Centre interuniversitaire de recherche
en économie quantitative

CIREQ

Cahier 01-2014

Capital-Labor Substitution, Structural Change and the Labor Income Share

Francisco ALVAREZ-CUADRADO
Ngo Van LONG and
Markus POSCHKE

CIREQ

Cahier 01-2014

Capital-Labor Substitution, Structural Change and the Labor Income Share

Francisco ALVAREZ-CUADRADO, Ngo Van LONG and Markus POSCHKE

CIREQ, Université de Montréal
C.P. 6128, succursale Centre-ville
Montréal (Québec) H3C 3J7
Canada
Téléphone : (514) 343-6557
Télécopieur : (514) 343-7221
cireq@umontreal.ca
http://www.cireqmontreal.com

Dépôt légal - Bibliothèque nationale du Canada, 2014, ISSN 0821-4441
Dépôt légal - Bibliothèque et Archives nationales du Québec, 2014
ISBN-13 : 978-2-89382-659-2

Capital-Labor Substitution, Structural Change and the Labor Income Share*

Francisco Alvarez-Cuadrado, Ngo Van Long and Markus Poschke

CIREQ and *Department of Economics, McGill University*

Abstract

Long run economic growth goes along with structural change. Recent work has identified explanatory factors on the demand side (non-homothetic preferences) and on the supply-side, in particular differential productivity growth across sectors and differences in factor proportions and capital deepening. This paper documents that there have also been differential trends in labor and capital income shares across sectors in the U.S. and in a broad set of other industrialized economies, and shows that a model where the degree of capital-labor substitutability differs across sectors is consistent with these trends. The interplay of differences in productivity growth and in the substitution elasticity across sectors drive both the evolution of sectoral factor income shares and the shape of structural change. We evaluate the empirical importance of this mechanism and the other mechanisms proposed in the literature in the context of the recent U.S. experience. We find that differences in productivity growth rates between manufacturing and services have been the most important driver of structural change. Yet, differences in substitution elasticities are key not only for understanding the evolution of sectoral and aggregate factor income shares, but also for the shape of structural change. Differences in capital intensity and non-homothetic preferences have hardly mattered for the manufacturing-services transition.

JEL Classification: O40, 041, O30

Key words: Structural change, labor income share, capital-labor substitution

*We would like to thank Cristiano Cantore, Rui Castro, Berthold Herrendorf, Joseph Kaboski, Diego Restuccia, Roberto Samaniego, Gregor Smith and Akos Valentinyi and seminar participants at the Workshop on Structural Change and Macroeconomic Dynamics, Cagliari (May 2012), the American Economic Association 2013 Meeting in San Diego, the 10th Meeting of German Economists Abroad (Konstanz 2013) and at the University of British Columbia for very helpful comments.

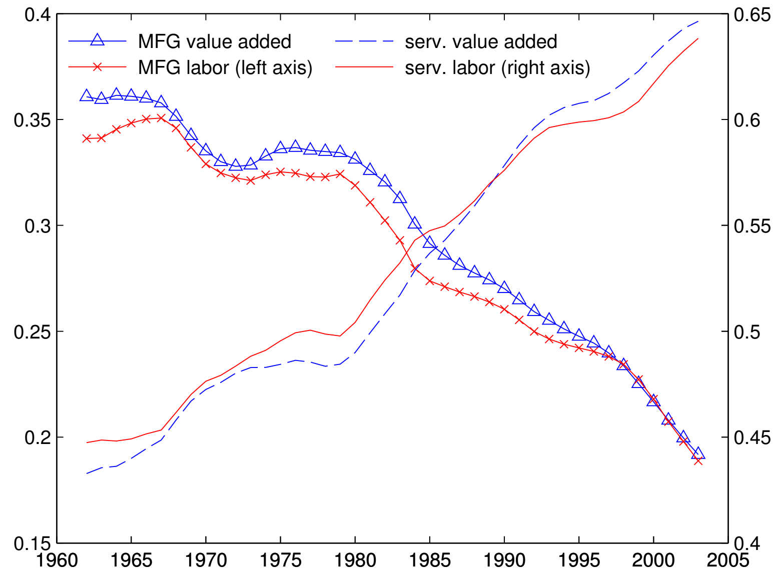
1 Introduction

In two recent contributions, Elsby, Hobijn and Şahin (2013) and Karabarbounis and Neiman (forthcoming) have documented the decline in the labor income share in the United States and in other countries. In this paper, we document that this decline was much more pronounced in manufacturing than in services, and propose an explanation that is consistent with these sectoral differences and with observed structural change from manufacturing to services over the period 1960 to 2005. The key element for explaining the observed differential evolution of sectoral labor income shares are differences in the substitutability of capital and labor across sectors. Such differences also affect what we call the "shape" of structural change – the extent of the reallocation of labor from manufacturing to services, compared to the reallocation of capital. At the same time, these differences imply that sectoral factor income shares also respond to other drivers of structural change that have been proposed in the literature, like differences in sectoral productivity growth rates. We therefore analyze the evolution of sectoral labor income shares and structural change jointly. Doing so provides insights about the sectoral production structure of the economy that are not available when studying either phenomenon in isolation. This information then allows us to quantify the importance of different channels as drivers of structural change and of change in the labor income share.

Structural change, i.e. the reallocation of economic activity across the three broad sectors agriculture, manufacturing and services, has long been known to accompany modern economic growth. (See e.g. Kuznets 1966.) As a consequence, agriculture now employs less than 2% of the workforce in the U.S., while the reallocation between manufacturing and services is still in full swing. It is the object of both policy discussion and academic interest (see e.g. Herrendorf, Rogerson and Valentinyi forthcoming, Buera and Kaboski 2012*b*). Figure 1 shows the evolution of the fraction of labor employed and the fraction of value added produced in the manufacturing and services sectors from 1960 to 2005 using data from Jorgenson's (2007) 35-sector KLEM data base. Structural change is clearly evident.

Several theoretical channels have been proposed as drivers of structural change, the most prominent being non-homothetic preferences (Kongsamut, Rebelo and Xie 2001) on the demand side and differences across sectors in productivity growth (Ngai and Pissarides 2007) or in capital intensity (Acemoglu and Guerrieri 2008) on the supply side. According to the first channel, the rise of the service sector may be due to a higher income elasticity of services demand: in a growing economy, consumers increase the share of their expenditure on ser-

Figure 1: The structural transformation in the United States, 1960-2005

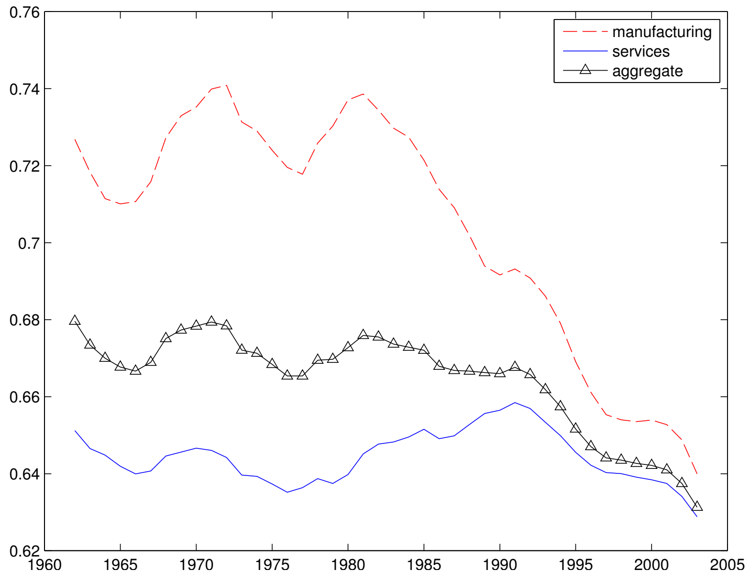


Source: Jorgenson's (2007) 35-sector KLEM data base.

vices. The second channel, also related to "Baumol's cost disease", implies that sectors with rapid productivity growth can shed resources, which are then employed in slower-growing sectors. The third channel is similar in spirit, but the difference lies in the contribution of investment to output across sectors. Given the patterns of capital intensity and sector-specific technological change, both of these channels require manufacturing and services output to be gross complements in consumption. All these channels can generate structural change in terms of both inputs and outputs. However, as we show below, none of them can generate the observed movements in factor income shares.

While aggregate factor income shares have long thought to be constant, recent work has demonstrated that in the last few decades, the labor income share has declined substantially. This is illustrated in Figure 2 using data from Jorgenson (2007). The trend shown here is in line with the highly detailed analysis by Elsby et al. (2013) for the U.S., and with the findings by Karabarbounis and Neiman (forthcoming) for a broad set of countries. Figure 2 also shows that the aggregate pattern is driven by differential developments at the sectoral level: Jorgenson's KLEM data reveal that while in 1960, the labor share of income in manufacturing exceeded the aggregate labor income share, while the reverse was true for services, this pattern has changed substantially over the last 45 years. In this time, the labor share of income in manufacturing has declined substantially, and the one in services slightly. The decline has been pronounced; the labor income share in manufacturing has declined on average by 2.1 percentage points per decade, and the aggregate one by 1.2 percentage points. Section 2 describes the data underlying this pattern in more detail and shows that correcting income shares following Valentinyi and Herrendorf (2008) leaves these broad patterns intact. More than that, we show using EUKLEMS data that the labor income share has declined in all but three of a set of 16 industrialized economies, with the labor income share in manufacturing declining by even more in most of them.[1]

Figure 2: The labor income share in the U.S.



Sources: Jorgenson's (2007) 35-sector KLEM data base. The figure shows 5-year moving averages.

Unfortunately, the conventional theories of structural changes mentioned above are unable to account for the changes in income shares. The key reason for this is that they assume that sectoral production functions are Cobb-Douglas.[2] In this case, if the elasticity of output with respect to capital differs across sectors, the aggregate labor income share may change with structural change, but *sectoral* labor income shares are forced to be constant. As we show below, potential changes in the aggregate income share that could be ascribed to struc-

[1]Karabarbounis and Neiman (forthcoming) have shown some similar results. However, they provide less country-level detail and do not contrast the evolution of the labor income share in manufacturing with that in services. In earlier work, Blanchard (1997) and Caballero and Hammour (1998) have documented medium term variation in the labor income share in some continental European economies and linked them to labor market rigidities.

[2]Kongsamut et al. (2001) assume instead that sectoral production functions are proportional, which is similarly restrictive.

tural change alone are quite limited. These theories thus have no chance of replicating the observed changes in labor income shares.

To address this issue, we propose a theory where manufacturing and services differ in the substitutability of capital and labor. The intuition is as follows: technological progress driving modern growth motivates capital accumulation. The evolution of factor income share depends on the substitution elasticity in a sector, and on whether effective capital (driven by capital accumulation) or effective labor (driven by growth in labor-augmenting productivity) becomes relatively more abundant. For instance, if the two inputs are gross complements in production and capital per unit of effective labor declines, then the wage to rental ratio falls more than proportionally, resulting in a decline of the labor income share.[3] Cross-sectoral differences in the substitutability of capital and labor also have implications for structural change. Notably, they affect what we call the "shape" of structural change: the extent of capital versus labor reallocation. This occurs because with differences in substitutability, the sector with higher substitutability (the "flexible sector") moves towards using the factor that becomes more abundant more intensively. One set of conditions under which the model can replicate the patterns in the data is the following: i) capital and labor are gross complements in production, ii) the manufacturing sector is the more flexible sector, and iii) effective labor becomes abundant relative to capital.

To take this new channel seriously, it is however necessary to explore its interaction with other potential drivers of structural change. Therefore, we extend the model to include the main drivers of structural change that have been proposed in the literature, i.e. not only difference in substitutability, but also non-homothetic preferences, differences in capital intensity and differences in the speed of technical change across sectors. We then calibrate the model to the U.S. experience and use it to assess the relative importance of these different potential sources of structural change.

This quantitative exercise yields rich results. The two most salient ones are the almost complete unimportance of non-homotheticities in preferences for structural change from manufacturing to services[4] and the quantitatively dominant role of differences in productivity growth rates across sectors. Differences in substitutability of inputs are somewhat important for the amount of structural change. More importantly, they are essential for capturing the observed changes in labor income shares, and for the shape of structural change.

[3]A similar mechanism is at work in the literature on capital-skill complementarity, for instance Krusell, Ohanian, Ríos-Rull and Violante (2000). In that context, the gross complementarity (substitutability) between equipment and skilled (unskilled) labor leads to an increase in the skill premium in response to capital accumulation.

[4]This may well be different for structural change out of agriculture.

A comparison of predictions of our calibrated model with CES production functions with a version with Cobb-Douglas production functions in the two sectors brings out the key results clearly: while a model with Cobb-Douglas production functions and differences in productivity growth across sectors can account well for both the amount and the shape of structural change, it is by construction hopeless at matching the evolution of factor income shares. A model with CES production functions and the right combination of cross-sectoral differences in factor substitutability and in productivity growth, in contrast, can capture those, too. These two cross-sectoral differences interact, and only considering the additional evidence from labor income shares allows distinguishing whether Cobb-Douglas or CES production functions provide a better fit.[5]

Apart from the recent work on the labor income share cited above, our paper is also closely related to recent work on structural change, in particular Buera and Kaboski (2009, 2012*a*, 2012*b*), Herrendorf, Herrington and Valentinyi (2013) and Swiecki (2013). Buera and Kaboski (2009) conduct a first quantitative evaluation of two potential drivers of structural change: productivity growth differences and non-homothetic preferences. One of their findings is that conventional models have trouble fitting the "shape" of structural change. However, this mainly appears to be a concern for the period before 1960, and the movement out of agriculture. After that, asymmetries in the sectoral reallocation of capital, labor and value added are more limited, and we address them in our CES framework. Buera and Kaboski (2012*a*, 2012*b*) also analyze the "Rise of the Service Economy," but focus on skill differences across different segments of the service sector and on differences in scale across sectors, respectively. Swiecki (2013) analyzes a set of four drivers of structural change that partly overlaps with those we consider. Like the two papers just cited, he does not consider implications for factor income shares. In line with our results, he stresses the importance of differences in sectoral growth rates for structural change. In addition, he concludes that non-homotheticities matter mostly for structural change out of agriculture. Trade, a factor that we abstract from, matters only for some individual countries. Finally, Herrendorf, Herrington and Valentinyi (2013) estimate CES production functions for the aggregate U.S. economy and for the sectors agriculture, manufacturing and services. Differently from our approach, they do so by sector, not jointly, and do not focus on the implications for the evolution of aggregate or sectoral factor income shares. Overall, our work appears to be the

[5] Ríos-Rull and Choi (2009) document the evolution of the labor income share over the business cycle and evaluate the ability of a variety of real business cycle models to account for its response to productivity shocks. As in our case, their preferred specification includes a CES technology where capital and labor are gross complements.

first contribution linking the evolution of factor income shares and structural change.

In the next section, we provide more detail on the evolution of sectoral factor income shares. In Section 3, we illustrate the workings of the mechanism in a simple model. At that stage, to clarify the mechanism, we assume that apart from the difference in substitutability, sectors are completely symmetric. In Section 4, we evaluate the power of the mechanism quantitatively. In doing so, we also allow for the other three drivers of structural change discussed above, and thus allow for multiple dimensions of heterogeneity between sectors. Finally, Section 5 concludes, while the appendices contain additional derivations and information on data sources.

2 The evolution of factor income shares in manufacturing and services

Recent literature has documented in detail the decline in the aggregate labor income share in the United States (Elsby et al. 2013) and across countries (Karabarbounis and Neiman forthcoming). In this section, we document that this decline was to a very large extent driven by a decline of sectoral labor income shares, in particular of that in manufacturing. We start by showing this for the U.S., and then add evidence for a cross-section of 16 other developed countries.[6]

2.1 Sectoral labor income shares in the United States

Figure 2 showed the evolution of the labor income share in manufacturing and services in the United States from 1960 to 2005. Over this period, the aggregate labor income share declined by 1.2 percentage points per decade, the one in manufacturing by 2.1 percentage points per decade, and that in services by 0.5.

The measure of the labor income share shown in Figure 2 is computed as total labor compensation in a sector divided by the sum of the value of capital services and labor compensation in that sector. We call this measure "naive" because it ignores links across industries. For example, food manufacturing uses inputs from the transportation industry which in turn are produced using capital, labor and other intermediate inputs. Therefore, the true labor income share in the production of manufacturing value added also depends on "naive" labor income shares in sectors producing intermediate inputs used in manufacturing.

[6] Elsby et al. (2013) draw a similar conclusion for the U.S. since 1987. Karabarbounis and Neiman (forthcoming) also show cross-industry developments (their Figure 5), but provide less country-level detail and no value for a broad service sector.

We thus also compute a measure that takes these links into account, following Valentinyi and Herrendorf (2008) (see the Appendix A.1 for details).

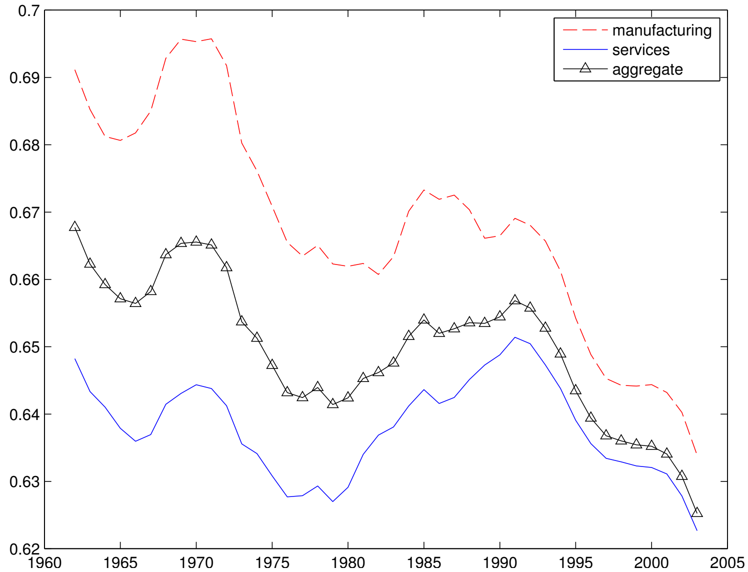
To do so, we use data from Jorgenson's (2007) 35-sector KLEM database. These data are based on a combination of industry data from the BEA and the BLS and are described in detail in Jorgenson, Gollop and Fraumeni (1987), Jorgenson (1990) and Jorgenson and Stiroh (2000). They cover 35 sectors at roughly the 2-digit SIC level from 1960 to 2005. The raw data are accessible at `http://hdl.handle.net/1902.1/10684`. The data contain, for each industry and year, labor compensation, the value of capital services, and the value of intermediate inputs by source industry. These add up to the value of gross output in that industry. Knowing the input-output structure allows computing the labor income share in production of sectoral value added.[7]

Results for the labor income share in value added are shown in Figure 3. Results using this measure are different in details, but the overall patterns remain unchanged. The main difference is that by this measure, the labor income share in manufacturing, while still exceeding that in services, is not as high as by the naive measure, since it takes into account that manufacturing value added also uses inputs from other sectors, with lower naive labor income shares. By this measure, the labor income share in manufacturing has fallen by 1.4 percentage points per decade, the one in services by 0.6 percentage points per decade, and the aggregate one by 1 percentage points per decade. The last two values are very close to the counterparts for the naive labor income share. Only the manufacturing number changes somewhat. However, the qualitative pattern is clearly maintained, with a decline in the aggregate labor income share driven largely by a decline in the labor income share in manufacturing.

Could the change in the aggregate labor income share be purely due to structural change? After all, the labor income share in manufacturing is higher than that in services, so that structural change from manufacturing to services will reduce the aggregate labor income share. A simple calculation shows that this channel is quantitatively minor. The aggregate labor income share is a value-added weighted average of the sectoral labor income shares. The share of manufacturing in value added has declined by almost 25 percentage points over the sample period (see Figure 1). Given a gap of 4.3 percentage points between the initial labor income shares in manufacturing and in services, this implies that structural change could account for a change in the aggregate labor income share of 1.075 percentage

[7]For the definitions of our sectors manufacturing and services, see Appendix A.2. Numbers reported here for the aggregate economy refer to the aggregate of manufacturing and services. Results are not sensitive to details of sector definition like the treatment of utilities, government or mining.

Figure 3: Non-naive labor income share, by sector, 1960-2005



Sources: Jorgenson's (2007) 35-sector KLEM data base. The figure shows 5-year moving averages.

points over the period 1960 to 2005. This amount corresponds approximately to the average change in the labor income share over a single decade. Thus, structural change on its own can account for less than a quarter of the observed change in the aggregate labor income share.

2.2 Other countries

Changes in the labor income share have not been limited to the United States. In this section, we provide evidence from 16 other developed economies. We do so using EU KLEMS data. The March 2011 data release, available at `http://www.euklems.net`, contains data up to 2007 for 72 industries. We again aggregate those up to manufacturing and services. While this data contains a lot of industry detail, it does not contain an input/output structure, so we are limited to computing the naive labor income share, computed as compensation of employees over value added. However, we have seen above that developments in the U.S. have been qualitatively and even quantitatively similar for both measures of the labor income share. Karabarbounis and Neiman (forthcoming) have also used this data to document industry-level changes in the labor income share, but have not studied the manufacturing-services division.

We define sectors as for the U.S.. We use countries with at least 15 observations.[8] This leaves us with 16 countries.

Table 1: The labor income share by sector and country, 1970-2007

	Manufacturing		Services		Aggregate			
country	level	change	level	change	level	change	SC	N
AUS	0.707	-2.5	0.561	-0.8	0.601	-1.9	-0.8	38
AUT	0.719	-6.0	0.589	-4.1	0.633	-5.0	-0.5	38
BEL	0.673	1.0	0.537	3.2	0.581	2.0	-0.7	38
DNK	0.751	-1.7	0.561	2.3	0.615	0.8	-0.4	38
ESP	0.609	0.5	0.584	-4.6	0.587	-3.0	-0.2	38
FIN	0.645	-5.3	0.568	-2.9	0.597	-4.0	-0.2	38
FRA	0.700	0.7	0.611	-4.9	0.632	-3.7	-0.4	38
GER	0.755	-0.9	0.551	-1.1	0.629	-1.8	-1.0	38
GRC	0.748	-3.4	0.410	0.8	0.505	-1.7	-1.6	38
HUN	0.612	-10.9	0.570	2.4	0.584	-2.0	-0.0	13
ITA	0.720	-0.7	0.625	-3.6	0.655	-3.0	-0.4	38
JPN	0.578	-0.1	0.562	-5.1	0.566	-3.1	-0.1	37
NLD	0.680	-2.0	0.644	-2.0	0.654	-2.0	-0.2	38
PRT	0.681	-2.4	0.466	4.5	0.523	2.8	-0.1	37
SWE	0.756	-7.9	0.559	-1.0	0.625	-3.6	-0.4	38
UK	0.758	1.3	0.659	-0.7	0.691	-1.2	-0.8	38

Note: N denotes the number of observations. The level is the sample average for a country. Changes are in units of percentage points per decade. SC = change in the value added share of manufacturing × (average LISM − average LISS). This is how much change in the aggregate labor income share could be explained by structural change alone. The series for Hungary excludes the first three observations (1992-1994), over which the labor income share in manufacturing collapses from 82 to 66% in three years.

The evolution of the LIS in these countries is shown in Table 1. The aggregate labor income share declines in all but three countries, by 2.1 percentage points per decade on average. The labor income share in manufacturing declines in all but 4 countries, by 2.5 percentage points per decade on average. The labor income share in services declines in all but five of the countries in our sample, by 1.5 percentage points per decade on average.[9]

[8] This essentially implies excluding transition economies. We also exclude Korea, because it starts the manufacturing to services transition only part-way through the sample. We also exclude Ireland and Luxembourg due to data problems. The remaining 16 countries are Australia, Austria, Belgium, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, the Netherlands, Portugal, Spain, Sweden and the United Kingdom.

[9] These trends are statistically significant at the 5% level in all, all but one, and all but two of the cases,

Overall trends are thus similar to those observed in the U.S., and even somewhat stronger. The overall ranking of sectors is also similar, with substantially stronger declines of the labor income share in manufacturing.

Structural change by itself cannot even account for 10% of the observed change in the labor income share in most countries. It could explain more than half of the change in the aggregate labor income share in only 3 countries, and more than a quarter in only four. This is of course not surprising, given that there are substantial dynamics in the sectoral labor income shares. In 10 of the 16 countries, the labor income share in manufacturing declines by more than the one in services, with the aggregate sandwiched in between.

2.3 The manufacturing labor income and employment shares

Over time, both the employment share and the labor income share in manufacturing decline. Figure 4 plots them together for all the countries in our sample, with the fraction of employment in manufacturing on the x-axis and the labor income share in manufacturing on the y-axis. For ease of reading and to reduce clutter, the data is shown for four groups of countries. Time is implicitly present in the graph, as over time, manufacturing employment declines, moving each economy from the right to the left.[10] Table 2 shows average per decade changes in the labor income shares and in employment in manufacturing by country.

It is clear from both the table and the figure that in most countries, the employment and labor income shares in manufacturing have moved in the same direction since 1970. This is also clear from the regression lines estimated on country groups that are drawn in the figure. All of them are significant at the 5% level. For individual countries, the association is positive for all countries but three, and is significant at the 5% level for all but two of these. The relationship is also highly significant ($p < 1\%$) in the entire sample. In the entire sample, a regression including country fixed effects shows that for every reduction in the employment share in manufacturing by 1 percentage point, the labor income share in manufacturing has declined by 0.26 percentage points.

To summarize, there have been substantial declines in the labor income share across countries, driven mainly by declines in the labor income shares within sectors. These developments have taken place at the same time as structural change out of manufacturing. This

respectively.

[10]Figure 6 in the Appendix shows the same with the aggregate labor income share. The patterns are very similar and slightly stronger.

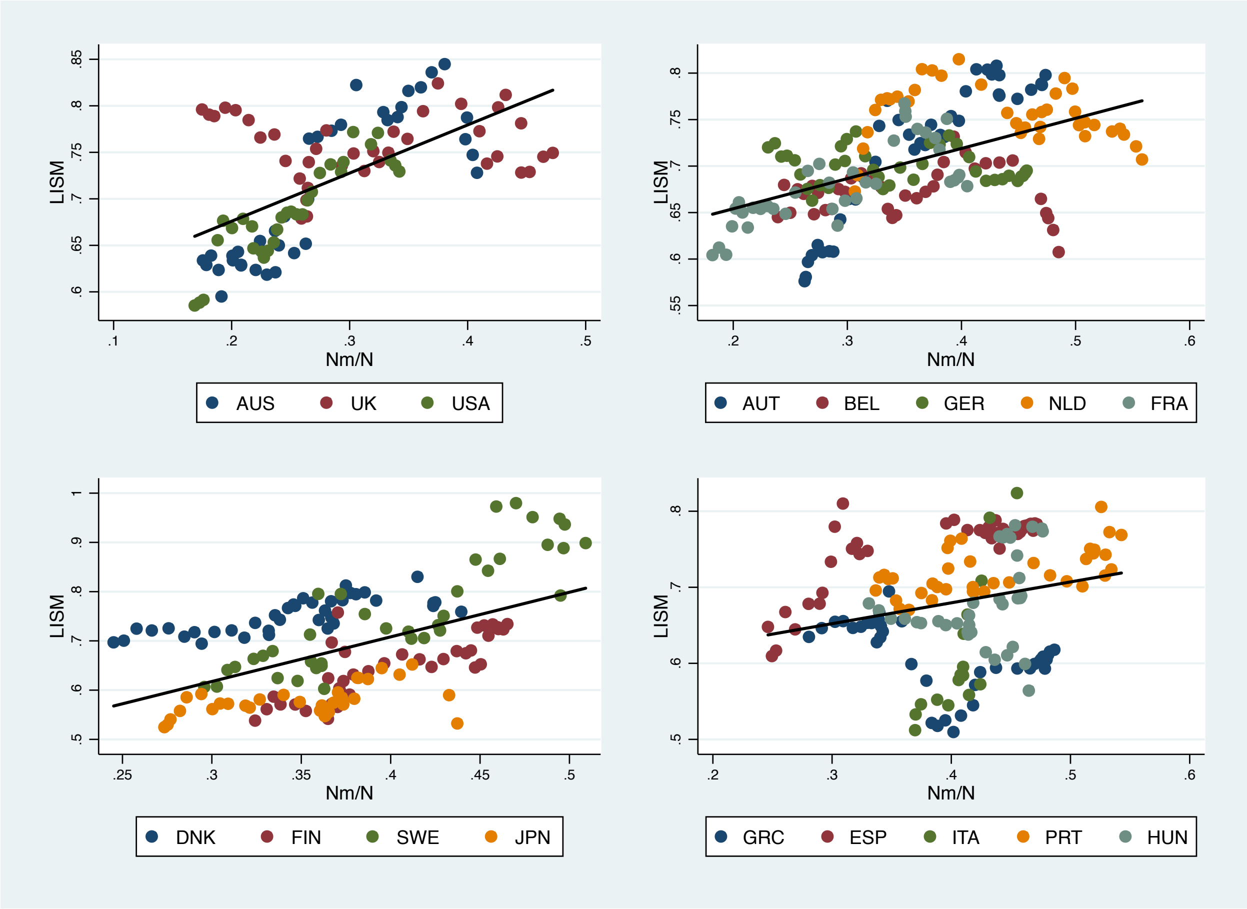


Figure 4: The joint evolution of the labor income share in manufacturing (LISM) and the employment share of manufacturing, 1970-2007.

suggests that there may be a common driver for both phenomena. In the model we present below, structural change and the change in sectoral labor income shares are driven by similar forces and occur together on the growth path of an economy. While each phenomenon is important in its own right, there are substantial gains from joint analysis. In particular, the evolution of sectoral labor income shares contains information that has hitherto been disregarded in the structural change literature, so that integrating their evolution in the analysis can yield additional insights on structural change.

3 The elasticity of substitution, the labor income share and structural change

In this section, we show how CES sectoral production functions can help to fit the data patterns documented in the previous section, and then explore the theoretical implications

Table 2: The joint evolution of the labor income share and the employment share of manufacturing, 1970-2007

	Change in... (pts per decade)				
country	L_m/L	LISM	LISS	LIS	N
AUS	-6.3	-2.5	-0.8	-1.9	38
AUT	-5.7	-6.0	-4.1	-5.0	38
BEL	-6.8	1.0	3.2	2.0	38
DNK	-5.3	-1.7	2.3	0.8	38
ESP	-5.6	0.5	-4.6	-3.0	38
FIN	-3.6	-5.3	-2.9	-4.0	38
FRA	-6.1	0.7	-4.9	-3.7	38
GER	-6.8	-0.9	-1.1	-1.8	38
GRC	-6.0	-3.4	0.8	-1.7	38
HUN	-5.7	-10.9	2.4	-2.0	13
ITA	-5.3	-0.7	-3.6	-3.0	38
JPN	-4.8	-0.1	-5.1	-3.1	37
NLD	-6.0	-2.0	-2.0	-2.0	38
PRT	-3.3	-2.4	4.5	2.8	37
SWE	-5.8	-7.9	-1.0	-3.6	38
UK	-8.0	1.3	-0.7	-1.2	38

Note: N denotes the number of observations. All changes are in units of percentage points per decade, for the period 1970-2007. The series for Hungary excludes the first three observations (1992-1994), over which the labor income share in manufacturing collapses from 82 to 66% in three years.

of cross-sector differences in the elasticity of substitution between capital and labor for the evolution of factor income shares and for structural change in a growing economy. To focus on this channel only, we entirely abstract from other drivers of structural change in this section. In Section 4, we allow for three more drivers of structural change and evaluate the relative importance of all four channels, in a model that is disciplined to also match the evolution of sectoral factor income shares.

3.1 The elasticity of substitution and the labor income share: What does it take to match the data?

Consider a competitive economy with two productive sectors, $i = 1, 2$. Each sector produces output combining capital, K_i, labor, L_i, and sector-specific labor-augmenting productivity,

A_i, according to a constant elasticity of substitution (CES) production function,[11]

$$Y_i = \left[\alpha_i K_i^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \alpha_i)(A_i L_i)^{\frac{\sigma_i - 1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i - 1}} \tag{1}$$

where $\alpha_i \in (0, 1)$ is a distributional parameter which determines the importance of each input and $\sigma_i \in (0, \infty)$ is the elasticity of substitution between capital and labor. This elasticity was first introduced by Hicks in *The Theory of Wages* (1932) in an attempt to explore the functional distribution of income in a growing economy. It is defined as

$$\sigma_i \equiv \frac{\partial k_i}{\partial \omega} \frac{\omega}{k_i}, \tag{2}$$

where k_i is the capital-labor ratio in sector i and ω is the ratio of factor prices, which is common across sectors.

We can use this production technology to evaluate what is required to accommodate the empirical patterns described in the previous section and to explore their implications for existing theories of structural change. Note that previous analyses of structural change have typically been restricted to Cobb-Douglas technologies in both sectors, i.e. $\sigma_1 = \sigma_2 = 1$ (see e.g. Ngai and Pissarides 2007, Acemoglu and Guerrieri 2008, Buera and Kaboski 2009). In this case, under perfect competition in factor markets, the capital income share $s_{iK} = \alpha_i$ and the labor income share $s_{iL} = 1 - \alpha_i$ are constant, although potentially different across sectors. The aggregate labor income share, s_L, can still vary, but only as a result of changes in the sectoral composition of the economy. This aggregate share is a weighted average of sectoral shares,

$$s_L = \frac{p_1 Y_1}{Y}(1 - \alpha_1) + \frac{p_2 Y_2}{Y}(1 - \alpha_2), \tag{3}$$

where p_i is the sectoral output price and Y is aggregate output.

How well can a model with sectoral Cobb-Douglas technologies account for the evolution of the aggregate labor income share? A simple accounting exercise yields results in line with those presented in the previous section. In the U.S. economy, the average labor income share in manufacturing exceeded that in services by about 3 percentage points over the period 1960 to 2005. At the same time, the manufacturing share of value added has declined by roughly 25 percentage points over the last fifty years. Hence, structural change alone can generate a decrease in the aggregate labor income share of the order of 0.75 percentage

[11] Since we will explore the implications of this model for a growing economy we restrict technological change to be labor-augmenting in this section only. It is well known from one-sector growth models that when the production function is not Cobb-Douglas, this restriction is required for the existence of a balanced growth path.

points. This is barely 13% of the decrease observed in the data.[12] This analysis illustrates one important limitation of existing theories of structural change. Models of structural change that rely on sectoral Cobb-Douglas technologies, such as Ngai and Pissarides (2007) or Acemoglu and Guerrieri (2008), are not consistent with the evolution of either the sectoral or the aggregate labor income share. At the same time, models of structural change where sectoral technologies are identical up to a constant, such as Kongsamut et al. (2001), are not consistent with the fact that sectoral labor income shares have evolved differently.

If sectoral production functions are CES, in contrast, sectoral factor income shares can change over time. In this case, the ratio of factor income shares implied by (1) is

$$\frac{s_{iK}}{s_{iL}} = \frac{\alpha_i}{1-\alpha_i}\left(\frac{K_i}{A_i L_i}\right)^{\frac{\sigma_i - 1}{\sigma_i}} \tag{4}$$

which, in principle, can reconcile the variation in aggregate and sectoral factor income shares. Specifically, if capital and labor are gross complements, i.e. $\sigma_i < 1$, as most empirical evidence suggests (see León-Ledesma, McAdam and Willman (2010) for a recent review of empirical estimates), then the labor income share in sector i decreases (increases) as long as the amount of capital per unit of effective labor in sector i decreases (increases). The intuition is simple: If capital and labor are gross complements in production, a decrease in capital per unit of effective labor induces a more than proportional decrease in the ratio of the wage to the rental rate, and therefore a reduction in the sectoral labor income share. In this case, the aggregate labor income share may decrease not only as a result of structural change, but also due to declining sectoral labor income shares.[13]

Differences in the evolution of sectoral income shares can then be due to differences in the growth of $K_i/(A_i L_i)$, to differences in σ_i, or both. This suggests that one should allow for differences in the sectoral elasticities of substitution. We therefore go on to analyze a two-sector model with cross-sectoral differences in the elasticity of substitution.

3.2 The elasticity of substitution, the labor income share, and structural change: A simple two-sector model

Time is continuous and final output is produced under perfect competition combining the output of two intermediate-good sectors, $Y_1(t)$ and $Y_2(t)$, according to a Cobb-Douglas

[12] The analysis in the previous section uses values for the sectoral labor income shares in 1960, where the gap in these shares across sectors was larger. In this analysis we use the average values for the sectoral labor income shares over the whole sample period. This explains the difference in results.

[13] A falling labor income share is also consistent with $\sigma_i > 1$ and rising $K_i/(A_i L_i)$, as in Karabarbounis and Neiman (forthcoming). However, this scenario runs counter to the fact that virtually all estimates of aggregate σ reported in León-Ledesma et al. (2010) lie below 1.

technology,

$$Y(t) = (Y_1(t))^{\gamma} (Y_2(t))^{(1-\gamma)} \tag{5}$$

where $\gamma \in (0,1)$ is the elasticity of output with respect to the first intermediate good, $Y_1(t)$. The two intermediate goods are produced competitively according to the CES production function (1). Sectoral production combines capital and labor, $K_i(t)$ and $L_i(t)$, with a common level of labor-augmenting productivity, A, that increases at the exogenous rate $g(A)$. We assume labor to be constant, and normalize L to one.[14]

Let the instantaneous utility function take the familiar CRRA specification, with constant intertemporal elasticity of substitution of consumption $1/(1-\mu) > 0$. Then, the discounted life-time welfare of the representative household is

$$\int_0^{\infty} e^{-\beta t} \left(\frac{C(t)^{\mu}}{\mu} \right) dt \tag{6}$$

where β is the rate of time preference and per capita and aggregate variables coincide due to our normalization of the labor force.

Capital accumulates out of final output and depreciates at a rate δ. The aggregate resource constraint requires that the sum of consumption, $C(t)$, and gross investment, $I(t)$, be equal to output of the final good. Therefore, the law of motion of capital becomes

$$\dot{K}(t) = I(t) - \delta K(t) \equiv Y(t) - C(t) - \delta K(t), \tag{7}$$

where the dot denotes the change in a variable.

Finally, both inputs are fully utilized,

$$\begin{aligned} L_1(t) + L_2(t) &= 1, \\ K_1(t) + K_2(t) &= K(t). \end{aligned} \tag{8}$$

In order to isolate the effect of differences in the sectoral elasticity of substitution on structural change, we impose a high level of symmetry in our model and allow only this elasticity to differ across sectors. Other potential drivers of structural change are not active: First, preferences are homothetic. Second, the sectoral levels of labor-augmenting productivity are equal across sectors. Third, we also equate the distributional parameters α_i across sectors. Finally, our Cobb-Douglas aggregator (5) implies that both sectoral outputs are required for the production of the final good and therefore for the accumulation of capital.

[14]See Alvarez-Cuadrado and Long (2011) for a version of this model with constant population growth. The implications for structural change remain unchanged.

These assumptions rule out the sources of structural change explored by Kongsamut et al. (2001), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008), allowing us to focus on the novel mechanism introduced by sectoral differences in capital-labor substitutability. We will relax these assumptions in the numerical analysis in Section 4.

The definition of equilibrium is standard. An equilibrium consists of a set of paths for input and intermediate goods prices, employment and capital allocations, and consumption and saving choices such that firms maximize profits, the representative household maximizes life-time welfare, and markets clear. Since markets are complete and competitive, the competitive equilibrium is equivalent to the solution of a social planner's problem of maximizing lifetime utility of the representative household (6) subject to (8), initial conditions for all the state variables, and the aggregate resource constraint, obtained by replacing (1) for both sectors in (5) and combining it with (7).

We break down the solution of our problem into two steps. First, given the vector of state variables at any point in time, $(K(t), L, A(t))$, the allocation of factors across sectors is chosen to maximize final output, (5). This is the static problem. Second, given factor allocations at each date, the time paths of the capital stock and consumption are chosen to maximize life-time welfare. The solution to the dynamic problem characterizes this process.

3.2.1 The static problem

Let us denote the rental rate, the wage rate, the prices of the two intermediate goods, and the price of the final good by $R \equiv r + \delta$, w, p_1, p_2 and P respectively.[15] We solve for the demand functions for the intermediate goods under perfect competition by maximizing output (5) subject to the zero profit condition $p_1Y_1 + p_2Y_2 = PY$. The optimality conditions for this problem imply the following demand functions and final good price. We normalize P to one.

$$Y_1 = \gamma \frac{P}{p_1} Y \tag{9}$$

$$Y_2 = (1-\gamma) \frac{P}{p_2} Y$$

$$P = p_1 \frac{Y_1}{Y} + p_2 \frac{Y_2}{Y} = \gamma p_1 + (1-\gamma) p_2 \equiv 1 \tag{10}$$

At any point in time, free mobility of capital and labor implies the equalization of the marginal value products across sectors,

$$p_1 \alpha \left(\frac{Y_1}{K_1}\right)^{\frac{1}{\sigma_1}} = p_2 \alpha \left(\frac{Y_2}{K_2}\right)^{\frac{1}{\sigma_2}} = R \tag{11}$$

[15]We drop time indicators when there is no risk of ambiguity.

$$p_1 (1-\alpha) \left(\frac{Y_1}{L_1}\right)^{\frac{1}{\sigma_1}} A^{\frac{\sigma_1 - 1}{\sigma_1}} = p_2 (1-\alpha) \left(\frac{Y_2}{L_2}\right)^{\frac{1}{\sigma_2}} A^{\frac{\sigma_2 - 1}{\sigma_2}} = w. \tag{12}$$

The solution to the static problem amounts to the determination of the sectoral allocations of capital and labor. It will prove useful to define the shares of capital and labor allocated to sector 1 as

$$\kappa (t) \equiv \frac{K_1 (t)}{K (t)} \quad \text{and} \quad \lambda (t) \equiv \frac{L_1 (t)}{L} = L_1 (t) . \tag{13}$$

In order to obtain analytical results we restrict sector 1 to be Cobb-Douglas and set $\sigma_1 = 1 \neq \sigma_2$. We will relax this assumption in Section 4. Combining (9), (11), (12) and (13) we reach

$$\phi (\kappa, \lambda, k, A) = \kappa - (1-\kappa)^{\frac{1}{\sigma_2}} \frac{\gamma}{1-\gamma} \left(\frac{Y_2}{K}\right)^{\frac{\sigma_2 - 1}{\sigma_2}} = 0 \tag{14}$$

$$\psi (\kappa, \lambda, k, A) = \lambda - (1-\lambda)^{\frac{1}{\sigma_2}} A^{\frac{1-\sigma_2}{\sigma_2}} (K)^{\frac{\sigma_2 - 1}{\sigma_2}} \frac{\kappa}{(1-\kappa)^{\frac{1}{\sigma_2}}} = 0, \tag{15}$$

where

$$\left(\frac{Y_2}{K}\right)^{\frac{\sigma_2 - 1}{\sigma_2}} = (1-\alpha) \left(\frac{(1-\lambda) A}{K}\right)^{\frac{\sigma_2 - 1}{\sigma_2}} + \alpha (1-\kappa)^{\frac{\sigma_2 - 1}{\sigma_2}} . \tag{16}$$

Combining (14), (15) and (16) we reach, after some tedious manipulation, the following relationship between the shares of capital and labor in sector 1.

$$\lambda = \lambda(\kappa) = \frac{\gamma (1-\alpha) \kappa}{\kappa - \alpha\gamma} \quad \text{with} \quad \frac{d\lambda}{d\kappa} = -\left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\lambda(\kappa)}{\kappa}\right)^2 < 0 \tag{17}$$

(See Appendix C.1 for a complete derivation.)

Furthermore, since $\lambda \leq 1$ and $\kappa \leq 1$, equation (17) determines the following feasibility regions for the sectoral factor allocations.

$$\lambda \in [\underline{\lambda}, 1], \quad \text{where} \quad 0 < \underline{\lambda} \equiv \frac{\gamma(1-\alpha)}{1-\alpha\gamma} < 1 \tag{18}$$

$$\kappa \in [\underline{\kappa}, 1], \quad \text{where} \quad 0 < \underline{\kappa} \equiv \frac{\alpha\gamma}{1-\gamma+\alpha\gamma} < 1 \tag{19}$$

Equation (17) shows that the fraction of labor used in a sector is a decreasing function of the fraction of capital in that same sector. As a result, an increase in the fraction of capital allocated to one sector leads to a reduction in the fraction of labor allocated to that sector. Structural change thus has a "shape" in this model: labor and capital allocations do not move in parallel. This pattern of sectoral reallocation differs from that obtained in the previous literature where a sector was either increasing its fractions of both capital and labor, or decreasing both. The reason is that in previous models of structural change, sectoral reallocations result from changes in relative sectoral output prices. As a consequence, the fractions of both inputs used in a sector move in the same direction as this relative price. In our model in contrast, structural change is driven by changes in the relative input price, the ratio of the wage per unit of effective labor to the rental rate. Cross-sectoral differences in the elasticity of substitution then allow the more flexible sector to absorb a larger fraction of the relatively cheap input and to release some of the relatively expensive one.

Now we are in a position to evaluate how factor allocations respond to changes in the state variables.

Proposition 1. *For the sake of exposition, let sector 1 be the more flexible sector, i.e.* $\sigma_2 < \sigma_1 = 1$*. Then the fraction of capital allocated to the more flexible sector 1,* κ*, increases as the economy's aggregate capital-labor ratio increases, while the fraction of labor in sector 1,* λ*, decreases. Similarly, the fraction of capital allocated to the more flexible sector decreases with the level of labour-augmenting productivity, while its fraction of labor increases. In particular,*

$$\frac{\partial\kappa}{\partial K} = \frac{(1-\sigma_2)}{\sigma_2 G(\kappa) K} > 0 \tag{20}$$

$$\frac{\partial\lambda}{\partial K} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\lambda(\kappa)}{\kappa}\right)^2 \frac{(\sigma_2-1)}{\sigma_2 G(\kappa) K} < 0 \tag{21}$$

$$\frac{\partial\kappa}{\partial A} = \frac{(\sigma_2-1)}{\sigma_2 G(\kappa) A} < 0 \tag{22}$$

$$\frac{\partial\lambda}{\partial A} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\lambda(\kappa)}{\kappa}\right)^2 \frac{(1-\sigma_2)}{\sigma_2 G(\kappa) A} > 0, \tag{23}$$

where

$$G(\kappa) \equiv \left[\frac{1}{\sigma_2\,(1-\lambda(\kappa))} + \frac{1}{\lambda(\kappa)}\right]\left(\frac{\lambda(\kappa)}{\kappa}\right)^2\left(\frac{\alpha}{1-\alpha}\right) + \left[\frac{1}{\kappa} + \frac{1}{\sigma_2(1-\kappa)}\right]. \tag{24}$$

The inequality signs in (20)-(23) are reversed when sector 1 is the less flexible one, i.e. $\sigma_2 > \sigma_1 = 1$.

Proof. Combining (15) with (17) and taking logarithms we reach

$$\frac{1-\sigma_2}{\sigma_2}\ln K = \frac{1}{\sigma_2}\ln(1-\lambda(\kappa)) - \ln\lambda(\kappa) + \frac{1-\sigma_2}{\sigma_2}\ln A + \ln\kappa - \frac{1}{\sigma_2}\ln(1-\kappa). \tag{25}$$

The result is obtained differentiating (25) using (17). ■

Remarks on Proposition 1: (i) As $\kappa \to \underline{\kappa}$, $\lambda(\kappa) \to 1$ and hence $G(\kappa) \to \infty$. This implies that as $\kappa \to \underline{\kappa}$, both $\frac{\partial \kappa}{\partial K}$ and $\frac{\partial \lambda}{\partial K} \to 0$. (ii) As the aggregate capital-labor ratio increases, the capital-labor ratio in the less flexible sector 1 grows at a lower (albeit positive) rate compared to the more flexible sector 2 (see Appendix C.2 for a proof).

In order to understand the logic of Proposition 1, let us first concentrate on the effects of capital accumulation on the fractions of capital and labor allocated to each sector. As the economywide capital-labor ratio increases, the ratio of the wage to rental rate increases. As a result, the more flexible sector tends to substitute away from the now more expensive input, labor, towards the relatively cheaper one, capital, at a higher rate than the less flexible sector 2 is able to do. Consequently the fraction of capital allocated to the more flexible sector –sector 1– increases while its share in labor employment decreases. Given full employment, the converse is true for the less flexible sector 2. Increases in the level of labor-augmenting productivity A raise the wage-rental rate ratio, but still lower the effective cost of using labour, i.e. they reduce the wage per unit of effective labor. As a result, the more flexible sector 1 takes advantage of this cost reduction by moving to a more labor-intensive mode of production.

Finally, notice that structural change takes place despite the assumption that the production function for final output is Cobb-Douglas, i.e. sectoral reallocations take place even when the expenditure shares of the two intermediate goods remain constant.

3.2.2 The dynamic problem

Now we turn to the characterization of the solution for the dynamic problem. With labor-augmenting productivity growing at a constant exogenous rate, which we denote by $g(A)$, it is convenient to rewrite the original problem in terms of the normalized variables

$$c(t) \equiv \frac{C(t)}{A(t)\,L(t)} = \frac{C(t)}{A(t)}, \qquad \chi(t) \equiv \frac{K(t)}{A(t)\,L(t)} = \frac{K(t)}{A(t)}, \tag{26}$$

which represent consumption and capital per unit of effective labor. fUsing these normalized variables, the objective function (6) becomes

$$\int_0^\infty e^{-\beta t}\left(\frac{(c(t)\,A(t))^\mu}{\mu}\right)dt = \int_0^\infty e^{-(\beta-\mu g(A))t}\left(\frac{c(t)^\mu}{\mu}\right)dt, \tag{27}$$

where we have normalized the initial level of productivity, $A(0)$, to one.

Combining (7) and (26), we reach the following law of motion for capital per unit of effective labor.

$$\dot{\chi} = \frac{Y}{K}\chi - c - (\delta + g(A))\,\chi = f(\chi) - c - (\delta + g(A))\,\chi \tag{28}$$

The derivation of $f(\chi) \equiv A\pi(\kappa(\chi))\chi$ is provided in Appendix C.4.

This optimization problem reduces to the standard optimal growth problem, since $f(\chi)$ is a strictly concave and increasing function of χ with $f(0) = 0$.[16]

Define

$$\rho \equiv \beta - \mu g(A)$$

and assume that ρ is positive. Let ψ be the shadow price of χ. The Hamiltonian for this problem is

$$H = \frac{c^{\mu}}{\mu} + \psi\left[f(\chi) - c - (\delta + g(A))\chi\right]$$

The necessary conditions are

$$c^{\mu-1} = \psi \tag{29}$$

$$\dot{\psi} = \psi\left[\rho + \delta + g(A) - f'(\chi)\right] \tag{30}$$

together with the transversality condition, $\lim_{t\to\infty} \chi\psi \exp(-\rho t) = 0$. The interpretation of these conditions is standard. Combining (29) and (30), we reach the familiar consumption Euler equation

$$\dot{c} = \frac{c}{1-\mu}\left[f'(\chi) - \rho - \delta - g(A)\right]. \tag{31}$$

Together with (28), the initial condition $\chi(0) = \chi_0$, and the transversality condition it fully describes the dynamic evolution of the economy.

As always, a "balanced growth path" (BGP) is a trajectory along which the growth rates of consumption and capital are constant.[17] Let us denote the growth rate of any variable X_i by $g(X_i)$ and its level along the balanced growth path as X_i^*. Then we have the following characterization of the BGP.

Proposition 2. *There is a unique (non-trivial) BGP that satisfies*

$$f'(\chi^*) = \rho + \delta + g(A), \tag{32}$$

$$c^* = f(\chi^*) - (\delta + g(A))\chi^*, \tag{33}$$

$$\chi^* = (\gamma(1-\alpha))^{\frac{\sigma_2}{\sigma_2-1}} \frac{(1-\kappa^*)^{\frac{1}{\sigma_2-1}}}{(\kappa^* - \alpha\gamma)\left(\kappa^*\left(1-\gamma(1-\alpha)\right) - \alpha\gamma\right)^{\frac{1}{\sigma_2-1}}}, \tag{34}$$

and

$$\lambda^* = \frac{\gamma(1-\alpha)\kappa^*}{\kappa^* - \alpha\gamma}. \tag{35}$$

[16] See Woodland (1982, p. 17-18 and 52-59) for a proof. In Appendix C.5 we outline the main arguments of this proof.

[17] This definition is equivalent to those of Kongsamut et al. (2001) and Acemoglu and Guerrieri (2008). The former define a "generalized balanced growth path" as a trajectory along which the real interest rate is constant, while the latter require a constant consumption growth rate.

Moreover, the (constant) growth rates of the relevant variables are

$$g(Y^*) = g(K^*) = g(Y_i^*) = g(K_i^*) = g(A)\,;\; g(L_i^*) = 0 \qquad \text{for } i = 1, 2.$$

The steady state associated with this BGP is locally saddle-path stable.

Proof. Imposing the BGP condition and given the properties of $f(\chi)$, the Euler equation (31) pins down the unique (non-trivial) level of capital per unit of effective labor, χ^*. Along this BGP, the level of consumption per unit of effective labor and the fractions of capital and labor allocated to sector 1 are determined combining χ^* with (17), (28) and (34) respectively. The growth rates immediately follow from (26), and the stationarity of the sectoral allocations. Finally, with χ being a sluggish variable while c is free to jump instantaneously, in order for the linear system associated with (28) and (31) to have a unique stable adjustment path (i.e., be saddle path stable) we require that it have one negative (stable) and one positive (unstable) eigenvalue. It can be easily verified that the concavity of $f(\chi)$ ensures that the Jacobian associated with (28) and (31) is indeed negative, and therefore we have two distinct real eigenvalues of opposite sign. ■

Proposition 2 has several interesting implications. First, since the growth rate of labor-augmenting productivity is equal across sectors, both sectors grow at the same rate along the BGP. The aggregate economy grows at the same rate. Second, the steady state fractions of employment and capital are strictly positive in both sectors. As opposed to the model explored in Acemoglu and Guerrieri (2008), where the fractions of employment and capital in the sector that sheds resources vanish asymptotically, both sectors reach the BGP with non-trivial shares of employment and capital in this model. Third, it is worth noticing that along the BGP, the capital-output ratio and the rental rate are constant, and, as a result, so is the share of capital in national income, while the wage rate grows with productivity at a rate $g(A)$. Finally, as in Acemoglu and Guerrieri (2008), once the economy reaches the BGP the process of sectoral reallocation comes to an end. This is the case since along such a path capital per unit of effective labor, χ, is constant and therefore the incentives for sectoral reallocations induced by capital deepening and technological change perfectly cancel out, since they are exactly equal but work in opposite directions. This becomes clear once one notices that (20)-(23) imply that $\frac{\partial \lambda}{\partial k}\frac{K}{\lambda} = -\frac{\partial \lambda}{\partial A}\frac{A}{\lambda}$ and $\frac{\partial \kappa}{\partial k}\frac{K}{\kappa} = -\frac{\partial \kappa}{\partial A}\frac{A}{\kappa}$.[18]

During the transition to a balanced growth path, differences in capital-labor substitutability across sectors can thus lead to structural change. For instance, during a transition "from

[18]The same conclusion could be reached in terms of the ratio of the wage per unit of effective labor to the rental rate that drives sectoral reallocations. Once χ is constant, this ratio is also constant, so structural change stops. This is clear from (14), (15) and (16).

below", along which capital, K, grows faster than effective labor, AL, the more flexible sector will substitute towards capital, the input that becomes relatively abundant. Hence, the more flexible sector will become more capital intensive, and the less flexible sector more labor intensive. As a consequence, the capital-labor ratio will grow more in the more flexible sector. The opposite occurs during a transition "from above".

Furthermore, if capital and labor are gross substitutes in the more flexible sector, the ratio of the capital to the labor income share, which is proportional to $[K_i/(AL_i)]^{\frac{\sigma_i-1}{\sigma_i}}$, will increase in that sector. With Cobb-Douglas production and thus a constant factor income shares in the other sector, the aggregate labor income share declines.

Conversely, if the more flexible sector has a Cobb-Douglas production function, i.e. $\sigma_1 = 1$ and $\sigma_2 < 1$, a transition from below will imply constant factor income shares in the flexible sector, and a declining capital income share in the less flexible sector. The latter occurs because the capital-labor ratio grows less quickly in the less flexible sector, while factor prices are the same. In both scenarios, the converse is true when capital grows less quickly than effective labor. Differences in capital-labor substitutability can thus induce structural change, and also lead to differential evolution of sectoral factor income shares.

4 Dissecting the U.S. experience

Since several drivers of structural change have already been proposed in the literature, we conduct a joint quantitative analysis of the effect of cross-sectoral differences in capital-labor substitutability and of three additional drivers of structural change in this section. This will lead to insights about the drivers of both structural change and the observed changes in labor income shares. Note that while, as discussed above, the other drivers of structural change do not lead to (sufficiently large) changes in aggregate factor income shares, they do lead to factor reallocation which, combined with differences in α_i, can affect the evolution of these aggregate shares. Conversely, differences in σ_i affect both the evolution of factor income shares and the amount and shape of structural change.

In order to accommodate the different drivers of structural change, we drop the strong symmetry across sectors imposed for expositional purposes in the previous section. To do so, we allow for differences across sectors not only in substitutability, but also in productivity growth and in capital intensity. We also allow preferences to be non-homothetic. We then calibrate the model to the recent U.S. experience to assess the relative importance of these different factors.

Since our period of analysis is restricted by the availability of data on factor income shares from 1960 onwards, we conduct our analysis in a two-sector environment, abstracting from agriculture. In 1960, agriculture already accounted for only 6% of employment and an even smaller fraction of value added. In terms of sectoral reallocations the contribution of agriculture over our sample period is small, accounting for barely 10% of the reallocations of labor and 8% of the changes in sectoral composition of value added. As a result of its limited contribution in terms of structural change we have decided to leave agriculture out of our analysis.[19]

4.1 Model

We model a closed economy in discrete time. It is populated by a representative infinitely-lived household with preferences given by

$$\max \sum_{t=0}^{\infty} \beta^t \ln \left(\gamma c_{mt}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(c_{st}+s\right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

The subscripts m and s denote manufactures and services, respectively, so c_i stands for per capita consumption of value-added produced in sector $i, i \in \{m, s\}$. ε is the elasticity of substitution between the two consumption goods. The term s introduces a non-homotheticity. The empirically relevant case is $s > 0$, which can be interpreted as households having an endowment of services. As a consequence, the income elasticity of demand for services is larger than that for manufactures. This introduces the possibility of demand-driven structural change, as in Kongsamut et al. (2001).

Sectoral outputs are produced according to general CES technologies

$$Y_m = D_m \left[\alpha_m (B_m K_m)^{\frac{\sigma_m-1}{\sigma_m}} + (1-\alpha_m)\left(A_m L_m\right)^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{\sigma_m}{\sigma_m-1}} \tag{36}$$

$$Y_s = D_s \left[\alpha_s (B_s K_s)^{\frac{\sigma_s-1}{\sigma_s}} + (1-\alpha_s)\left(A_s L_s\right)^{\frac{\sigma_s-1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s-1}} . \tag{37}$$

As opposed to the model explored in the previous section, our modeling of services and manufactures allows for a rich set of asymmetries. First, we introduce sector-specific elasticities of substitution and distributional parameters, σ_i and α_i respectively. Second, we allow both the initial levels of labor- and capital-augmenting productivity, A_i and B_i, and their growth rates, $g(A_i)$ and $g(B_i)$, to differ across sectors. Third, we allow for non-homothetic preferences, which introduce an additional difference between the services and manufacturing

[19]See Jensen and Larsen (2005) for the solution of an N-sector model with exogenous saving and differences in sectoral elasticities of factor substitution.

sectors. Finally, we follow most of the literature on structural change by assuming that capital is only produced in the manufacturing sector, so services are fully consumed. Therefore, using upper-case letters to denote aggregate variables, $Y_s = C_s$ and $Y_m = C_m + I$.

As in the previous section, factors are fully utilized which, normalizing the labor endowment to one, implies

$$\frac{L_s}{L} + \frac{L_m}{L} \equiv l_m + l_s = 1 \tag{38}$$

$$l_m k_m + l_s k_s = l_m k_m + (1 - l_m)\, k_s = k, \qquad \text{where } k_i \equiv \frac{K_i}{L_i}. \tag{39}$$

In the following, we will use lower-case variables to denote per capita quantities, except for k_i, which stands for the capital-labor ratio in sector i.

We choose manufactures to be the numeraire and denote the price of services by p_s. Since markets are competitive, production efficiency requires equating marginal revenue products across sectors, so

$$\alpha_m B_m^{\frac{\sigma_m - 1}{\sigma_m}} \left(\frac{Y_m}{K_m}\right)^{\frac{1}{\sigma_m}} = p_s \alpha_s B_s^{\frac{\sigma_s - 1}{\sigma_s}} \left(\frac{Y_s}{K_s}\right)^{\frac{1}{\sigma_s}} = R \tag{40}$$

$$(1 - \alpha_m)\, A_m^{\frac{\sigma_m - 1}{\sigma_m}} \left(\frac{Y_m}{L_m}\right)^{\frac{1}{\sigma_m}} = p_s (1 - \alpha_s)\, A_s^{\frac{\sigma_s - 1}{\sigma_s}} \left(\frac{Y_s}{L_s}\right)^{\frac{1}{\sigma_s}} = w. \tag{41}$$

As a consequence, the following relationship between the sectoral capital-labor ratios emerges.

$$k_s = \left(\frac{1 - \alpha_m}{1 - \alpha_s}\frac{\alpha_s}{\alpha_m}\left(\frac{A_m}{B_m}\right)^{\frac{\sigma_m - 1}{\sigma_m}}\right)^{\sigma_s} \left(\frac{A_s}{B_s}\right)^{1 - \sigma_s} k_m^{\frac{\sigma_s}{\sigma_m}} \equiv \varrho k_m^{\frac{\sigma_s}{\sigma_m}}. \tag{42}$$

Using this notation, (40) implies that the relative price of services is given by

$$p_s = \frac{\alpha_m}{\alpha_s}\frac{B_m^{\frac{\sigma_m - 1}{\sigma_m}}}{B_s^{\frac{\sigma_s - 1}{\sigma_s}}} \frac{\left[\alpha_m B_m^{\frac{\sigma_m - 1}{\sigma_m}} + (1 - \alpha_m)(A_m / k_m)^{\frac{\sigma_m - 1}{\sigma_m}}\right]^{\frac{1}{\sigma_m - 1}}}{\left[\alpha_s B_s^{\frac{\sigma_s - 1}{\sigma_s}} + (1 - \alpha_s)(A_s / (\varrho k_m^{\sigma_s/\sigma_m}))^{\frac{\sigma_s - 1}{\sigma_s}}\right]^{\frac{1}{\sigma_s - 1}}}. \tag{43}$$

Household optimization in turn requires equating the marginal rate of substitution between the two consumption goods to their relative price in every period, implying relative demands given by

$$\frac{c_s + s}{c_m} = \left(\frac{1 - \gamma}{\gamma}\right)^{\varepsilon} p_s^{-\varepsilon}. \tag{44}$$

Given a solution to the dynamic problem and state variables k, A_m, A_s, B_m, B_s, equations (42), (43) and (44) pin down p_s, k_m and l_m. Their counterparts for services, k_s and l_s, then follow from equations (38) and (39).

The solution to the household's dynamic problem, stated in terms of choosing c_m, implies the Euler equation

$$\frac{1}{c_{mt}} = \frac{\beta}{c_{m,t+1}}(MPK_{m,t+1} + 1 - \delta), \tag{45}$$

where MPK_{it} denotes the physical marginal product of capital in sector i at time t. The law of motion of capital is given by

$$K_{m,t+1} = Y_{mt} - C_{mt} + (1-\delta)K_t - K_{s,t+1}. \tag{46}$$

It is useful to rewrite this in terms of capital-labor ratios:

$$k_{m,t+1} = \frac{1}{l_{m,t+1}}[Y_{mt} - C_{mt} + (1-\delta)K_t - (1 - l_{m,t+1})k_{s,t+1}] \tag{47}$$

$$= \frac{1}{l_{mt+1}}\left[Y_{mt} - C_{mt} + (1-\delta)K_t - (1 - l_{mt+1})\varrho_{t+1}\, k_{mt+1}^{\frac{\sigma_s}{\sigma_m}}\right] \tag{48}$$

Since the marginal product of capital in manufacturing in the future depends on the labor allocation in that future period, the dynamic problem is not independent of the static problem.[20] The dynamic problem can thus be represented as a system of three equations:

$$\frac{c_{m,t+1}}{c_{m,t}} = h_1(k_{m,t+1}) \tag{49}$$

$$k_{m,t+1} = h_2(l_{m,t+1}, k_{m,t}, l_{m,t}, k_t, c_{m,t}) \tag{50}$$

$$l_{m,t} = h_3(c_{m,t}, k_{m,t}), \tag{51}$$

where exogenous variables are omitted from the function arguments for the sake of clarity, and h_1 to h_3 are functions.

An equilibrium in this economy consists of sequences of allocations $\{c_{m,t}, c_{s,t}, k_{m,t}, k_{s,t}, k_t, l_{m,t}, l_{s,t}\}_{t=0}^{\infty}$ and prices $\{p_{s,t}, w_t, r_t\}_{t=0}^{\infty}$ such that equations (40) to (45) and (48) hold and feasibility is satisfied.

Before proceeding to our quantitative results, two technical remarks are in order. First, our model allows for capital-augmenting technical progress which, as is well-known, is not consistent with balanced growth. In general, many models of structural change are not consistent with balanced growth. This is true for Kongsamut et al.'s (2001) model of demand-driven structural change, except in a special case, and also for Acemoglu and Guerrieri's (2008) model of technology-driven structural change, except in the limit. This is no coincidence. In their forthcoming chapter on structural change in the Handbook of Economic

[20] This does not depend on whether we use the law of motion of k or k_m in the dynamic problem. Key is that the future marginal product of capital in manufacturing that appears in the Euler equation (45) depends on the future labor allocation.

growth, Herrendorf et al. (forthcoming) state (p. 4): "It turns out that the conditions under which one can simultaneously generate balanced growth and structural transformation are rather strict, and that under these conditions the multi-sector model is not able to account for the broad set of empirical regularities that characterize structural transformation. ... we think that progress in building better models of structural transformation will come from focusing on the forces behind structural transformation without insisting on exact balanced growth." As the quantitative results below show, our model features approximate balanced growth.

Second, the absence of exact balanced growth requires the imposition of a different terminal condition for the dynamic system given in equations (49) to (51). We proceed by a) choosing a finite time horizon T (100 years in the results shown below), b) imposing that consumption growth is constant at the end of that horizon: $c_{T+1}/c_T = c_T/c_{T-1}$ (this implies a solution to equation (49) for $t = T$, which in turn allows solving equations (50) and (51) for that period), and c) check that the specific horizon T that we chose does not affect results.

4.2 Calibration

4.2.1 Data

We calibrate the model to U.S. data over the period 1960-2005. The data we use to obtain factor income shares and allocations is from Jorgenson (2007) and has been discussed above. We convert capital services reported there into capital stock figures using the average long-run rental rate from the model. To calibrate preferences, we also require information on consumptions shares and the relative price, which we take from Herrendorf, Rogerson and Valentinyi (2013).

4.2.2 Preferences

Herrendorf, Rogerson and Valentinyi (2013, Figures 9 and 10) show that despite an increase in the relative price of services, the ratio c_s/c_m has not fallen, but increased slightly over the last 65 years. With $s \geq 0$, Leontief preferences between manufacturing and services ($\varepsilon = 0$) provide the best approximation to this trend. This is, of course, in line with Herrendorf, Rogerson and Valentinyi's (2013) estimates.[21]

Given $\varepsilon = 0$, we calibrate s using the growth rates of quantities and prices of manufacturing and services consumption, in value added terms, over the 1960 to 2005 period. This

[21]Below, we show that results are very similar when calibrating the model with a larger ε of 0.5; the route taken by Buera and Kaboski (2009) to avoid Leontief preferences.

results in a value for s of around 20% of first period services consumption. We then obtain a value of γ from the relative weight of manufacturing and services consumption in the initial period. Since the ratio of manufacturing to services consumption does not change much, this value is not very sensitive to the time period we use for calibrating it.

4.2.3 Technology

For reference, each sector's CES production function is given by

$$Y_{it} = D_i \left[(1-\alpha_i)(A_{it}N_{it})^{\frac{\sigma_i - 1}{\sigma_i}} + \alpha_i (B_{it}K_{it})^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}},$$

where $D_i, \alpha_i, A_{it}, B_{it}, \sigma_i$ and the growth rates of A_{it} and B_{it} are all allowed to differ across sectors.[22] It is clear that the levels of D, A, B and α in a given sector sector cannot all be identified separately. We therefore normalize D_i and B_{i0} to 1 in each sector. We set A_{i0} to match the initial labor income share in a sector given observed output per worker in that sector. We also set the initial capital-labor ratio for each sector from the data.[23] We set the depreciation rate δ to 5% per year and the discount factor β to 0.94.

At this point, eight parameters remain to be calibrated: the two elasticities of substitution between capital and labor, σ_m and σ_s, four growth rates of A_m, A_s, B_m, B_s, and α_m and α_s. We set their values to match eight data moments, all computed for the period 1960 to 2005: the change in the capital income share to labor income share ratio in each sector, the average labor income share in each sector, the change in the fraction of labor and capital (respectively) employed in manufacturing, the average fraction of labor employed in manufacturing, and the aggregate growth rate of output per worker, measured in units of the aggregate consumption good. All these data moments are computed from Jorgenson's (2007) data.[24]

We choose these targets because of their information content regarding the model parameters. While all eight remaining parameters affect the values of all targets, some relationships are particularly strong. Thus, given σ_i, α_m and α_s strongly affect the average factor income

[22] As recommended by León-Ledesma et al. (2010), we use a normalized version of this production function when we change σ_i in the decomposition below. See below for more details.

[23] One way to understand the need to impose initial conditions on the sectoral allocations is to contrast our solution method with that of a two sector model with a well-defined steady state. In this latter case, the initial conditions for sectoral allocations are determined in such a way that the system eventually converges to its steady state. In our case, in the absence of a steady state, we pin down the initial values of the sectoral allocations from the data. Thereafter, these allocations are endogenously determined. This is also the approach followed by Acemoglu and Guerrieri (2008).

[24] To avoid excessive influence of the first and last observation, we compute the growth rates between averages for the first and last 5 years.

shares in the two sectors. The relative growth rates of A_i and B_i drive the change in relative income shares in a sector. The relative growth rate of A_m and A_s contribute strongly to the pace of structural change. The growth rate of A_m then determines overall output growth. The substitution elasticities drive the shape of structural change (strength of labor versus capital reallocation). The initial labor allocation also depends on the elasticities, but also strongly on the values of α_i.[25]

Calibration results are shown in Table 3. The model can reproduce both the differential pattern in changes in the labor income share shown in Figure 3 and the amount and shape of structural change observed in the data. It also matches average levels of the labor income shares and the labor allocation well, and fits aggregate output growth exactly. It does not fit the average labor income share in services exactly, as there is a tension between doing so and fitting the average labor allocation. In addition, the model replicates very closely the average and changes in the capital and output allocations, which were not targeted in the calibration. As a consequence, it also replicates the level and changes of the aggregate labor income share rather closely.

Calibrated parameters are shown in Panel B of Table 3. Key parameters coming from the calibration are those for the sectoral substitution elasticities and the growth rates. The calibration yields substitution elasticities below but close to unity in manufacturing, and substantially below unity in services. Cross-sector differences in growth rates are large. Our results suggest positive labor-augmenting technical change and negative capital-augmenting technical change. Both growth rates are larger in the manufacturing sector. As discussed below, to understand the implications of non-unitary substitution elasticities for structural change, it is not sufficient to know the growth rates; differences in the elasticities are also key.

Estimates of substitution elasticities below 1 are in line with previous estimates at the aggregate level compiled in León-Ledesma et al. (2010). Because all of these estimates are at the aggregate level, it is impossible to compare our estimates for sectoral factor-specific productivity growth rates to those from this literature. Herrendorf, Herrington and Valentinyi (2013) is the only paper we are aware of that estimates CES production functions at the sectoral level. They also obtain estimates of the elasticities below unity, and also estimate manufacturing to be the more flexible sector. Their estimate for σ_m is 0.8, very close to ours. Their estimate for services is 0.75, slightly higher than ours. Calibrating all

[25] The Jacobian of model moments with respect to parameters, evaluated at our selected parameter values, has full rank.

Table 3: Calibration: Targets and model moments and parameters.

Panel A: Targets and model moments				
	Data		Model	
	Manufacturing	Services	Manufacturing	Services
calibration targets:				
$\overline{LIS_i}$ (%)	66.7	63.8	66.4	65.1
$g(KIS_i/LIS_i)$ (% p.a.)	0.6	0.3	0.6	0.3
$\mathrm{d}(L_m/L)$	-22.4		-23.9	
$\mathrm{d}(K_m/K)$	-21.3		-20.7	
$\overline{L_m/L}$	34.2		34.5	
$g(Y/L)$ (% p.a.)	1.8		1.8	
not targeted:				
$\overline{LIS}$ (aggregate, %)	64.8		65.6	
$g(KIS/LIS)$ (agg., % p.a.)	0.5		0.4	
$\overline{K_m/K}$	33.2		33.3	
$\overline{Y_m/Y}$	35.1		34.1	
$\mathrm{d}(Y_m/Y)$	-24.5		-22.9	

Panel B: Parameters				
sector-specific:	Manufacturing	Services	*general:*	
$g(A_i)$ (% p.a.)	7.8	1.1	δ	0.05
$g(B_i)$ (% p.a.)	-1.6	-5.9	β	0.94
σ_i	0.776	0.571	ϵ	0
α_i	0.358	0.488	γ	0.314

Note: The data period used is 1960 to 2005. Averages are geometric means over this period. Absolute changes are percentage point changes over the entire period. Growth rates are computed using averages of the first and last 5 years. For readability, annual percentage changes are given for output growth and the growth in relative income shares.

parameters jointly, instead of estimating sector by sector, thus makes a difference.[26]

To understand what drives the calibration results for σ_i and the growth rates, it is useful to consider how the model can match observed patterns in factor income shares and the shape of structural change. The key observations here are that in the data, the labor income share

[26]When inserting their estimates in our model, the model predicts too much change in the labor income share in manufacturing, and too little (in fact, positive) change in services. The latter is due to the higher substitution elasticities in services estimated by Herrendorf, Herrington and Valentinyi (2013). The model also predicts almost no change in the share of capital employed in manufacturing.

declines in both sectors, and more so in manufacturing, and that structural change looks similar, whether expressed in terms of the allocation of employment or that of capital. Given limited importance of non-homotheticity and α differences (see the decomposition below), the amount and shape of structural change and the relative evolution of the labor income shares in the model depend on the substitution elasticities and on relative productivity growth in the two sectors.

Consider first the evolution of factor income shares. This is determined by

$$\frac{s_{itK}}{s_{itL}} = \frac{\alpha_i}{1-\alpha_i}\left(\frac{B_{it}K_{it}}{A_{it}L_{it}}\right)^{\frac{\sigma_i-1}{\sigma_i}},$$

which is a version of equation (4) that allows for capital-augmenting productivity. From here it is clear that if $\sigma_i < 1$ and capital per worker grows in both sectors, A needs to grow faster than B in both sectors for the labor income share to decline in both of them. The decline in the labor income share thus is driven by the increasing scarcity of effective capital relative to effective labor. The larger σ_i, the larger $g(A_i/B_i)$ needs to be to induce a given change in the labor income share. In line with this, $g(A_m/B_m) > g(A_s/B_s)$ in our calibration, given that $\sigma_m > \sigma_s$. What would happen if σ_i and $g(A_i/B_i)$ in the two sectors were more similar? To answer this question, we reduce σ_m and $g(A_m/B_m)$ in such a way as to keep the change in the labor income share and the growth of output per worker in manufacturing unaffected. Given a background of increasing scarcity of effective capital relative to effective labor, as required to match the evolution of factor income shares, reducing σ_m makes manufacturing less flexible and thus pushes firms in the sector to retain more capital. At the same time, a smaller difference between $g(A_m)$ and $g(B_m)$ reduces the speed at which capital becomes more scarce, and thus pushes firms in the other direction. Quantitatively, the first channel dominates, making manufacturing progressively more capital-intensive. This leads to a worse fit in terms of the shape of structural change, with too little change in the fraction of capital used in manufacturing. With our calibrated parameters, in contrast, these forces are balanced in the right way.[27] [28]

[27] Above we remarked that a rising capital income share is also consistent with $\sigma > 1$ and rising $BK/(AL)$, as in Karabarbounis and Neiman (forthcoming). In our calibration, we also searched the region with $\sigma_i > 1$ but could not find parameter combinations with a good fit to the data.

[28] The finding of negative capital-augmenting technical change is surprising at first sight. However, note that such estimates are not uncommon in the literature. For instance, both Antras (2004) and Herrendorf, Herrington and Valentinyi (2013) obtain negative estimates for $g(B)$. In our setting, with both sectors exhibiting substitution elasticities below unity, negative $g(B)$ is necessary for matching the observed declining labor income shares, combined with the observed level of output growth. The reason is that given observed growth in capital intensity in both sectors, a substantial difference between $g(A)$ and $g(B)$ is required in both sectors. At the same time, these growth rates need to take levels consistent with output growth of 1.8%. It turns out that these requirements are only met with $g(B) < 0$.

4.3 The benchmark time path

Figure 5 compares the time path of our simulated economy with the data. The top right panel shows output per capita, which grows at an average and almost-constant rate of 1.8% per year. The interest rate (top left panel) is also almost constant: after an initial transition of 18 years, during which it increases by 1.3 percentage points, it becomes almost constant, falling only 0.11 percentage points over the remaining 28 years of our simulated data. The economy thus exhibits "approximate balanced growth".[29]

The two bottom panels show structural change and the change in factor income shares. The left panel shows that the labor income share declines in both sectors, but more sharply in manufacturing, driven by the larger discrepancy between the growth rates of A and B in that sector. Since this is a calibration target, the model reproduces the observed declines in the labor income share well and also matches their average level rather well. The aggregate labor income share declines from 68.2% to 63.8% in the model, compared to a fall from 68.4% to 63.0% in the data. While this moment was not targeted, it is clearly closely related to three of our calibration targets.

The right panel depicts structural change generated in the model. It shows that the calibrated model fits the data both in terms of the amount of structural change generated and in terms of its composition: the reallocation of labor and capital look strikingly similar both in the model and in the data, apart from some medium-run fluctuations which the model is not designed to capture. The model also fits the data well in terms of the level and change of the share of output in manufacturing; it falls by 22.9 percentage points, versus 24.5 in the data.[30]

4.4 Decomposition of structural change

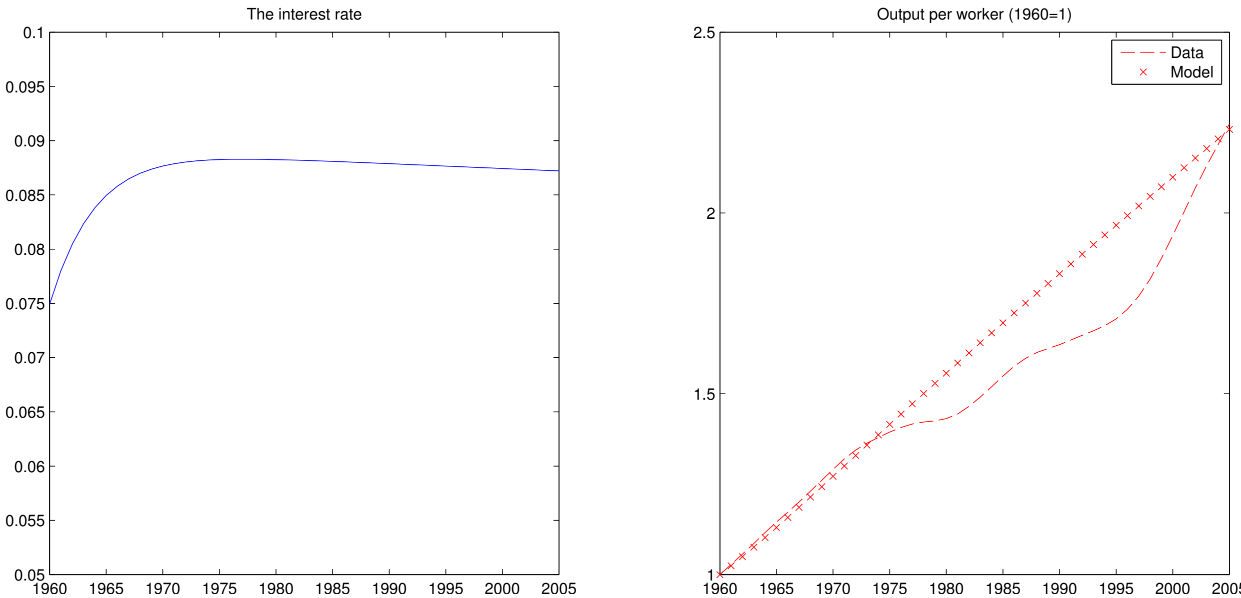
In the following, we assess the relative importance of different drivers of structural change. To do so, we explore how results change once we separately disable the four drivers of structural change in the model. In addition, we compare results to a case with Cobb-Douglas production functions in both sectors in the next subsection.

To eliminate the non-homotheticity, we set s to zero. To eliminate differences in α_i, we set these parameters in both sectors to their average value. To eliminate differences in

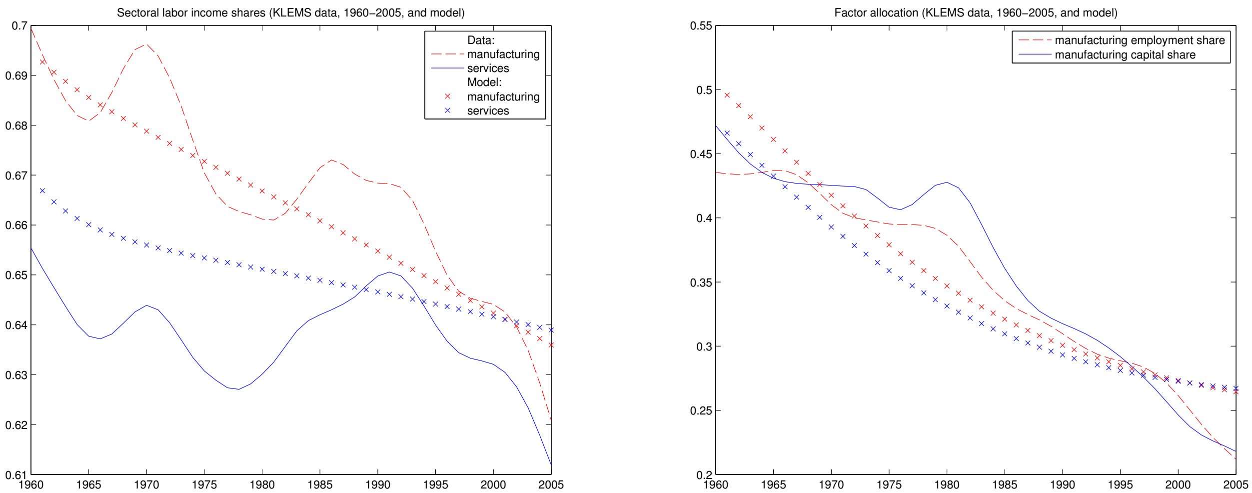
[29] The initial capital stock appears to be too high. This is similar to Acemoglu and Guerrieri (2008), who find a small decline in the interest rate in the first years, followed by stabilization.

[30] Note that this was not targeted. Since the output share depends not only on the input shares but also on productivity and prices, it is not obvious that it should fit so well.

Figure 5: Structural change in the benchmark economy



(a) The rental rate and aggregate output per capita



(b) Structural change and factor income shares in the benchmark economy

Notes: Parameters are given in Table 3. Data is smoothed using an HP-Filter with smoothing parameter 6.25.

productivity growth, we set $g(A_m) = g(A_s)$ and $g(B_m) = g(B_s)$ and choose growth rates such that aggregate output growth and the mean difference between $g(A_i)$ and $g(B_i)$ remain unchanged.

Finally, to eliminate differences in σ_i, we set σ in both sectors to the average, 0.673.

Table 4: Structural change: counterfactuals

	dLIS$_m$	dLIS$_s$	dLIS (aggregate)	d(L_m/L)	d(K_m/K)	d(Y_m/Y)
data	-5.7	-2.5	-5.4	-22.4	-21.3	-24.5
benchmark	-6.0	-3.1	-4.4	-23.9	-20.7	-22.9
$s = 0$	-6.1	-3.2	-4.5	-23.3	-20.2	-22.3
common α	-6.1	-3.1	-6.0	-25.4	-22.7	-24.5
common $g(A)$	-16.0	-31.9	-22.9	10.5	-4.8	2.2
common σ, a)	-8.9	-2.5	-4.1	-22.9	-16.3	-20.8
common σ, b)	-5.8	-3.3	-6.1	-25.1	-20.5	-23.9

Notes: LIS stands for "labor income share" and d for the absolute change between the first and last years of the sample. All changes are in percentage points. Except for the parameters that are equated, all parameters as in the benchmark, except for line 7. Line 4: α in both sectors is set to the average of α_m and α_s in Table 3. Line 5: $g(A_m) = g(A_s) = 0.0385$ and $g(B_m) = g(B_s) = -0.0435$. This value implies that the growth rate of y is the same as in the benchmark economy, and the average distance between $g(A_i)$ and $g(B_i)$ is preserved. Line 6: $\sigma_m = \sigma_s = 0.673$. This is the average of σ_m and σ_s. Line 7: Recalibration, under the restriction of common σ_i. This results in a common σ of 0.866, $g(A_m)$ of 9.5%, $g(B_m)$ of -3.4%, $g(A_s)$ of 1.8%, $g(B_s)$ of -8.2%, α_m of 0.303 and α_s of 0.444.

When computing results for this case, we work with a normalized version of the CES:

$$Y_{it} = \bar{Y}_i \left[\overline{1-\theta}_i \left(\frac{A_{it}N_{it}}{\bar{A}_i\bar{N}_i} \right)^{\frac{\sigma_i-1}{\sigma_i}} + \bar{\theta}_i \left(\frac{B_{it}K_{it}}{\bar{B}_i\bar{K}_i} \right)^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}},$$

where variables with a bar denote the geometric sample average of the underlying series, and $\bar{\theta}_i$ and $\overline{1-\theta}_i$ refer to the average sectoral capital and labor income shares, respectively. In terms of the production function above, this involves setting D_i to $\bar{Y}_i$, α_i to $\bar{\theta}_i$ and $1-\alpha_i$ to $\overline{1-\theta}_i$ (the two terms sum approximately to one even with geometric averages of the income shares). León-Ledesma et al. (2010) and Cantore and Levine (2012) strongly recommend using this normalization. Temple (2012) discusses the interpretation in detail. Herrendorf, Herrington and Valentinyi (2013) also use it when estimating sectoral production functions. The normalization chosen matters for exercises where the substitution elasticity is changed. The normalization advocated by the literature and chosen here implies that as σ_i is changed, isoquants are tangential at the average capital-labor ratio in the data, and output at that capital-labor ratio does not change. With this normalization, equating σ_i across sectors thus does not affect sectoral output at the sample average.

Decomposition results are reported in Table 4. The table shows the effect of these changes both on structural change and on changes in sectoral and aggregate labor income shares. All changes worsen the fit of the model, though only slightly for setting s to zero.

It is clear that two channels, non-homothetic preferences and differences in capital intensity, hardly matter. Setting s to zero reduces structural change slightly, as now the income elasticity of services does not exceed that of manufacturing anymore. However, with low s, the importance of this demand-driven structural change channel à la Kongsamut et al. (2001) is minor. Eliminating differences in α even leads to *more* structural change. The reason is that in the data, capital intensity as measured by α is actually slightly higher in services than in manufacturing. Different capital intensities thus lead to a small movement of inputs towards manufacturing, in line with Acemoglu and Guerrieri (2008). Eliminating these differences then implies an acceleration of structural change towards services.

Differences in σ and in growth rates clearly have effects of a different calibre. Differences in growth rates are the most important contributor to structural change. Without them, the remaining channels (differences in the elasticity of substitution, in capital intensity and the non-homotheticity) generate only one quarter as much structural change in terms of changes in the capital allocation, while the labor allocation would move in the opposite direction. The reason for this is that with common growth rates, the less flexible sector (services) shifts towards more capital intensive production, given that A grows faster than B. Manufacturing must then move towards more labor-intensive production. The difference in σ again governs the shape of structural change. The share of output produced in manufacturing increases slightly in this scenario. By this metric, structural change thus is entirely driven by cross-sectoral differences in productivity growth rates.

Finally, while differences in the elasticity of substitution are not the main driver of structural change, they are key for getting both changes in the labor income shares and the shape of structural change right. Consider first the line labelled "common σ, a)". This line contains results obtained when changing σ, but keeping the remaining parameters as in the normalized benchmark. Equating σ affects the amount of structural change somewhat – the decline in the value added share of manufacturing is reduced by about 10% – but more importantly, strongly affects the shape of structural change. Without σ differences, manufacturing, with its larger difference between $g(A)$ and $g(B)$ and its larger $g(A)$, shifts towards more capital-intensive production. Combined with an overall movement out of manufacturing, this implies that manufacturing sheds capital substantially more slowly than it sheds labor – differently from the data. In addition, eliminating the differences in σ accentuates differences in the evolution of labor income shares, with the larger difference between $g(A)$

and $g(B)$ in manufacturing driving a much faster decline in the labor income share in that sector.

Similar patterns arise when the model is recalibrated under the restriction of common σ in both sectors (see the line labelled "common σ, b)"). Parameters for this case are given in the notes to Table 4. Here, too, common σ combined with the growth patterns implies uneven structural change: manufacturing sheds more labor than capital, differently from the data.[31]

To match observed patterns, two things are thus required: growth rate differences across sectors are key to generate structural change, and differences in σ_i are essential to generate changes in the labor income share, and to generate the right shape of structural change at the same time.

4.5 Another look at the Cobb-Douglas production function

It is clear from Section 3 that a model with Cobb-Douglas production functions at the sectoral level cannot match the evolution of sectoral labor income shares, and quantitatively cannot match the evolution of the aggregate labor income share either. It can, of course, still generate factor reallocation and thereby output reallocation. How well can it account for these, and which drivers of structural change matter most? Let each sector's production function be

$$Y_i = K_i^{\alpha_i}(A_i L_i)^{1-\alpha_i},$$

and allow α_i and $g(A_i)$ to vary across sectors.

For symmetry with above, we calibrate α_i to match average factor income shares, resulting in α_m of 0.333 and α_s of 0.362. Leaving $\varepsilon, \gamma, s, \delta$ and β as before, we then still need to calibrate $g(A_i)$. To do so, we use two targets, the growth rate of aggregate output per worker of 1.8% per year, and the change in the fraction of employment in manufacturing over the period 1960 to 2005, which is 22.4%. Doing so, we find $g(A_m)$ of 6.22% and $g(A_s)$ of -1.7%.

It is illustrative to compare these results to the CES case. The productivity growth rate in manufacturing under Cobb-Douglas lies close to a naive α_m-weighted average of the capital- and labor-augmenting growth rates from the CES case. In services, the difference is a bit larger but again, the Cobb-Douglas productivity growth rate lies in between the capital- and labor-augmenting ones, and closer to the latter – the factor with the larger output elasticity.

[31] This is qualitatively similar to what occurs when we simulate the model with Herrendorf, Herrington and Valentinyi's (2013) parameter estimates; see also footnote 26.

Table 5: Structural change: counterfactuals with Cobb-Douglas production functions

	dLIS$_m$	dLIS$_s$	dLIS (aggregate)	d(L_m/L)	d(K_m/K)	d(Y_m/Y)
data	-5.7	-2.5	-5.4	-22.4	-21.3	-24.5
CES benchmark	-6.0	-3.1	-4.4	-23.9	-20.7	-22.9
Cobb-Douglas:						
best fit	0.0	0.0	-0.6	-22.4	-21.7	-22.2
$s = 0$	0.0	0.0	-0.6	-21.5	-20.8	-21.3
common α	0.0	0.0	0.0	-23.8	-23.8	-23.8
common $g(A)$	0.0	0.0	-0.1	-2.4	-2.4	-2.4

Notes: LIS stands for "labor income share" and d for the absolute change between the first and last years of the sample. All changes are in percentage points. Line 5: α in both sectors is set to the average of α_m and α_s. Line 6: Common $g(A)$ is chosen such that the growth rate of aggregate output per worker is unchanged. This implies a value of $g(A)$ of 1.79%.

Decomposing structural change into the effects from different sources, results are starker than in the CES case: with Cobb-Douglas production functions, different productivity growth rates are responsible for almost the entirety of structural change. The non-homotheticity in preferences, being small, contributes only very little. Since the capital income share in services is higher than that in manufacturing, this is a channel that acts against the other two and leads to the movement of factors *into* the service sector. Again, this channel is not very powerful, as the difference in factor intensities is small.

Another thing that is clear from the Cobb-Douglas results is that with this production function, structural change looks similar, no matter which outcome measure we consider: the change in the allocation of labor, capital, or value added across sectors. With a Cobb-Douglas production function, only differences in α lead to small differences between structural change in terms of capital and labor inputs.

Of course, sectoral factor income shares do not change with productivity growth when sectoral production functions are Cobb-Douglas. The aggregate labor income share does change a little bit: as structural change moves labor out of manufacturing, the sector with the initially higher labor income share, the aggregate labor income share declines. However, this change in the aggregate income share, driven only by structural change, is weak. This is in line with results in Section 2, which showed that the bulk of the change in the aggregate labor income share was due to within-sector changes in the labor income share, and not due to composition effects.

Comparing results with CES and Cobb-Douglas production functions, it becomes clear that CES production functions are key not only for understanding the evolution of sectoral labor income shares, but also that of the aggregate labor income share, and the shape of structural change. Although the Cobb-Douglas specification can match the shape of structural change in the data, this is due to a coincidence: the effects of differential productivity growth rates and different substitution elasticities in manufacturing and services make structural change similar in terms of both capital and labor, and thus make it look consistent with the Cobb-Douglas specification. Yet, the evolution of labor income shares shows that this is just a coincidence, and that differences in g and σ are required to match the data. Explaining the evolution of sectoral factor income shares thus is not only a goal in itself, but also provides insights about the structure of production.

4.6 Robustness

We also compute results for an economy where preferences over manufacturing and services consumption are not Leontief. Just as Buera and Kaboski (2009), we consider a value of ε of 0.5. While far away from Leontief, this still implies that manufacturing and services output clearly are gross complements in consumption. We recalibrate the model in this setting, and conduct the same decomposition as above.

Table 6 shows calibration results. The calibration for higher ε clearly fits less well. While the model replicates changes in the factor income shares and structural change well, it proved impossible to get levels of factor income shares and the initial labor allocation to fit as closely as in our benchmark calibration. In particular, the average labor income share in services does not fit well. This also affects the fit of the aggregate labor income share. Note that changes in both the labor income share and in allocations do fit rather well, though. Parameters are overall similar; the manufacturing sector is more flexible, and growth rate patterns are as above, driven again by the same data patterns.

Broadly speaking, decomposition results are similar to those in the Leontief case. Nonhomotheticity of preferences hardly affects changes in labor income shares or structural change. Common α here has some effect, though this is also due to the fact that in this calibration, the average labor income share in services is off by 8 percentage points. Differences in σ and in growth rates again prove to the be most important determinants of changes in labor income shares and structural change. Eliminating differences in growth rates essentially eliminates structural change in terms of value added; the value added share of manufacturing falls by only half a percentage point. There is still substantial factor reallocation because of

Table 6: Calibration for $\varepsilon = 0.5$.

	Data		Model	
Panel A: Targets and model moments				
	Manufacturing	Services	Manufacturing	Services
calibration targets:				
$\overline{LIS_i}$ (%)	66.7	63.8	65.7	71.9
$g(KIS_i/LIS_i)$ (% p.a.)	0.6%	0.3%	0.60%	0.31%
$\mathrm{d}(L_m/L)$	-22.4		-22.6	
$\mathrm{d}(K_m/K)$	-21.3		-20.9	
$\overline{L_m/L}$	34.2		36.5	
$g(Y/L)$ (% p.a.)	1.8%		1.78%	
not targeted:				
$\overline{LIS}$ (aggregate, %)	64.8		69.5	
$g(KIS/LIS)$ (agg., % p.a.)	0.5		0.3	
$\overline{K_m/K}$	33.2		44.1	
$\overline{Y_m/Y}$	35.1		38.6	
$\mathrm{d}(Y_m/Y)$	-24.5		-21.9	

Panel B: Parameters

sector-specific:	Manufacturing	Services	*general:*	
$g(A_i)$ (% p.a.)	9.8	0.8	δ	0.05
$g(B_i)$ (% p.a.)	-2.6	-6.9	β	0.94
σ_i	0.875	0.534		
α_i	0.341	0.279		

Note: The data period used is 1960 to 2005. Averages are geometric means over this period. Absolute changes are percentage point changes over the entire period. Growth rates are computed using averages of the first and last 5 years. For readability, annual percentage changes are given for output growth and the growth in relative income shares.

σ differences, though: Since overall, growth rates are such that efficiency units of labor become more abundant relative to capital, the more flexible sector (manufacturing) substitutes towards that more abundant input. Without structural change on average, this implies that labor flows into manufacturing, while the fraction of capital used in the sector declines.

Eliminating differences in σ in this setting affects not only the shape but also the amount of structural change. As above, a larger difference between $g(A)$ and $g(B)$ in manufacturing implies that if σ is equal, labor moves towards services more quickly than capital does. With

Table 7: Structural change: counterfactuals ($\varepsilon = 0.5$)

	dLIS$_m$	dLIS$_s$	dLIS (aggregate)	d(L_m/L)	d(K_m/K)	d(Y_m/Y)
data	-5.7	-2.5	-5.4	-22.4	-21.3	-24.5
best fit	-6.1	-2.8	-2.4	-22.6	-20.9	-21.9
$s = 0$	-6.1	-2.9	-2.7	-21.6	-19.6	-20.8
common α	-5.0	0.3	-0.4	-24.5	-20.6	-23.4
common $g(A)$	-11.8	-44.1	-25.6	14.4	-18.9	-0.6
common σ	-15.2	-2.6	-6.0	-17.5	-6.0	-13.8

Notes: LIS stands for "labor income share" and d for the absolute change between the first and last years of the sample. All changes are in percentage points. Except for the parameters that are equated, all parameters as in Table 6. Line 4: α in both sectors is set to the average of α_m and α_s in Table 6. Line 5: $g(A_m) = g(A_s) = 0.0462$ and $g(B_m) = g(B_s) = -0.0543$. This value implies that the growth rate of y is the same as in the benchmark economy, and the average distance between $g(A_i)$ and $g(B_i)$ is preserved. Line 6: $\sigma_m = \sigma_s = 0.705$. This is the average of σ_m and σ_s.

structural change taking place at the same time, this results in a decline of the fraction of capital used in manufacturing that is slower than the decline of the fraction of labor employed in manufacturing. The changes in sectoral labor income shares are in line with this. Differences in growth rates tend to lead to a faster decline of the labor income share in manufacturing. The higher substitution elasticity in manufacturing counteracts this; eliminating the σ difference then makes the labor income share in manufacturing decline much faster than that in services. The largest difference between this case and the benchmark is that equating σ affects not only the shape of structural change, but also how much structural change takes place overall: the decline in the share of value added produced in the manufacturing sector drops from 22.9 percentage points in the benchmark calibration to 13.8 percentage points here.

Overall, these results are in line with our benchmark results above. We can thus conclude that while growth rate differences across sectors are the primary driver of structural change from manufacturing to services, differences in σ are key for understanding the behavior of sectoral labor income shares, and the shape of structural change. They may also to some extent contribute to the amount of structural change that is observed.

5 Conclusion

In this paper, we have established a connection between two important developments that have been taking place over the last half century: large reallocations of resources and production from manufacturing to services, and a decline in the aggregate labor income share. We have documented that most of the decline in the aggregate labor share, which has occurred in several developed countries, does not result from sectoral reallocations but rather from declines in the labor income shares at the sectoral level, particularly in manufacturing. We propose a novel mechanism for structural change consistent with both developments: sectoral differences in the elasticity of substitution. We then explore the joint implications of this mechanism for structural change and for the evolution of factor income shares. Using data on sectoral factor income shares implies restrictions on sectoral production functions that turn out to be important not only for tracing the evolution of sectoral and aggregate factor income shares, but also for reproducing the shape of structural change. The essence of our mechanism is simple. As the relative abundance of inputs of production changes with the process of economic growth, so does their relative price. In the presence of sectoral differences in the elasticity of substitution, this induces a process of factor reallocation with relatively abundant factors moving towards relatively flexible sectors, i.e. sectors with a relatively high elasticity of substitution. Incorporating this mechanism into a standard model of structural change that allows for non-homothetic preferences, sector-specific technological change and differences in capital intensity, we evaluate the empirical relevance of the determinants of the process of structural change for the U.S. since 1960. We find that differences in sector-specific productivity growth have been the most important driver of the transition from manufacturing to services, with differences in capital intensity and non-homothetic preferences playing a minor role. Differences in the elasticity of substitution at the sectoral level are not only important for understanding the decline in sectoral and aggregate labor income shares, but also for the pattern of reallocation of capital and labor between sectors: the "shape" of structural change.

In this context, it is natural to wonder what kept the U.S. labor income share roughly constant for most of the last century. One possibility is that, as a result of structural change, sectoral changes canceled out in the aggregate. The careful estimates of sectoral labor income shares constructed by Valentinyi and Herrendorf (2008) show that in 1997, the labor income share in non-agriculture was 50% larger than that in agriculture, a difference of 21 percentage points. Caselli and Coleman II (2001) document a decrease in U.S. agricultural employment of 40 percentage points between 1880 and 1960. Over this period, the fraction of value added produced in agriculture declined by 21 percentage points. Combining these pieces of evidence, structural change out of agriculture by itself would generate an increase in the aggregate labor income share of the order of 4.5 percentage points. This development would have counteracted a decline in the labor income share driven by manufacturing. In view of these calculations, it is possible that at the aggregate level, changes in the labor income shares in manufacturing and agriculture cancelled each other. This would be in line with Keynes's (1939) observation (cited in Elsby et al. 2013, p. 13) that the "remarkable constancy" of the aggregate labor share is "a bit of a miracle".

Another possibility, along the lines suggested by Acemoglu (2002), is that the mix of capital- and labor-augmenting technological change was different in the past, leading sector-level changes in income shares to balance at the aggregate level. At this stage, these are conjectures; they constitute interesting topics for future work.

References

Acemoglu, D. (2002), 'Directed technical change', *Review of Economic Studies* **69**(4).

Acemoglu, D. and Guerrieri, V. (2008), 'Capital Deepening and Non-Balanced Economic Growth', *Journal of Political Economy* **116**(3), 467–498.

Alvarez-Cuadrado, F. and Long, N. (2011), 'Capital-labor substitution, structural change and growth', *CIRANO Scientific Series* **2011s-68**.

Antras, P. (2004), 'Is the US aggregate production function Cobb-Douglas? New estimates of the elasticity of substitution', *Contributions to Macroeconomics* **4**(1), 1–34.

Blanchard, O. (1997), 'The Medium Run', *Brookings Papers on Economic Activity* **2**(4), 89–158.

Buera, F. J. and Kaboski, J. P. (2009), 'Can Traditional Theories of Structural Change Fit the Data?', *Journal of the European Economic Association* **7**(2-3), 469–477.

Buera, F. J. and Kaboski, J. P. (2012*a*), 'Scale and the Origins of Structural Change', *Journal of Economic Theory* **147**(2), 684–712.

Buera, F. J. and Kaboski, J. P. (2012*b*), 'The Rise of the Service Economy', *American Economic Review* **102**(6), 2540–69.

Caballero, R. J. and Hammour, M. L. (1998), 'Jobless Growth: Appropriability, Factor Substitution, and Unemployment', *Carnegie-Rochester Conference on Public Policy* **48**, 51–94.

Cantore, C. and Levine, P. (2012), 'Getting normalization right: dealing with "dimensional constants" in macroeconomics', *Journal of Economic Dynamics and Control* **36**, 1931–1949.

Caselli, F. and Coleman II, W. J. (2001), 'The u.s. structural transformation and regional convergence: A reinterpretation', *Journal of Political Economy* **109**(3).

Elsby, M. W. L., Hobijn, B. and Şahin, A. (2013), 'The Decline of the U.S. Labor Share', *Brookings Papers on Economic Activity* .

Herrendorf, B., Herrington, C. and Valentinyi, A. (2013), 'Sectoral technology and structural transformation', *mimeo, ASU* .

Herrendorf, B., Rogerson, R. and Valentinyi, A. (2013), 'Two Perspectives on Preferences and Structural Transformation', *American Economic Review* **103**(7), 2752–2789.

Herrendorf, B., Rogerson, R. and Valentinyi, A. (forthcoming), 'Growth and Structural Transformation', *Handbook of Economic Growth* .

Hicks, J. R. (1932), *The Theory of Wages*, Macmillan, London.

Jensen, B. S. and Larsen, M. E. (2005), 'General Equilibrium Dynamics of Multi-Sector Growth Models', *Journal of Economics* **10**, 17–56.

Jorgenson, D. W. (1990), Productivity and Economic Growth, *in* 'Fifty Years of Economic Measurement: The Jubilee Conference on Research in Income and Wealth', University of Chicago Press, Chicago, IL.

Jorgenson, D. W. (2007), '35 Sector KLEM'. `http://hdl.handle.net/1902.1/10684`.

Jorgenson, D. W., Gollop, F. M. and Fraumeni, B. M. (1987), *Productivity and U.S. Economic Growth*, Harvard University Press, Cambridge, MA.

Jorgenson, D. W. and Stiroh, K. J. (2000), 'Raising the Speed Limit: U.S. Economic Growth in the Information Age', *Brookings Papers on Economic Activity* **1**, 125–211.

Karabarbounis, L. and Neiman, B. (forthcoming), 'The Global Decline of the Labor Share', *Quarterly Journal of Economics* .

Keynes, J. M. (1939), 'Relative Movements of Real Wages and Output', *Economic Journal* **49**, 34–51.

Kongsamut, P., Rebelo, S. T. and Xie, D. (2001), 'Beyond Balanced Growth', *Review of Economic Studies* **68**(4), 869–882.

Krusell, P., Ohanian, L. E., Ríos-Rull, J.-V. and Violante, G. L. (2000), 'Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis', *Econometrica* **68**(5), 1029–1053.

Kuznets, S. (1966), *Modern economic growth: rate, structure, and spread*, Yale University Press, New Haven, CT.

León-Ledesma, M. A., McAdam, P. and Willman, A. (2010), 'Identifying the Elasticity of Substitution with Biased Technical Change', *The American Economic Review* **100**(4), 1330–1357.

Ngai, L. R. and Pissarides, C. (2007), 'Structural Change in a Multi-Sector Model of Growth', *American Economic Review* **97**(1), 429–443.

Ríos-Rull, J. V. and Choi, S. (2009), 'Understanding the Dynamics of Labor Share: The Role of Noncompetitive Factor Prices', *Annals of Economics and Statistics* **95/96**, 251–78.

Swiecki, T. (2013), 'Determinants of Structural Change', *mimeo, University of British Columbia* .

Temple, J. (2012), 'The calibration of CES production functions', *Journal of Macroeconomics* **34**, 294–303.

Valentinyi, Ákos. and Herrendorf, B. (2008), 'Measuring factor income shares at the sectoral level', *Review of Economic Dynamics* **11**(4), 820 – 835.

Woodland, A. D. (1982), *International Trade and Resource Allocation*, North-Holland, Amsterdam.

Appendix

A Data

A.1 Computation of labor income shares

This section closely follows Valentinyi and Herrendorf (2008). Let the number of commodities be M and the number of industries be I. Index commodities by m and industries by i. From the 35-sector KLEM data, we compute the following objects:

- The "use matrix" $\mathbf{B}$: this is an $M \times I$ matrix with representative element (m, i) that states how much output of commodity m is required to produce 1$ of output of industry i.
- The "make matrix" $\mathbf{W}$: this is an $I \times M$ matrix with representative element (i, m) that states for industry i which share of commodity m it produces.
- The final expenditure vector $\mathbf{y}$: this length-I column vector states the amount of output of each industry i that serves as final expenditure.

In our setting, we have no commodity level information and have to assume that each industry produces a single commodity. Therefore, $M = I$.

The final expenditure vector $\mathbf{y}$ can be computed by subtracting each industry's output that is used as an intermediate input in another industry from the total value of its output. The use matrix $\mathbf{B}$ be can be computed by dividing the value of intermediate inputs from industry M used in industry I by the value of gross output in industry I. When each industry produces a single, unique commodity, the make matrix $\mathbf{W}$ simply is the $I \times I$ identity matrix.

Let the column vector of sectoral shares of labor income in the value of gross output be α_l and the column vector of the sectoral shares of capital income in the value of gross output be α_k. Let the $I \times 1$ sector identification vector with elements $\mathbb{1}_{i=j}$ for a sector j be $\mathbf{1}_j$ and let $\mathbf{y}_j \equiv \mathbf{y}'\mathbf{1}_j$. Following Valentinyi and Herrendorf (2008), we then obtain the share of labor income in value added in sector j as

$$\frac{\alpha_l' \mathbf{W}(\mathbf{I} - \mathbf{BW})^{-1}\mathbf{y}_j}{(\alpha_l' + \alpha_k')(\mathbf{I} - \mathbf{BW})^{-1}\mathbf{y}_j}, \tag{52}$$

A.2 Sector classification.

U.S. data: Using data from Jorgenson's (2007) 35-sector data base, we construct sectors as follows:

Manufacturing: Sectors 7-27: Food and kindred products, Tobacco, Textile mill products, Apparel, Lumber and wood, Furniture and fixtures, Paper and allied, Printing, publishing and allied, Chemicals, Petroleum and coal products, Rubber and misc plastics, Leather, Stone, clay, glass, Primary metal, Fabricated metal, Non-electrical machinery, Electrical machinery, Motor vehicles, Transportation equipment & ordnance, Instruments, Misc. manufacturing

Services: Transportation, Communications, Electric utilities, Gas utilities, Trade, Finance Insurance and Real Estate, Other Services

Results from Section 2 are not sensitive to excluding Utilities from Services or to including Mining and Construction with Manufacturing, or Government in Services.

Cross-country data: Using EU-KLEMS data from `http://www.euklems.net`, we define manufacturing analogously to the U.S. data. Again, as there, we include utilities (sector E), wholesale and retail trade (G), hotels and restaurants (H), transport, storage and communication (I), financial intermediation (J), real estate etc. (K) in services. We exclude public administration and defence (L), education (M), health and social work (N), other community services (O), private households (P).

A.3 The measurement of industry factor income shares.

Jorgenson et al. (1987) contains a detailed description of measurement of industry-level factor income shares. The following are some key features. Labor compensation is measured as wage and salary income plus supplements, including employers' contribution to Social Security and unemployment compensation contributions by employers. Annual measures are computed using time actually worked. Earnings of the self-employed are split between capital and labor compensation assuming an after-tax rate of return that matches that of corporate businesses.

B Additional Tables and Figures

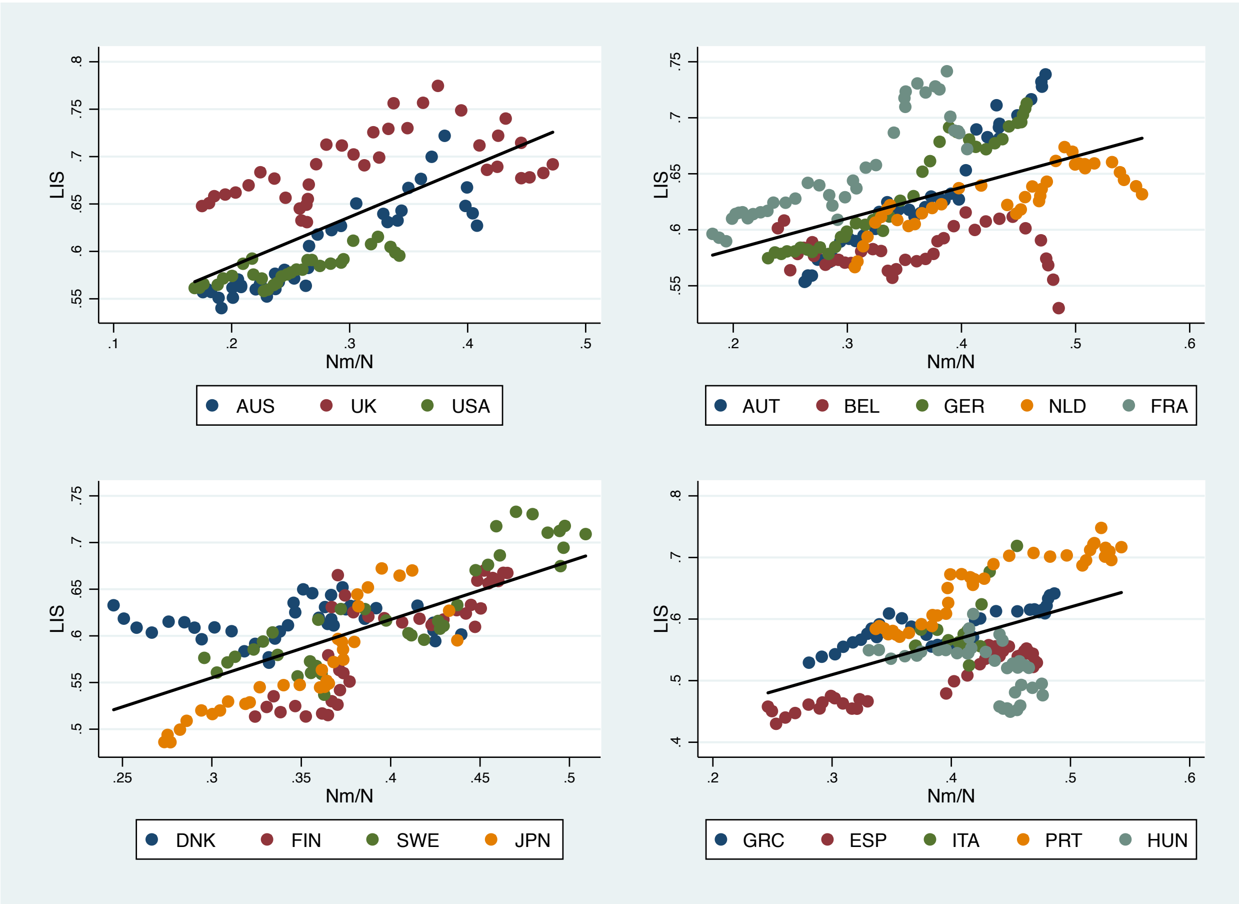


Figure 6: The joint evolution of the aggregate labor income share (LIS) and the employment share of manufacturing

C Details on Section 3

C.1 Derivation of $\lambda(\kappa)$

Combining (14) and (16) we obtain

$$\kappa = (1-\kappa)^{1/\sigma_2} \left(\frac{\gamma}{1-\gamma}\right) \left[(1-\alpha)(1-\lambda)^{\frac{\sigma_2-1}{\sigma_2}} \left(\frac{K}{A}\right)^{\frac{1-\sigma_2}{\sigma_2}} + \alpha(1-\kappa)^{\frac{\sigma_2-1}{\sigma_2}}\right] \tag{53}$$

Re-arranging (15)

$$(K/A)^{\frac{1-\sigma_2}{\sigma_2}} = \frac{(1-\lambda)^{\frac{1}{\sigma_2}}}{\lambda} \frac{\kappa}{(1-\kappa)^{1/\sigma_2}} \tag{54}$$

Re-arranging (53)

$$\kappa - (1-\kappa)^{1/\sigma_2} \left(\frac{\gamma}{1-\gamma}\right) \alpha(1-\kappa)^{\frac{\sigma_2-1}{\sigma_2}} = (1-\kappa)^{1/\sigma_2} \left(\frac{\gamma}{1-\gamma}\right) (1-\alpha)(1-\lambda)^{\frac{\sigma_2-1}{\sigma_2}} (K/A)^{\frac{1-\sigma_2}{\sigma_2}} \tag{55}$$

Substitute (54) into (55)

$$\frac{\kappa}{(1-\kappa)^{1/\sigma_2}} - \left(\frac{\gamma}{1-\gamma}\right) \alpha(1-\kappa)^{\frac{\sigma_2-1}{\sigma_2}} = \left(\frac{\gamma}{1-\gamma}\right) (1-\alpha)(1-\lambda)^{\frac{\sigma_2-1}{\sigma_2}} \frac{(1-\lambda)^{\frac{1}{\sigma_2}}}{\lambda} \left(\frac{\kappa}{(1-\kappa)^{1/\sigma_2}}\right)$$

or

$$1 - \left(\frac{\gamma}{1-\gamma}\right) \alpha(1-\kappa)^{\frac{\sigma_2-1}{\sigma_2}} \left(\frac{(1-\kappa)^{1/\sigma_2}}{\kappa}\right) = \left(\frac{\gamma}{1-\gamma}\right)(1-\alpha)\left(\frac{1-\lambda}{\lambda}\right)$$

or

$$1 - \left(\frac{\gamma}{1-\gamma}\right) \alpha \left(\frac{1-\kappa}{\kappa}\right) = \left(\frac{\gamma}{1-\gamma}\right)(1-\alpha)\left(\frac{1-\lambda}{\lambda}\right)$$

or

$$1 - \left(\frac{\gamma}{1-\gamma}\right) \alpha \left(\frac{1}{\kappa}-1\right) = \left(\frac{\gamma}{1-\gamma}\right)(1-\alpha)\left(\frac{1}{\lambda}-1\right)$$

Thus

$$\lambda = \frac{\gamma(1-\alpha)\kappa}{\kappa-\alpha\gamma} \quad \text{with} \quad \frac{d\lambda}{d\kappa} = -\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\lambda}{\kappa}\right)^2 < 0 \tag{56}$$

C.2 Derivation of Proposition 1

Equation (56) shows that λ and κ always move in opposite direction. Furthermore since $\lambda \leq 1$ and $\kappa \leq 1$, (56) determines the range of permissible values for κ is $[\underline{\kappa}, 1]$ and for λ is $[\underline{\lambda}, 1]$ where

$$0 < \underline{\lambda} \equiv \frac{\gamma(1-\alpha)}{1-\alpha\gamma} < 1$$

and

$$0 < \underline{\kappa} \equiv \frac{\alpha\gamma}{(1-\gamma)+\alpha\gamma} < 1$$

Combining (54) and (56) and taking logs we reach,

$$\frac{1-\sigma_2}{\sigma_2}\ln\left(\frac{K}{A}\right)=\frac{1}{\sigma_2}\ln(1-\lambda(\kappa))-\ln\lambda(\kappa)+\ln\kappa-\frac{1}{\sigma_2}\ln(1-\kappa)$$

This relationship is monotone increasing (iff $\sigma_2<1$): an increase in K leads to an increase in κ :

$$\left(\frac{1-\sigma_2}{\sigma_2}\right)\left(\frac{dK}{K}-\frac{dA}{A}\right)=G\left(\kappa\right)d\kappa \tag{57}$$

where

$$G(\kappa)\equiv\left[\frac{1}{\sigma_2\left(1-\lambda(\kappa)\right)}+\frac{1}{\lambda(\kappa)}\right]\left(\frac{\lambda(\kappa)}{\kappa}\right)^2\left(\frac{\alpha}{1-\alpha}\right)+\left[\frac{1}{\kappa}+\frac{1}{\sigma_2(1-\kappa)}\right]>0$$

where $\lambda(\kappa)$ is given by (56).

C.3 Remarks on Proposition 1

Notice that $K_1=\kappa K/\lambda$ and therefore $\hat{K}_1=\widehat{\kappa}-\widehat{\lambda}+\hat{K}=\left[\frac{K}{\kappa}\frac{\partial\kappa}{\partial K}-\frac{K}{\lambda}\frac{\partial\lambda}{\partial K}+1\right]\hat{K}$. We now show that the expression inside the squared brackets is positive.

$$\frac{K}{\kappa}\frac{\partial\kappa}{\partial K}-\frac{K}{\lambda}\frac{\partial\lambda}{\partial K}+1=\frac{(1-\sigma_2)}{\kappa\sigma_2G(\kappa)}+\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\lambda(\kappa)}{\kappa}\right)\frac{(\sigma_2-1)}{\kappa\sigma_2G(\kappa)}+\frac{\kappa\sigma_2G(\kappa)}{\kappa\sigma_2G(\kappa)}$$

$$=\frac{1}{\kappa\sigma_2G(\kappa)}+\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\lambda(\kappa)}{\kappa}\right)\frac{1}{\kappa\sigma_2G(\kappa)}+\frac{\kappa G(\kappa)-1-\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\lambda(\kappa)}{\kappa}\right)}{\kappa G(\kappa)}$$

Using (24)

$$\kappa G(\kappa)=\left[\frac{\kappa}{\sigma_2\left(1-\lambda(\kappa)\right)}\right]\left(\frac{\lambda(\kappa)}{\kappa}\right)^2\left(\frac{\alpha}{1-\alpha}\right)+\left[\frac{\kappa}{\sigma_2(1-\kappa)}\right]+\left(\frac{\lambda(\kappa)}{\kappa}\right)\left(\frac{\alpha}{1-\alpha}\right)+1$$

So

$$\kappa G(\kappa)-1-\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\lambda(\kappa)}{\kappa}\right)=\kappa\left[\frac{1}{\sigma_2\left(1-\lambda(\kappa)\right)}\right]\left(\frac{\lambda(\kappa)}{\kappa}\right)^2\left(\frac{\alpha}{1-\alpha}\right)+\left[\frac{\kappa}{\sigma_2(1-\kappa)}\right]>0$$

The analysis concerning the capital-labor ratio in the other sector, $K_2=\left(\frac{1-\kappa}{1-\lambda}\right)K$, is similar.

C.4 Derivation of $f(\chi)$

This appendix provides some details on the derivation of $f(\chi) \equiv A\pi(\kappa(\chi))\chi = \dfrac{Y}{K}\chi$.

Using (26) we can rewrite (15) as,

$$\frac{(1-\kappa)^{\frac{1}{\sigma_2}}}{\kappa}\frac{\lambda}{(1-\lambda)^{\frac{1}{\sigma_2}}} = \chi^{\frac{\sigma_2-1}{\sigma_2}}$$

which, after using (17) to replace λ, gives the following one-to-one relationship between χ and κ

$$\chi = (\gamma(1-\alpha))^{\frac{\sigma_2}{\sigma_2-1}}\frac{(1-\kappa)^{\frac{1}{\sigma_2-1}}}{(\kappa-\alpha\gamma)\left(\kappa\left(1-\gamma(1-\alpha)\right)-\alpha\gamma\right)^{\frac{1}{\sigma_2-1}}} \tag{58}$$

where $\chi'(\kappa) < 0$ (respectively $\chi'(\kappa) > 0$) for all $\kappa \in (\underline{\kappa}, 1)$ if $\sigma_2 > 1$ (respectively $\sigma_2 < 1$). Furthermore, it is worth noticing that when $\sigma_2 > 1$, $\chi(1) = 0$ and $\lim_{\kappa\to\underline{\kappa}}\chi(\kappa) = \infty$, and when $\sigma_2 < 1$, $\chi(\underline{\kappa}) = 0$ and $\lim_{\kappa\to 1}\chi(\kappa) = \infty$.

Combining (5) and (16), the agregate output-capital ratio is given by,

$$\frac{Y}{K} = \left[(\lambda\chi^{-1})^{1-\alpha}(\kappa)^{\alpha}\right]^{\gamma}\left[(1-\alpha)\left((1-\lambda)\chi^{-1}\right)^{\frac{\sigma_2-1}{\sigma_2}} + \alpha((1-\kappa))^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{(1-\gamma)\sigma_2}{\sigma_2-1}}$$

which can be expressed using (14), (16), (17), and (58) as

$$\frac{Y}{K} = A\pi(\kappa) \tag{59}$$

where

$$A \equiv \left[\frac{1-\gamma}{\gamma}\right]^{\frac{(1-\gamma)\sigma_2}{\sigma_2-1}}\left(\gamma\left(1-\alpha\right)\right)^{-\frac{(1-\alpha)\gamma}{\sigma_2-1}}, \tag{60}$$

$$\pi(\kappa) \equiv \frac{\kappa^{(\sigma_2-\gamma)/(\sigma_2-1)}\left(\kappa\left(1-\gamma(1-\alpha)\right)-\alpha\gamma\right)^{\frac{\gamma(1-\alpha)}{\sigma_2-1}}}{(1-\kappa)^{\frac{1-\alpha\gamma}{\sigma_2-1}}}, \tag{61}$$

Notice that when $\sigma_2 > 1$ (respectively, $\gamma < \sigma_2 < 1$), $\pi(\kappa)$ is an increasing (respectively decreasing) function defined over the interval $[\underline{\kappa}, 1]$, with $\pi(\underline{\kappa}) = 0$ (respectively $\pi(\kappa) = \infty$) and $\lim_{\kappa\to 1}\pi(\kappa) = \infty$ (respectively $\pi(1) = 0$).

Finally, the definition of $f(\chi)$ follows from the fact that (58) defines κ as an implicit function of χ, $\kappa(\chi)$.

C.5 Properties of $f(\chi)$

This appendix determines relevant properties of the function $f(\chi)$.The following approach is an adaptation of the results in the book *International Trade and Resource Allocation* by A.D. Woodland, in particular pages 17-18 and 52-59.

There are n sectors. Let y_j be the output of sector j. The production function is $y_j = f^j(x_j)$ where x_j is a vector of primary inputs (e.g. capital, labor). Assume that $f^j(.)$ is linear homogeneous, continuous, quasi-concave, and increasing.

Let v be the vector of factor endowments (e.g., $v = (K, L)$). Define the production possibility set by

$$Q(v) \equiv \left\{ y : 0 \leq y_j \leq f^j(x_j), x_j \geq 0 \, , j = 1, ..., n; \sum\nolimits_{j=1}^{n} x_j \leq v \right\}$$

Then it can be shown that the production possibility set $Q(v)$ is non-empty, convex and compact in v. (Woodland, page 52).

Given the price vector $p = (p_1, p_2, ..., p_n)$, define the GNP function (Woodland, page 49):

$$G(p, v) = \max_y \{py : y \in Q(v)\}$$

Then it can be shown that (Woodland, page 58):

"$G(p, v)$ is a non-negative function of $p > 0$ and $v > 0$. It is non-decreasing, linear homogeneous, and concave in v; and non-decreasing, linear homogeneous, and convex in p." (p. 58).

In our problem, the corresponding idea is: instead of considering the sum $p_1y_1 + p_2y_2 + ... + p_ny_n$, we consider a final good production function that has as its inputs the sectoral output vector $y = (y_1, ..., y_n)$.

$$y_0 = F^0(y_1, ..., y_n)$$

Then, assuming that $F^0(y)$ is homogenous of degree 1, and increasing, we can define the function

$$F(v) = \max_y \left\{ F^0(y) : y \in Q(v) \right\}$$

Then using the type of arguments in Woodland, we can conclude that $F(v)$ is non-decreasing, linear homogeneous, and concave in v.

Then, if $v = (K, ML)$ where ML is effective labour, we have

$$y_0 = F(K, ML) = (ML)F\left[\frac{K}{ML}, 1\right] = (ML)F[\chi, 1] \equiv (ML)f(\chi)$$

where $f(\chi)$ is concave and increasing.

D Details on Section 4

Time is continuous and we model a closed economy populated by a representative households with preferences given by,

$$v = \ln\left(u(c_m, c_s)\right) \quad \text{where} \quad u(c_m, c_s) = \left(\gamma c_m^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(c_s + s\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{62}$$

where the variables and parameters are described in the body of the paper and instantaneous utility is discounted at a rate β. Sectoral outputs are produced according to the general CES technologies given by (36) and (37) and since factors are fully utilized conditions (38) and (39) apply.

We choose manufactures to be the numeraire and denote the price of services by p_s. Since markets are competitive, production efficiency requires equating marginal revenue products across sectors, so

$$\alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} \left(\frac{Y_m}{K_m}\right)^{\frac{1}{\sigma_m}} = r = p_s \alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} \left(\frac{Y_s}{K_s}\right)^{\frac{1}{\sigma_s}} \tag{63}$$

$$\left(1-\alpha_m\right) A_m^{\frac{\sigma_m-1}{\sigma_m}} \left(\frac{Y_m}{L_m}\right)^{\frac{1}{\sigma_m}} = w = p_s \left(1-\alpha_s\right) A_s^{\frac{\sigma_s-1}{\sigma_s}} \left(\frac{Y_s}{L_s}\right)^{\frac{1}{\sigma_s}}. \tag{64}$$

As a consequence, the following relationship between the sectoral capital-labor ratios emerges.

$$k_s = \left(\frac{1-\alpha_m}{1-\alpha_s}\frac{\alpha_s}{\alpha_m}\left(\frac{A_m}{B_m}\right)^{\frac{\sigma_m-1}{\sigma_m}}\right)^{\sigma_s} \left(\frac{A_s}{B_s}\right)^{1-\sigma_s} k_m^{\frac{\sigma_s}{\sigma_m}} \equiv \varrho k_m^{\frac{\sigma_s}{\sigma_m}}. \tag{65}$$

where $\varrho \equiv \left(\frac{1-\alpha_m}{1-\alpha_s}\frac{\alpha_s}{\alpha_m}\left(\frac{A_m}{B_m}\right)^{\frac{\sigma_m-1}{\sigma_m}}\right)^{\sigma_s} \left(\frac{A_s}{B_s}\right)^{1-\sigma_s}$ and using (39) the economy-wide capital-labor ratio can be written as

$$k = l_m k_m + l_s k_s = l_m k_m + \left(1-l_m\right) \varrho k_m^{\frac{\sigma_s}{\sigma_m}}. \tag{66}$$

Using this notation, the relative price is given by

$$p_s = \frac{\alpha_m}{\alpha_s}\frac{B_m^{\frac{\sigma_m-1}{\sigma_m}}}{B_s^{\frac{\sigma_s-1}{\sigma_s}}} \frac{\left[\alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} + (1-\alpha_m)(A_m/k_m)^{\frac{\sigma_m-1}{\sigma_m}}\right]^{\frac{1}{\sigma_m-1}}}{\left[\alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} + (1-\alpha_s)(A_s/(\varrho k_m^{\frac{\sigma_s}{\sigma_m}}))^{\frac{\sigma_s-1}{\sigma_s}}\right]^{\frac{1}{\sigma_s-1}}}, \tag{67}$$

On the household side the representative agent equates the marginal rate of substitution between the two goods to their relative price, so

$$\frac{v_s}{v_m} = \frac{(1-\gamma)}{\gamma}\left(\frac{c_s+s}{c_m}\right)^{-\frac{1}{\varepsilon}} = p_s. \tag{68}$$

Let's define the following variables,

$$x_s \equiv \frac{p_s\left(c_s+s\right)}{c_m} = \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(p_s\right)^{1-\varepsilon} \qquad \text{and} \qquad x_m \equiv \frac{c_m}{c_m} = 1 \tag{69}$$

and then, total expenditure, including the non-homothetic element, relative to consumption expenditure on manufactures is given by,

$$X = x_s + x_m = \frac{p_s\left(c_s+s\right)+c_m}{c_m} \equiv \frac{c}{c_m} \tag{70}$$

Now we are in a position to determine the static allocation of resources across sectors. Combining (67) and (70) we can express consumption of services as,

$$c_s = \frac{x_s c_m}{p_s} - s = x_s \frac{c}{X}\frac{\alpha_s}{\alpha_m}\frac{B_s^{\frac{\sigma_s-1}{\sigma_s}}}{B_m^{\frac{\sigma_m-1}{\sigma_m}}}\frac{D_s}{D_m}\frac{\left[\alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} + (1-\alpha_m)(A_m/k_m)^{\frac{\sigma_m-1}{\sigma_m}}\right]^{\frac{1}{1-\sigma_m}}}{\left[\alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} + (1-\alpha_s)(A_s/(\varrho k_m^{\frac{\sigma_s}{\sigma_m}}))^{\frac{\sigma_s-1}{\sigma_s}}\right]^{\frac{1}{1-\sigma_s}}} - s \tag{71}$$

since service output is fully consumed,

$$c_s = y_s = D_s\left[\alpha_s\left(B_s \varrho k_m^{\frac{\sigma_s}{\sigma_m}}\right)^{\frac{\sigma_s-1}{\sigma_s}} + (1-\alpha_s)\,A_s^{\frac{\sigma_s-1}{\sigma_s}} + \right]^{\frac{\sigma_s}{\sigma_s-1}} l_s \tag{72}$$

Combining the previous two expressions we can solve for l_m as a function of current values of the endogenous variables, k_m and c_m, given the exogenous levels of labor- and capital-augmenting productivity. This labor allocation together with the full employment condition for capital given by (66), determines the current sectoral allocation of resources.

Now we turn to the dynamic evolution of the economy. The Euler equation for this problem is given by

$$-\hat{v}_m = \alpha_m D_m B_m^{\frac{\sigma_m-1}{\sigma_m}}\left[\alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} + (1-\alpha_m)\,A_m^{\frac{\sigma_m-1}{\sigma_m}} k_m^{\frac{1-\sigma_m}{\sigma_m}}\right]^{\frac{1}{\sigma_m-1}} - (\delta+\beta) \tag{73}$$

since,

$$v_m = \frac{\gamma u^{\frac{1}{\varepsilon}} c_m^{-\frac{1}{\varepsilon}}}{u} = \frac{u_m}{u} \tag{74}$$

and preferences are homogeneous of degree 1,

$$u = u_m c_m + u_s\left(c_s+s\right) = u_m c_m + p_s u_m\left(c_s+s\right) = \left(c_m + p_s\left(c_s+s\right)\right)u_m = c u_m \tag{75}$$

where the second equality uses (68) and (74) and the last one uses (70).

Combining these last two results with the Euler equation we reach,

$$-\hat{v}_m = \hat{c} = \alpha_m D_m B_m^{\frac{\sigma_m - 1}{\sigma_m}} \left[\alpha_m B_m^{\frac{\sigma_m - 1}{\sigma_m}} + (1 - \alpha_m) A_m^{\frac{\sigma_m - 1}{\sigma_m}} k_m^{\frac{1 - \sigma_m}{\sigma_m}} \right]^{\frac{1}{\sigma_m - 1}} - (\delta + \beta) \tag{76}$$

that using equation (70) could be expressed in terms of c_m that is convenient for the numerical solution given that manufacturing output is the only source of capital accumulation

The per capita capital stock evolves according to,

$$\dot{k} = D_m \left[\alpha_m (B_m k_m)^{\frac{\sigma_m - 1}{\sigma_m}} + (1 - \alpha_m) A_m^{\frac{\sigma_m - 1}{\sigma_m}} \right]^{\frac{\sigma_m}{\sigma_m - 1}} l_m - c_m - \delta k \tag{77}$$

that using the full employment condition for capital, (66), can be expressed in terms of k_m. Equations (45) and (48) in the main text are the discrete-time counterparts of these last two equations.

Récents cahiers de recherche du CIREQ
Recent Working Papers of CIREQ

Si vous désirez obtenir des exemplaires des cahiers, vous pouvez les télécharger à partir de notre site Web http://www.cireqmontreal.com/cahiers-de-recherche

If you wish to obtain copies of the working papers, you can download them directly from our website, http://www.cireqmontreal.com/cahiers-de-recherche

02-2013 Amarante, M., "Conditional Expected Utility", février 2013, 19 pages

03-2013 Benchekroun, H., F. Taherkhani, "Adaptation and the Allocation of Pollution Reduction Costs", mai 2013, 33 pages

04-2013 Bossert, W., H. Peters, "Single-Basined Choice", juin 2013, 15 pages

05-2013 Okasha, S., J.A. Weymark, W. Bossert, "Inclusive Fitness Maximization : An Axiomatic Approach", mai 2013, 17 pages

06-2013 Ehlers, L., B. Klaus, "House Allocation via Deferred-Acceptance", juillet 2013, 13 pages (2013-05)

07-2013 McCausland, W.J., A.A.J. Marley, "Bayesian Inference and Model Comparison for Random Choice Structures", juillet 2013, 26 pages (2013-06)

08-2013 Cardia, E., P. Gomme, "The Household Revolution : Childcare, Housework, and Female Labor Force Participation", juillet 2013, 44 pages (2013-07)

09-2013 Dutta, R., S. Horan, "Inferring Rationales from Choice : Identification for Rational Shortlist Methods ", mars 2013, 51 pages

10-2013 Dutta, R., R. Ishii, "Coordinating by Not Committing : Efficiency as the Unique Outcome", août 2013, 38 pages

11-2013 Amarante, M., "A Representation of Risk Measures", octobre 2013, 8 pages (2013-08)

12-2013 Amarante, M., "A Characterization of Exact Non-atomic Market Games", octobre 2013, 8 pages (2013-09)

13-2013 Dufour, J.-M., L. Khalaf, M. Voia, "Finite-Sample Resampling-Based Combined Hypothesis Tests, with Applications to Serial Correlation and Predictability", octobre 2013, 23 pages

14-2013 Zhang, H.J., J.-M. Dufour, J.W. Galbraith, "Exchange Rates and Commodity Prices : Measuring Causality at Multiple Horizons", septembre 2013, 41 pages

15-2013 Gaudet, G., P. Lasserre, "The Taxation of Nonrenewable Natural Resources", octobre 2013, 41 pages (2013-10)

16-2013 Benchekroun, H., G. Gaudet, "On the Effects of Mergers on Equilibrium Outcomes in a Common Property Renewable Asset Oligopoly", octobre 2013, 33 pages (2013-11)

17-2013 Andersson, T., L. Ehlers, "An Algorithm for Identifying Agent-k-Linked Allocations in Economies with Indivisibilities", novembre 2013, 7 pages