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1 Introduction

The burning of fossil fuels generates emissions that harm the environment not only in the present but also in the future, because emissions add to a pollution stock which decays only very slowly. As has been pointed out by many authors, the first best policy measure is to impose at each point of time a tax on emissions that equals the capitalized value of the stream of marginal damages of emissions (see e.g. Hoel, 2011, Ploeg and Withagen, 2012). When the first best measure cannot be implemented because of political constraints, there are a variety of policy measures that at first sight might seem to approximate the first best measure. However, in some cases, a careful analysis would reveal that some policies that seemingly would do the job will actually turn out to have an adverse effect on the environment, contrary to the good intention of the policy makers. This outcome is known as the Green Paradox (Sinn, 2008a,b, 2012).

The possibility of a Green Paradox outcome has been shown to exist under a wide variety of circumstances. Sinn (2008a,b) pointed out that the announcement of a steeply rising path of carbon tax can induce owners of oil and coal reserves to extract their resources more quickly, resulting in a worse climate outcome in the short and the medium term.[1] Hoel (2008) demonstrated that technical progress in the backstop technology that produces at constant cost a clean energy which is a perfect substitute for fossil fuels would have a similar effect on the supply behavior of resource-extracting firms.[2] Grafton et al. (2012) showed that an increase in a time-independent ad valorem rate of subsidy on biofuels can result in a Green Paradox outcome if the supply curve for biofuels is sufficiently concave. Ploeg and Withagen (2012) showed that whether a Green Paradox arises may depend on whether extraction costs increase sharply as the size of the remaining stock diminishes. Long and Stähler (2012) demonstrated that, if both fossil fuels and non-fossil fuels are being used concurrently, a fall in production cost of non-fossil fuels may generate income effects leading to a fall in the interest rate, which in turn induces greater current extraction rate of fossil fuels, and possibly greater cumulative extraction.[3]

All the above models assume that if several types of fuels are available, they are *perfect substitutes*. While perfect substitution is often a useful assumption to simplify the analysis, one must admit that at the present level of technology, biofuels cannot entirely replace petroleum in a number of uses, e.g., in aviation. Efforts are being made to improve the substitutability of biofuels for petroleum. This paper therefore relaxes the assumption of perfect substitutability.[4] This allows us to ask a number of interesting questions. Does a

[1] For some early analyses of responses of intertemporal extraction plan to anticipations of taxation or expropriation, see Long (1975) and Long and Sinn (1985). For an overview of recent contributions to the Green Paradox literature, see van der Werf and di Maria (2011).

[2] See also Strand (2007). Though early models of substitute production of Heal (1976) and Hoel (1978,1983) did not deal with CO_2 emissions, they contained all the ingredients from which one can deduce a Green Paradox result. Welsch and Stähler (1990) provided an early treatment of the dynamic supply response of the Green-Paradox type.

[3] There is a large literature on technological changes in the context of exhaustible resources. See for example Pittel and Bretschger (2011), and Acemoglu et al. (2012). However, Long and Stähler (2012) were the first to focus on the endogenously generated interest rate effect.

[4] Our model is consistent with empirical facts concerning biofuel production. The main producing countries for transport biofuels are the U.S., Brazil and the EU. Brazil and the U.S. produced 55 and 35 percent,

technological change that makes biofuels a *closer substitute* to petroleum benefit or harm the environment in the near term? If it harms the environment, we call this a technology-induced Green Paradox (as distinct from tax/subsidy induced Green Paradox).[5]A related question is whether imperfect substitutability increase or reduce the likelihood of a Green Paradox outcome induced by raising the subsidy rate on biofuels or by increasing the base-year rate of an advalorem tax on fossil fuels.

When the prices for fossil fuels and non-fossil fuels are not identical because of imperfect substitutability, the analysis can become tedious, because now there are several prices to consider, and they will be changing over time.To facilitate the analysis, this paper defines the concept of a "reduced-form demand function" for fossil fuels. This function incorporates the parameters of the demand and supply functions of the clean energy. Using this reduced form demand function, we are able to analyse the possibility of a Green Paradox outcome caused by a technological change by identifying its direct effect (usually "pro-green"), and its indirect effect (usually "anti-green"). The direct effect is defined as the change in the quantity demanded keeping the price of fossil fuels constant (while allowing the price of non-fossil fuels to change to clear that market). The indirect effect arises from intertemporal optimization behavior of owners of fossil fuel stocks. It works through the change in the equilibrium price path of fossil fuels.

We assume that the non-fossil energy is produced under increasing marginal cost. This assumption reflects the reality that renewable substitutes such as biofuels are produced using different grades of land (Chakravorty et al., 2011). In fact, the greater use of biofuels may, by moving up the supply curve of land, increase the unit cost of production of the renewable substitutes. By and large, the existing analyses of biofuel subsidies use a static framework. In the static context, many authors have identified mechanisms for increased carbon emissions that could result from biofuel subsidies. For example, a common argument is that the production process of biofuels is not environmentally friendly because it involves the use of inputs with high carbon contents.

The present paper, by showing how the equilibrium price path of fossil fuels responds to increased substitutability, complements the existing literature with a dynamic mechanism that arises even if a technical change in favor of substitutability occurs only once. The main contribution is a delimitation of cases when a Green Paradox outcome occurs and when it does not.[6]

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respectively, of the world's ethanol in 2009 while the EU produced 60 percent of the total biodiesel output. There are a wide range of policies that encourage the substitution away from fossil fuels, especially for motor vehicles. Government mandates for blending biofuels into vehicle fuels have been introduced in no less than 17 countries, and in many states and provinces within these countries. Typical mandates require blending 10–15 percent ethanol with gasoline or blending 2–5 percent biodiesel with diesel fuel. Recent targets have encouraged greater adoption rate of biofuels in various countries (UNEP, 2009, page 15-16).

[5]Subsidies and tax exemptions have complemented quantity-based policies such as setting targets and blending quotas. In the US, the total value of biofuels supports in 2008 was estimated to be between \$9.2 and 11.07 billion. These include consumption mandates, tax credits, import barriers, investment subsidies and general support to the sector such as public research investment. See Koplow (2007, pp. 29-31).

[6]Fischer and Salant (2012) examined the Green Paradox in the presence of a subsidy for renewable resources. However, unlike our model, they assumed (a) perfect substitutablity, and (b) constant unit cost of renewable in any given period. They allow unit cost to fall over time through knowledge accumulation.

It will be shown that for the case of a system of linear demand functions, an increase in substitutability can result in a Green Paradox outcome, if the existing degree of substitutability is already high. On the other hand, starting from a very low degree of substitutability, a small increase in substitutability cannot generate a Green Paradox outcome.

I also examine the case where the fossil fuel producers form a cartel and act as a Stackelberg leader while biofuel producers are followers. It is found that for certain range of parameter values, there is a Green Paradox outcome induced by an increase in substitutability.

2 A brief review of the related literature

There is a large literature that connects the dynamic analysis of non-renewable fossil resources with climate change damages. Sinclair (1992, 1994) pointed out that climate change policies must aim at delaying the extraction of oil, and argued that the carbon tax, expressed in ad valorem term, must decline over time to encourage owners of fossil fuels to defer extraction. This result, however, depends in part on the assumptions that (a) damages appear multiplicatively in the production function, (b) the pollution stock does not decay, and (c) capital and oil are substitutable inputs in a Cobb-Douglas production function.[7]

Recently, Groth and Schou (2007) confirm Sinclair's declining tax result using a similar model, allowing endogenous growth. Ulph and Ulph (1994), specifying damages as an additive term in the social welfare function, and assuming exponential decay of pollution, find that the time profile of the optimal per unit carbon tax has an inverted U shape. Hoel and Kverndokk (1996) specify a model of economic exhaustion that includes rising extraction costs. They show that carbon tax peaks before the peak in atmospheric carbon. Consistent with Ulph and Ulph (1994), they find that in the optimal long run the carbon tax approaches zero. They also consider the case where there is a backstop technology that produces a substitute at constant cost. Farzin and Tahvonen (1996), assuming the decay rate of pollution to be non-linear in the stock, show that the optimal carbon tax can take a variety of shapes. Similarly, Tahvonen (1997) obtains eleven different tax regimes, depending on initial sizes of the stock of CO_2 concentration and the stock of fossil fuels.

Contrary to the above models which focussed on the optimal carbon tax, the key point of the Green Paradox literature is that the optimal carbon tax cannot be implemented given the political economy that exists in most countries. In particular, this literature argues that climate change policies that superficially may seem to be second-best measures could cause environmental damages, if the response of owners of fossil fuel stocks is to hasten their extraction (for example, to avoid high future carbon taxes). Models that depict this adverse response to anticipation of taxes or substitute production include Sinn (2008a,b), Hoel (2008), Di Maria et al. (2008), Gerlagh and Liski (2008), Smulders et al. (2009), and Eichner and Pethig (2010).[8] According to Strand (2007), a technological international

[7] Heal (1985) and Sinn (2008a,b) also model damages from GHGs emissions as a negative externality in production. Most papers however specify damages as an additive term in the social welfare function.

[8] As pointed out in Hoel (2008), prior to 2008, "there is little work making the link between climate policies and exhaustible resources when policies are non-optimal or international agreements are incomplete". He mentioned a few exceptions: Bohm (1993), Hoel (1994) in a static framework.

agreement that makes carbon redundant in the future may increase current emissions. Hoel (2008, 2011b) assumes that carbon resources remain cheaper than the substitute and analyses the situation where different countries have climate policies of different ambition levels. He shows that, in the absence of an efficient global climate agreement, climate costs may increase as a consequence of improved technology of substitute production.

Gerlagh (2011), in studying the effect of an improvement in the backstop technology, makes a distinction between a Weak Green Paradox and a Strong Green Paradox. The former is said to arise when current emissions increase as a result of an improvement in the backstop technology. The latter arises when the net present value of the stream of all future damages increases. He shows that both the Weak and the Strong Green Paradox arise in the benchmark model with constant extraction cost and unlimited supply of the backstop energy. Assuming linear demand, he finds that increasing extraction costs counteract the Strong Green Paradox, while a rising marginal cost of the substitute may reduce the likelihood of both the Weak and the Strong Green Paradox. Ploeg and Withagen (2012) address the case of stock-dependent marginal extraction costs while retaining the assumption that the substitute is available in unlimited supply at a constant marginal cost. They show that if first-best policies are not feasible, a Green Paradox occurs if the cost of backstop decreases, provided that the backstop remains expensive such that the non-renewable resource stock is eventually exhausted. By contrast, if the backstop becomes so cheap that physical exhaustion will not take place, thethen there is no Green Paradox outcome.

Grafton et al. (2012) emphasise the facts that biofuels are already available, but the expansion of biofuel output is possible only with increasing costs. They assume that biofuels and fossil fuels are *perfect substitutes.* Using a framework where *both* types of fuels are simultaneously consumed in the first phase, they find conditions under which a Green Paradox outcome will *not* occur, as well as conditions under which it will occur. Their main focus was on the effect of a biofuel subsidy on the date of exhaustion of the fossil fuel resources. A striking result was that in the case of a linear demand for energy, together with (i) a zero extraction cost for fossil fuels and (ii) an upward-sloping linear marginal cost of biofuels, a biofuel subsidy will have *no effect* on the amount of fossil fuels extracted at each point of time, even though it will reduce fuel prices. The increase in quantity of fuel demanded is exactly matched by an increased output of biofuels, leaving the extraction rate unchanged. Thus, the time at which the stock of fossil fuels is exhausted is unchanged. Their result stands in sharp constrast to the inevitable Green Paradox in the model of Hoel (2008) which assumes that the supply curve of the renewable resource is horizontal. When the assumption of zero extraction cost is replaced by the assumption of a positive constant marginal extraction cost, a biofuel subsidy will result in a longer time over which the fossil fuel stock is exhausted. In this second case there is no Green Paradox outcome: the subsidy works as intended; it delays the exhaustion time. Finally, Grafton et al. consider the case where the marginal cost of biofuel production is strictly increasing and strictly concave. In this situation along the equilibrium price path, as the price of energy rises gradually, the rate of the supply increase (per dollar increase in energy price) is *greater* when the price is higher. Consequently, fossil fuel firms, in anticipation of the greater expansion of the substitute in the later stage, respond by increasing their extraction at an earlier date. In this case, a Green Paradox outcome is obtained.

3 A model of imperfect substitutability between fossil fuels and renewable energy

We consider an economy with three goods: fossil fuels, denoted by x, renewable energy, denoted by y, and a numeraire good, denoted by z. Assume that y is a perfectly clean source of energy. In contrast, the consumption of fossil fuels generates emissions which contribute to a stock of pollution.

For simplicity, we assume that the demands for these goods come from the utility maximization of a representative consumer. Goods x and y are *imperfect substitutes* and thus command different prices.

The stock of pollution at time t is denoted by S_t. The rate of change in S_t is assumed to be equal to x_t. This simplifying assumption may be justified on the ground that the rate of natural decay of GHG pollution is very slow. Then

$$S_t = S_0 + \int_0^t x_\tau d\tau$$

Fossil fuels are extracted from a resource stock R_t:

$$\dot{R}_t = -x_t,\ R(0) = R_0,\ R_t \geq 0. \tag{1}$$

Then cumulative emissions from time zero to time t is

$$\int_0^t x_\tau d\tau = R(0) - R_t$$

and the stock of pollution is linearly related to the stock of exhaustible resources,

$$S_t = S_0 + R(0) - R_t$$

Note that R_t will be falling over time, causing S_t to rise over time, until the resource stock is exhausted.

We assume that the representative consumer has a separable net utility function

$$U(x_t, y_t, z_t) - C(S_t) \tag{2}$$

where $C(S_t)$ represents the damages caused by pollution. Because of the pollution externalities embodied in the net utility function (2), it is widely thought that policies that encourage replacing fossil fuels by a clean energy should be promoted. One often hears arguments in favour of the subsidization of public and private R&D activities that would increase the degree of substitutability of clean energy for fossil energy.

In this paper, we do not model R&D activities. Instead, we wish to find out whether an exogenous technical progress that increases the substitutability is good or bad for the environment, given that first best policies are not available.

3.1 Assumptions on demand

For simplicity, we abstract from the income effect, and assume that U is quasi-linear: $U(x_t, y_t, z_t) = u(x_t, y_t) + z_t$. This assumption implies that any income change will impact only the demand for the numeraire good. As usual, it is assumed that income is sufficiently large such that the consumption of the numeraire good is strictly positive. The function $u(x_t, y_t)$ is strictly concave.

We assume that the marginal utility of any good is finite even when its consumption is zero.

Let P_{1t} and P_{2t} denote the consumer prices of good x_t and y_t respectively. Consumer's maximization then leads to the following first order conditions that characterize an interior maximum:

$$u_1(x_t^d, y_t^d) = P_{1t}$$
$$u_2(x_t^d, y_t^d) = P_{2t}.$$

where u_1 and u_2 stand for the partial derivatives of u with respect to x and y respectively. The superscript d in x_t^d and y_t^d indicate that these are quantities demanded. From the FOCs, we obtain the demand functions

$$x_t^d = X^d(P_{1t}, P_{2t}, \mu) \tag{3}$$

$$y_t^d = Y^d(P_{1t}, P_{2t}, \mu) \tag{4}$$

where we have used μ as a parameter representing the degree of substitutability. We assume that each of these demand functions is decreasing in its own price, and increasing in the price of the other good (its imperfect substitute). Furthermore, we assume that an increase in substitutability will reduce the demand for a good if its price is higher than the price of the substitute:

$$\frac{\partial x_t^d}{\partial \mu} < 0 \text{ if } P_{1t} > P_{2t}$$

$$\frac{\partial y_t^d}{\partial \mu} < 0 \text{ if } P_{2t} > P_{1t}$$

These assumptions are satisfied for the standard linear quadratic preferences.

3.2 Assumptions on the supply of renewable energy

Let y_t^s denote the quantity of renewable energy supplied at time t. We assume that marginal cost is positive and increasing in output level, y_t^s. The producers of renewables are price takers. They produce at the level that equates their marginal cost to the producer price P_{2t}^f:

$$MC(y_t^s) = P_{2t}^f$$

The superscript f indicates that this is the price the firms receive per unit sold. The difference between the producer price and the consumer price, $P_{2t}^f - P_{2t}$, is the subsidy on renewables.

Let us assume that there is a constant ad valorem subsidy rate, denoted by $s \geq 0$, so that

$$P_{2t}^f = (1 + s)P_{2t} \equiv hP_{2t}$$

We call h the "subsidy factor". Note that $h \geq 1$.

We can then derive the supply function for renewables:

$$y_t^s = G(hP_{2t})$$

where G in the inverse of the marginal cost function MC. Clearly, $G' > 0$ because we assumed an upward sloping MC function.

3.3 Supply of fossil fuels and the choke price

We assume that there are a large number of identical resource-extracting firms, each owning a deposit of fossil fuels. The deposits are homogeneous and are of identical size. The aggregate fossil resource stock at time zero (the present time) is denoted by R_0. Resource-extracting firms operate under perfect competition. They perfectly forecast the price path of the fossil fuel. Assume the marginal cost of extraction is constant, $c \geq 0$. Define the net price of the fossil fuel as $P_{1t} - c$. As long as the net price rises at the rate of interest r, individual fossil fuel producers are willing to supply any amount. In equilibrium, however, the industry's supply of the fossil fuel at any time t must equal the demand for it. There will be a time T when the price of fossil fuel is so high that the demand for fossil fuel becomes zero, and from that time onwards only renewable energy is consumed, in quantity $\overline{y}$ and at the price $\overline{P}_2$. Such a high price for fossil fuel is called the "choke price" for fossil fuel, and we denote it by $\overline{P}_1$. The price $\overline{P}_1$ is implicitly defined by the following three equations, which determined $(\overline{P}_1, \overline{P}_2, \overline{y})$:

$$u_1(0, \overline{y}) = \overline{P}_1$$

$$u_2(0, \overline{y}) = \overline{P}_2$$

$$\overline{y} = G(h\overline{P}_2)$$

Let T be the time at which the aggregate fossil resource stock is exhausted. Then $P_{1T} = \overline{P}_1$. From the Hotelling Rule, the present value of the net price is the same at all points of time in the interval $[0, T]$:

$$(P_{1t} - c)e^{-rt} = \left(\overline{P}_1 - c\right) e^{-rT} \text{ for all } t \leq T$$

This means that if we know T, we can calculate P_{1t} as follows

$$P_{1t} = c + \left(\overline{P}_1 - c\right) e^{-r(T-t)} \equiv \phi(\overline{P}_1, T, t)$$

Then the time of exhaustion T must satisfy the following equation, which requires that total consumption of fossil fuel from time 0 to time T must be equal to the total resource stock:

$$\int_0^T X^d(P_{1t}, P_{2t}, \mu)dt = R_0$$

where, in this equation, P_{1t} is given by the function $\phi(\overline{P}_1, T, t)$ specified above. But what about P_{2t}? Since the market must clear, the demand for the renewable energy at any time t must equal its supply. Thus

$$Y^d(\phi(\overline{P}_1, T, t), P_{2t}, \mu) = G(hP_{2t})$$

This equation shows that P_{2t} can be expressed as a function of $\phi(\overline{P}_1, T, t)$, which is the equilibrium P_{1t} along the Hotelling path. This observation leads us to a simple reformulation, using the concept of the "reduced form demand function for fossil fuel." This will be made clear in the next section.

3.4 The reduced-form demand function for fossil fuels

We assume that the market for renewables clears at each point of time: the quantity demanded equals the quantity supplied. Then

$$Y^d(P_{1t}, P_{2t}, \mu) = G(hP_{2t})$$

This relationship allows us to solve for the equilibrium consumer price *function* for renewable energy, which we denote by $\pi_2(.)$,

$$P_{2t} = \pi_2(P_{1t}, \mu, h) \tag{5}$$

In other words, given the subsidy factor h and given the fossil fuel price P_{1t}, we can deduce the price P_{2t} that would clear the market for renewables. Clearly,

$$\frac{\partial \pi_2}{\partial P_{1t}} > 0 \text{ and } \frac{\partial \pi_2}{\partial h} < 0$$

where the first inequality reflects the fact that the two goods are substitutes rather than complements. The second inequality simply means that an increase in the subsidy factor for renewables will reduce the equilibrium consumer price for renewables, at any given price of fossil fuels.

Substituting (5) into equation (3), we obtain the demand function for fossil fuels, given that the market for renewables clears:

$$x_t^d = X^d(P_{1t}, \pi_2(P_{1t}, \mu, h), \mu) \equiv D^r(P_{1t}, \mu, h). \tag{6}$$

We call $D^r(P_{1t}, h, \mu)$ the *reduced form demand* for good x. Notice that the function D^r does not contain the price P_{2t} as an argument. This does not mean that the demand for fossil fuels is independent of the price of non-fossil fuels. Rather, the market-clearing P_{2t} which is conditional on P_{1t}, has been used.

Clearly D^r is decreasing in P_{1t}:

$$\frac{\partial D^r}{\partial P_{1t}} = \frac{\partial X^d}{\partial P_{1t}} + \frac{\partial X^d}{\partial P_{2t}} \frac{\partial \pi_2}{\partial P_{1t}} < 0.$$

We assume that D^r is decreasing in μ, at least for those values of P_{1t} high enough so that $P_{1t} > P_{2t} = \pi_2(P_{1t}, \mu, h)$. In particular, assume that at any P_1 near the choke price $\overline{P}_1$, a small increase in substitutability will reduce the demand for fossil fuels:

$$\frac{\partial D^r(\overline{P}_1, \mu, h)}{\partial \mu} < 0. \tag{7}$$

This assumption can be verified in the linear quadratic utility function case.[9]

From our earlier definition of the choke price for fossil fuels, $\overline{P}_1$, it holds that

$$0 = D^r\left(\overline{P}_1, h, \mu\right).$$

From the properties of the function D^r, we deduce that an increase in μ will reduce the choke price:

$$\frac{d\overline{P}_1}{d\mu} = -\frac{\frac{\partial D^r(\overline{P}_1,\mu,h)}{\partial \mu}}{\frac{\partial D^r}{\partial P_1}} < 0.$$

3.5 The two phases

We consider an equilibrium path consisting of two phases:

Phase 1: Both fossil fuels and renewable energy are simultaneously supplied.

Phase 2: Only renewable energy is supplied, because the fossil fuel stock has been exhausted.

Let T be the time at which Phase 1 ends and Phase 2 begins. At time T, the fossil fuel stock is just exhausted. We must determine T endogeneously. At time T, we have $x(T) = 0$.

Thus T must satisfy the equation

$$\int_0^T D^r(P_{1t}, \mu, h)dt - R_0 = 0$$

where $P_{1t} = \phi(\overline{P}_1, T, t)$.

We wish to show that there is a range of value for μ such that, at any given μ in this range, a small increase in it will lead to an earlier exhaustion date. The following fact is obvious:

Fact 1: *If there exists a real interval (μ^*, μ^{**}) such that the exhaustion time T is decreasing in μ then a small increase in μ in this interval will cause the pollution stock to be higher for all $t < T$.*

4 Effect of an increase in substitutability on the exhaustion time

What are the forces that determine the net effect of an increase in the substitutability parameter μ on the time of exhaustion T? We have assumed (and this can be verified for the linear quadratic case) that an increase in μ will lower the fossil fuel choke price $\overline{P}_1$.

At the same time, the inequality (7) indicates that an increase in μ rotates the (reduced form) demand curve for fossil fuel in the anti-clockwise direction. Does an increase in μ generate, on average, greater demand for fossil fuels over the time interval T? To answer this question, it is useful to decompose the effect of an increase in μ on fossil fuel consumption into a direct effect and an indirect effect.

[9]It can also be shown that D^r in increasing in h. This implies that a subsidy for renewables will, at constant P_{1t}, reduce the demand for fossil fuels.

Direct Effect: The anti-clockwise rotation of the (reduced form) demand curve for fossil fuels implies that at any given price P_{1t} sufficiently high, the quantity demanded is smaller than before. This direct effect is captured by the term D_μ which is generally negative, at least for high P_{1t}. The direct effect is generally "pro-Green": an increase in substitutability reduces demand for fossil fuels, at any given sufficiently high price P_{1t}.

Indirect Effect: Since the increase in μ (substitutability) lowers the fossil fuel choke price $\overline{P}_1$, it follows that, holding T constant, the price P_{1t} must fall for all $t < T$. A fall in P_{1t} increases the quantity demanded. The indirect effect, captured by the term $\frac{\partial D^r}{\partial P_{1t}} \frac{\partial P_{1t}}{\partial \mu}$, is positive, i.e. it is "anti-Green".

The total effect on fossil fuels consumption at any time t is the sum of the direct effect and the indirect effect at that time. In general, if the function $D^r(\phi, \mu, h)$ is not restricted beyond the usual assumption that the demand curve is downward sloping, the total effect can be positive at some points of time and negative at some other points of time. Therefore, to find the effect of an increase in μ on the exhaustion time T, one has to compute the cumulative total effect, over the interval $[0, T]$. If this cumulative total effect is positive, it means that the exhaustion time is brought closer to the present, which is a Green Paradox outcome. A more formal analysis follows.

To find the net effect of a change in μ on the exhaustion time T, we define the function

$$\Omega(T, \mu, h) = \int_0^T D\left(P_{1t}, \mu, h\right) dt - R_0 = 0 \tag{8}$$

where $P_{1t} = \phi(\overline{P}_1(\mu), T, t)$.

Then

$$\frac{dT}{d\mu} = -\frac{\Omega_\mu}{\Omega_T} = \frac{-\int_0^T \left[D^r_{P_1} \frac{\partial P_{1t}}{\partial \mu} + D^r_\mu\right] dt}{D^r(\overline{P}_1, \mu) + \int_0^T \left[D^r_{P_1} \frac{\partial P_{1t}}{\partial T}\right] dt} \tag{9}$$

The denominator is positive. Thus we can state the following Proposition.

Proposition 1: (Necessary and sufficient condition for a Green Paradox). *A small increase in substitutability will bring the resource exhaustion date closer to the present if and only if the cumulative sum of the indirect effect (anti-Green) outweighs the cumulative sum of the direct effect (generally pro-Green):*

$$\int_0^T \left[D^r_{P_1} \frac{\partial P_{1t}}{\partial \mu} + D^r_\mu\right] dt > 0 \tag{10}$$

Without a more explicit specification of the reduced form demand function D^r, we cannot determine whether a Green Paradox outcome will arise from an increase in substitutability.

4.1 Parameterizing substitutability: the linear quadratic case

For illustrative purpose, consider the following linear-quadratic formulation.

Assume that

$$U(x, y, z) = \theta x - \frac{b}{2}x^2 + \theta y - \frac{b}{2}y^2 - \mu xy + z$$

where $\mu \geq 0, \theta > 0$ and $b > 0$. To ensure that U is concave, we assume that $b > \mu$. In the limiting case where $\mu = b$, the two types of fuels are perfect substitutes.

The above utility function implies that for given (x, y), if the goods become closer substitute (μ increases) then the utility decreases.[10] On the other hand, for environmental reasons, an increase in substitutability gives the economy the potential to increase welfare by a well designed change in the composition of demand to reduce environmental damages.

At each point of time t, the consumer faces the budget constraint

$$P_{1t}x_t + P_{2t}y_t + z_t = M_t$$

where M_t is the total expenditure allocated to period t. Assume that M_t is sufficiently great, so that z_t is always positive. Then the consumer's FOCs with respect to x_t and y_t are

$$\theta - bx_t - \mu y_t - P_{1t} \leq 0,\ x_t \geq 0,\ x_t\left[\theta - bx_t - \mu y_t - P_{1t}\right] = 0 \tag{11}$$

$$\theta - by_t - \mu x_t - P_{2t} \leq 0,\ y_t \geq 0,\ y_t\left[\theta - by_t - \mu x_t - P_{2t}\right] = 0 \tag{12}$$

It is straightforward to derive the demand functions

$$x_t^d = \max\left\{0, \frac{(\theta - P_{1t})b - (\theta - P_{2t})\mu}{b^2 - \mu^2}\right\} \tag{13}$$

$$y_t^d = \max\left\{0, \frac{(\theta - P_{2t})b - (\theta - P_{1t})\mu}{b^2 - \mu^2}\right\} \tag{14}$$

We will focus on the phase where both types of fuels are demanded in positive amounts. Note that if $P_{1t} \geq P_{2t}$ an increase in substitutability will reduce the demand for fossil fuels:[11]

$$\frac{\partial x_t^d}{\partial \mu} = \frac{-(b-\mu)^2(\theta - P_{2t}) - 2\mu b(P_{1t} - P_{2t})}{\left(b^2 - \mu^2\right)^2}$$

Renewable energy is produced by perfectly competitive firms, under increasing marginal cost. Its supply is denoted by y^s. Assume that the marginal cost of producing y^s is $A + By^s$, where $0 \leq A < \theta$ and $B > 0$.Then the supply of renewable energy satisfies the condition that marginal revenue, hP_{2t} , is equated to marginal production cost, $A + By^s$,

$$hP_{2t} = A + By_t^s \tag{15}$$

which gives

$$y_t^s = \frac{hP_{2t} - A}{B} > 0 \text{ if } hP_{2t} \geq A.$$

Recall our assumption that the market for renewables clears at each point of time. Equating y_t^d with y_t^s, we obtain the equilibrium P_{2t} for a given P_{1t}

$$\frac{(\theta - P_{2t})b - (\theta - P_{1t})\mu}{b^2 - \mu^2} = \frac{hP_{2t} - A}{B}$$

[10] We could add a scale parameter that depends on μ, so that when μ increases, the scale parameter also change in such a way that, keeping current consumption constant, utility rises. We refrain from doing this in order to keep the analysis simple.

[11] Note, however, that if P_{1t} is much lower than P_{2t}, then an increase in μ will increase x_t.

It is convenient to define

$$\lambda = \frac{A}{\theta} < 1$$

Then

$$P_{2t} = \frac{(b-\mu)\,(B\theta + (b+\mu)\lambda\theta)}{bB + (b^2-\mu^2)h} + \frac{B\mu P_{1t}}{bB + (b^2-\mu^2)h} \equiv \pi_2(P_{1t}, \mu, h) \tag{16}$$

where π_2 is increasing in P_{1t}, decreasing in h, and decreasing in μ:

$$\frac{\partial \pi_2}{\partial \mu} = -\frac{B\theta h(b-\mu)^2 + 2\mu AbB^2}{\left(bB + (b^2-\mu^2)h\right)^2} < 0 \tag{17}$$

Thus, for a *given* P_{1t}, an increase in substitutability reduces the market clearing price P_{2t}.

Remark: Along the Hotelling path, P_{1t} will be rising, and so will P_{2t}, as equation (16) indicates.

$$\frac{\partial P_{2t}}{\partial P_{1t}} = \frac{B\mu}{bB + (b^2-\mu^2)h} \tag{18}$$

In particular, this response is greater, the higher is degree of substitutability. We record this result as Fact 2:

Fact 2: *The increase in the price of renewables in response to the increase in the price of the exhaustible resource along the Hotelling path is itself an incresasing function of the substitutability parameter μ.*

Notice that, given market clearance, the gap between the renewable energy price and the fossil fuel price can be expressed as follows,

$$P_{2t} - P_{1t} = \frac{B(b-\mu)\,(1 + (b+\mu)\lambda)\,\theta}{bB + (b^2-\mu^2)h} - \left[\frac{h(b-\mu)^2 + (b-\mu)B}{bB + (b^2-\mu^2)h}\right] P_{1t} \tag{19}$$

Thus the price gap $P_{2t} - P_{1t}$ decreases as P_{1t} increases.

Substituting (16) into (13) we obtain the "reduced form demand function" for fossil fuels:

$$x_t^d = W - VP_{1t} \equiv D^r(P_{1t}, \mu, h) \tag{20}$$

where

$$V(h,\mu) \equiv \frac{bh + B}{(b^2-\mu^2)h + bB} > 0$$

$$W(h,\mu) \equiv \frac{(b-\mu)h\theta + \mu\lambda\theta + B\theta}{(b^2-\mu^2)h + bB} > 0$$

Let us find the choke price for fossil fuels, given that non-fossil fuels are available. Setting $D^r(P_{1t}, \mu, h) = 0$, we obtain the "choke price" for fossil fuels:

$$\overline{P}_1 = \frac{W}{V} = \frac{(b-\mu)h\theta + \mu\lambda\theta + B\theta}{bh + B} = \theta\left(1 - \frac{\mu(h-\lambda)}{bh + B}\right)$$

Note that $\overline{P}_1$ is decreasing in μ and increasing in B, where $1/B$ measures the steepness of the marginal cost curve of renewable fuels.

Equation (20) can be used to draw the reduced form demand curve for fossil fuels in a Marshallian-type diagram where P_{1t} is measured along the vertical axis, and x_t on the horizontal axis. Here W is the intercept on the horizontal axis, while vertical intercept, W/V, is the "choke price" for fossil fuels. The slope of this demand curve is $1/V$.

Since $h \geq 1 > \lambda$, an increase in substitutability will lower the choke price $\overline{P}_1$, as expected.

When P_{1t} equals the choke price $\overline{P}_1$, then $x_1^d = 0$, and the equilibrium price of non-fossil fuels then attains its steady-state price $\overline{P}_2$ as defined below:[12]

$$\overline{P}_2 = \frac{B\theta + b\lambda\theta}{bh + B}.$$

How does an increase in substitutability affect the reduced form demand function for fossil fuels? Since

$$V_\mu = \frac{2\mu h\,(bh + B)}{\left[h(b^2 - \mu^2) + bB\right]^2} \geq 0$$

it follows that an increase in substitutability makes the residual demand curve flatter.

We now show that there exists a unique threshold $\widetilde{\mu}$, where $0 < \widetilde{\mu} < b$, such that an increase in μ will move the quantity intercept of the reduced form demand curve to the left if $\mu \in (0, \widetilde{\mu})$ and to the right if $\mu \in (\widetilde{\mu}, b)$. From the definition of $W(h, \mu)$, we obtain

$$W_\mu = \frac{-(h\theta - \lambda\theta)\,(h(b^2 - \mu^2) + bB) + 2\mu h\,((b - \mu)h\theta + \mu\lambda\theta + B\theta)}{\left[h(b^2 - \mu^2) + bB\right]^2} \tag{21}$$

The sign of the numerator is the sign of the expression

$$g(\mu) = -h(h - \lambda)\mu^2 + 2h(bh + B)\mu - b(bh + B)(h - \lambda)$$

Since $h \geq 1 > \lambda$, the function $g(\mu)$ is quadratic and concave in μ. It is negative at $\mu = 0$ and positive at $\mu = b$. Consequently, there exists a unique value $\widetilde{\mu}$ in $(0, b)$ such that for all $\mu \in (0, \widetilde{\mu})$, a marginal increase in μ will shift the quantity intercept to the left, and for all $\mu \in (\widetilde{\mu}, b)$, a marginal increase in μ will shift the quantity intercept to the right.

The threshold value $\widetilde{\mu}$ is given by

$$\widetilde{\mu} = \frac{(bh + B) - \sqrt{(bh + B)(bh + B - \frac{b(h-\lambda)^2}{h})}}{h - \lambda} \tag{22}$$

It follows from equation (21) that keeping P_{1t} constant, a marginal increase in μ will unambiguously reduce the quantity of fossil fuels demanded (a pro-green effect) if μ is in $(0, \widetilde{\mu})$. In contrast, if μ is in $(\widetilde{\mu}, b)$, then $W_\mu > 0$ and a marginal increase in μ will increase the quantity of fossil fuels demanded if P_{1t} is low, and reduce it if P_{1t} is high. Therefore, for any given $\mu \in (\widetilde{\mu}, b)$, there exists a corresponding positive "pivot price" $\widetilde{P}_1(\mu)$ such that for at any given price P_{1t} below this pivot price, a marginal increase in μ will increase the demand for fossil fuels.[13]

[12] Interestingly, $\overline{P}_2$ is independent of μ.

[13] It can also be shown that for any given $P_{1t} \geq 0$, an increase in the subsidy rate for renewables will result in a lower demand for fossil fuels. It follows that an increase in the subsidy rate for renewables will reduce the demand for fossil fuels at any given price, and make the slope of the demand curve flatter.

For $\mu \in (\widetilde{\mu}, b)$, the pivot price is

$$\widetilde{P}_1(\mu) \equiv \frac{W_\mu(\mu)}{V_\mu(\mu)} = \frac{g(\mu)\theta}{2\mu h\,(bh+B)} < \overline{P}_1 \tag{23}$$

Thus, at any P_{1t} in the interval $\left(\widetilde{P}_1(\mu), \overline{P}_1\right)$, an increase in μ will reduce the demand for fossil fuels.

5 Sufficient Conditions for a Green Paradox outcome caused by an increase in substitutability

Let us investigate the possibility of a Green Paradox outcome when substitutability increases, under the assumption of linear demand arising from a linear quadratic utility function.There are two ways to proceed with the analyisis. The first method makes use of Proposition 1 (in section 4) and consists of determining whether the necessary and sufficient condition (10) is satisfied. The second method consists of evaluating the market clearing equation (8) directly, which allows us to solve for T as a function of μ. The first approach is useful because it is applicable also to the case of non-linear reduced form demand functions.

5.1 Approach 1: evaluating the direct effect and the indirect effect

Let us first evaluate the direct effect at any given point of time. For the case of linear reduced form demand, the direct effect is

$$\begin{aligned} D^r_\mu &= W_\mu - V_\mu P_{1t} \\ &= W_\mu - V_\mu\left[c + \left(\overline{P}_1(\mu) - c\right)e^{-r(T-t)}\right] \end{aligned}$$

This term is negative if $\mu < \widetilde{\mu}$. If $\mu > \widetilde{\mu}$, this term is positive (pro-green) for low values of P_{1t} and negative (anti-green) for high values of P_{1t}.

On the other hand, the indirect is unambiguously anti-green:

$$D^r_{P_1}\frac{\partial P_{1t}}{\partial \mu} = e^{-r(T-t)}\frac{(h-\lambda)\theta}{(b^2-\mu^2)h + bB} > 0$$

Combining the direct effect and the indirect effect, and defining $\gamma = c/\theta$,we obtain the following Proposition.

Proposition 2: *Under the linear quadratic formulation, the necessary and sufficient condition for a Green Paradox outcome (a marginal increase in substitutability hurts the environment) is that μ is sufficiently high such that*

$$f(\mu) = -h(h-\lambda)\mu^2 + 2h(bh+B)(1-\gamma)\mu - b(bh+B)(h-\lambda) > 0$$

Remark: The function $f(\mu)$ is quadratic and concave in μ.It is negative at $\mu = 0$ and positive at $\mu = b$ provided that γ is sufficiently small such that

$$B\,(h+\lambda) + 2bh\lambda > 2h(bh+B)\gamma \tag{24}$$

Therefore, if this condition holds, there exists a unique value μ^* in $(0, b)$ such that for all $\mu \in (0, \mu^*)$, a marginal increase in μ will reduce the exhaustion time, bringing climate change damages closer to the present. Note that $0 < \widetilde{\mu} < \mu^*$. In fact,[14]

$$\mu^* = \frac{(bh+B)(1-\gamma) - \sqrt{(bh+B)((bh+B)(1-\gamma)^2 - \frac{b(h-\lambda)^2}{h})}}{h-\lambda}$$

Numerical Examples

Setting $h = 1, b = 1, \lambda = 0.5, \gamma = 0.1,$ $B = 0.1$, and $r = 0.05$. Then $\mu^* \simeq 0.30$. Thus a Green Paradox outcome will occur when there is a marginal increase in substitutability, where $\mu \in (0.30, 1)$. For example, if initially $\mu = 0.50$, and $R_0/\theta = 417.88$ then competitive firms will take 500 years to exhaust the stock. Let there be a technological progress such that the substitutability increases by 4%, i.e. the new μ is 0.52. We find that the time of exhaustion falls by 1.2%, i.e. T is now 494 years (making exhaustion occur 6 years earlier).

Let us consider a smaller γ. Say $\gamma = 0.02$. Then $\mu^* \simeq 0.27$. Thus a smaller extraction cost facilitates a Green Paradox outcome

What about the subsidy factor h? Keeping all other parameters as specified in the base line scenatio, but let $h = 1.5$. Then $\mu^* \simeq 0.43$, i.e. a Green Paradox outcome is less likely.

Consider now a higher marginal cost intercept of renewables, A. An increase in A is equivalent to an increase in λ. Let the new λ be $\lambda = 0.6$. Then $\mu^* \simeq 0.23$, i.e.a Green Paradox outcome occurs for a wider range of μ.

An increase in B (the steepness of the supply curve) also makes a Green Paradox outcome is more likely. If $B = 5$, then $\mu^* \simeq 0.28$.

5.2 Approach 2: direct computation of the exhaustion time

Using the linear reduced form demand function D^r, we can compute the exhaustion time directly from the equation

$$\int_0^T (W - VP_{1t})\, dt = R_0$$

where

$$P_{1t} = c + \left(\overline{P}_1 - c\right) e^{-r(T-t)} = c + \left(\frac{W}{V} - c\right) e^{-r(T-t)}$$

Then the condition that total demand for fossil fuels over the time interval $[0, T]$ must equal the total supply reduces to a simple equation:

$$\int_0^T (W - VP_{1t})\, dt = (W - cV)\left(T - \frac{1 - e^{-rT}}{r}\right) = R_0 \qquad (25)$$

This equation allows us to compute T, once W, V, r and R_0 are specified. Since W and V are dependent on μ, we can determine how T responds to an increase in μ.

Define

$$G(\mu) = W(\mu) - cV(\mu)$$

[14] The condition that $f(b) > 0$ is sufficient for the root μ^* to be real.

$$F(T) = T - \frac{1 - e^{-rT}}{r}$$

Note that $G(\mu) > 0$ because we have assumed that c is lower than the choke price.Then the exhaustion time T can be obtained from

$$G(\mu)F(T) = R_0$$

Proposition 3: *The effect of a marginal increase in substitutability on the exhaustion time is*

$$\frac{dT}{d\mu} = \frac{-F(T)G'(\mu)}{G(\mu)F'(T)}$$

and the necessary and sufficient condition for a marginal increase in substitutability to reduce exhaustion time is $G'(\mu) > 0$, i.e.

$$W_\mu - cV_\mu > 0 \tag{26}$$

Proof: Use the facts that

$$F'(T) = 1 - e^{-rT} > 0$$

and

$$G(\mu)F'(T)dT + F(T)G'(\mu)d\mu = dR_0.$$

Remark: The condition (26) is identical to $f(\mu) > 0$. Since $V_\mu > 0$, a necessary condition for a Green Paradox outcome is $W_\mu > 0$.

Condition $W_\mu - cV_\mu > 0$ is equivalent to

Corollary: *If condition (26) holds, the stock of pollution at each point of time is increasing in the subsitutability parameter μ for $\mu \in (\mu^*, b)$.*

Proof: The equilibrium time path of price is

$$P_{1t} = c + \left(\overline{P}(\mu) - c\right) e^{-r(T(\mu)-t)}$$

Since $x_t(\mu) = W - VP_{1t}(\mu)$ and

$$S_t = S_0 + \int_0^t x_\tau d\tau$$

It follows that

$$\frac{dS_t(\mu)}{d\mu} = \int_0^t \frac{dx_\tau}{d\mu} d\tau$$

where

$$\frac{dx_\tau}{d\mu} = D_P^r \frac{dP_{1t}(\mu)}{d\mu} = -V \frac{dP_{1t}(\mu)}{d\mu}$$

$$= -V \left[e^{-r(T(\mu)-t)} \frac{d\overline{P}}{d\mu} + \left(\overline{P}(\mu) - c\right)\left(-r\frac{dT}{d\mu}\right) \right] < 0$$

Therefore the stock S_t increases with μ if $\mu \in (\mu^*, b)$.

6 Monopoly

What happens if the fossil firm is a monopolist? The monopolist chooses the price path of price P_{1t} to maximize its stream of discounted profits, knowing the reduced form function

$$x_t = D^r(P_{1t})$$

This formulation implies that the monopolist is a leader in the market for fuels. He knows that the price of non-fossil fuels will adjust to his announced price of fossil fuel so that the market clears. Formally, let T_m denote the monopolist's exhaustion time.The monopolist seeks the terminal time T_m and the price path P_{1t} defined over $[0, T_m]$ to maximize

$$\int_0^{T_m} (P_{1t} - c)\, D^r(P_{1t}) e^{-rt} dt$$

subject to

$$\dot{R}_t = -D^r(P_{1t})$$

and

$$R_{T_m} \geq 0.$$

The following results are obtained: The monopolist's planned exhaustion time T_m is determined by

$$\frac{1}{2}(W - Vc)\left(T_m - \frac{1 - e^{-rT_m}}{r}\right) = R_0 \tag{27}$$

Thus, from equations (25) and (27), we can see that

(i) the threshold level of substitutability beyond which a Green Paradox outcome occurs is the same under monopoly as under perfect competition,

(ii) the relationship between the monopolist's exhaustion time T_m and the competitive exhaustion time T_c satisfies the following equation:

$$\frac{\left(T_m - \frac{1-e^{-rT_m}}{r}\right)}{\left(T - \frac{1-e^{-rT}}{r}\right)} = 2$$

We can state the following Proposition:

Proposition 4: *The threshold level of substitutability beyond which a Green Paradox outcome occurs is the same under monopoly as under perfect competition. The monopolist takes a longer time to exhaust the stock R_0, and the response of T_m to an increase in substitutability when $\mu \in (\mu^*, b)$ is of the same sign as the response of T_c to an increase in substitutability. In absolute value, a given increase in μ decreases the monopolist's exhaustion time by more than under perfect competition.*

A Numerical Example

We set $h = 1, b = 1, \lambda = 0.5, \gamma = 0.1,\ B = 0.1$, and $r = 0.05$. Then a Green Paradox outcome will occur if $\mu \in (\mu^*, 1)$, where $\mu^* \simeq 0.30$. Let $R_0/\theta = 417.88$. We have found that that if $\mu = 0.50$, then under perfect competition, competitive firms will take 500 years to exhaust the stock. Under the same parameter values, the monopolist will exhaust the stock in 980 years. Suppose there is a technological progress such that substitutability increases by 4%, i.e. the new μ is 0.52. Under perfect competition, the time of exhaustion falls by 1.2%, i.e. T_c is now 494 years (making exhaustion occur 6 years earlier). Under monopoly, the time of exhaustion falls by 1.22% (T_m falls from 980 to 968, making exhaustion occur 12 years earlier).

7 Concluding Remarks

This paper explores the possibility of a Green Paradox associated with an increase in the extent to which non-fossil fuels can be substituted for fossil fuels. We have shown that a technological change that increases marginally the degree of substitutability may cause fossil fuels producers to anticipate lower demand in the future, and to react by increasing current extraction, leading to higher near-term emissions and accelerating climate change damages. Such a Green Paradox outcome is more likely to occur if the existing degree of substitutability is moderate or high. In fact, if the current degree of substitutability is near zero, then there will be no Green Paradox outcome associated with a marginal increase in substtitutability.

APPENDIX: Proof of Proposition 4

Let ψ_t denote the co-state variable. The Hamiltonian is

$$H = (P_{1t} - c)\, D^r(P_{1t}) - \psi_t D^r(P_{1t})$$

The necessary conditions are

$$(P_{1t} - c - \psi_t)\, D^r_P + D^r = 0$$

and

$$\dot{\psi}_t = r\psi_t.$$

Simple manipulation yields

$$(P_t - c - \psi_0 e^{rt})(-V) + (W - VP_t) = 0$$

$$W + V\left(c + \psi_0 e^{rt}\right) = 2VP_t$$

Then

$$P_t = \frac{W}{2V} + \frac{1}{2}\left(c + \psi_0 e^{rt}\right)$$

When $P = \overline{P} = \frac{W}{V}$, we have

$$W + V\left(c + \psi_0 e^{rT_m}\right) = 2W$$

So

$$c + \psi_0 e^{rT_m} = \frac{W}{V}$$

$$\psi_0 = \left(\frac{W}{V} - c\right) e^{-rT_m}$$

$$P_t = \frac{W}{2V} + \frac{1}{2}\left(c + \psi_0 e^{rt}\right) = \frac{W}{2V} + \frac{1}{2}\left(c + \left(\frac{W}{V} - c\right) e^{-rT_m} e^{rt}\right)$$

Then exhaustion implies

$$\int_0^{T_m} (W - VP_t)\, dt = \int_0^{T_m} \left(W - \frac{W}{2} - \frac{V}{2}\left(c + \left(\frac{W}{V} - c\right) e^{-rT_m} e^{rt}\right)\right) dt = R_0$$

Thus

$$\frac{1}{2}(W - Vc)\left(T_m - \frac{1 - e^{-rT_m}}{r}\right) = R_0.$$

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