The American System of Economic Growth

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Abstract

The early history of industrialization in the United States—famously known as "The American System of Manufactures"—exhibited four key features: the substitution of specialized intermediate inputs for skilled work in assembling final goods, the freedom with which knowledge has long been shared in the United States, a learning technology that leverages existing mechanical know-how in human capital accumulation, and increasing returns to intermediate inputs in processing final goods. Our endogenous growth model embodies these components and utilizes historical time series data on labor force "operatives" to calibrate the model's parameters. Our simulation matches the 1.88% average per capita product growth in the United States from 1860 to date, without using per capita product data in the calibration. The simulation predicts that growth will peak in 2026 and converge to 1.63% — a growth slowdown rooted from the beginning in the economization of skilled labor inherent in the American System. By 2000, simulated per capita product is 2.6 times larger than a counterfactual in which the American System of manufactures never existed.
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1 Introduction

Real output per capita in the United States has kept remarkably close to its trend since 1860. This can obscure that fact that the growth rate demonstrates a clear secular pattern, rising until mid-20th Century and falling after that.[1] This pattern of growth took place as the United States made the long transition to a fully industrialized economy.[2] Motivated by the historical narrative and unique features of what became known as "The American System of Manufactures" our model explains this pattern and proposes a reason for the current slowdown.

The "American System of Manufactures" was the name given to the novel method of production in the United States that used interchangeable parts as a way to economize on the craft skill necessary to "fit" components into final goods. To assist this "assembling" that replaced the more time-consuming and skill-intensive "fitting" also required the invention and diffusion of specialized machines. The usefulness of knowledge and skill moved over time from finishing tasks in the later stages of production to the creation of intermediate, specialized machines in the early stages of production. In our model, this can result in a net economization of knowledge and a reduction in the rate at which it is rational to accumulate human capital. Growth might slow.

Another, less emphasized, feature of American industrial history was the sharing of technical information within and across industries. Compared to Great Britain, the flow of knowledge was far freer and contributed greatly to the development of the system of standardization and reduction in reliance on craft skill.

The model has two final goods that are perfect substitutes in consumption. We refer to the final goods as "conventional" or "advanced" depending on how intermediate inputs are combined – or *processed* – into final goods. Human capital enhances the productivity of labor in the processing of conventional intermediates into final goods – this is the *fitting* – but does *not* raise labor productivity in the processing — or *assembling* — of advanced intermediates. Unskilled, or *innate*, labor is just as productive as skilled labor in processing intermediates using the advanced technology. Katz and Margo (2014) call such workers "operatives" — workers with limited skill that assembled intermediate inputs into final goods.

Individuals in the model accumulate new knowledge as specialized human capital by allocating time to learning as in Lucas (1988), Goodfriend and McDermott (1995, 1998), and Lucas and Moll (2014). The productivity of learning time depends on the individual's human capital, and is enhanced by a learning externality based on the variety of intermediate

[1] See Gordon (2016) p. 16 for the pattern of growth; Jones (2005, 1995) notes its steady nature.

[2] Hounshell (1984), Rosenberg (1972), Mayr and Post, eds (1981).

goods. To use Romer's (1990) terminology, our intermediate goods are both excludable and rivalrous in final-good production, but they are neither in the learning technology, reflecting the sharing of technical know-how that is a hallmark of the American System.

Early in the history of the American System, human capital was naturally predominantly occupied in industries using *conventional* techniques. It was this stock of knowledge – which was accustomed to the difficulties of fitting custom intermediates into final goods – that was used to create the first, new, simple specialized machine tools designed to substitute for craft skill in the fitting stage of final-good production. Our view is that this reservoir of knowledge in conventional technologies *spilled over* to reduce the cost and increase the productivity of the new machine tools, which we identify with advanced intermediate goods. This important feature of the model provides an explanation for why the advanced technology could begin at small scale. Conventional intermediate inputs were limited in terms of potential productivity. These limitations are spelled out in Section 2, where we review the evolution of industry in the United States in the 19th Century.

As population grows and individuals accumulate specialized human capital, more firms find it profitable to enter intermediate-good production whose fixed costs can be spread over a larger volume of output. In this way, the model exhibits increasing returns along the lines of Romer (1987, 1990). This scale effect lifts output and real wages. As human capital increases, intermediate firms take advantage of knowledge spillovers to introduce machine tools that save on skilled effort, raising the wage in advanced processing more than in conventional processing. Effort moves from conventional to advanced processing. This result — that advanced processing gradually replaces conventional processing — diminishes the incentive to accumulate knowledge because the advanced technology economizes on human capital in the processing stage.

We simulate a dynamic version of the model to analyze both transitional and steady-state growth. The principle message is that our American System model succeeds well in reconciling the growth facts mentioned at the outset. The model simulation matches the average 1.88% per capita product growth rate since 1860 in the United States — without using per capita product data in the calibration.

The simulation achieves nearly constant long-run growth to date because of two countervailing features of the American System acting through human capital accumulation. The incentive to accumulate human capital is *reinforced* by the rising range of specialized intermediate goods via the *specialization spillover* in the learning technology. However, the incentive to accumulate human capital is *reduced* as the American System shifts work from conventional to advanced processing which lowers the share of skilled work in total work and where advanced processing exhibits smaller returns to human capital accumulation than conventional processing.

Going forward, the simulation predicts that long-run per capita product growth will peak in 2026 and eventually fall to 1.63%, as the share of unskilled work rises to 50%. In so doing, our simulated American System model offers a new perspective on the growth slowdown discussed in Fernald (2015), Jones (2002), Goldin and Katz (2008), Cowen (2011), Fernald and Jones (2014), Gordon (2016), Acemoglu and Restrepo (2018) and Brynjolfsson et al. (2018).

In Section 2, we present a historical account of the revolutionary nature of the American System of Manufactures, drawing on the insights of historians of technology. We also marshal evidence to support our assumptions about the importance of knowledge sharing in the American economy, especially compared to that of Great Britain at the time.

In Section 3 we introduce the conventional and advanced goods-producing technologies. The learning technology is presented in Section 4. Sections 5 and 6 derive equilibria in the conventional and advanced processing sectors, separately. The infinitely-lived, representative household's optimization problem is framed in Section 7. Section 8 links the conventional and advanced sectors by imposing wage equality, and examines the overall momentary equilibrium allocation of work effort. In Section 9, we analyze the conditions for a maximum to the household's intertemporal problem and use phase plane analysis as in Romer (1986) to characterize the transitional dynamics of the endogenous growth model.

In Section 10 we calibrate the model's six parameters using historical data. Three parameters are taken from the literature. The other three are based also on a time series of the share of operatives in the total work force reported in the *Historical Statistics of the United States* (Carter et al., eds 2006). In Section 11, we simulate our model of the American System from 1860 forward through the 20th century and beyond. In Section 12, we point out that the latest incarnation of the American System of advanced processing technology facilitates the self-processing of goods and services by individuals alone. The substitution of such non-market processing for conventional processing imparts a growing downward bias to standard measures of aggregate GDP and hours worked. Section 13 concludes the paper.

2 The American System and Knowledge Sharing

The "American System of Manufactures" was so named by the British after the technology was exhibited at the Crystal Palace Exhibition in London in 1851. The American display of "interchangeable parts," in particular in the manufacture of firearms, was recognized to greatly simplify not only their assembly but also their maintenance and repair in battle.[3] Soon after, the British organized parliamentary representatives to visit the United States to observe the new manufacturing techniques. Their reports entitled *The American System of Manufactures: the Reports of the Committee on Machinery of the United States 1855, and the Special Reports of George Wallis and Joseph Whitworth 1854* "clearly and emphatically called attention to certain unique technical developments in the United States and urged their adoption in Great Britain…"[4] In what follows, we motivate the foundations of our model by reviewing historical accounts of the American System.

As Rosenberg (1972) put it: "A major feature of the new American technology may… be simply stated: it eliminated or at least substantially reduced the very costly fitting activities which were an inseparable aspect of the older handicraft system. The new machines were labor-saving in that they simply eliminated the need for the highly labor-intensive stage of fitting. A crucial difference between the old and new techniques is the difference between fitting and assembling. The parliamentary committee [mentioned above] was so amazed at this feature that it enclosed the word "assemble" in quotation marks whenever the word was used throughout the report."[5]

Rosenberg continued: "Interchangeable components, the elimination of dependence upon handicraft skills, and the abolition of extensive fitting operations were, in turn, all aspects of a system whose central characteristic was the design and utilization of highly specialized machinery. . . .Interchangeability, it should be understood, is critical not only because it permits assembly without fitting, but because it makes possible a much higher degree of specialization than would otherwise be possible…" [6]

In his Introduction to the *Report*, Rosenberg pointed out: "What must be recorded, however, is that after the system had been planted in Great Britain it failed to flourish as it had in the United States. Its adoption was halting and fitful. Handicraft methods showed a remarkable capacity to survive, methods to assure precision workmanship—such as the use of limit gauges—were adopted very slowly, and in engineering workshops the shift from general purpose machinery to more specialized machines was not nearly as rapid or as extensive as in the United States."[7]

[3]The precision manufacturing process for guns using interchangeable parts was first put into operation by Honoré Blanc's French system in 1785. The revolution in France prevented its adoption. However, Thomas Jefferson, then United States Minister to France, observed the idea and encouraged American gunsmiths to adopt the new system. See Winchester (2018), pp. 86-92, and Chapters 3 and 4.

[4]Rosenberg (1969), ed., p. 1, and Hounshell (1984), Chapter 1, pp. 15-65 and Appendix 1: "The Evolution of the Expression the American System of Manufactures," pp. 331-336.

[5]Rosenberg (1972), p. 94.

[6]Rosenberg (1972), pp. 95-6.

[7]Rosenberg (1969), p. 72, and footnotes therein, and Burke (1978), pp. 146-51.

Rosenberg explained the resistance to the adoption of the American System of manufactures abroad: "...The Birmingham metal trades were capable of producing any of a wide range of articles which could be produced by highly skilled and ingenious craftsmen working only with tools and the simplest machinery such as the stamping machine which was basic to both the buckle and button trades...New products could be introduced with relative ease so long as they involved only relatively small departures from known craft skills and exploited the considerable virtuosity of these skills... In an important sense such craft skills are dead ends, for several reasons. They generated attitudes and traditions characterized by a preoccupation with qualitative aspects of the final product which were at best irrelevant and at worst hostile to the solution of problems associated with raising resource productivity. The accumulated skills and traditions constituted an obstacle to the learning process which is a prerequisite to the acquisition of new and radically different techniques..." [8]

"The Birmingham metal trades", Rosenberg continued, "were superbly successful in exploiting human skills. They lacked the capacity, however, for incorporating these skills in machinery. Moreover, the highly self-contained nature of the organization of these trades tended to cut them off from contact with other industries—particularly the producers of capital goods—from whom they might have learned and borrowed. These trades, by the middle of the nineteenth century, had gone about as far as possible given their reliance upon the speed, strength, precision and dexterity of the human hand. Further technical progress involved a recourse to completely different techniques for ways of achieving form and precision in the shaping of materials. But Birmingham's industrial history had left it ill-prepared in the novel engineering techniques and broader range of mechanical skills upon which further technical progress depended."[9]

Scale economies were important to the early applications of interchangeable parts in firearms, lock-making, and clocks. However, it was far from clear that the precision, standardization, and specialization of machinery at the heart of the American System could be so improved over time to be used ever more widely in the production of goods and services.[10] Hence, historians have identified other prominent factors unique to the United States that help account for the emergence of the American System of manufactures and its ultimate spreading throughout the economy.

Ferguson (1962) summarizes the foundations of the American System of manufactures as follows: "First, the information that came from Europe was essential, and it was used freely and without prejudice...Second, the stream of travelers going to Europe to obtain

[8]Rosenberg (1969), pp. 78-9.
[9]Rosenberg (1969), p. 79.
[10]Rosenberg (1972), Hounshell (1984), and Winchester (2018).

mechanical and engineering information was important... Third... geography, unlimited natural resources, economy and political climate of the new country all had a powerful influence upon mechanical developments... Fourth, the intelligence, ability and self-reliance of mechanics... was certainly an important factor in the development of the tradition of "know-how..." A final ingredient, found in the United States but not in Europe, was the freedom with which knowledge was shared and exchanged.[[11]] Closed shops and mills seem to have been few and far between in the United States..."[12]

Evidence from patent records indicates that practical know-how was widely distributed in the early United States. Sokoloff and Khan (1990) report that "... Overall, the modest proportions of specialized patentees imply that much of the human capital tapped by inventors could be applied to problems in a wide variety of economic activities. Most of the skills and knowledge needed for patentable invention appear to have been either of general applicability or easily acquired..."[13] They also point out that "One of the implications of these results is that the skills and knowledge necessary for patentable invention at the beginning of industrialization were widely dispersed among the population..."[14]

Khan and Sokoloff (2001) explain that "... the U.S. patent system was a significant factor behind the rapid technological progress and great prosperity that the nation enjoyed... The broad spectrum of the U.S. population involved in inventive activity received much comment, as did the wide range of industries to which American inventors had made contributions. Many suggested that America's distinctive patent laws were especially favorable to invention, and it was no coincidence that Britain, after nearly a quarter century of study by a series of parliamentary committees, approved a major overhaul of its patent system in 1852 to make it more like that of its competitor across the Atlantic (Dutton, 1984)"[15]. Not only did "... U.S. institutions perform well in stimulating inventive activity. Not only did they enhance the material incentive to inventors of even humble devices with grants of monopoly privileges for limited duration, but they also encouraged a market for technology and the diffusion of technological knowledge."[16] [17]

Sawyer (1954) summarizes social behavior that helped to facilitate the adoption of the American System of manufacturing technology: "Experts commenting on our machinery and

[11] See Mokyr (2010), pp. 38 – 44.

[12] Ferguson (1962), p. 15.

[13] Sokoloff and Khan (1990), p. 372.

[14] Sokoloff and Khan (1990), p. 377.

[15] Khan and Sokoloff (2001) p. 234. Also see Khan (2005), Chapter 2.

[16] Khan and Sokoloff (2001), pp. 134-5.

[17] "Before the seventeenth century, the typical objective of the grant of an industrial "patent" was not to stimulate "invention" in modern sense of the term; rather it was to elicit the migration of foreign artisans into the grantor's dominion, and establish therein a craft that was already known elsewhere." David (2008), p. 13.

industrial or industrial commissions studying our techniques of standardized manufacturing sustain the prevailing themes of the general traveler... these reports turn to differences in the nature and diffusion of education in America; the absence of rigidities and restraints of class and craft; the freedom from hereditary definitions of the tasks or hardened ways of going about them; the high focus on personal advancement and drives to high material welfare; and the mobility, flexibility, adaptability of Americans and their boundless belief in progress. These and closely related patterns are linked directly to economic behavior and economic results—to initiative, originality, system effort, and boldness; the 'eager resort to machinery' and productive use of small capital, at a time when small capital was decisive; the ceaseless search and ready adoption of the new and more efficient; the intense responsiveness to shifting opportunities and expanding horizons; the go-aheadism that visitors from all categories often placed as the root of the 'immense drive' of American manufacturing." [18]

Amazingly, American cultural technological advantages were still apparent in the 1950s. Referencing Sawyer's paper above, Ferguson (1979) notes that "... The American workman's traits of initiative, drive, and originality, eager resort to machinery, a 'go-ahead' spirit, and a striving for efficiency so often noted in the 19th century were observed also in the 20th century by post-World War II 'productivity teams' from Europe that visited the United States to learn the secret of America's astounding war record of industrial production." [19]

Boorstin (1965) provides an overview of the emergence of the American System: "The purpose of the Interchangeable System, Eli Whitney himself explained, was 'to substitute correct and effective operations of machinery for that skill of the artist which is acquired only by long practice and experience; a species of skill which is not possessed in this country to any considerable extent. The unheralded Know-how Revolution produced a new way, not only of making things, but of making the machines that make things. It was a simple but far-reaching change, not feasible in a Europe rich in traditions, institutions, and vested skills...' [T]he scarcity of craft skills set the stage for a new nearly craftless way of making things. And this prepared a new concept of material plentitude and of the use and expendability of things which would be called the American Standard of Living."[20]

"The new 'Uniformity System' broke down the manufacture of a gun or of any other complicated machine to the separate manufacture of each of its component pieces. Each piece could then be made independently and in large quantities, by workers who lacked the skill to make a whole machine...What Whitney had to offer, he well knew, was not skill but know-how: a general organizing competence to make anything... Specialized machinery was

[18] Sawyer (1954), pp. 376-7.
[19] Ferguson (1979), p. 11.
[20] Boorstin (1965), p. 3 and Chapter 5.

required for an unspecialized people." [21]

"A premium on general intelligence. In the old world, to say a worker was unskilled was to say he was unspecialized, which meant his work had little value. In America, the new system of manufacturing destroyed the antithesis between skilled and unskilled. Lack of artisan skill no longer prevented a man from making complex products. Old crafts became obsolete. In America, too, a 'liberal'—that is an unspecialized— education... was useful to all... While neighboring nations in Europe had jealously guarded their techniques from one another, here individual laborers from England, France, and Germany learned from one another and freely mingled their techniques."[22]

Katz and Margo (2014) describe the advent of "operatives" in much the same terms as the historians above: "Although special purpose, sequentially implemented machinery displaced artisans from certain tasks in production, the machines could not run on their own—they required 'operatives.' Operatives were less skilled than the artisans they displaced in the sense that an artisan could fashion a product from start to finish, while the operative could perform a smaller set of tasks aided by machinery...But operatives were not without skills—rather, it is more accurate to say that the skills they acquired were those necessary to operate productively the machinery to which they were assigned (Bessen, 2012). Further, skilled workers (engineers and mechanics) were still needed to install and maintain the equipment, as well as design it (and assist in its manufacture) in the first place (Goldin and Katz, 1998)." [23]

Atack et al. (2019) are in the process of investigating the exhaustive *Hand and Machine Labor Study* commissioned by the US Department of Labor in 1894 and completed in 1899.[24] This study made detailed comparisons of tasks and productivity that followed the conversion from hand fitting methods to machine-assisted assembling methods. Like Rosenberg (1972) referenced above, they find that the new methods allowed much greater specialization of workers.

In what follows, we construct a model that captures the revolutionary nature of the switch in industrial technology described in this section.

3 Technology and Effort Allocations

Final good output in the model consists of two non-storable, perfectly substitutable consumption goods, produced by distinct technologies. The production of the goods differ in

[21] Boorstin (1965), pp. 31-33.
[22] Boorstin (1965), pp. 33-34.
[23] Katz and Margo (2014), p. 17.
[24] See US Department of Labor, 1899.

that one — the advanced processing technology – economizes on specialized human capital in the processing stage of production.

The conventional technology for processing final goods is:

$$Y_c = \left(e_{sc}\bar{h}N\right)^{1-\alpha} \int_0^{M_c} (x_c(i))^\alpha \, di \tag{3.1}$$

where $0 < \alpha < 1$. Conventionally processed final-good output is Y_c. The quantity of each intermediate input i designed for conventional processing is $x_c(i)$. These inputs are processed into the final good with *effective labor*, which is the product of the number of workers N, average human capital $\bar{h}$,[25] and e_{sc}, skilled effort in conventional processing. The limit in the integral M_c refers to the range of different intermediate goods that are used with this technology. The final-good firms are perfectly competitive and take prices and wages as given.

The advanced technology is similar, except in one key respect: assembling inputs into final goods does *not* require human capital:

$$Y_a = (e_{ua}N)^{1-\alpha} \int_0^{M_a} (x_a(i))^\alpha \, di \tag{3.2}$$

Unskilled work effort is e_{ua}. All other variables are defined analogously to those in (3.1). Final-good output processed with the advanced technology is Y_a. The quantity of each intermediate input is $x_a(i)$ and M_a is the range of intermediate inputs. Like conventional firms, advanced firms are price-takers.

Intermediate-good production is monopolistically competitive. Each intermediate is produced by a different monopolistic competitor using only effective labor. The cost function for producing intermediate inputs designed for conventional processing is:

$$V_c(x_c) = v_0 + v_1 x_c \tag{3.3}$$

where $V_c(x_c)$ is the cost in units of effective labor and v_0 and v_1 are constants. Let e_{ic} be skilled work effort devoted to producing intermediate inputs designed for conventional processing. Then:

$$M_c V_c(x_c) = e_{ic}\bar{h}N \tag{3.4}$$

[25] Although this is a representative agent model, we make a distinction between the economy's average human capital $\bar{h}$ – which the individual does not regard as something he can influence over time – and the individual's own human capital h, which he believes he can influence by his own learning effort. This distinction is maintained through the optimization problem.

The demand for effective labor must equal the supply. Call e_c the total specialized work effort involved in the production of conventionally-processed output. It follows that:

$$e_c = e_{sc} + e_{ic} \tag{3.5}$$

Now consider the advanced technology. The cost function for producing intermediate inputs is:

$$V_a(x_a) = \frac{v_0 + v_1 x_a}{S} < v_0 + v_1 x_a \tag{3.6}$$

where:

$$S \equiv \left[f\left(\frac{\bar{e}_c \bar{h}}{\bar{e}_{ua}} \right) \right]^{1-\alpha} > 1 \tag{3.7}$$

and:

$$f\left(\frac{\bar{e}_c \bar{h}}{\bar{e}_{ua}} \right) \equiv 2\left(1-\alpha\right) \left(\frac{\bar{e}_c \bar{h}}{\bar{e}_{ua}} \right) - \tau > \tag{3.8}$$

with $\tau > 1$. The function S in (3.7) exceeds unity for all relevant values of effort and human capital.

The expression S in the cost function expresses a key feature of the model — that a human capital *knowledge spillover* from conventional processing reduces the cost of producing advanced inputs. This spillover is more effective the larger is the ratio $\frac{\bar{e}_c \bar{h}}{\bar{e}_{ua}}$. We locate the source of the external effect in work performed using the *conventional* technology $e_c h$ because the skilled tasks there occur in *both* the input stage and the processing stage. This experience helps workers learn how to create *advanced* inputs -- specialized machine tools -- because they are familiar with the difficulty of skilled fitting and the nature of the assembling that must be done. As noted in Section 2, workers using the conventional technology perform *all* the tasks, from start to finish, which gives them the necessary insight to innovate at low cost to replace skill with machines. We also documented in Section 2 that the *sharing* of technological know-how was, and is, a key feature of American industrial culture. Finally, Ferguson (1962) noted that the knowledge from Great Britain — conventional in character — was essential for the new system of interchangeable parts. The unpriced spillover raises the productivity of human capital in producing intermediate goods for advanced processing.

The spillover is asymmetric: it affects the cost of new, advanced inputs, but not the cost of conventional inputs. This relative advantage arises precisely because we assume, consistent with the historical narratives in Section 2, that the conventional technology is subject to technological limits that are overcome by the advanced technology. The form of the expression S in (3.7) and (3.8) is chosen because it makes the analysis of momentary

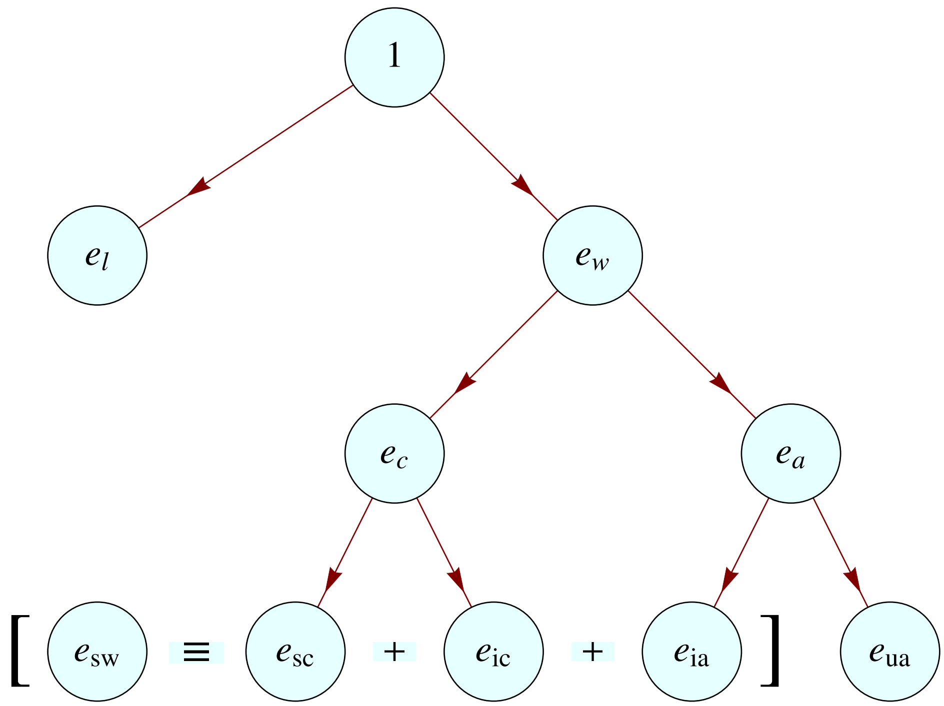

Figure 1: Effort Allocation as a Percentage of the Fixed Total

and dynamic equilibrium tractable.

Effective labor engaged in producing advanced intermediates equals the supply:

$$M_a V_a (x_a) = e_{ia} \bar{h} N \tag{3.9}$$

where e_{ia} is effort producing advanced inputs. Total effort in the advanced sector is e_a:

$$e_a = e_{ua} + e_{ia} \tag{3.10}$$

Figure 1 illustrates how the representative individual allocates one unit of time among all activities. In the top tier, the first constraint facing the agent is:

$$1 = e_w + e_l \tag{3.11}$$

where e_w is total work effort and e_l is effort devoted to learning in order to accumulate human capital. In the second tier, work is split between production of conventional and advanced final output:

$$e_w = e_c + e_a \tag{3.12}$$

The third tier shows how work effort is split between intermediate input production and final-good processing within the two sectors: $e_{sc} + e_{ic}$ and $e_{ua} + e_{ia}$. We refer to the sum of the effort engaged in tasks that benefit from human capital – and shown in the third tier in Figure 1 — as *skilled work effort*: $e_{sw} \equiv e_{sc} + e_{ic} + e_{ia}$.

The two final goods are perfect substitutes in consumption and output is non-storable, so aggregate consumption is $C = Y_c + Y_a$. Population and the number of workers are the same, so per capita consumption and output are given by: $c = \frac{C}{N} = \frac{Y_c}{N} + \frac{Y_a}{N} = y_c + y_a$.

The share of intermediate inputs in final goods processing α is the same in the conventional and advanced processing. Firms producing inputs for conventional processing M_c goods are different from those producing inputs for advanced processing M_a goods – and, once produced, the intermediate quantities x_c and x_a are not interchangeable.

4 Learning Technology

Individuals devote time to learning e_l to accumulate human capital. We assume the following accumulation technology:

$$\dot{h} = L^{\gamma} h^{1-\gamma} e_l - \eta h \tag{4.1}$$

where $0 < \gamma < 1$ and η is the population growth rate. Population growth η enters (4.1) because we assume that it is necessary to spend learning time to educate the newly born. The variable L is defined as:

$$L \equiv \frac{M_c + kM_a}{N \, \bar{e}_w} \tag{4.2}$$

where $k > 0$. The productivity of learning time is enhanced by own human capital h and a positive *specialization spillover* represented by L^{γ} that captures the degree of economic specialization as in Goodfriend and McDermott (1995, 1998). Our intermediate goods, to use the terminology of Romer (1990), are excludable and rivalrous in final-goods production, but they are neither in the learning technology. We assume decreasing returns to learning time with respect to own human capital because limited human capabilities make it increasingly difficult for an individual to increase own specialized knowledge. Economic development, however, raises the degree of specialization and offsets the decreasing returns to accumulation making constant growth possible.

The parameter γ controls the productivity of the specialization spillover L^{γ} relative to own human capital. The parameter k weights the degree that learning productivity is enhanced by intermediate inputs designed for advanced processing M_a relative to intermediate inputs designed for conventional processing M_c. Finally, the weighted range of specialized inputs is deflated by aggregate hours worked $N\bar{e}_w$ to reflect the negative effect of congestion on the learning externality.[26]

[26] For a recent example of negative congestion externalities, which operate on utility from local amenities,

5 Conventional Processing

Conventional final-good firms purchase inputs and work effort on competitive markets. Producers of the intermediate inputs are monopolistically competitive. In this section, we take as given total work effort e_c. Then, we show how to determine wages, prices, the range of specialized intermediates, and the division of e_c between e_{sc} and e_{ic}.

Conventional final-good processing firms hire workers as long as the payment for a unit of effective labor w_c — which we call the *base wage* — is equal to the marginal product of effective labor. Exploiting the symmetry of intermediate firms, this yields:

$$w_c = \frac{\partial Y_c}{\partial \left(e_{sc}\bar{h}N\right)} = (1-\alpha)\left(\frac{e_{sc}\bar{h}N}{x_c}\right)^{-\alpha} M_c \tag{5.1}$$

The actual wage of a worker w_s is the product the base wage w_c and that worker's human capital, h:

$$w_s = w_c h \tag{5.2}$$

We call this the *skilled wage* of a worker with human capital h.

Conventional final-good processing firms choose the quantity of each intermediate input x_c that equates its marginal product from (3.1) to its price p_c:

$$p_c = \alpha\left(\frac{e_{sc}\bar{h}N}{x_c}\right)^{1-\alpha} \tag{5.3}$$

Profits of each intermediate firm are:

$$\pi_c = p_c x_c - w_c V_c(x_c) \tag{5.4}$$

where the cost of production $V_c(x_c)$ in (3.3) is in terms of effective labor. Monopolistically competitive producers take the base wage w_c as given and substitute (3.3) and (5.3) into (5.4) to maximize profit by taking the derivative with respect to x_c, yielding the mark-up:

$$p_c = \left(\frac{1}{\alpha}\right) v_1 w_c \tag{5.5}$$

Symmetry means that each of the M_c intermediate firms charges a relative price that is a proportional mark-up of $\frac{1}{\alpha}$ over marginal cost $v_1 w_c$.

Appendix A shows that the momentary free entry, zero-profit equilibrium in the conventional intermediate sector is unique and stable. Zero-profit firm output is found by

see Desmet et al. (2018).

substituting from (3.3) and (5.5) into the profit expression (5.4), setting the result to zero, and solving for x_c:

$$x_c^* = x^* \equiv \frac{\alpha v_0}{(1-\alpha)v_1} \tag{5.6}$$

In the producer equilibrium, labor is allocated proportionally to the two stages of production:[27]

$$e_{sc} = (1-\alpha)\, e_c \tag{5.7}$$

$$e_{ic} = \alpha e_c \tag{5.8}$$

Free entry of intermediate firms means that the number of such firms is endogenous. In our set-up, the range M_c corresponds to the number of different intermediate firms. In equilibrium, the range is:[28]

$$M_c = \left(\frac{\alpha\,(1-\alpha)}{v_0}\right) e_c \bar{h} N \tag{5.9}$$

Scale in the form of the total amount of e*ffective labor* $e_c\bar{h}N$ increases the degree of specialization M_c and the base wage in (5.1).

In equilibrium, the base wage is:[29]

$$w_c = B\left(e_c \bar{h} N\right)^{1-\alpha} \tag{5.10}$$

so the skilled wage is:

$$w_s = w_c h = B\left(e_c \bar{h} N\right)^{1-\alpha} h \tag{5.11}$$

In these expressions, B is defined as:

$$B \equiv \frac{\alpha}{v_0}\,(1-\alpha)^{2-\alpha}\,(x^*)^{\alpha} \tag{5.12}$$

The base wage in (5.10) *rises* with an increase in $\bar{h}$ or N, unlike the effect in (5.1), where M_c is held fixed. Since M_c will rise when scale increases – as new intermediate firms enter the market – the effect on the base wage turns from negative to positive. The effect of h on *skilled* wages is even stronger, since there is a direct productivity effect that must be added to the scale effect.

[27] To derive these expressions, combine the markup (5.5) with the price and wage equations (5.3) and (5.1). Then use (3.4) and (3.5) and insert the value for x^*.

[28] To derive this expression, begin with (3.4) and (3.3) and substitute in (5.6) and (5.8).

[29] To derive (5.10), use (5.6) and (5.9) to substitute for x_c and M_c in (5.1).

6 Advanced Processing

Advanced final-good processing firms maximize profit by choosing e_{ua} and x_a to equate the marginal products of unskilled work effort and intermediate inputs to their given price and wage. Using (3.2) we have:

$$w_u = \frac{\partial Y_a}{\partial \left(e_{ua} N\right)} = (1-\alpha)\left(\frac{e_{ua}N}{x_a}\right)^{-\alpha} M_a \tag{6.1}$$

$$p_a = \alpha \left(\frac{e_{ua}N}{x_a}\right)^{1-\alpha} \tag{6.2}$$

where w_u is the wage of those the performing the processing – or assembling – task that does not require skill and p_a is the price of advanced inputs.

Workers that perform tasks that do require skill – that is, the production of intermediate inputs — earn the skilled wage, which is the product of the *advanced base wage* w_a and one's own human capital h. That is, the skilled wage here is $w_a h$. All workers possess the same amount of human capital and can perform all tasks equally well, so they must earn the same wage if both kinds of tasks are done:

$$w_u = w_a h \tag{6.3}$$

The base wage for the advanced technology is therefore $w_a = \frac{w_u}{h}$. Intermediate firms in advanced processing take the base wage w_a as given and choose the price p_a to maximize profits:

$$\pi_a = p_a x_a - w_a V_a\left(x_a\right) \tag{6.4}$$

Substituting (3.6) into (6.2) and (6.4) and differentiating yields the profit-maximizing mark-up:

$$p_a = \left(\frac{1}{\alpha}\right)\left(\frac{1}{S}\right) v_1 w_a \tag{6.5}$$

The mark-up for advanced intermediate producers is smaller than for the conventional conventional producers because $S > 1$. As with conventional firms, there is a stable, free-entry, zero-profit equilibrium in which all firms produce $x_a = x^*$ given in (5.6).[30] Likewise, the

[30] Substitute (3.6) and (6.5) into (6.4) and set profits to zero, then solve for x.

allocation of work effort is split proportionally:[31]

$$e_{ua} = (1-\alpha)\, e_a \tag{6.6}$$

$$e_{ia} = \alpha e_a \tag{6.7}$$

The equilibrium range of advanced inputs depends on scale, but also on the knowledge spillover:[32]

$$M_a = S\left(\frac{\alpha\,(1-\alpha)}{v_0}\right) e_a \bar{h} N \tag{6.8}$$

Since $S > 1$, for the same scale as measured by effective labor, the advanced sector would have a greater variety of inputs. This increased degree of specialization from the new system was emphasized by Rosenberg (1972) and later noted by Atack et al. (2019) as a conclusion of the *Hand and Machine Labor* study of 1899.

The equilibrium wage for unskilled labor in the advanced sector is:[33]

$$w_u = w_a h = SB\,(e_a N)^{1-\alpha}\, h \tag{6.9}$$

where B is given by (5.12). Human capital h raises the unskilled wage proportionally since employers of final-good assemblers must compete with intermediate firms who are hiring the same workers into skilled positions. The knowledge spillover S also raises the wage proportionally; because it reduces the cost of intermediates, increasing input variety M_a and labor productivity.

7 Household Optimization

The infinitely-lived representative household maximizes:

$$\int_0^\infty N(t)\, u(c(t))\, e^{-\rho t} dt = \int_0^\infty u(c(t))\, e^{-(\rho-\eta)t} dt$$

by choosing per capita consumption c, the four work effort allocations e_{sc}, e_{ic}, e_{ua}, and e_{ia}, and learning time e_l, where $u(c)$ is instantaneous utility, ρ is the subjective rate of discount, and exogenous population growth is η. We normalize initial population $N(0)$ and household

[31] These are derived as follows. Begin with the markup (6.5) and then substitute in (6.3), (6.2), and (6.1). Then use (3.9) for M_a and (5.6) for x.

[32] M_a is determined from (3.6), (3.9), (6.7) and the definition of x^*.

[33] This is derived by substituting (6.8), (5.6), and (6.6) into (6.1).

size to unity.

Household maximization is subject to a time budget constraint:

$$1 = e_{sc} + e_{ic} + e_{ua} + e_{ia} + e_l \tag{7.1}$$

and to a resource budget constraint:

$$c = w_c h \left(e_{sc} + e_{ic}\right) + w_u e_{ua} + w_a h e_{ia} \tag{7.2}$$

Individuals consider base wages – w_c and w_a – to be given along with the wage for unskilled tasks w_u. Accumulating h allows them to increase their wage – but only in three of the four occupations.

The first-order conditions for the optimization problem are set out in Appendix B. In this dynamic problem, there are two differential equations. First, the learning technology, (4.1) and (4.2), governs the change in h over time. Second, the shadow utility price λ of human capital must change constantly to equate the cost and benefit of accumulating human capital:

$$\dot{\lambda} = \left(\rho - \eta\right)\lambda - \frac{\partial \mathcal{H}}{\partial h} = \left(\rho - \eta\right)\lambda - \lambda\left(\left(1-\gamma\right)L^{\gamma}h^{-\gamma}e_l - \eta\right) - \theta_1\left(w_c\left(e_{sc} + e_{ic}\right) + w_a e_{ia}\right) \tag{7.3}$$

Finally, we require the transversality condition:

$$\lim_{t\to\infty} \lambda\left(t\right) h\left(t\right) e^{-(\rho-\eta)t} = 0 \tag{7.4}$$

The representative household takes economy-wide averages $\bar{h}$ and $\bar{e}_w$ as given. In what follows, we impose aggregate consistency so that the representative household's choices match economy-wide averages: $\bar{h} = h$ and $\bar{e}_w = e_w$.

8 Momentary Equilibrium

In this section, we describe the momentary equilibrium of the economy taking as given human capital h, population N, and work effort e_w. Labor is mobile and all tasks are performed in equilibrium so wages must be equalized: $w_c h = w_s = w_u = w_a h$.[34]

Figure 2 illustrates the momentary equilibrium. The locus labeled "w_s" shows the conventional skilled wage as a function of work effort in the *advanced* sector e_a. It is derived

[34]This follows from the first-order conditions (B.2), (B.3), (B.4), and (B.5) from the household's problem in Appendix B; along with the assumption of an interior solution.

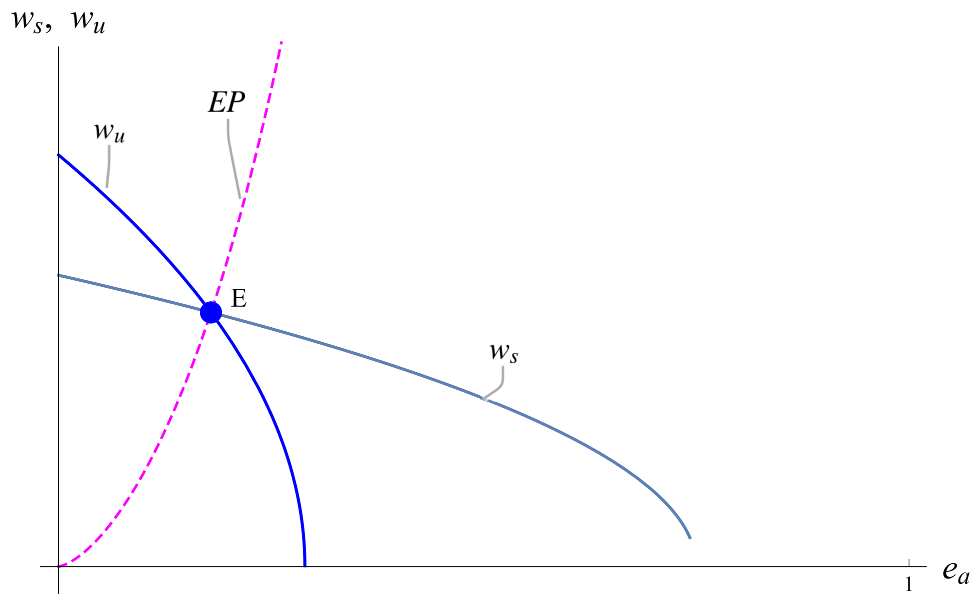

Figure 2: Momentary Equilibrium in the Labor Market

from (5.11) after substituting $e_w - e_a$ for e_c from (3.12), yielding:

$$w_s = B\left(\left(e_w - e_a\right)h\right)^{1-\alpha} N^{1-\alpha}h \tag{8.1}$$

where B is given in (5.12). The w_s locus is downward sloping and concave to the origin; as work effort leaves the conventional technology, the skilled wage falls there due to increasing returns to scale.

The locus labeled "w_u" shows the unskilled wage as a function of work effort in the advanced sector; it is derived from (6.9) after substituting $e_w - e_a$ for e_c, $e_{ua} = (1-\alpha)\,e_a$ from (6.6), and the definitions of S and $f\left(\frac{e_c h}{e_{ua}}\right)$ in (3.7) and (3.8) to yield:

$$w_u = B\left(2\left(e_w - e_a\right)h - \tau e_a\right)^{1-\alpha} N^{1-\alpha}h \tag{8.2}$$

The w_u locus is also downward sloping and concave to the origin. The unskilled wage falls as e_a increases, even though this sector, too, is subject to increasing returns to scale. The reason is that, due to the knowledge spillover $f\left(\frac{e_c h}{e_{ua}}\right)$ in (3.8), the cost advantage of producers of intermediates for the advanced technology falls, reducing the demand for labor.

The momentary equilibrium is Point E in Figure 2. Equations (8.1) and (8.2) can be used to show that $\frac{\partial w_u}{\partial e_a} < \frac{\partial w_s}{\partial e_a} < 0$ where they cross, which means that the loci can only cross once. This also means that the equilibrium at E is stable.

To examine what happens to labor allocations when h increases, it is useful to consider the ratio:

$$r(h) = \frac{h}{f\left(\frac{e_c h}{e_{ua}}\right)} \tag{8.3}$$

where $f\left(\frac{e_c h}{e_{ua}}\right)$ is given by (3.8). The function $r(h)$ is the *relative productivity of work* in conventional processing relative to advanced processing. From the production functions (3.1) and (3.2) we know that h only improves worker productivity *directly* in conventional processing. However, it confers an *input cost advantage* to unskilled work in advanced processing via the knowledge spillover in (3.6) – an effect that works just like a productivity advantage in advanced processing.[35] Given that the elasticity of $f(\ldots)$ with respect to h is greater than 1, we know that $r'(h) < 0$ so an increase in h *reduces* the relative productivity of a worker using the conventional technology.

The ratio of wages is:[36]

$$\frac{w_u}{w_s} = \left(\frac{e_a}{r(h)\, e_c}\right)^{1-\alpha} = \left(\frac{2e_c - \frac{\tau e_a}{h}}{e_c}\right)^{1-\alpha} \tag{8.4}$$

The second equality reveals that at the original labor allocation, an increase in h raises the w_u curve more than the w_s curve, so the new equilibrium lies to the northeast of Point E in Figure 2. In the new equilibrium, wages are higher and the share of advanced output e_a rises. The locus labeled "EP" shows the series of equilibria that result from a continuous increase in h.[37]

The ratio of wages when $e_a = 0$ depends only on α: $\left.\frac{w_u}{w_s}\right|_{e_a=0} = 2^{1-\alpha}$, where $0 < \alpha < 1$. When e_a is small, then, there is a wage advantage in favor of firms utilizing advanced processing. In this way, the American System could be introduced initially at small scale by competitive firms in a free market economy, even though the advanced technology would benefit over time from increasing returns to scale and specialization.

Equate the wages in (8.4) to see that *in equilibrium* the ratio of work effort in the advanced sector relative to the conventional sector is:

$$\left.\frac{e_a}{e_c}\right|_{w_u=w_s} = \frac{h}{\tau} = r(h) \;\; \Rightarrow \;\; f\left(\frac{e_c h}{e_{ua}}\right) = \tau \tag{8.5}$$

The ratio $\frac{e_a}{e_c}$ is proportional to h, where the factor of proportionality is $(1/\tau)$. In equilibrium,

[35] That is, we could have put S in the production function, not in the cost function, and it would lead to the same solution.

[36] The first equality comes from using (5.11) and (6.9) along with the definitions of S, and $f(\ldots)$ and $r(h)$. The second equality uses(8.2) and (8.1).

[37] The equation of the EP locus is: $w = BN^{1-\alpha}\tau^{2-\alpha}\left[\frac{e_a^{2-\alpha}}{(e_w - e_a)}\right]$. This "expansion path" is drawn for a given value of e_w. In general, however, e_w will change as h rises.

labor allocations e_c and e_{ua} continually adjust to clear the market in such a way that the spillover function $f(\ldots)$ is always equal to the constant $\tau > 1$. From now on, we only consider positions of momentary equilibrium. The ratio (8.5) and the constraint (3.12) mean that sectoral effort e_c and e_a are proportional to total effort e_w:

$$e_c = \left(\frac{1}{1+\frac{h}{\tau}}\right) e_w \tag{8.6}$$

$$e_a = \left(\frac{\frac{h}{\tau}}{1+\frac{h}{\tau}}\right) e_w \tag{8.7}$$

Then, from (5.7), (5.8), (6.6), and (6.7) we find that each effort allocation is also proportional to e_w:

$$e_{sc} = (1-\alpha)\left(\frac{1}{1+\frac{h}{\tau}}\right) e_w \tag{8.8}$$

$$e_{ic} = \alpha\left(\frac{1}{1+\frac{h}{\tau}}\right) e_w \tag{8.9}$$

$$e_{ua} = (1-\alpha)\left(\frac{\frac{h}{\tau}}{1+\frac{h}{\tau}}\right) e_w \tag{8.10}$$

$$e_{ia} = \alpha\left(\frac{\frac{h}{\tau}}{1+\frac{h}{\tau}}\right) e_w \tag{8.11}$$

The momentary equilibrium of per capita output, given e_w, h, and N, is derived by imposing symmetry of inputs on (3.1) and (3.2):

$$y = \frac{Y_c + Y_a}{N} = \frac{(e_{sc}hN)^{1-\alpha} M_c (x^*)^{\alpha}}{N} + \frac{(e_{ua}N)^{1-\alpha} M_a (x^*)^{\alpha}}{N} \tag{8.12}$$

This can be written more compactly as:[38]

$$y = B\,(he_w)^{2-\alpha}\left(\frac{N}{1+\frac{h}{\tau}}\right)^{1-\alpha} \tag{8.13}$$

where B is given in (5.12). Expression (8.13) shows that per capita output y benefits from increasing returns to e_w, h, and N, separately, all working through scale via increasingly specialized intermediate goods. We do not yet know, however, how the state h and N affect

[38]Substitute (5.9) and (6.8) for M_c and M_a, (8.8) and (8.10) for e_{sc} and e_{ua}, and set the equilibrium knowledge spillover term in (3.6) to $\tau^{1-\alpha} > 0$.

e_w over time.[39]

Certain key ratios depend only on h and the parameters:

$$\frac{y_a}{y} = \frac{\frac{h}{\tau}}{1+\frac{h}{\tau}} \tag{8.14}$$

$$\frac{M_a}{M_c} = \frac{h}{\tau^\alpha} \tag{8.15}$$

$$\frac{e_{sw}}{e_w} = \frac{1+\alpha\frac{h}{\tau}}{1+\frac{h}{\tau}} \tag{8.16}$$

As noted from Figure 1, we define $e_{sw} \equiv e_{sc} + e_{ic} + e_{ia}$ to be the labor engaged in skilled tasks.

We do not need to analyze the dynamics to see that as h rises, the advanced technology increases its share of output; the variety of advanced intermediate inputs increases relative to conventional inputs; and skilled labor declines as a share of the total.

9 Dynamic Equilibrium and Growth

We combine the first-order conditions set out in Appendix B with the conditions of momentary equilibrium to determine the equilibrium allocation of time between work and learning. This split depends on the state of the economy described by per capita human capital h and the utility value of an individual's human capital z, where $z \equiv \lambda h$ and λ is the shadow price of human capital introduced in Section 7.

The *specialization spillover* in learning productivity L^γ in (4.1) and (4.2) is a function of h and the fixed cost of producing inputs v_0:[40]

$$L^\gamma = h^\gamma A\Omega\left(\frac{h}{\tau}\right) \tag{9.1}$$

[39]Wage income must add up to the value of output: $Y = B\left(e_w hN\right)^{2-\alpha}\left(\frac{1}{1+\frac{h}{\tau}}\right)^{1-\alpha} = w_s e_w N$. Aggregate product is found by multiplying per capita output y in (8.13) by N resulting in the first equality above, where B is given by (5.12). Aggregate wage income is found by adding wages paid to skilled work $w_s e_{sw} N$ and wages paid to unskilled work $w_u e_{ua} N$, where $w_s = w_u$ in equilibrium and $e_{sw} + e_{ua} = e_w$. These imply that aggregate wage income is $w_s e_w N$. Substituting for the wage with (5.11), and using (8.6) yields the second equality above.

[40]Substitute (3.4) and (3.9), using x^* in (5.6) for x, to eliminate M_c and M_a in (4.2), then substitute (8.9) and (8.11) for the labor allocations.

where:

$$A \equiv \left[\frac{\alpha(1-\alpha)}{v_0}\right]^{\gamma} \tag{9.2}$$

$$\Omega\left(\frac{h}{\tau}\right) \equiv \left(\frac{1+k\left(\frac{h}{\tau}\right)}{1+\left(\frac{h}{\tau}\right)}\right)^{\gamma} \tag{9.3}$$

and k is the weight in (4.2). Equilibrium work effort e_w and learning e_l are:[41]

$$e_w = \frac{1}{zA\Omega\left(\frac{h}{\tau}\right)} \tag{9.4}$$

$$e_l = 1 - \frac{1}{zA\Omega\left(\frac{h}{\tau}\right)} \tag{9.5}$$

The dynamic system is given by the two motion equations:[42]

$$\dot{h} = h\left(A\Omega\left(\frac{h}{\tau}\right) - \frac{1}{z} - \eta\right) \tag{9.6}$$

$$\dot{z} = z\left[\rho - \eta + \gamma A\Omega\left(\frac{h}{\tau}\right)\right] - \left[\gamma + \Phi\left(\frac{h}{\tau}\right)\right] \tag{9.7}$$

where:

$$\Phi\left(\frac{h}{\tau}\right) \equiv \frac{1+\alpha\left(\frac{h}{\tau}\right)}{1+\left(\frac{h}{\tau}\right)} \tag{9.8}$$

and $\Phi'\left(\frac{h}{\tau}\right) < 0$.

To achieve a maximum, the representative household must select an initial $z(0)$ for a given initial $h(0)$, follow the (h, z) system of first-order differential equations (9.6) and (9.7) thereafter, and satisfy transversality (7.4) at infinity.

We use the h and z motion equations to derive two stationary loci in (h, z) space. Figure 3 illustrates. The HH locus, where $\dot{h} = 0$, is found by setting (9.6) to zero and solving for z:

$$z_H(h) = \frac{1}{A\Omega\left(\frac{h}{\tau}\right) - \eta} \tag{9.9}$$

The motion of h relative to this locus is *unstable*: if h is above the locus, it keeps increasing; if h is below the HH locus, it falls. The ZZ locus, where $\dot{z} = 0$, is found by setting (9.7) to

[41] These are found by using (B.1), (B.4), and (B.6) with (9.1) and $z \equiv \lambda h$.

[42] Use (4.1), (9.1), and (9.5) to obtain the equation for $\dot{h}$. The motion equation $\dot{z}$ is derived in Appendix C. We assume that $z \geq 1/A\Omega\left(\frac{h}{\tau}\right)$. If this is not true, then $e_w = 1$ and $e_l = 0$. Then, $\dot{h} = -\eta z$ and $\dot{z} = (\rho - \eta) z - \Phi\left(\frac{h}{\tau}\right)$.

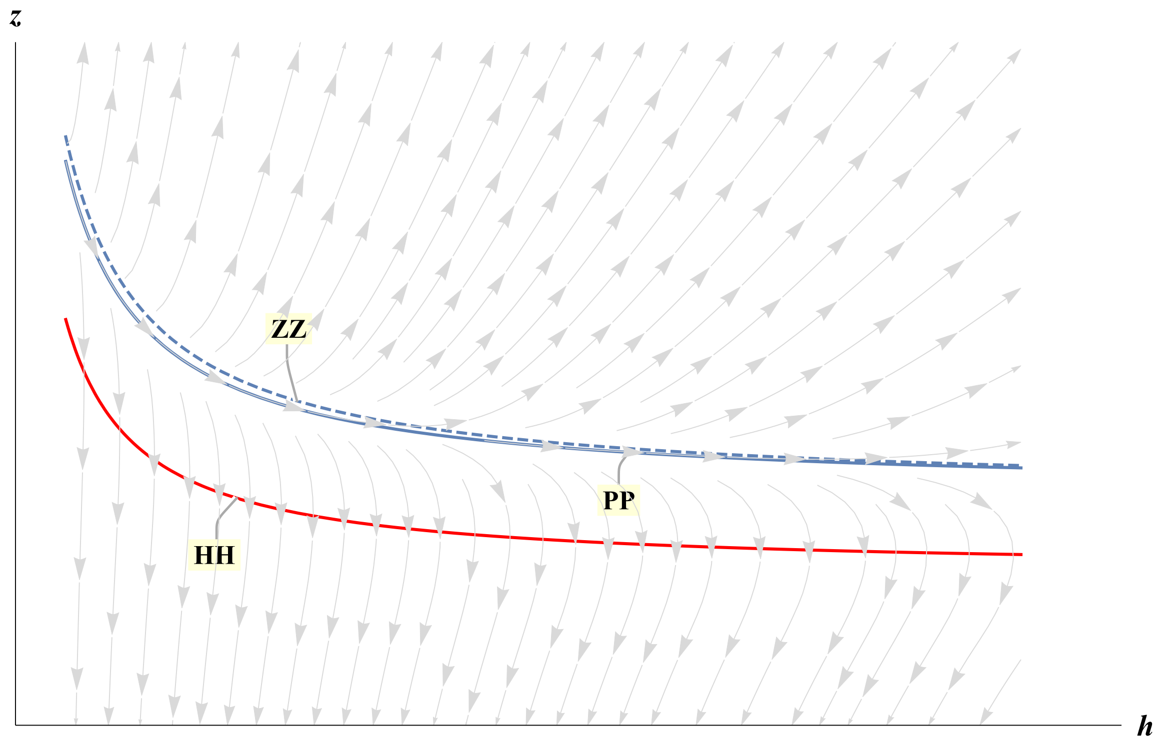

Figure 3: Perpetual Growth in h

zero:

$$z_Z(h) = \frac{\gamma + \Phi\left(\frac{h}{\tau}\right)}{\rho - \eta + \gamma A \Omega\left(\frac{h}{\tau}\right)} \tag{9.10}$$

The motion of z relative to the ZZ locus is also unstable.

Different growth regimes are possible, depending on the values of the parameters and the initial value of human capital, $h(0)$. We focus on the regime that guarantees perpetual growth in per capita human capital because, as we show in Appendix D, it is the only growth regime consistent with the historical record. The configuration of the HH locus (9.9) and the ZZ locus (9.10) that supports perpetual growth is pictured in Figure 3, a phase plane similar to that in Romer (1986). The ZZ stationary locus (dashed) lies above the HH stationary locus for all h. Between the two loci, z is falling and h is rising. The initial $z(0)$ must be chosen so that the optimal path – labeled PP – navigates between these two loci forever, but it gets arbitrarily close to the ZZ locus as h gets large. If it did not, the (z, h) pair would go to either infinity or zero (in finite time), violating transversality (7.4). Perpetual growth is supported if ZZ lies above HH for all values of $h > h(0)$, since in that case there exists a path along which z approaches a constant and h grows forever. Appendix D derives the conditions under which this is the case.

We use the result that z converges to a constant to find the long-run, steady-state values for the growth rates of per capita output and human capital, and the allocations of labor. The definition of $\Omega\left(\frac{h}{\tau}\right)$ in (9.3) and of $\Phi\left(\frac{h}{\tau}\right)$ in (9.8) imply:

$$\lim_{h\to\infty}\Omega\left(\frac{h}{\tau}\right)=k^{\gamma} \tag{9.11}$$

$$\lim_{h\to\infty}\Phi\left(\frac{h}{\tau}\right)=\alpha \tag{9.12}$$

Using these limit values, as h tends to infinity, the ZZ locus (9.10) becomes horizontal at:

$$\hat{z}_Z=\frac{\alpha+\gamma}{\rho-\eta+\gamma Ak^{\gamma}} \tag{9.13}$$

In the long run, z converges to $\hat{z}_Z$ and h growth converges to a constant rate from above. To find the growth rate to which h converges, substitute (9.11) and (9.13) into (9.6) to yield:

$$\tilde{g}_h=\frac{\alpha Ak^{\gamma}-(\rho-\eta)}{\alpha+\gamma}-\eta \tag{9.14}$$

Substitute the same limit values into (9.4) to see that total work effort converges to the constant:

$$\tilde{e}_w=\frac{\rho-\eta+\gamma Ak^{\gamma}}{(\alpha+\gamma)\,Ak^{\gamma}} \tag{9.15}$$

The growth of conventional output y_c in the long run is due only to population growth η via increasing returns to specialization:[43]

$$\tilde{g}_{yc}=(1-\alpha)\,\eta \tag{9.16}$$

The growth of advanced output converges to:[44]

$$\tilde{g}_{ya}=\tilde{g}_h+(1-\alpha)\,\eta=\frac{\alpha Ak^{\gamma}-(\rho-\eta)}{\alpha+\gamma}-\alpha\eta \tag{9.17}$$

Advanced sector growth exceeds that of the conventional sector and eventually absorbs all of the growth of human capital, and even though output produced conventionally never stops growing, it becomes an ever smaller share of total output.

[43] To see this, note that $y_c=w_se_c=B\,(e_ch)^{2-\alpha}\,N^{1-\alpha}$ by (5.11). By (8.5), however, we can write this as $y_c=B\,(e_a\tau)^{2-\alpha}\,N^{1-\alpha}$. In the limit as $h\to\infty$, e_a becomes constant at e_w by (8.7) so growth in y_c is due only to N via specialization.

[44] Use $y_a=w_ue_a$, substituting from (6.9) and noting from (8.7) that e_a converges to the constant work effort in (9.15).

10 Calibration

We must calibrate five primary model parameters – α, η, ρ, γ, and k – and one composite parameter, A, to simulate our model. Based on Jones (2011), p. 18, the share of intermediate goods in final-goods processing is set at $\alpha = 0.50$; annual population growth in the United States is set to $\eta = 0.013$; and utility time preference is $\rho = 0.04$.

The parameter $0 < \gamma < 0$ in (4.1) controls the strength of the positive specialization spillover in (4.2). We calibrate γ to eliminate the scale effect of population growth on per capita product growth — the weak scale effect in Jones (2005), pp. 193-95. The calibrated γ is chosen so that the scale effect in (8.13) is offset by the time cost to educate the newly born in (4.1).

To calibrate γ, proceed as follows. Differentiate (9.17) with respect to η, setting the result to zero, and solving to get $\gamma_a = \frac{1}{\alpha} - \alpha$, the value of γ that would eliminate the population scale effect on per capita output growth in a purely advanced economy. Differentiate (F.12) in Appendix F to find the γ that would eliminate the population scale effect in a purely conventional economy, $\gamma_c = 1 - \alpha$. Calibrate γ as a weighted average of γ_a and γ_c, where the weights are the fractions of output produced by conventional and advanced processing, respectively:

$$\gamma = \left(\frac{y_c}{y}\right)\gamma_c + \left(\frac{y_a}{y}\right)\gamma_a \tag{10.1}$$

Substituting (8.14) and (8.10), and γ_c and γ_a from above, the γ that eliminates the effect of population growth on per capita output growth is:

$$\gamma = 1 - \alpha + \frac{1}{\alpha}\left(\frac{e_{ua}}{e_w}\right) \tag{10.2}$$

where the ratio $\frac{e_{ua}}{e_w}$ is the ratio of unskilled work effort in advanced final-goods processing to economy-wide total work effort.

In order to calibrate γ in (10.2), and later the parameters A and k, we utilize *operative share* data given in our Table 1, "Operatives in the Total Work Force," taken from the *Historical Statistics of the United States*, Millennium Edition.[45] As described earlier, operatives were a new class of worker developed in the American System – a worker with limited skill that assembled intermediate inputs into final goods. We take the share of operatives in the data to correspond to the ratio of unskilled to total work in our model:

[45] See Carter et al., eds (2006). Occupations in the table from which the operative data are taken are classified into 13 groups: professionals, farmers, proprietors, managers and officials, clerical workers, sales workers, craft workers, operatives, domestic service workers, other service workers, farm laborers, laborers, and unclassified.

Table 1: Operatives in the Total Work Force

Year	*Workers*	*Operatives*	*Operative Share*
1860	8,160,752	975,641	0.120
1870	12,004,238	1,527,567	0.127
1880	16,478,927	2,273,682	0.138
1890	NA	NA	NA
1900	27,554,085	3,824,089	0.139
1910	36,236,003	5,809,146	0.160
1920	40,113,274	6,352,824	0.158
1930	NA	NA	NA
1940	47,584,238	8,586,887	0.180
1950	56,973,749	11,489,205	0.202
1960	63,870,595	11,843,724	0.185
1970	76,270,515	13,601,511	0.178
1980	97,378,407	14,334,201	0.147
1990	115,083,094	13,373,392	0.116

Source: *Historical Statistics of the United States*

Millennium Edition (2006, On Line, Table Ba1033-1046, page 2-133)

$Operative\ Share\,(t) \equiv \frac{e_{ua}(t)}{e_w(t)}$. We calibrate γ using the 1860 operative share 0.12 in Table 1 to substitute into (10.2) to yield $\gamma = .74$. We use the earliest operative share to avoid downward bias. Operative work was concentrated for decades in manufacturing where reported operatives with limited skills were identified clearly as working to assemble final goods with interchangeable parts and specialized machines. Such reported operative share data does correspond closely to the model's aggregate theoretical ratio of unskilled processing labor to total work. However, our theoretical model regards operative work more broadly – to encompass also the portion of unskilled work outside of manufacturing where individuals make use of sophisticated intermediate good technology (such as communication devices, computers, and artificial intelligence). From our broader theoretical perspective, as American System technology diffused and became more sophisticated, operative shares as typically reported in Table 1 increasingly underestimate the theoretical ratio of unskilled processing work to total work in (10.3). So we choose to calibrate γ with the earliest operative share in Table 1.

There are two remaining parameters to calibrate – the parameter k introduced in (4.2) that weights the degree that learning productivity is enhanced by M_a relative to M_c – and the composite parameter A introduced in (9.1) and (9.2) that, in conjunction with α and γ, calibrates the *fixed cost* of producing intermediate goods, v_0 in (3.3) and (3.6).

Our (A, k) calibration is restricted to the *perpetual growth region* northeast of $PG1$ and $PG2$ in Figure 4. These loci are explained in Appendix D. The two loci are positioned using the calibrations for α, η, ρ, and γ from above and, in the case of $PG1$, an initial condition for $\frac{h(0)}{\tau}$ that is found by dividing (8.10) by e_w and solving for $\frac{h}{\tau}$:

$$\frac{h}{\tau} = \frac{\frac{e_{ua}}{e_w}}{1 - \alpha - \frac{e_{ua}}{e_w}} \tag{10.3}$$

To position $PG1$, we use $\alpha = 0.5$, and the 1860 operative share, 0.12, in (10.3), to yield the initial condition $\frac{h(0)}{\tau} = 0.31$. This tells us that the relative productivity advantage of the new, American System of Manufactures in 1860 was on the order of 3.[46]

Appendix D shows that the $PG1$ locus shifts left over time as h grows, so all (A, k) calibrations bounded below by $PG1$ and $PG2$ in Figure 4 will support perpetual growth for all time. In Appendix E, we describe how we find the policy function and the time paths for output and labor allocations for an arbitrary (A, k) parameter pair.

To calibrate (A, k) we work with the dynamic system in z and $\frac{h}{\tau}$ based on (9.7) and (9.6), the (10.3) link between the state $\frac{h}{\tau}$ and the operative share, and the four parameters already set.[47] Our methodology searches for an (A, k) pair that best matches two *simulated*

[46] This follows from the meaning of $r(h)$ in (8.3) and its equilibrium value (8.5).

[47] The dynamic system in z and h/τ behaves identically to the system in z and h discussed in Section 9 and pictured in Figure 3, since τ is a constant and $\dot{h}/h$ depends only on h/τ. That is, letting $q \equiv h/\tau$, we

operative shares to two time-dated *historical* operative shares from Table 1. We restrict the operative share data in Table 1 by discarding observations that are outliers, inconsistent with the theory expressed in (8.10) that growing per capita human capital raises the share of unskilled work. That leaves us with enough operative share data to produce 26 separate (A, k) calibrations.[48]

The 26 separate (A, k) calibrations are found as follows: (1) select two operative-share observations, one for a *base year*, say 1860, and another for a *target year,* say 1910; (2) determine the initial condition on the state $\frac{h(0)}{\tau}$ by substituting the base-year historical operative share into (10.3); (3) pick an arbitrary (A, k) pair and simulate the model; (4) measure the squared deviation of the simulated target-year operative share from the historical target-year operative share; (5) repeat by selecting values over 2,306 (A, k) points in a grid in Figure 4, bounded below by *PG1* and *PG2*, to a maximum of $k = 3.3$ and $A = .08$; (6) choose the (A, k) pair that determines the lowest squared deviation. We do not hit the historical target-year operative share exactly, but only because our grid is not infinitely fine. We come very close.

In Figure 4, the small dots show the 26 calibrated (A, k) points fitted through each of the 26 pairs of time-dated operative shares. All are well within the region bounded below by *PG1* and *PG2*. We also show three *aggregate* (A, k) calibrations based on the 26 separate (A, k) calibrations. The first aggregate calibration, shown as *G_med* in Figure 4, is the *median* of the separate 26 values for A and k and is given by $(A = .056,\ k = 2.53)$.[49] The second aggregate calibration, shown as *G_ave, averages* the 26 primary values for A and k. This point is given by $(A = .057,\ k = 2.49)$. The third aggregate calibration, shown as *G_19,* averages the three (A, k) calibrations based only on the operative shares in the *19th Century*: 1860/1870, 1860/1880, and 1870/1880. The *G_19* pair is the point $(A = .059,\ k = 2.55)$.

Before turning to the simulation of the calibrated model, it is worth emphasizing that our methodology for calibrating the six model parameters α, η, ρ, γ, k, and A relies on matching simulated operative shares to actual operative share data — our calibration utilized no data on actual output or output growth in the United States.

---

see from (9.6) that $\dot{q}/q = \dot{h}/h = A\Omega(q) - (1/z) - \eta$.

[48]We discard the consistently falling operative share data after 1950, which, as noted, we believe reflect an increasing underestimate of its theoretical counterpart. This leaves operative shares in Table 1 from 1860 to 1950 to calibrate A and k, which gives us 8 data points and $\frac{8^2-8}{2} = 28$ time-dated operative share pairs to fit the boundary conditions. For the same reason, we also discard two other operative share pairs, (1880, 1900) and (1910, 1920), where due to temporary factors or measurement error the latter pair shows an outright operative share decline and the former shows essentially no growth in the operative share even over a span of two decades.

[49]To calculate the median value of k, the 26 primary values are sorted by magnitude then the middle two values (positions 13 and 14) are averaged. The median A was found in the same way.

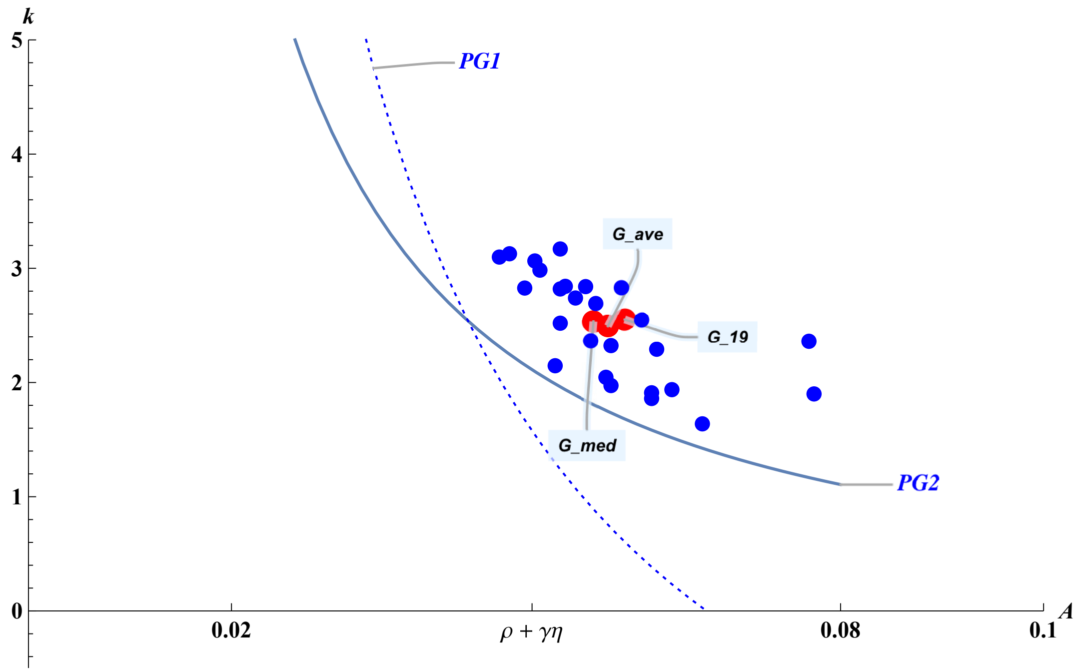

Figure 4: Calibration of k and A from Time-Dated Operative Shares

11 Simulating Transitional Growth

In this section we explain how our model can account for near-constant growth in the United States even as the economy was in transition from a largely conventional technology to one that was exclusively advanced. The evidence is in Figure 5, which shows the log of per capita output in the US and Great Britain from 1860 to 2016. Two points deserve mention. First, the actual path in the US is very close to the dashed trend line over the entire history -- growth is near-constant at 1.88 percent. For Great Britain, there is much more variability around the trend line (not shown). Second, in keeping with the historical narrative in Section 2, we see that over the second half of the 19th Century and well into the 20th, Great Britain grew considerably slower than the US – the growth rate was 1.32 percent from 1860 to the present.[50]

There is no closed-form expression for per capita output growth during the transition, so we utilize the numerical method described in Appendix G to compute and simulate growth paths. Figure 6 shows separately the simulated growth paths of output per capita from 1860 to 2200 for the 26 primary calibrations and the 3 aggregate calibrations represented by the 29 (A, k) points in Figure 4. We simulate the aggregate calibrations in Figure 6 labeled *G_med*, *G_ave*, and *G_19* using (10.3) and the operative share in 1860 for the initial condition for

[50]The data is from The Maddison Project Database 2018 (see Bolt et al. (2018)). The growth rates reported in this section are $100 * \hat{\beta}_1$, where $\hat{\beta}_1$ is estimated from the OLS regression $\ln y = \beta_0 + \beta_1 t + \epsilon$.

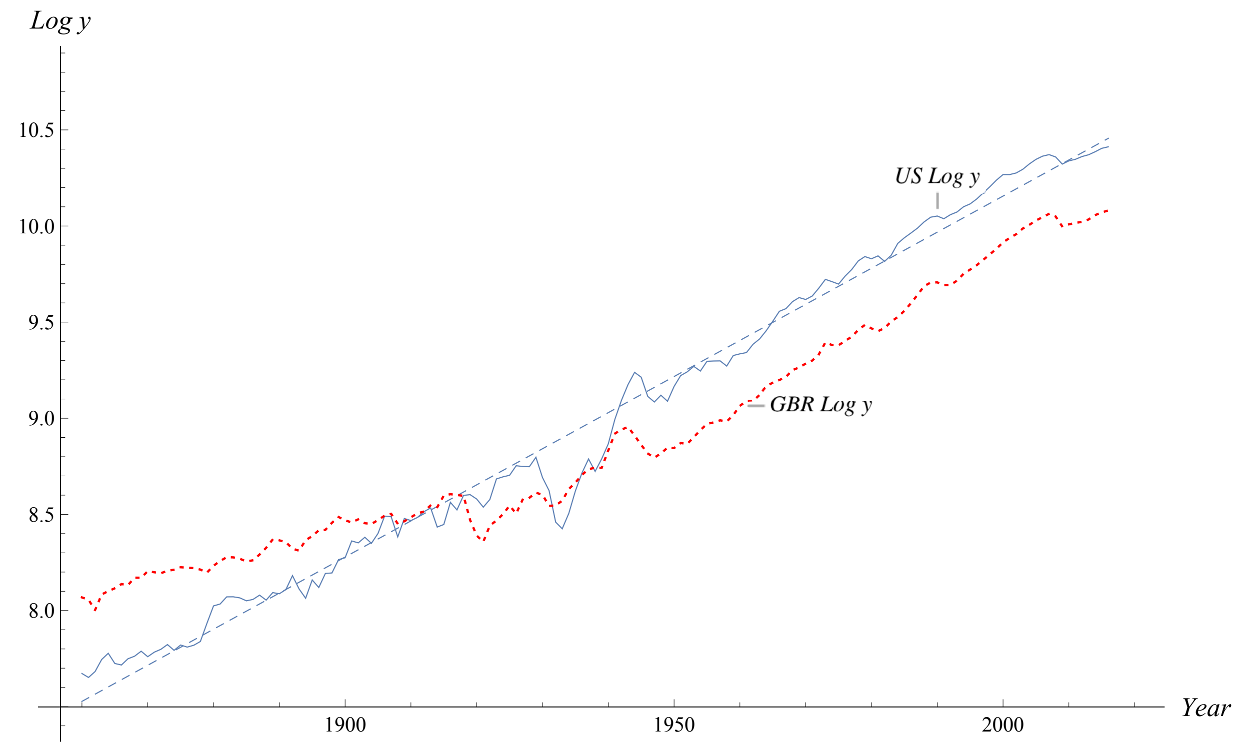

Figure 5: Per Capita GDP in the US and Great Britain, 1860 - 2016 (Natural Logarithm)

h/τ.

The spread of the 26 primary calibrated per capita growth paths in Figure 6 reflects measurement error associated with the different paired operative shares for each primary (A, k) calibration. We chose the *G_med* aggregate as our baseline calibration of the model's parameters to use in simulating the model because *G_med* moderates the influence of outliers while still using information from all of our operative share historical data.

The parameters of our *G_med* baseline calibration are displayed in first three rows of Table 2. The lower portion of Table 2 presents the *G_med* simulated growth rates and effort allocations periodically from 1860 to 2200 and in the limit of time. Figure 7 reproduces the calibrated baseline *G_med* path of per capita output growth from Figure 6 drawn against various historical benchmarks: the whole period of 1860 - 2016 (the Grand Mean), the period before the Great Depression (1860 - 1930), and the period after the Second World War (1950 - 2016), where the last two periods are subdivided into equal-length sub-periods.

The average value of the *G_med* simulated growth path g_y shown in Figures 6 and 7 is 1.88% from 1860 to 2016[51]. Over the same period, the time trend based on realized per capita product in the United States was also 1.88%. It is remarkable that the simulation matches actual growth so well — especially considering that our calibration uses no data on actual

[51]The average of the numerical continuous function $g_y(t)$ is $\frac{1}{(2016-1860)} \int_{1860}^{2016} g_y(t)\, dt = 0.01884$.

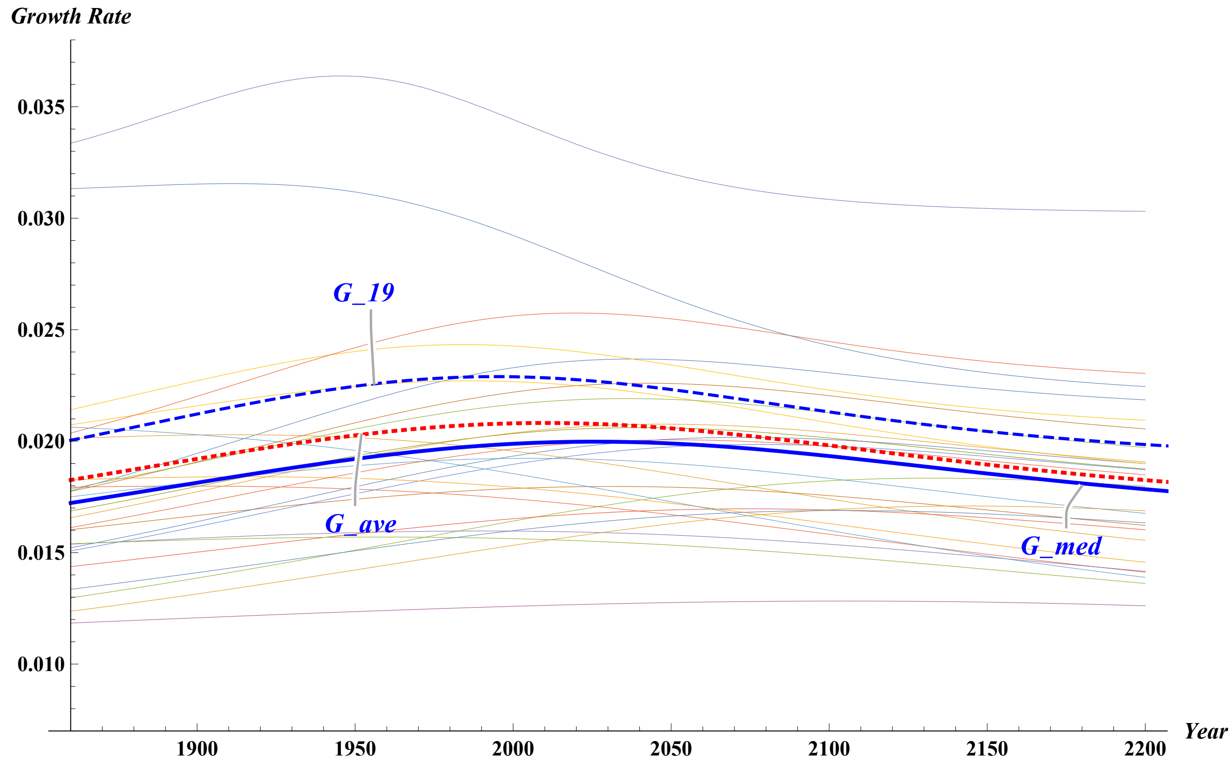

Figure 6: Simulated Paths of Per Capita Product Growth

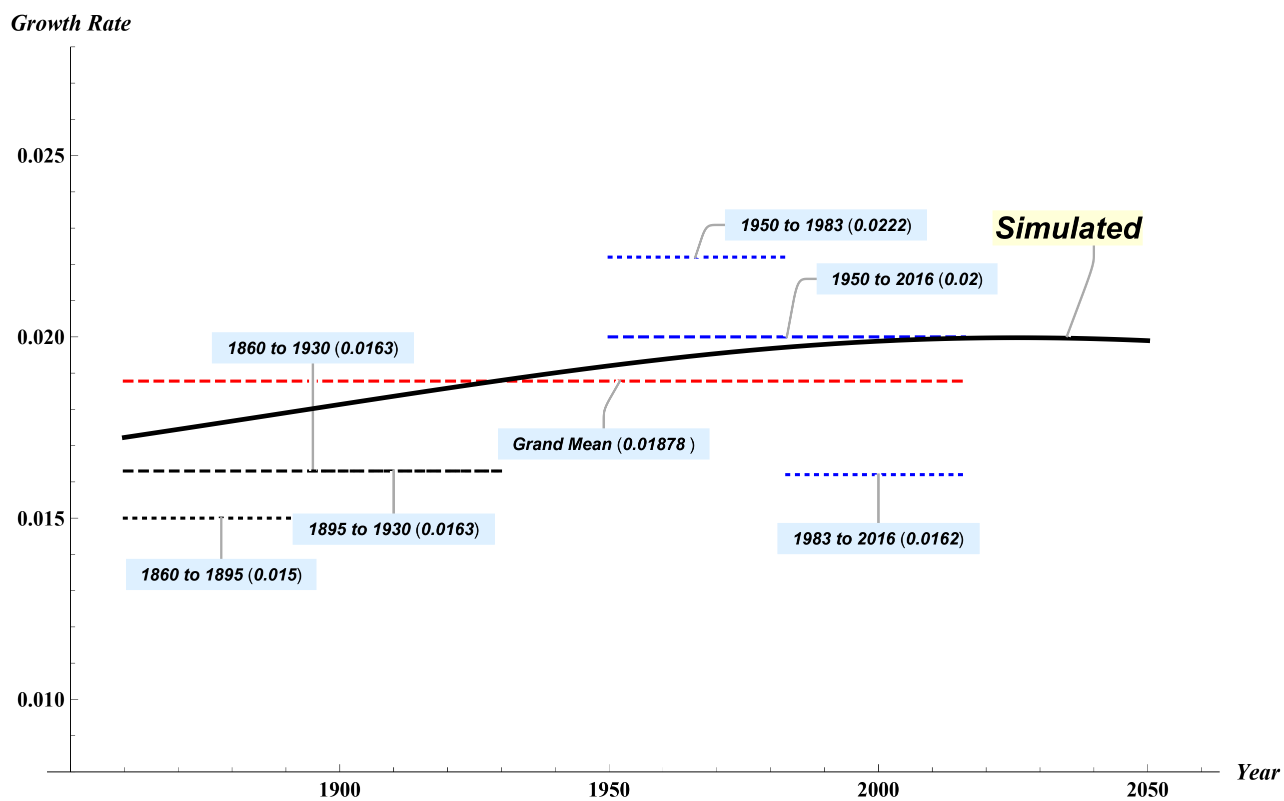

Figure 7: U.S. Growth Rates and the Simulated Growth Path

Table 2: *G_med* Baseline Parameters and Simulated Paths

Calibrations			
$\alpha =$	0.5	$\gamma =$	0.74
$\eta =$	0.013	$A =$	0.0556
$\rho =$	0.04	$k =$	2.5339

		Simulations						
		1860	1900	1950	2000	2100	2200	∞
(1)	g_y	1.72	1.81	1.92	1.99	1.93	1.78	1.63
(2)	g_{ya}	2.31	2.40	2.49	2.48	2.19	1.88	1.63
(3)	g_{yc}	1.54	1.56	1.54	1.44	1.08	0.82	0.65
(4)	g_h	0.77	0.84	0.95	1.04	1.10	1.06	0.98
(5)	$\frac{y_a}{y}$	0.24	0.30	0.40	0.53	0.77	0.91	1
(6)	$\frac{e_{ua}}{e_w}$	0.12	0.15	0.20	0.26	0.38	0.45	0.5
(7)	$\frac{e_{sw}}{e_w}$	0.88	0.85	0.80	0.74	0.62	0.55	0.5
(8)	e_w	0.71	0.71	0.72	0.73	0.76	0.78	0.79
(9)	e_l	0.29	0.29	0.28	0.27	0.24	0.22	0.21
(10)	e_{ua}, e_{ia}	0.08	0.11	0.14	0.19	0.29	0.35	0.40
(11)	e_{sc}, e_{ic}	0.27	0.25	0.21	0.17	0.09	0.04	0
(12)	e_{sw}	0.62	0.60	0.57	0.54	0.47	0.43	0.40

Rates in Rows (1) - (4) are continuous multiplied by 100.

output from the United States. The simulation achieves nearly constant long-run growth to date because of two countervailing features of the American System acting through the engine of economic growth, human capital accumulation. The *first* feature of the American System *raises* the growth rate of human capital for two reasons. Learning productivity in (9.1) and (9.2) is increasingly enhanced by the *specialization spillover* L^γ in the learning technology as the ranges of both intermediate goods in (4.2) grow with human capital accumulation. Learning productivity is further enhanced because the calibrated $k > 1$ and the ratio of advanced intermediates is $\frac{M_a}{M_c} = h/\tau^\alpha$ in equilibrium, as we noted in (8.15). As $\frac{M_a}{M_c}$ rises with h, there is an additional boost to learning productivity through (4.2). As row (4) of Table 2 shows, these two features of the American System work to increase the growth rate of human capital per capita rapidly from 0.77% in 1860 to 1.04% in 2000.

The *second* feature of the American System works to *slow* the growth of per capita product. This is the decline in z – the utility value of human capital – that reduces e_l through (9.5). The fall in z along the optimal path is clearly shown in Figure 3. Our conjecture is that this decline is due to the shift to advanced processing, which raises the share of unskilled work effort, reducing the incentive for individuals to accumulate human capital. We see in rows (6) and (7) of Table 2, and in (8.10) and (8.16), that there is a major realignment of labor to tasks: skilled-task work falls and unskilled tasks become more common. The features of the American System acting to slow growth become stronger over time and cause human capital growth to peak sometime in the mid-21st Century, according to the simulation in row (4) of Table 2.

One prominent feature of our simulation is the pronounced hump-shaped path of per capita output growth g_y shown in Figures 6 and 7. The *simulated* paths follows the pattern of rising, then falling, *actual* growth from 1860 to 2016. Actual growth was more variable, especially after 1950, but there is an unmistakable decline in the rate of growth after 1983 which is also present in our simulated growth path, although it appears somewhat later. The simulated hump-shaped growth rate can be seen in row (1) of Table 2 where growth is 1.72% in 1860, peaks at 1.99% in 2000, falls back to 1.78% by 2200, and ultimately falls *below* its starting point to 1.63%.

We explore the underlying reasons for the hump-shaped growth path using Figure 8. This figure shows simulated growth paths for y, y_a, and y_c from 1860 to 2200. Early on, when h was small, equation (8.14) means that advanced processing is a very small part of the economy. As human capital is accumulated at increasing rates as discussed above, market size expands, and advanced intermediate firms take increasing advantage of the knowledge spillover from conventional skills to produce lower priced intermediate goods for advanced processing. Figure 8 shows that the advanced sector grows fast as firms raise wages in

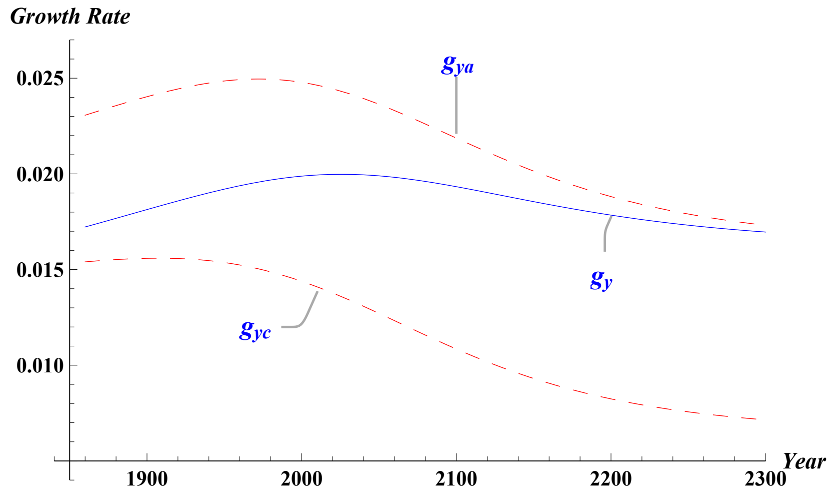

Figure 8: Simulated Growth of Conventional and Advanced Sector Per Capita Product

unskilled processing, and attract work effort away from the conventional processing. On the other hand, conventional sector growth begins to slow from the start as employment shifts to the advanced sector. We see in Figure 8 the net effect of this first growth phase is to *increase* the growth rate of aggregate per capita output.

As the share of advanced processing in aggregate output continues to expand, however, the economy enters its *second growth phase* in 2026, when growth in per capita product peaks and begins to *decrease*. The shift from conventional to advanced processing slows the growth of per capita product by reducing aggregate returns to human capital. That is, taking into account the proportional effect of h on M_a and M_c, (8.12) shows that conventional processing exhibits *increasing* returns to human capital of degree $2-\alpha$ while advanced processing shows only *constant* returns. The ongoing shift from conventional to advanced processing reduces aggregate increasing returns to human capital.

Eventually, according to (8.14) and row (5) of Table 2, advanced processing diffuses throughout the economy. Aggregate increasing returns to human capital converge from above to constant returns, and the share of specialized work in total work, row (7) of Table 2, converges to $\alpha = 0.5$. The simulation predicts that the American System will enter a *third growth phase* early in the 22nd Century, at the inflection point where growth stops falling at an *increasing rate* from its 2.00% peak in 2026 and begins falling at a *decreasing rate* as it begins to converge to the predicted perpetual 1.63% growth rate given in row (1) of Table 2.

Our simulated American System model offers an alternative perspective on the growth slowdown documented in Fernald (2015) and in the views of past and future growth in Jones (2002), Goldin and Katz (2008), Cowen (2011), Fernald and Jones (2014), Gordon (2016), Acemoglu and Restrepo (2018), and Brynjolfsson et al. (2018). The American System model

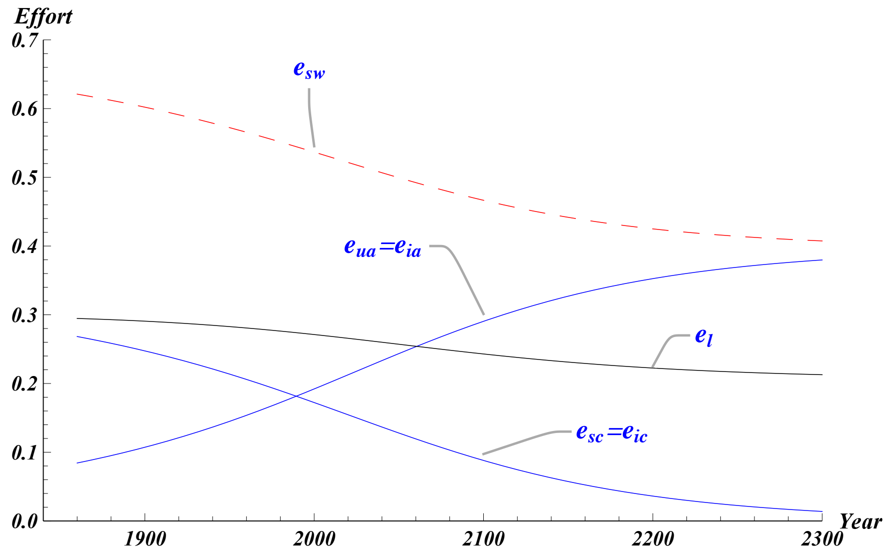

Figure 9: Simulated Effort Allocations

foresees a permanent growth slowdown rooted from the beginning in the economization of skilled labor inherent in the American System of endogenous economic growth. In other words, our calibrated simulated prediction of an eventual slowdown of per capita output growth in the U.S. has been "hiding in plain sight" for over a century.

Figure 9 shows the progress of the simulated labor allocations over time. Skilled tasks e_{sw} decline as unskilled tasks e_{ua} rise. Eventually, they are the same (because $\alpha = .5$) and fully engaged with the advanced technology. Learning time e_l stays virtually flat for nearly a century (see Table 2, as well) before declining. That means that human capital growth, which is increasing over that century in the simulation, does so because of the rise of productivity from the specialization spillover L^γ.

The simulation results may appear to be at odds with recent trends in labor and education, mainly that the share of the labor force with a college education has increased significantly in the last 40 years. However, *within* the group of college-educated workers, since 2000 the fraction of high-skill occupations has *fallen* while the fraction of low-skill (and middle-skill) occupations has *increased*.[52] There is also evidence that time spent learning is already falling. The fraction of 18 - 24 year olds enrolled in college – and the fraction of high-school students starting college – is slightly lower today than in 2010.[53] Finally, there is support for the idea that a college education is, for the average student, not as rigorous as it was in the last century and may, therefore, be a poor measure of the accumulation of

[52]See Autor (2019), specifically Figs. 5 and 8A, and adjoining discussion. Also see Beaudry et al. (2016) on the reversal in demand for cognitive skill.

[53]See the *Digest of Educational Statistics*, National Center for Education Statistics, Institute of Education Sciences, Department of Education: https://files.eric.ed.gov/fulltext/ED580954.pdf.

knowledge.[54]

Our simulation results are sensitive to the calibration we choose, but the general shape of the growth path is not. This is evident in Figure 6: all of the paths show some form of the hump that was analyzed above. The *G_ave* calibration for k and A produces a higher simulated average growth rate of 1.98%; and the year of highest growth comes sooner, around 2010 (not 2026), peaking at 2.05%. In the very long run, growth slows to 1.70% (not (1.63%).

We also calibrated (A, k) by a different method, but one that still relies only on matching operative shares. Choose a base year for the initial condition for (h/τ) then search over the grid to find the pair that minimizes $\sum_{1860}^{1950} [ops\,(t) - opa\,(t)]^2$ where $ops\,(t)$ is the simulated operative share and $opa\,(t)$ is the actual operative share. Using all eight years, one by one, for the initial condition, we took the median of the eight (A, k) calibrations. This resulted in $A = 0.0493$ and $k = 3.266$. Simulated per capita product growth on average over 1860 - 2016 was a bit lower (at 1.82%) than our baseline calibration (1.88%), but peak growth is 2.16% in 2074, which is higher and later than our baseline peak (2.00% in 2026). The growth rate converges to 1.94% in the limit. This calibrated growth path also shows a humped shape and a growth slowdown in relation to the peak, but not relative to the average to date.

To sum up, the growth slowdown is a consequence of the economization of skilled work inherent from the beginning in the American System of economic growth. The knowledge spillover is a powerful force in the model. It is the reason that rising human capital causes the advanced sector to expanded ceaselessly and move labor into tasks that do not require as much skill as before. This reduces the incentive to accumulate more human capital and slows the expansion of knowledge, output, and wages.

12 Internet: Self-Processing, Measurement, and the Counterfactual

The Internet – and the devices that have sprung up to enable its use – can be thought of as a more direct form of the advanced sector of our model. Production and consumption take place nearly simultaneously. In other words, this latest incarnation of American System advanced processing — the new information technology —facilitates the *self-processing* of goods and especially services such as health and financial services, and entertainment, by individuals alone with little special skill. A worker buys advanced intermediate inputs, like access to the Internet and associated applications, with income earned from skilled work

[54]See Arum and Roksa (2011). The College Assessment of Learning data (now CLA+ data) is available at https://cae.org/research/use-of-cae-data/.

producing advanced inputs. Then, she pays herself an implicit wage – in direct utility – for the time spent processing goods for self consumption.

To the extent that self-processing becomes more common, we would expect to see two related developments. First, self-processing would be associated with a decline of the measured labor market participation as individuals shift work from the market to the home. Second, measured output would likewise fall short of actual output (and consumption) and the short-fall would grow over time, as the processing of intermediate-Internet goods moved out of the market. This could be the beginning of a historic reversal in the age-old shift of work from home to market production. As emphasized in Goodfriend and McDermott (1995) pre-industrial urbanization allowed increasing returns to scale in cities and efficiency gains via outsourcing portions of the production of clothing, food, and shelter. Industrialization enabled more efficiency gains, and labor market participation rates rose as household services were increasingly produced and purchased outside the home. In the future as output and consumption increasingly take the form of intangible services that can be delivered electronically, it is not hard to imagine that much production will return to the home. We have already begun to observe the emergence of these trends. As noted by Fernald et al. (2017), aside from demographics, the most important determinant of the decline in labor force participation in the last 10 years has been leisure time, which "includes a large amount of TV watching and other video-based entertainment". Aguiar et al. (2017) make a similar point.

Finally, we employ our model to compare the actual path of output to a counterfactual history in which advanced processing technology had never been adopted in the United States. Appendix F solves for an economy in which only the conventional technology is in use. In such an economy, per capita output growth is given by $g_{yc} = (1-\alpha)\,\eta + (2-\alpha)\,g_{hc}$, where g_{hc} is the growth in human capital and is given by $g_{hc} = \frac{A-\rho-\gamma\eta}{1+\gamma}$.[55]

We assume that the calibrations for α, η, ρ, γ, and A are unchanged; and k becomes irrelevant in the counterfactual since M_a does not exist. In particular, we assume that the fixed cost of producing intermediates, represented by A, and the sharing of knowledge represented by γ, would have been the same if the American System had never been introduced. There are no transitional dynamics in the counterfactual conventional-processing only economy. Using our parameter values in the expressions above gives constant perpetual growth of per capita product of $g_{yc} = 0.017$ or 1.17% per annum from 1860 forward.

Figure 10 compares the log of per capita product from the simulation of the American System model (labeled $\ln y_{sim}$) to that of the counterfactual conventional-processing only economy (labeled $\ln y_{con}$). These paths are constrained to begin at the historical trend value

[55] See Equations (F.12) and (F.11) in Appendix.

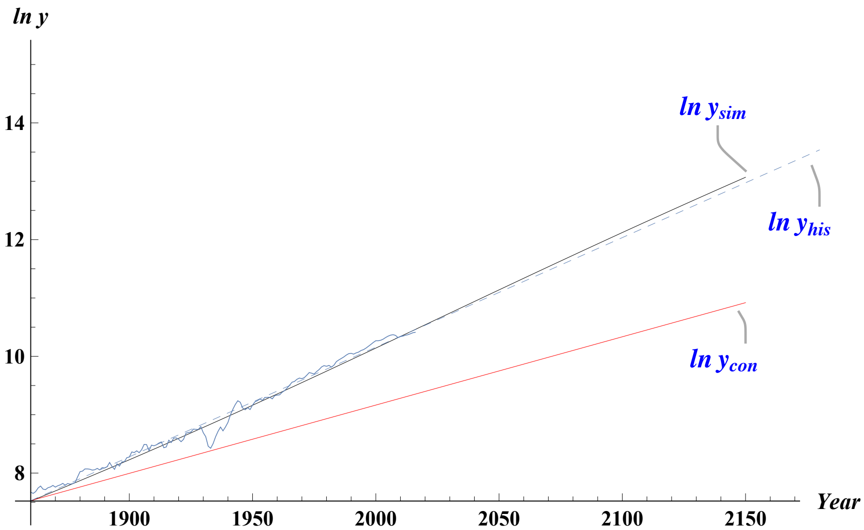

Figure 10: Simulated, History, and Counterfactual

of the log of per capita output in 1860 and are run to the year 2150. Figure 10 also displays the actual historical path of the log of y until the present and the 1.88% trend line (labeled $\ln y_{his}$) taken from Figure 5, but extended to 2200. The counterfactual $\ln y_{con}$ path, growing at 1.17% per annum, falls ever farther below the simulated $\ln y_{sim}$ path. By 2000, per capita product of the simulated American System is *2.6 times* as large as the counterfactual conventional-processing only economy, and by 2100 it is more than *6 times* as large.

We saw earlier that Great Britain, despite its lead in manufacturing and real output per capita, was not quick to adopt the innovations of the American System in the second half of the 19th Century. To date, its growth has averaged 1.32%. However, if we only measure growth between 1860 and 1913, the growth rate was even lower than our counterfactual at .089%. Over that period, the US grew at 1.67%. After 1950, it appears as though globalization has eliminated the advantage enjoyed by the US. The growth rate in the US averaged 2.00% and in Great Britain 1.98%.

13 Conclusion

Our paper has exploited the complementarity between economic history and economic theory encouraged by (Romer, 1996a,b). Historical evidence has been used in two distinct senses to inform our model. The historical narrative of the American System of Manufactures summarized in Section 2 served as the basis for specifying the theoretical model. The availability of operative share data going back to 1860 was crucial in calibrating the model.

Our principal finding may be summarized this way: what looks like steady-state per capita product growth to date has been the result of two countervailing features of the American System working through human capital accumulation. Increasing specialization, especially with respect to expanding advanced processing, enhances the specialization spillover in learning productivity, and so tends to raise growth. According to the model simulation, the net result has been to raise the growth rate somewhat from 1.72% in 1860 to 1.99% in 2000 as the model expands the share of unskilled in total work from 12% to 26% in 2000. On the other hand, as the American System shifts work effort to advanced processing, it reduces the degree of aggregate increasing returns to human capital from $2 - \alpha$ to 1, and raises the share of unskilled work effort in total work. Both factors tend to lower the productivity of human capital and to slow growth.

By interpreting nearly balanced growth to date as part of the longer-run industrial transition to a lower perpetual growth steady state, our model implies a new perspective on the widely-discussed growth slowdown. Going forward, the model simulation predicts that the American System will eventually drive out conventional processing and push the unskilled share of work up to 50%. As it does so, the growth-slowing features of the American System overtake the growth-enhancing features so that per capita product growth peaks at 2.00% in 2026, falls back to 1.93% in 2100, and eventually converges from above to 1.63%. In other words, our model predicts a long-run growth slowdown that has been "hiding in plain sight" — rooted from the beginning in the economization of skilled labor inherent in the American System of economic growth.

Another insight is that the American System raised average per capita growth to date from 1.17% to 1.88% so that by 2000 per capita product is 2.6 times larger than the counterfactual in which the American System of manufactures never existed. The American System succeeded relative to the conventional counterfactual by exploiting the (1) *specialization spillover* by which advanced intermediates had a profound impact on learning; and (2) the *knowledge spillover* by which the sharing of know-how reduced the relative cost of intermediates designed to eliminate skilled work in processing final goods.

We think our historically-grounded American System model of economic growth can provide a useful quantitative foundation within which to explore a variety of factors that have been advanced to help understand and explain growth. In particular, the important roles played in the American System model by skill-saving intermediate goods in processing final goods, learning and specialization spillovers in human capital accumulation, increasing returns to scale via specialization in intermediate goods, and the sharing of know-how on the relative cost of skill-saving intermediate inputs, all suggest that our American System model would be a fruitful environment within which to evaluate the past and future public policy for growth and welfare.

Appendices

A Zero-Profit Equilibrium in the Intermediate Sector

Given e_c, h, and N, the EQ locus in Figure 11 shows the momentary profit-maximizing x_c that a conventional intermediate-good firm would produce for a given range M_c of firms producing conventional inputs. To derive the EQ locus use (5.3), (5.1) and (5.2) to substitute for p_c and w_c in the markup condition (5.5); then eliminate e_{sc} with (2.5), and substitute for e_{ic} using (3.4) and (3.3) to yield:

$$x_c = \frac{\alpha^2 \left(e_c hN - v_0 M_c\right)}{\left(1 - \alpha + \alpha^2\right) v_1 M_c} \tag{A.1}$$

where (5.9) shows that the numerator in (A.1) is positive, and the EQ locus is downward sloping.

Given e_c, h, and N, the ZP locus in Figure 11 shows the momentary zero-profit combinations x_c and M_c in the conventional intermediate-good sector. To derive the ZP locus set profits in (5.4) to zero, and substitute for p_c, w_c, e_{sc}, and e_{ic} as above to yield:

$$x_c = \frac{\alpha e_c hN - v_0 M_c}{v_1 M_c} \tag{A.2}$$

where (5.9) shows that the numerator in (A.2) is positive, and the ZP locus is also downward sloping.

The ZP locus crosses the EQ locus once from above. Conventional intermediate firm's profits are positive below and negative above the ZP locus. Hence, the momentary free-entry zero-profit monopolistically competitive equilibrium in the conventional intermediate

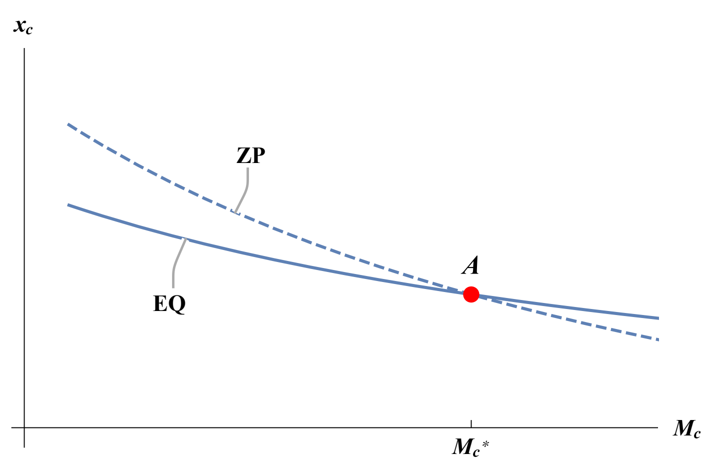

Figure 11: Monopolistically Competitive Equilibrium

sector is unique and stable at point A in Figure 11. Using the corresponding equations from Section 3 and Section 6 one can show likewise that the momentary equilibrium in the advanced intermediate sector is unique and stable.

B Intertemporal Optimization

The Hamiltonian for the problem is:

$$\begin{aligned}\mathcal{H} &= u(c) + \lambda\left(L^{\gamma} h^{1-\gamma} e_l - \eta h\right) + \\ &+ \theta_1\left(w_c h\left(e_{sc} + e_{ic}\right) + w_u e_{ua} + w_a h e_{ia} - c\right) + \\ &+ \theta_2\left(1 - e_{sc} - e_{ic} - e_{ua} - e_{ia} - e_l\right)\end{aligned}$$

where λ is the co-state, shadow price of h, we attach the constraints (7.1) and (7.2) with Lagrangian multipliers θ_1 and θ_2, and utility is assumed to be logarithmic: $u(c) = \ln c$.

The static FOC's are as follows.

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \Rightarrow \; \frac{1}{c} = \theta_1 \tag{B.1}$$

$$\frac{\partial \mathcal{H}}{\partial e_{sc}} = 0 \Rightarrow \; \theta_1 w_c h = \theta_2 \tag{B.2}$$

$$\frac{\partial \mathcal{H}}{\partial e_{ic}} = 0 \Rightarrow \; \theta_1 w_c h = \theta_2 \tag{B.3}$$

$$\frac{\partial \mathcal{H}}{\partial e_{ua}} = 0 \Rightarrow \; \theta_1 w_u = \theta_2 \tag{B.4}$$

$$\frac{\partial \mathcal{H}}{\partial e_{ia}} = 0 \Rightarrow \; \theta_1 w_a h = \theta_2 \tag{B.5}$$

$$\frac{\partial \mathcal{H}}{\partial e_l} = 0 \Rightarrow \; \lambda L^{\gamma} h^{1-\gamma} = \theta_2 \tag{B.6}$$

C Derivation of $\dot{z}$

To derive (9.7), use (7.3), along with (B.1), (9.1) ,and wage equalization, to get:

$$\frac{\dot{\lambda}}{\lambda} = \rho - \eta - \left((1-\gamma) A\Omega \left(\frac{h}{\tau}\right) e_l - \eta\right) - \frac{w_c}{c\lambda} e_{sw} \tag{C.1}$$

Multiply and divide the last term by h, use (7.2) for c, and the definition of e_{sw} to get:

$$\frac{\dot{\lambda}}{\lambda} = \rho - \eta - \left((1-\gamma) A\Omega \left(\frac{h}{\tau}\right) e_l - \eta\right) - \left(\frac{e_{sw}}{e_w}\right) \frac{1}{z} \tag{C.2}$$

The last term can be expressed as $\Phi\left(\frac{h}{\tau}\right)\frac{1}{z}$, which can be seen from (9.8) and (8.16) in the text. Adding (9.6) to (C.2) yields:

$$\frac{\dot{z}}{z} = \rho - \eta - \left((1-\gamma) A\Omega\left(\frac{h}{\tau}\right) e_l - \eta\right) - \Phi\left(\frac{h}{\tau}\right)\frac{1}{z} + A\Omega\left(\frac{h}{\tau}\right) - \frac{1}{z} - \eta \tag{C.3}$$

From here, we use (9.5) to get (9.7) in the text.

D Restricting (A, k) to Support Perpetual Growth

Two conditions on the model's parameters are necessary to ensure perpetual growth. $PG1$ guarantees that for the *initial* value of human capital $h(0)$ the ZZ locus lies above the HH locus. $PG2$ makes sure that the *endpoint* of ZZ, as $h \to \infty$, lies above the endpoint of HH. In this appendix we show that if these two conditions are satisfied, then the ZZ lies above HH for *all* $h > h(0)$.

To find $PG1$, set $h = h(0)$, equate (9.9) to (9.10) and solve for k to get:

$$k_{PG1} = \left(1 + \frac{1}{\frac{h(0)}{\tau}}\right)\left(\frac{(\rho-\eta) + \left[\gamma + \Phi\left(\frac{h(0)}{\tau}\right)\right]\eta}{A\Phi\left(\frac{h(0)}{\tau}\right)}\right)^{(1/\gamma)} - \frac{1}{\frac{h(0)}{\tau}} \tag{D.1}$$

where $\Phi\left(\frac{h}{\tau}\right)$ is defined in (9.8). *PG1* requires that $k > k_{PG1}$ in (D.1).

To find $PG2$, note that as h tends to infinity, the HH locus (9.9) becomes horizontal at:

$$\hat{z}_H = \frac{1}{Ak^{\gamma} - \eta} \tag{D.2}$$

Equate (D.2) and (9.13) in the text and solve for k to get:

$$k_{PG2} \equiv \left(\frac{\rho - \eta + (\alpha+\gamma)\eta}{\alpha A}\right)^{\frac{1}{\gamma}} \tag{D.3}$$

$PG2$ requires that $k > k_{PG2}$ in (D.3).

The $PG1$ and $PG2$ loci are shown in Figure 12. These are the same as those shown in Figure 4 in the text, with the only difference that in Figure 12, we write $PG1(h)$ as an explicit function of h. For a given value h_0, $PG1(h_0)$ shows (A, k) calibrations such that ZZ and HH *intersect* at $h = h_0$ in the phase plane. For (A, k) calibrations *above* $PG1(h_0)$ in Figure 12, ZZ lies *above* HH for h_0, while below it, ZZ is *beneath* HH at h_0. $PG2$, which does not depend on h, shows (A, k) pairs such that ZZ and HH intersect as $h \to \infty$ in the phase plane. For (A, k) points above $PG2$, ZZ ends up above HH as $h \to \infty$, while below

$PG2$, ZZ is beneath HH as $h \to \infty$ in the phase plane. Therefore, *only* for calibrations above $PG1\,(h_0)$ *and* $PG2$ — which we say reside in "Region P" in Figure 12 — does ZZ lie above HH for both h_0 and as $h \to \infty$. Such a calibration supports the motion depicted in Figure 3.

The above statements are sufficient to establish the proposition – that perpetual growth only occurs in Region P — since we can prove that ZZ and HH *never cross for* $h > h_0$ *for* (A, k) *calibrations in Region P.*

To show this, begin with an (A, k) calibration like Point Q or Point S and an initial $h = h_0$. We know that ZZ is greater than HH for either calibration at h_0 so $\dot{h} > 0$. As h rises, use (D.1), (9.11) and (9.12), to show that $PG1\,(h)$ rotates *counterclockwise,* as its intercept with $PG2$ moves *left*, in such a way that as $h \to \infty$, $PG1\,(h)$ becomes indistinguishable from $PG2$. The arrows show the movement in $PG1\,(h)$ as h rises. Therefore, for all $h > h_0$, there is no $PG1\,(h)$ locus that passes through either Q or S, so there is no $h > h_0$ for which ZZ and HH intersect. This is true for all calibrations in Region P when $h = h_0$.

We can make this argument for *any* initial $h \in (0, \infty)$. The reason is that for $h = 0$, the $PG1\,(h)$ locus coincides with the vertical line labeled CA in Figure 12. The equation of CA is $A = \rho + \gamma\eta$, which is derived by solving (D.1) for A and noting that $\Omega\,(0) = \Phi\,(0) = 1$. It follows that for any initial $h_1 > 0$, no matter how small, there is a Region P for which the argument above works. If $h_1 = 0$, Region P becomes bounded by CA on the left and $PG2$ below. If $h_1 = 1{,}000{,}000$, Region P is bounded below, effectively, by $PG2$ alone.

The above observations imply that in Region P per capita human capital growth will converge from above to a positive perpetual rate of growth. Hence, we search for (A, k) pairs in Region P in Figures 4 and 12 because model simulations using these calibrations have the potential to track historical per capita product growth.

There is one other region in (A, k) space in Figure 12 with the potential for the simulation to hit the historical per capita product growth path. This region is to the right of CA and $PG1\,(h_0)$ and to the left of $PG2$. Points like R support saddle-point growth of h. That is, Point R is in the region where (see above) ZZ is *above* HH at h_0 but ends up *below* HH as $h \to \infty$. That is, ZZ cuts HH from above for the calibration given by R, establishing a saddle-point equilibrium for a value $h_R > h_0$. It follows that at h_0, $\dot{h} > 0$ as h converges to h_R; but $\dot{h} \to 0$ as $t \to \infty$: human capital growth converges asymptotically from above to zero.

Recall that the $PG1\,(h)$ locus moves leftward and rotates counterclockwise as h grows. But h growth slows as h approaches the saddle-point equilibrium h_R, so $PG1\,(h)$ asymptotically approaches but never reaches Point R. In other words, per capita human capital growth is falling toward zero in this saddle-point regime. And if R is very close to $PG2$, falling

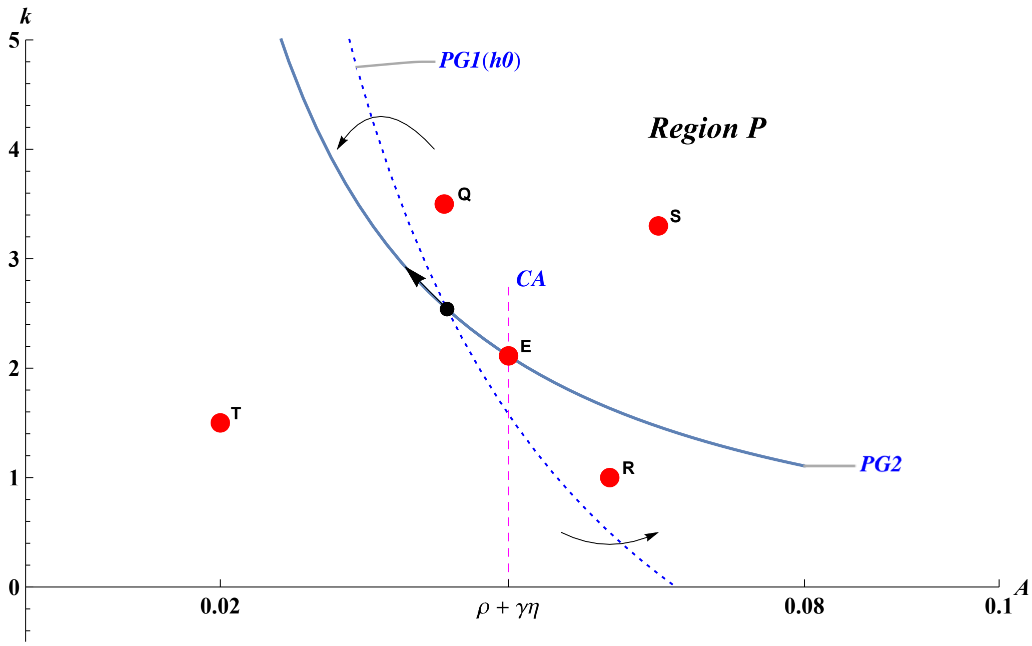

Figure 12: Growth Regimes in (k, A) Space

per capita human capital growth might be imperceptible, with the potential for the model simulation to track the nearly constant historic per capita product growth rate reasonably well.

To check this possibility, we searched for (A, k) pairs that mimic perpetual growth in the region around points like R. Using the methodology from Section 10, every calibration we tried between $PG1\left(h_0\right)$ and $PG2$ (26 primary and 3 aggregated calibrations) resulted in growth in per capita product that *fell* unmistakably over the 20th Century. In addition to this declining trend – which is not in the historical record – simulations based on these calibrations also show average growth that is between 1.6% and 1.7%, far below the actual historical average of 1.88%. We discard the saddle-point calibration because the simulated growth rates decline monotonically from 1860 to date and average only 1.65%, failing badly to track the historical record.

Finally, consider Point T. Since T is below $PG1\left(h_0\right)$ *and* $PG2$, it must be true that $\dot{h} < 0$. It is also true that h falls forever since $PG1\left(h\right)$ can never go through Point T. This follows from our observation above that, as $h \rightarrow 0$, the $PG1\left(h\right)$ locus moves rightward and clockwise until it coincides with the CA vertical line.

There is a region of Figure 12 – centered southwest of Point E but close to $PG2$ and CA – where a single calibration of (A, k) supports *two* equilibria of the dynamic system. One of these is unstable and one is saddle-point stable. This must be true because, as we vary h, we can trace out a region in which the $PG1\left(h\right)$ loci so generated cross one another. That is, there is a region where $PG1(h_1) = PG1(h_2)$ for $h_1 \neq h_2$. That region is confined to a relatively small area beneath $PG2$ and to the left of CA. In the phase plane, it corresponds to calibrations for which ZZ intersects HH twice, which is conceivable given their downward slope and convex curvature.

For these reasons, we confine our (A, k) calibrations to the region supporting perpetual growth of per capita human capital in Figures 4 and 12.

E Policy Function

Here we show how, for an arbitrary (A, k) pair, we eliminate time, and find the *policy function* $z = F\left(\frac{h}{\tau}\right)$ that corresponds to the PP path in the phase plane of Figure 3. We pointed out in Section 9 that the optimal path PP gets arbitrarily close to ZZ where it is horizontal at $\hat{z}_Z$ in (9.13) as h – and therefore $\frac{h}{\tau}$ – gets large. We construct the policy function numerically by working backwards from the point $\left(\hat{z}_Z, \frac{\hat{h}}{\tau}\right)$ where $\frac{\hat{h}}{\tau}$ is an arbitrarily large value. To eliminate time, we take the ratio of (9.7) and (9.6), where the latter is interpreted as a differential equation in $\frac{h}{\tau}$, to get $\frac{dz}{dt}/\frac{d\left(\frac{h}{\tau}\right)}{dt} = dz/d\left(\frac{h}{\tau}\right)$. The differential equation $dz/d\left(\frac{h}{\tau}\right)$ allows us to work backwards from the point $\left(\hat{z}_Z, \frac{\hat{h}}{\tau}\right)$ to trace out the optimal PP path. We solve for this path numerically to obtain the policy function $z = F\left(\frac{h}{\tau}\right)$.

Then, we substitute the policy function $F\left(\frac{h}{\tau}\right)$ for z in (9.6), which yields a differential equation in $\left(\frac{\dot{h}}{\tau}\right)$ that depends only on $\frac{h}{\tau}$, the six model parameters, and the initial value $\frac{h(0)}{\tau}$. We get $\frac{h(0)}{\tau}$ from (10.3) using operative share data from Table 1. This allows us to find the time path $\frac{h}{\tau} = W(t)$ using numerical methods. The time path of z is obtained by substitution as $z = F(W(t))$. With the state and co-state as functions of time, we can find all effort allocations, output levels, and growth rates – contingent on our arbitrary choice of (A, k).

F Counterfactual Conventional-Only Economy

We begin by specifying the constraints:

$$1 = e_c + e_l = e_{sp} + e_{ic} + e_l \tag{F.1}$$

$$c = w_b h e_c \tag{F.2}$$

The learning technology (4.2) is modified to:

$$\dot{h} = L_c^{\gamma} h^{1-\gamma} e_l - \eta h \tag{F.3}$$

where

$$L_c \equiv \frac{M_c}{\underline{e_c} N} \tag{F.4}$$

since $M_a = 0$. Learning productivity is still enhanced by the *knowledge spillover* L_c^γ from the range of conventional intermediate goods. Substituting (5.9) into (F.4) and noting that the ratio $\frac{S}{h}$ is a constant, we define:

$$A \equiv \left(\frac{L_c}{h}\right)^\gamma = \left(\frac{\alpha\left(1-\alpha\right)}{v_0}\right)^\gamma \tag{F.5}$$

First order condition (B.6) is replaced by:

$$\lambda L_c^\gamma h^{1-\gamma} = \theta_2 \tag{F.6}$$

We retain (B.1) and (B.2). The first-order conditions and the constraint (F.2) yield:

$$e_c = \frac{1}{zA} \tag{F.7}$$

It follows from the constraint (F.1) that:

$$e_l = 1 - \frac{1}{zA}$$

so that we can write the accumulation equation (F.3) as:

$$\dot{h} = \begin{cases} h\left(A - \frac{1}{z} - \eta\right) & if\ z \geq \frac{1}{A} \\ -\eta h & if\ z < \frac{1}{A} \end{cases} \tag{F.8}$$

where the inequality is the condition that determines whether or not e_l is zero. The arbitrage condition (7.3) is replaced with:

$$\dot{\lambda} = \left(\rho - \eta\right)\lambda - \frac{\partial \mathcal{H}}{\partial h} = \left(\rho - \eta\right)\lambda - \lambda\left(\left(1-\gamma\right)S^\gamma h^{-\gamma} e_l - \eta\right) - \theta_1 w_c e_w \tag{F.9}$$

To get the motion equation for z, we first note that $\frac{\dot{z}}{z} = \frac{\dot{\lambda}}{\lambda} + \frac{\dot{h}}{h}$. Using the first-order conditions, along with (F.8) and (F.9), allows us to express the motion of z as follows:

$$\dot{z} = \begin{cases} z[\rho - \eta + \gamma A] - (1+\gamma) & if\ z \geq \frac{1}{A} \\ \left(\rho - \eta\right) z - 1 & if\ z < \frac{1}{A} \end{cases}$$

The optimal policy for the representative household that satisfies the first-order conditions and transversality condition (7.4) is to set:

$$z(t) = z_c \equiv \frac{1+\gamma}{\rho - \eta + \gamma A} \tag{F.10}$$

There are no transitional dynamics in the conventional-only economy. Substituting z_c from (F.10) into (F.8) shows that the growth of per capita human capital h is constant in this dynamic equilibrium:

$$g_{hc} = \frac{A - \rho - \gamma\eta}{1+\gamma} \tag{F.11}$$

where $g_{hc} > 0$ for the parameter calibrations in Section 11.

Output per capita is given by $y_c = w_s e_c$, so substituting (F.10) into (F.7), which implies that equilibrium e_c is constant, means that equilibrium growth in w_s and y_c is identical and given from (5.11) as:

$$g_{yc} = (1-\alpha)\,\eta + (2-\alpha)\,g_{hc} \tag{F.12}$$

where g_{hc} is given in (F.11).

G Simulating the Growth Rate of Per Capita Product

This appendix shows how to construct the equation for the growth rate of per capita output that we use in the simulations.

Take the growth rate of y in (8.13):

$$g_y = (2-\alpha)\left(g_h + \frac{\dot{e}_w}{e_w}\right) + (1-\alpha)\left(\eta - \frac{\dot{Q}}{Q}\right) \tag{G.1}$$

where $Q \equiv 1 + \frac{h}{\tau}$.

To find a workable expression for (G.1), begin with g_h. For this, we use (9.6) and the policy function $z = F(h)$ introduced in Appendix E. This yields:

$$g_h = A\Omega\left(\frac{h}{\tau}\right) - \frac{1}{F(h)} - \eta \tag{G.2}$$

where Ω is given by (9.3). The policy function, recall, depends on the six calibrated parameters. From the definition of Q above, we have:

$$\frac{\dot{Q}}{Q} = \frac{g_h * \frac{h}{\tau}}{1 + \frac{h}{\tau}} \tag{G.3}$$

where we substitute g_h from (G.2).

To find $\frac{\dot{e}_w}{e_w}$ in (G.1), we begin with (9.4). This yields:

$$\frac{\dot{e}_w}{e_w} = -\frac{\dot{z}}{z} - \frac{\dot{\Omega}}{\Omega} \tag{G.4}$$

Use (9.7) to see that:

$$\frac{\dot{z}}{z} = \rho - \eta + \gamma A \Omega \left(\frac{h}{\tau}\right) - \frac{\gamma + \Phi\left(\frac{h}{\tau}\right)}{F(h)} \tag{G.5}$$

where $\Phi\left(\frac{h}{\tau}\right)$ is given by (9.8). The last term in (G.4) can be found by differentiating the expression for $\Omega\left(\frac{h}{\tau}\right)$ in (9.3). This yields:

$$\frac{\dot{\Omega}}{\Omega} = \gamma g_h \left(\frac{k * \left(\frac{h}{\tau}\right)}{k * \left(\frac{h}{\tau}\right) + 1} - \frac{\frac{h}{\tau}}{\frac{h}{\tau} + 1} \right) \tag{G.6}$$

Putting all these pieces into (G.1) gives us an expression for g_y as a function of h – and of all the parameters. Call this function:

$$g_y = \Gamma(P, h) \tag{G.7}$$

where P stands for the vector of parameters. We have found h as a function of time: $h = W(t)$, as noted in Section 10. Substituting that into the Γ function gives us:

$$g_y = G(P, t) \tag{G.8}$$

We use (G.8) to simulate the paths of g_y.

References

Acemoglu, Daron and Pascual Restrepo, "Artificial Intelligence, Automation, and Work," in Ajay Agarwal, Avi Goldfarb, and Joshua Gans, eds., *The Economics of Artificial Intelligence: An Agenda*, National Bureau of Economic Research, Inc, 2018, chapter 8.

Aguiar, Mark, Mark Bils, Kerwin Kofi Charles, and Erik Hurst, "Leisure Luxuries and the Labor Supply of Young Men," Working Paper 23552, National Bureau of Economic Research June 2017.

Arum, R. and J. Roksa, *Academically Adrift: Limited Learning on College Campuses*, Chicago, Illinois: University of Chicago Press, 2011.

Atack, Jeremy, Robert A. Margo, and Paul W. Rhode, ""Automation" of Manufacturing in the Late Nineteenth Century: The Hand and Machine Labor Study," *Journal of Economic Perspectives*, May 2019, *33* (2), 51–70.

Autor, David H., "Work of the Past, Work of the Future," *AEA Papers and Proceedings*, May 2019, *109*, 1–32.

Beaudry, Paul, David A. Green, and Benjamin M. Sand, "The Great Reversal in the Demand for Skill and Cognitive Tasks," *Journal of Labor Economics*, 2016, *34* (S1), S199–S247.

Bessen, James, "More Machines, Better Machines... or Better Workers?," *The Journal of Economic History*, 2012, *72* (1), 44–74.

Bolt, Jutta, Robert Inklaar, Herman de Jong, and Jan Luiten van Zanden, "Maddison Project Database 2018," 2018.

Boorstin, Daniel .J., *The Americans: The National Experience*, Vol. 2, Random House, 1965.

Brynjolfsson, Erik, Daniel Rock, and Chad Syverson, "Artificial Intelligence and the Modern Productivity Paradox: A Clash of Expectations and Statistics," in Ajay Agarwal, Avi Goldfarb, and Joshua Gans, eds., *The Economics of Artificial Intelligence: An Agenda*, National Bureau of Economic Research, Inc., 2018, chapter 1.

Burke, James, *Connections*, Boston: Little, Brown, and Company, 1978.

Carter, Susan B., Scott Sigmund Gartner, Michael Haines, Alan Olmstead, Richard Sutch, and Gavin Wright, eds, "The Historical Statistics of the United States, Millennial Edition Online," 2006.

Cowen, T., *The Great Stagnation: How America Ate All the Low-hanging Fruit of Modern History, Got Sick, and Will (eventually) Feel Better* New York, Dutton, 2011.

David, Paul A., "The Historical Origins of 'Open Science': An Essay on Patronage, Reputation and Common Agency Contracting in the Scientific Revolution," *Capitalism and Society*, October 2008, *3* (2), 1–106.

Desmet, Klaus, Dávid Krisztián Nagy, and Esteban Rossi-Hansberg, "The Geography of Development," *Journal of Political Economy*, 2018, *126* (3), 903–983.

Dutton, H.I., *The Patent System and Inventive Activity: During the Industrial Revolution 1750-1852*, Manchester University Press, 1984.

Ferguson, Eugene S., "On the Origin and Development of American Mechanical "Know-How"," *Mid-America Studies Association*, 1962, *3* (2), 2 – 16.

_ , "The American-ness of American Technology," *Technology and Culture*, 1979, *20* (1), 3–24.

Fernald, John G., "Productivity and Potential Output before, during, and after the Great Recession," in "NBER Macroeconomics Annual," Vol. 29, National Bureau of Economic Research, Inc, 2015, pp. 1–51.

_ **and Charles I. Jones**, "The Future of US Economic Growth," *The American Economic Review*, 2014, *104* (5), 44–49.

_ **, Robert E. Hall, James H. Stock, and Mark W. Watson**, "The Disappointing Recovery of Output after 2009," *Brookings Papers on Economic Activity*, 2017, *48* (1 (Spring), 1–81.

Goldin, Claudia and Lawrence F. Katz, "The Origins of Technology-Skill Complementarity," *The Quarterly Journal of Economics*, 1998, *113* (3), 693–732.

_ **and Lawrence Katz**, *The Race Between Education and Technology.*, Belknap Press for Harvard University Press, 2008.

Goodfriend, Marvin and John McDermott, "Early Development," *American Economic Review*, March 1995, *85* (2), 166–133.

_ **and** _ , "Industrial Development and the Convergence Question," *American Economic Review*, December 1998, *88* (5), 1277–89.

Gordon, R.J., *The Rise and Fall of American Growth: The U.S. Standard of Living since the Civil War* The Princeton Economic History of the Western World, Princeton University Press, 2016.

Hounshell, David A., *From the American System to Mass Production, 1800–1932: The Development of Manufacturing Technology in the United States*, Baltimore; Johns Hopkins University Press, 1984.

Jones, Charles I., "Time Series Tests of Endogenous Growth Models," *The Quarterly Journal of Economics*, 1995, *110* (2), 495–525.

_ , "Sources of U.S. Economic Growth in a World of Ideas," *American Economic Review*, March 2002, *92* (1), 220–239.

_ , "Growth and Ideas," in Philippe Aghion and Steven Durlauf, eds., *Handbook of Economic Growth*, Vol. 1 of *Handbook of Economic Growth*, Elsevier, 2005, chapter 16, pp. 1063–1111.

_ , "Intermediate Goods and Weak Links in the Theory of Economic Development," *American Economic Journal: Macroeconomics*, April 2011, *3* (2), 1–28.

Katz, Lawrence F. and Robert A. Margo, "Technical Change and the Relative Demand for Skilled Labor: The United States in Historical Perspective," in Leah Platt Boustan, Carola Frydman, and Robert A. Margo, eds., *Human Capital in History: The American Record*, NBER Chapters, National Bureau of Economic Research, Inc, April 2014, chapter 1, pp. 15–57.

Khan, B. Zorina, *"The Patent System in Europe and America", Chapter 2 in Democratization of Invention: Patents and Copyrights in American Economic Development, 1790-1920* , NBER Series on Long-Term Factors in Economic Development, Cambridge University Press, 2005.

_ **and Kenneth L. Sokoloff**, "History Lessons: The Early Development of Intellectual Property Institutions in the United States," *The Journal of Economic Perspectives*, 2001, *15* (3), 233–246.

Lucas, Robert E. Jr., "On the mechanics of economic development," *Journal of Monetary Economics*, July 1988, *22* (1), 3–42. available at http://ideas.repec.org/a/eee/moneco/v22y1988i1p3-42.html.

_ **and Benjamin Moll**, "Knowledge Growth and the Allocation of Time," *Journal of Political Economy*, 2014, *122* (1), 1–51.

Mayr, O. and R.C. Post, eds, *Yankee enterprise, the rise of the American system of manufactures: a symposium*, Washington, D.C.: Smithsonian Institution Press, 1981.

Mokyr, Joel, "The Contribution of Economic History to the Study of Innovation and Technical Change," in Bronwyn Hall and Nathan Rosenberg, eds., *Handbook of the Economics of Innovation*, 1 ed., Vol. 1, Elsevier, 2010, chapter 2, pp. 11–50.

Romer, Paul M., "Increasing Returns and Long-run Growth," *Journal of Political Economy*, October 1986, *94* (5), 1002–37.

_ , "Growth Based on Increasing Returns to Specialization," *American Economic Review*, October 1987, *98* (3), 56–62.

_ , "Endogenous Technological Change," *Journal of Political Economy*, October 1990, *98* (5), S71–102.

_ , "Why, indeed, in America? Theory, History, and the Origins of Modern Economic Growth," NBER Working Paper 5443, National Bureau of Economic Research, Inc 1996a.

_ , "Why, Indeed, in America? Theory, History, and the Origins of Modern Economic Growth," *The American Economic Review*, 1996b, *86* (2), 202–206.

Rosenberg, Nathan, "Introduction," in Nathan Rosenberg, ed., *The American System of Manufactures: The Report of the Committee on the Machinery of the United States 1855, and the Special Reports of George Wallis and Joseph Whitworth 1854*, University of Edinburgh Press, 1969, chapter 1.

_ , *Technology and American Economic Growth* The Academy library, New York: Harper & Row, 1972.

Sawyer, John E., "The Social Basis of the American System of Manufacturing," *The Journal of Economic History*, 1954, *14* (4), 361–379.

Sokoloff, Kenneth L. and B. Zorina Khan, "The Democratization of Invention During Early Industrialization: Evidence from the United States, 1790-1846," *The Journal of Economic History*, 1990, *50* (2), 363–378.

US Department of Labor, *Thirteenth Annual Report of the Commissioner of Labor, 1898: Hand and Machine Labor*, US Government, 1899.

Winchester, Simon, *The Perfectionists: How Precision Engineers Created the Modern World*, New York: Harper Collins, 2018.