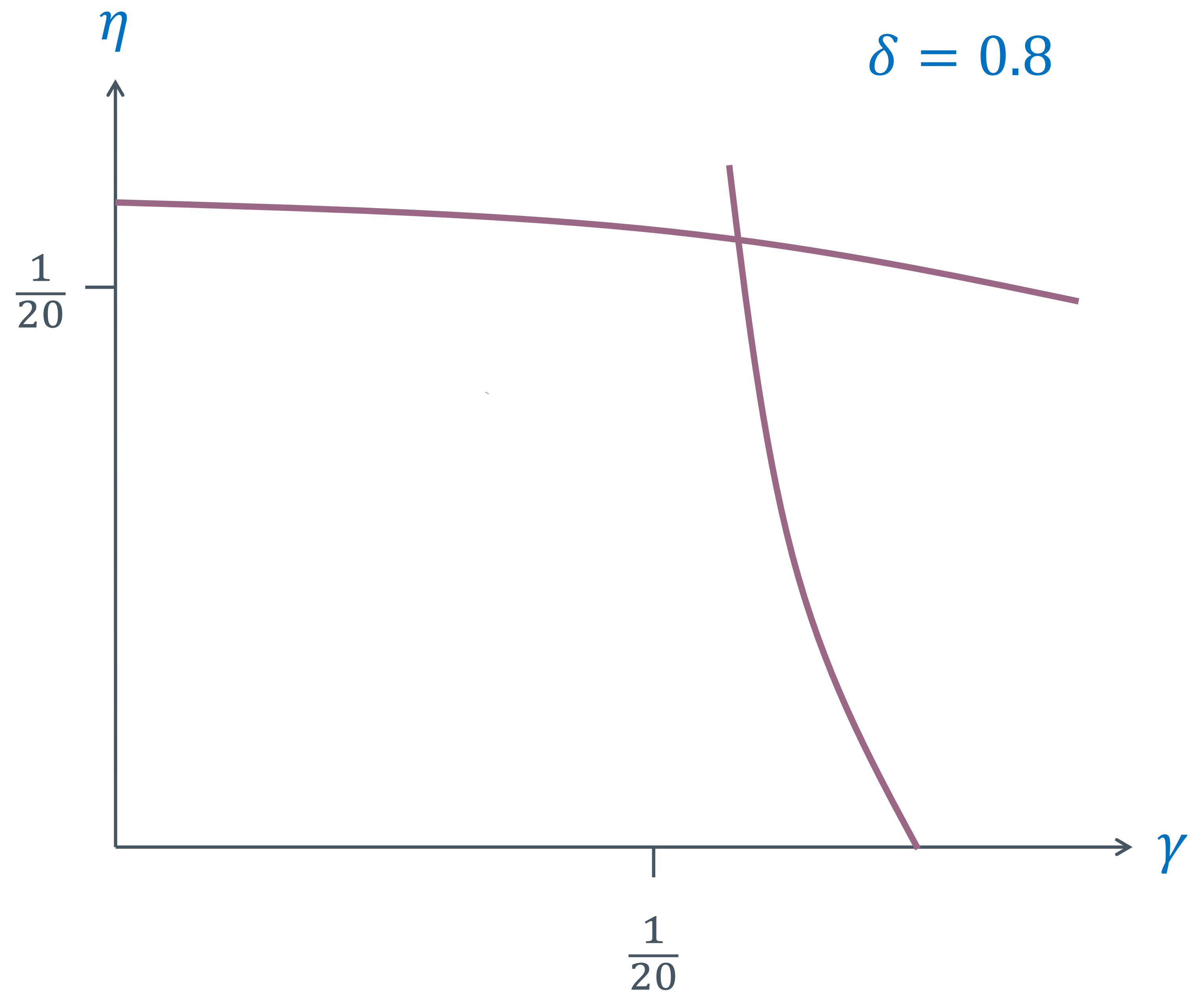
γ = correlation in q^*

Example 4

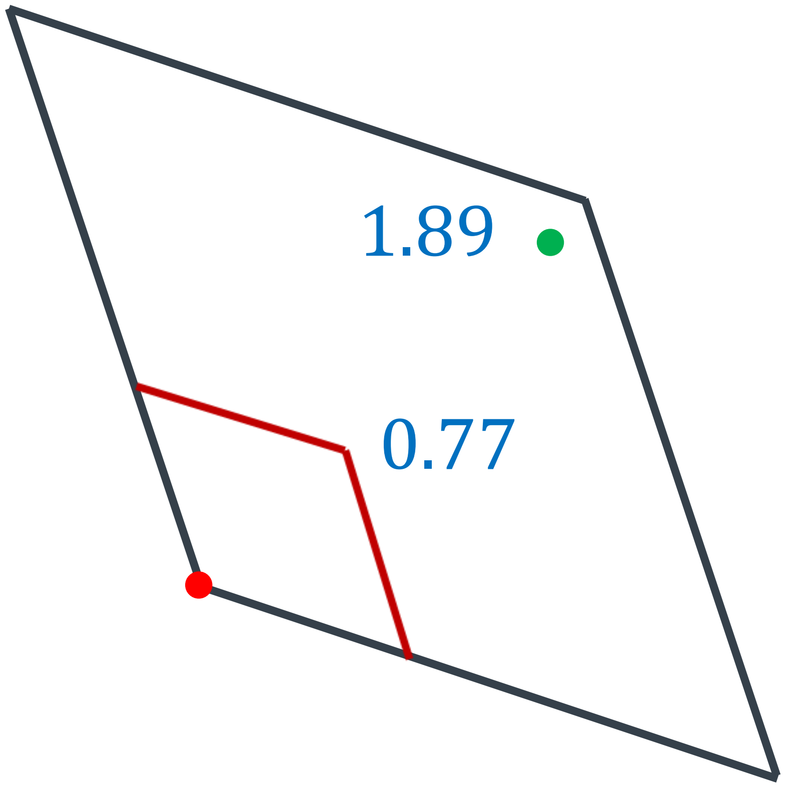
	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

$\frac{1}{3}(1-\gamma)$	$\frac{1}{6}\gamma$	$\frac{1}{6}\gamma$
$\frac{1}{6}\gamma$	$\frac{1}{3}(1-\gamma)$	$\frac{1}{6}\gamma$
$\frac{1}{6}\gamma$	$\frac{1}{6}\gamma$	$\frac{1}{3}(1-\gamma)$

$\left(\frac{1}{3}-\eta\right)^2$	$\frac{1}{9}-\eta^2$	$\frac{1}{9}-\frac{1}{3}\eta$
$\frac{1}{9}-\eta^2$	$\left(\frac{1}{3}+\eta\right)^2$	$\frac{1}{9}+\frac{1}{3}\eta$
$\frac{1}{9}-\frac{1}{3}\eta$	$\frac{1}{9}+\frac{1}{3}\eta$	$\frac{1}{9}$


$\delta = 0.8$
η
$\frac{1}{20}$
γ
$\frac{1}{20}$

Example 4



$\delta = 0.8$

$\eta = 0.02$
$\gamma = 0.02$