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Yves SPRUMONT

CIREQ, Université de Montréal
C.P. 6128, succursale Centre-ville
Montréal (Québec) H3C 3J7
Canada

téléphone : (514) 343-6557
télécopieur : (514) 343-5831
cireq@umontreal.ca
http://www.cireq.umontreal.ca

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Resource egalitarianism with a dash of efficiency

Yves Sprumont*

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Abstract

We study the problem of defining inequality-averse social orderings over the space of allocations in a multi-commodity environment where individuals differ only in their preferences. We formulate notions of egalitarianism based on the axiom that any dominance between the consumption bundles of two individuals should be reduced. This *Dominance Aversion* requirement is compatible with *Consensus,* a weak version of the Pareto principle saying that an allocation $\mathbf{y}$ is better than $\mathbf{x}$ whenever everybody finds that everyone's bundle at $\mathbf{y}$ is better than at $\mathbf{x}$.

We identify two families of multidimensional leximin orderings satisfying Dominance Aversion and Consensus. We also discuss weaker forms of egalitarianism based on a new definition of multidimensional Lorenz dominance.

JEL Classification: D63, D71

Keywords: multidimensional inequality, leximin, Lorenz dominance

1 Introduction

This paper is an attempt to define notions of egalitarianism that would be adequate for guiding collective choices in a multi-commodity context. Egalitarianism, as understood here, is the view that an unequal treatment of individuals is socially desirable only if it

*Département de Sciences Economiques and CIREQ, Université de Montréal, C.P. 6128, succursale Centre-ville, Montréal QC, H3C 3J7, Canada (yves.sprumont@umontreal.ca). I thank F. Maniquet, H. Moulin and W. Thomson for comments and discussions. This research was supported by the Social Sciences and Humanities Research Council of Canada.

is justified by some relevant difference between them. Equal treatment should prevail by default.

The specific problem we analyze is that of allocating commodities among individuals with no property rights, objectively equal needs, but possibly different preferences. Our goal is not just to select a fairest allocation but to construct a full ordering of all conceivable allocations, including those that are not efficient or even feasible. This is essential in order to make consistent social choices in an environment where constraints are changing. The issue is to identify which consumption inequalities are justified by differences in preferences. We want our social choice ordering to be averse to all unjustified inequalities in consumption.

1.1 Unidimensional egalitarian criteria

Egalitarianism is well understood in the benchmark case of a single commodity, say, wealth. Since all preferences coincide – more is better – individuals are fully identical and no inequality is justified. Equal division is therefore the best allocation. Moreover, it is desirable to reduce any existing inequality. Two formal interpretations of this idea are offered in the literature. In a society of n individuals, consider a wealth allocation $\mathbf{x} = (x_1, x_2, x_3, ..., x_n)$ where, say, individual 1 is strictly poorer than 2, $x_1 < x_2$. The *transfer principle* (Pigou, 1912, Dalton, 1920) states that any transfer of wealth from 2 to 1 which does not make 2 poorer than 1 – a so-called Pigou-Dalton transfer – is socially desirable: $(x_1 + t, x_2 - t, x_3, ..., x_n)$ is better than $\mathbf{x}$ whenever $x_1 < x_1 + t \leq x_2 - t < x_2$. The *inequality-aversion principle* (adapted from Hammond, 1976), says that reducing the wealth difference between 1 and 2 is desirable, even if this involves destroying wealth[1]: $(y_1, y_2, x_3, ..., x_n)$ is better than $\mathbf{x}$ whenever $x_1 < y_1 \leq y_2 < x_2$.

These principles are the cornerstones of the two fundamental egalitarian criteria for ranking allocations in the single-commodity case, *Lorenz dominance* and the *leximin ordering.* An allocation $\mathbf{y}$ Lorenz-dominates an allocation $\mathbf{x}$ if, for every number $k = 1, ..., n$, the k poorest individuals at $\mathbf{y}$ are jointly at least as rich as the k poorest individuals at $\mathbf{x}$. A result of Hardy, Littlewood and Pólya (1934) states that $\mathbf{y}$ Lorenz-dominates $\mathbf{x}$ if and only

[1] Hammond states this principle, which he calls *equity*, for utility rather than wealth. Our analysis in this paper is entirely ordinal: no utility information is available.

if $\mathbf{y}$ can be obtained from $\mathbf{x}$ by a sequence of Pigou-Dalton transfers and permutations of individual wealths[2]. The leximin ordering, which is an extension of the Lorenz dominance relation, deems $\mathbf{y}$ better than $\mathbf{x}$ if, for some number $k = 1, ..., n$, each of the $k-1$ poorest individuals has identical wealth at $\mathbf{x}$ and $\mathbf{y}$ while the kth poorest is richer at $\mathbf{y}$. Hammond (1976) showed that this is the only ordering satisfying the inequality-aversion principle, the standard anonymity condition (permuting individual wealths leads to an equally good allocation) and the Pareto principle (increasing one individual's wealth yields a better allocation). See d'Aspremont (1985) for a detailed discussion.

Thus, two simple principles lead naturally to two explicit criteria for ranking allocations. Very importantly, both criteria satisfy the Pareto principle. This principle, however, is not very demanding because all individuals have identical preferences.

1.2 Multidimensional egalitarian criteria

The picture is radically different in a multi-commodity context. A first stream of literature, initiated by Kolm (1977), ignores preferences. It aims at answering the question: when is an allocation more equal than another? One possibility is to declare an allocation $\mathbf{y}$ more equal than $\mathbf{x}$ if $\mathbf{y}$ can be reached from $\mathbf{x}$ through a sequence of permutations of individual consumption bundles and transfers between individuals, each transfer involving a single commodity and being a Pigou-Dalton transfer for the allocation of that commodity. Clearly, $\mathbf{y}$ is more equal than $\mathbf{x}$ in this sense if and only if the allocation of every commodity at $\mathbf{y}$ Lorenz-dominates the corresponding allocation at $\mathbf{x}$. Other multidimensional notions of Lorenz dominance were studied by Atkinson and Bourguignon (1982), Joe and Verducci (1993), and Koshevoy (1995, 1998). Complete rankings of allocations were proposed by Tsui (1995) and Gajdos and Weymark (2005). See Weymark (2006) for a survey. All these criteria rely on some extension of the transfer principle. To the best of our knowledge, neither Hammond's inequality-aversion principle nor the leximin ordering were generalized

[2] This holds if aggregate wealth is identical at both allocations. The result extends easily to arbitrary allocations: $\mathbf{y}$ (generalized-)Lorenz-dominates $\mathbf{x}$ if and only if $\mathbf{y}$ can be obtained from $\mathbf{x}$ trough a sequence of Pigou-Dalton transfers, permutations of individual wealths, and individual wealth increases. See Shorrocks (1983).

to the multidimensional context.

The criteria developed in this literature are useful in the absence of information on preferences but can lead to collective choices that are unappealing to a well-informed egalitarian planner. Consider two individuals with linear preferences over apples and bananas: one apple is worth two bananas to individual 1, two apples are worth one banana to individual 2. According to all the standard multidimensional inequality criteria, the allocation $\mathbf{x} = ((4, 2), (2, 4))$, where individual 1 consumes 4 apples and 2 bananas and individual 2 consumes 2 apples and 4 bananas, is less equal than the allocation $\mathbf{y} = ((3, 3), (3, 3))$. But $\mathbf{x}$ Pareto-dominates $\mathbf{y}$. When preferences are known, the proper goal of an egalitarian planner is not to reduce all inequalities but only those that are not justified by differences in preferences. In the current example, the differences in consumption at $\mathbf{x}$ are easily justifiable.[3]

A more recent stream of literature, reviewed by Maniquet (2007), seeks to define inequality-averse rankings of allocations that use information about individual preferences. The approach consists of combining weak multidimensional versions of the transfer principle with the Pareto principle (an allocation preferred to another by all individuals is socially preferable to it). An example is Fleurbaey's (2005, 2007) defense of the Pazner and Schmeidler (1978) egalitarian-equivalent ordering. Given an aggregate endowment of resources and an allocation $\mathbf{x}$, compute for each individual i the fraction $\lambda_i(\mathbf{x})$ of the aggregate endowment that makes him exactly indifferent to the bundle he receives, x_i. An allocation $\mathbf{y}$ is considered better than $\mathbf{x}$ if $(\lambda_1(\mathbf{y}), ..., \lambda_n(\mathbf{y}))$ dominates $(\lambda_1(\mathbf{x}), ..., \lambda_n(\mathbf{x}))$ according to the leximin criterion. The use of the leximin criterion implies a limited form of inequality aversion: if the consumption bundles of individuals 1 and 2 happen to be proportional to the aggregate endowment, then any transfer reducing the difference between their consumptions is desirable.

Yet, we contend that criteria such as the Pazner-Schmeidler ordering are not completely faithful to our view of egalitarianism: they may recommend unjustified inequalities. Consider again our two individuals with linear preferences over apples and bananas and

[3] Even in the absence of precise information on preferences, it is not obvious that reducing inequality is always desirable: is the allocation $((100, 100), (1, 100))$ really better than $((100, 99), (1, 101))$? Haven't we reduced the "wrong" inequality?

suppose that the aggregate endowment in society is $(18, 18)$. We argue that the allocation $\mathbf{x} = ((2, 8), (4, 10))$, where individual 1 consumes less of both commodities than 2, embodies unjustifiable inequality. By assumption, claims and needs are identical, so that any justification for differences in consumption must originate in differences in preferences. But *both individuals agree* that 2's bundle is better than 1's – and *any* individual with monotonic preferences would concur. Since society has no reason to overrule this consensus, there is no direct justification for the physical dominance between the bundles received by the individuals. This dominance should be reduced. Nevertheless, the Pazner-Schmeidler ordering prefers $\mathbf{x}$ to $\mathbf{y} = ((3, 9), (3, 9))$ since $(\lambda_1(\mathbf{x}), \lambda_2(\mathbf{x})) = (\frac{1}{3}, \frac{1}{3})$ leximin-dominates $(\lambda_1(\mathbf{y}), \lambda_2(\mathbf{y})) = (\frac{5}{18}, \frac{7}{18})$.

1.3 Egalitarianism with a dash of efficiency

The notion of egalitarianism proposed in this paper is based on the view that any dominance between the consumption bundles of two individuals is an unjustified inequality. Other forms of inequality, however, may be desirable. This view underlies the following multidimensional extensions of the Pigou-Dalton transfer principle and Hammond's inequality-aversion principle.

If $\mathbf{x} = (x_1, x_2, x_3, ..., x_n)$ is a multi-commodity allocation where individual 1 consumes less of all goods than 2, $x_1 < x_2$, the *dominance-reducing transfer principle* states that a transfer of commodities from 2 to 1 is desirable as long as 2 keeps consuming as much as 1 : $(x_1 + t, x_2 - t, x_3, ..., x_n)$ is better than $\mathbf{x}$ whenever $x_1 < x_1 + t \leq x_2 - t < x_2$. *Dominance aversion* says that reducing the consumption difference between the two individuals is always desirable: $(y_1, y_2, x_3, ..., x_n)$ is better than $\mathbf{x}$ whenever $x_1 < y_1 \leq y_2 < x_2$. We submit that these principles are the adequate basis for defining multidimensional egalitarianism, in a weak and a strong sense respectively, when individual preferences may differ but are monotonic.

Unfortunately, Fleurbaey and Trannoy (2003) showed that even the dominance-reducing transfer principle is incompatible with the Pareto principle. Consider again our earlier two-individual example with linear preferences. According to the dominance-reducing transfer principle, allocation $\mathbf{x} = ((11, 2), (12, 3))$, where individual 1 consumes 11 apples and 2 bananas and individual 2 consumes 12 apples and 3 bananas, is better than allocation $\mathbf{y} = ((9,0),(14,5))$ because the former is obtained from the latter by transferring 2 apples and 2 bananas from the richer individual to the poorer. If the individuals are selfish, the Pareto principle implies that society should be indifferent between allocation $\mathbf{y}$ and allocation $\mathbf{z} = ((3,12),(2,11))$. But the dominance-reducing transfer principle tells us that $\mathbf{z}$ is better than $\mathbf{w} = ((5,14),(0,9))$, which the Pareto principle deems equivalent to the allocation $\mathbf{x}$ we started from.[4]

One reaction to the incompatibility just described is that bundle dominance is, after all, a justifiable form of inequality: it cannot be avoided if we insist on the Pareto principle. Alternatively – and this is the route we take – one may argue that the dominance-reducing transfer principle and dominance aversion are fundamental criteria of fairness. The natural question is then whether they can be combined with weaker versions of the Pareto principle.

There are reasons for questioning the version used so far. The Pareto principle, as used in social choice theory, stipulates that a social alternative is better than another whenever all individuals prefer the former to the latter, two alternatives being equally good if they leave all individuals indifferent. In the particular context studied here, social alternatives are allocations. Strictly speaking, therefore, the Pareto principle remains silent as long as individual preferences over consumption bundles have not been extended to preferences over allocations. Of course, this is traditionally done as above through the (generally implicit) assumption of selfishness: each individual ranks allocations by applying her preference ordering to the bundles she receives at those allocations.

But selfishness is problematic for at least two reasons. First, it is not a testable restriction in a classic market exchange environment: individual choices reveal preferences over consumption bundles, not over allocations. If we want to rank allocations on the basis of recoverable information only, we should rely on individual preferences over consumption bundles without assuming that they extend to allocations in any particular way, selfish or other. Formally, this renders the Pareto principle inapplicable.

Second, there is some (mostly experimental) evidence that individual preferences over

[4] To keep the argument simple, we used an example involving indifferences. The incompatibility persists under the version of the Pareto principle based on strict preferences only.

allocations – if well defined – are probably not selfish. At least in some environments, individuals seem to care about inequality. Assuming inequality-averse individuals would not necessarily avoid the incompatibility between the dominance-reducing transfer principle and the Pareto principle, but it would definitely change the implications of the latter.

Given these difficulties, it may be wise to adopt a more neutral view of preferences: selfishness should not be ruled out, but it should not be assumed either. Since we must rank allocations without a full knowledge of individual preferences over them, we propose to replace the Pareto principle, which is inapplicable, with a condition that retains some of its spirit. This Pareto-like condition, which we call *consensus*, says that an allocation $\mathbf{y}$ is better than an allocation $\mathbf{x}$ whenever everyone finds everybody's bundle at $\mathbf{y}$ better than at $\mathbf{x}$.

A possible interpretation is the following. While we ignore the detail of preferences over allocations, we assume that individuals are not malevolent towards others but have poor information regarding their preferences. They could be selfish but are perhaps altruistic. If they are, then it is important to consider everyone's opinion when modifying the bundle allocated to any individual. Because of their limited information, individuals use their own preferences to evaluate the consumption of others. Consensus says that we should recommend any "sure Pareto improvement", namely, any social change that is a Pareto improvement for all possible non-malevolent but poorly informed extensions of individual preferences.

As it turns out, the consensus axiom is compatible with both the dominance-reducing transfer principle and dominance aversion: a dash of efficiency can be added to resource egalitarianism. Between the preference-free approach of the inequality measurement literature and the standard Paretian approach of the recent literature on inequality-averse social orderings, there is room for new definitions of multidimensional egalitarianism.

Our main results rely on our stronger distributional axiom, dominance aversion. Specifically, we propose and defend two multidimensional generalizations of the classic leximin ordering which satisfy dominance aversion and consensus. We also offer partial results about our weaker distributional axiom, the dominance-reducing transfer principle, and briefly discuss social orderings satisfying this principle and the consensus axiom.

2 The model and the main conditions

There is a fixed finite set of individuals, $N = \{1, ..., n\}$, $n \geq 2$, and a fixed finite number of goods, $m \geq 2$. The commodity space is $X = \mathbb{R}^m_+$. Each individual $i \in N$ has a continuous and strictly monotonic preference ordering R_i over X. Given the fixed preference profile $(R_1, ..., R_n)$, we seek to construct a social ordering $\mathbf{R}$ over the set of conceivable allocations X^N.

The properties of a social ordering we are chiefly concerned with are *Consensus* and *Dominance Aversion.* Consensus is a Paretian axiom. If P_i denotes the strict preference relation associated with R_i and $\mathbf{P}$ the strict social preference associated with $\mathbf{R}$, the standard Pareto principle, in its weak form, states that $(y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)$ whenever $y_i P_i x_i$ for all $i \in N$. As explained earlier, this may not be appropriate if preferences over allocations are not known to be selfish. Consensus is a weaker condition saying that an allocation is better than another if all individuals find that everyone gets a better bundle in the former than in the latter.

Consensus. For all $(x_1, ..., x_n)$, $(y_1, ..., y_n) \in X^N$, $[y_i P_j x_i$ for all $i, j \in N] \Rightarrow$ $[(y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)]$.

A slightly stronger condition requires, in addition, that $(y_1, ..., y_n)\mathbf{R}(x_1, ..., x_n)$ whenever $y_i R_j x_i$ for all $i, j \in N$. We call this variant *Full Consensus*.

A first interpretation of Consensus was offered in the introduction: society should support any change which is a Pareto improvement for every poorly informed but non-malevolent extension of preferences.

A second possible interpretation, fundamentally resourcist, goes as follows. Commodities have intrinsic value which individual preferences reflect approximately. A good social ordering should use these preferences to estimate the "correct" relative value of commodity bundles. If all individuals like a bundle better than another, then society should be confident that the former is indeed more valuable than the latter – it has no reason whatsoever to support the opposite view. If each bundle composing an allocation $(y_1, ..., y_n)$ is deemed more valuable than the corresponding component of another allocation $(x_1, ..., x_n)$, then society should prefer $(y_1, ..., y_n)$ over $(x_1, ..., x_n)$.

Using the notation $\geq, >, \gg$ for vector inequalities, a direct consequence of the axiom is that $(y_1, ..., y_n)\mathbf{P}(x, ..., x)$ whenever $y_i > x$ for all $i \in N$: more for all is always better, even if resources are (perhaps much) more unequally distributed. This shows that Consensus does limit the scope of egalitarianism.

Next we turn to our main distributional axiom. Dominance Aversion says that reducing bundle dominance is always desirable.

Dominance Aversion. For all $(x_1, ..., x_n)$, $(y_1, ..., y_n) \in X^N$ and all $i, j \in N$, $[x_i > y_i \geq y_j > x_j$ and $y_k = x_k$ for all $k \in N \setminus \{i, j\}] \Rightarrow [(y_1, ..., y_n)\mathbf{R}(x_1, ..., x_n)]$.

Let us repeat that preferences are the only source of heterogeneity in our model. Individuals are otherwise identical. They cannot be distinguished according to claims (as individual endowments are unspecified) or needs (as utilities are absent). In such a simplified context, we believe that Dominance Aversion is a radical but ethically correct corollary to the view that bundle dominance is unjustified when all preferences are strictly monotonic. It is the proper multidimensional extension of Hammond's inequality-aversion principle. The *Dominance-Reducing Transfer Principle*, which restricts the application of Dominance Aversion to those cases where no resources are lost when switching from $(x_1, ..., x_n)$ to $(y_1, ..., y_n)$, will be discussed in Section 5.

3 Consensual egalitarianism

We describe a first class of social orderings (which we sometimes just call orderings, for brevity) that satisfy the conditions just defined. Recall that the profile of individual preferences $(R_1, ..., R_n)$ is fixed. Let R be a continuous ordering over the commodity space which *agrees* with $R_1, ..., R_n$ in the sense that, for any two bundles $x, y \in X$, $[yP_ix$ for all $i \in N] \Rightarrow [yPx]$.[5] This ordering R is not a social ordering in the technical sense – it does not rank allocations. Rather, it expresses society's evaluation of the relative value of commodity bundles. This evaluation is consensual in the sense that it respects individual preferences whenever they do not conflict. There is a simple way to construct such an ordering: if

[5]Because R is continuous and $R_1, ..., R_n$ are strictly monotonic, it follows that, for all $x, y \in X$, $[yR_ix$ for all $i \in N] \Rightarrow [yRx]$.

$u_1, ..., u_n$ are arbitrary numerical representations of the individual preferences, a function $x \mapsto u(x) = f(u_1(x), ..., u_n(x))$ represents an ordering R agreeing with $R_1, ..., R_n$ if f is strictly increasing in all its arguments.

For any allocation $(x_1, ..., x_n) \in X^N$, denote by $(x_1^R, ..., x_n^R)$ any allocation obtained by rearranging the bundles $x_1, ..., x_n$ from worst to best according to R, so that $x_n^R R x_{n-1}^R R ... R x_1^R$.

Definition 1. A social ordering $\mathbf{R}$ is a *consensual Rawlsian* ordering if and only if there is a continuous ordering R on X which agrees with $R_1, ..., R_n$ such that, for all allocations $(x_1, ..., x_n), (y_1, ..., y_n) \in X^N$, $\left[y_1^R P x_1^R\right] \Rightarrow \left[(y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)\right]$. We say that $\mathbf{R}$ is based on R.

In words: a consensual Rawlsian social ordering evaluates an allocation on the basis of its worst component according to a (continuous) preference that agrees with individual preferences.

When there are more than two individuals, not every consensual Rawlsian ordering satisfies Dominance Aversion. This is because such an ordering need not pay attention at all to the components of an allocation that are not minimal according to the preference it is based on. The consensual leximin orderings avoid this difficulty. The *leximin extension* to X^N of an ordering R on X is the ordering $\mathbf{R}_{lex}(R)$ on X^N defined as follows: $(y_1, ..., y_n)\mathbf{R}_{lex}(R)(x_1, ..., x_n)$ if and only if either there exists $j \in N$ such that $y_i^R I x_i^R$ for all $i < j$ and $y_j^R P x_j^R$ (in which case we write $(y_1, ..., y_n)\mathbf{P}_{lex}(R)(x_1, ..., x_n)$) or else $y_i^R I x_i^R$ for all $i \in N$ (in which case we write $(y_1, ..., y_n)\mathbf{I}_{lex}(R)(x_1, ..., x_n)$).

Definition 2. A social ordering $\mathbf{R}$ is a *consensual leximin* ordering if and only if there is a continuous ordering R agreeing with $R_1, ..., R_n$ such that, for all allocations $(x_1, ..., x_n)$, $(y_1, ..., y_n) \in X^N$, $\left[(y_1, ..., y_n)\mathbf{P}_{lex}(R)(x_1, ..., x_n)\right] \Rightarrow \left[(y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)\right]$.

A social ordering $\mathbf{R}$ is a *simple* consensual leximin ordering if and only if it is the leximin extension of some ordering R agreeing with $R_1, ..., R_n$, that is, $\mathbf{R} = \mathbf{R}_{lex}(R)$.

Clearly, every consensual leximin ordering is a consensual Rawlsian ordering. While a consensual leximin ordering need not respect the indifference relation associated with the ordering R it is based on, a simple consensual leximin ordering necessarily does.

The rest of this section is an axiomatic study of the orderings introduced in Definitions

1 and 2. While the basic axioms we use are Consensus and Dominance Aversion, we do rely on additional properties. We do not regard these auxiliary properties as fundamental aspects of a definition of preference-sensitive multidimensional egalitarianism. They are disputable conditions that help us select salient examples of egalitarian orderings.

We begin with two requirements of internal coherence of the social ordering which do not impose any relationship between the social ordering and the profile of individual preferences.

The first is what we call *Intrinsic Dominance*. From a resourcist perspective, comparing two fully egalitarian allocations involves no issue of distributive justice at all. Therefore a statement such as $(y, ..., y)\mathbf{R}(x, ..., x)$ should reflect society's evaluation of the relative intrinsic values of the bundles x and y. The axiom says that if society deems each of the bundles $x_1, ..., x_n$ intrinsically at least as valuable as bundle x, then the allocation $(x_1, ..., x_n)$ should be at least as good as $(x, ..., x)$. An intrinsically richer society is preferable to a poorer one.

Intrinsic Dominance. For all $(x_1, ..., x_n) \in X^N$, $x \in X$, $[(x_i, ..., x_i)\mathbf{R}(x, ..., x)$ for all $i \in N] \Rightarrow [(x_1, ..., x_n)\mathbf{R}(x, ..., x)]$.

This axiom combines two ideas. First, like Consensus, it limits society's concerns for equality. This is best seen by considering the contraposition of the axiom. If an egalitarian allocation $(x, ..., x)$ is judged superior to another allocation $(x_1, ..., x_n)$, it cannot be just because the former is egalitarian: the common bundle x received by all individuals should be deemed intrinsically more valuable than at least one of the bundles composing the other allocation.

Second, in keeping with the resourcist view, Intrinsic Dominance assumes that the intrinsic value of consumption bundles is enough, in specific cases, to determine the social ranking of allocations: if each of the bundles $x_1, ..., x_n$ is judged intrinsically at least as valuable as x, then *any* allocation of these bundles to the individuals is at least as good as $(x, ..., x)$ – society need not worry about who gets what. Indeed, if $(x_i, ..., x_i)\mathbf{R}(x, ..., x)$ for all $i \in N$ and π is an arbitrary permutation on N, then $(x_{\pi(i)}, ..., x_{\pi(i)})\mathbf{R}(x, ..., x)$ for all $i \in N$ and Intrinsic Dominance implies $(x_{\pi(1)}, ..., x_{\pi(n)})\mathbf{R}(x, ..., x)$.

This does not mean that individual preferences cannot be taken into account. In particular, we emphasize that Intrinsic Dominance is compatible with the standard version of the weak Pareto principle based on the selfishness hypothesis. For instance, define for each individual i a numerical representation u_i of R_i and let $(y_1, ..., y_n)\mathbf{R}(x_1, ..., x_n)$ if and only if $\min_{i\in N} u_i(y_i) \geq \min_{i\in N} u_i(x_i)$. The social ordering $\mathbf{R}$ satisfies Intrinsic Dominance and the standard weak Pareto principle: $(y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)$ whenever $y_i P_i x_i$ for all $i \in N$.

Our second auxiliary condition is a weak form of continuity of the social ordering. *Weak Continuity* requires that the social ranking of fully egalitarian allocations be continuous.

Weak Continuity. For any $x, y \in X$ and any sequence (y^k) in X converging to y, $\left[(y^k, ..., y^k)\mathbf{R}(x, ..., x) \text{ for all } k\right] \Rightarrow \left[(y, ..., y)\mathbf{R}(x, ..., x)\right]$.

Since egalitarian allocations are in a simple one-to-one correspondence with bundles, a discontinuous social ranking of such allocations would seem at odds with continuous individual preferences.

We are now ready to state our first result.

Proposition 1. *If a social ordering satisfies Consensus, Dominance Aversion, Intrinsic Dominance, and Weak Continuity, then it is a consensual Rawlsian social ordering. Every simple consensual leximin social ordering satisfies these four axioms.*

The proof is in the Appendix. Proposition 1 is not a complete characterization of the simple consensual leximin orderings. In order to offer such a characterization, we introduce two further properties.

First, observe that the social evaluation of an allocation by a simple consensual leximin ordering depends only on the bundles that compose it, not on who consumes them.

Strong Symmetry. For all $(x_1, ..., x_n) \in X^N$ and every permutation π on N, $(x_1, ..., x_n)\mathbf{I}(x_{\pi(1)}, ..., x_{\pi(n)})$.

This axiom is a purely resourcist variant of the familiar property of anonymity. While Strong Symmetry is clearly a very demanding property – it rules out anonymously preference-sensitive orderings –, it is important to observe that all consensual Rawlsian orderings do satisfy a very closely related condition. If, according to R, the worst bundle in $(y_1, ..., y_n)$ is strictly better than the worst bundle in $(x_1, ..., x_n)$, then any consensual Rawlsian ordering $\mathbf{R}$

based on R deems $(y_1, ..., y_n)$ strictly better than $(x_1, ..., x_n)$, irrespective of who in each allocation receives this worst bundle. This means that for all $(x_1, ..., x_n)$, $(y_1, ..., y_n) \in X^N$ and every permutation π on N, $(y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)$ if and only if $(y_1, ..., y_n)\mathbf{P}(x_{\pi(1)}, ..., x_{\pi(n)})$. Extending this property to social indifference implies Strong Symmetry.

Simple consensual leximin orderings also satisfy a separability property. If $(x_1, ..., x_n) \in X^N$ and $i \in N$, denote by $x_{-i} \in X^{N\backslash\{i\}}$ the sub-allocation $(x_j)_{j\in N\backslash\{i\}}$ and, if $y_i \in X$, write $(y_i; x_{-i})$ for the allocation obtained from $(x_1, ..., x_n)$ by replacing x_i with y_i.

Internal Separability. Let $i \in N$ and let $x_i, x_i', y_i, y_i' \in X$ be such that $(x_i, ..., x_i)\mathbf{I}(x_i', ..., x_i')$ and $(y_i, ..., y_i)\mathbf{I}(y_i', ..., y_i')$. Then, for all $z_N, z_N' \in X^N$, $(x_i; z_{-i})\mathbf{R}(x_i'; z_{-i}') \Leftrightarrow (y_i; z_{-i})\mathbf{R}(y_i'; z_{-i}')$.

Because society deems bundle x_i intrinsically just as valuable as x_i', it may ignore individual i when comparing $(x_i; z_{-i})$ to $(x_i'; z_{-i}')$. Likewise, it may ignore individual i when comparing $(y_i; z_{-i})$ to $(y_i'; z_{-i}')$. Therefore the social ranking of the allocations $(x_i; z_{-i})$ and $(x_i'; z_{-i}')$ should be the same as the ranking of $(y_i; z_{-i})$ and $(y_i'; z_{-i}')$ since, ignoring i, the sub-allocations to be compared are the same in both cases, namely z_{-i} and z_{-i}'.

The axiom is related to Fleming's (1952) separability axiom. The essential difference is that individual i is ignored not because he is indifferent between x_i and x_i' and between y_i and y_i', but because society judges that the bundles he receives are equally valuable in both cases. If preferences are not selfish, Fleming's axiom is logically too demanding.

The following proposition characterizes the simple consensual leximin social orderings.

Proposition 2. *A social ordering satisfies Consensus, Dominance Aversion, Weak Continuity, Strong Symmetry, and Internal Separability if and only if it is a simple consensual leximin social ordering.*

Notice that Intrinsic Dominance does not appear in this proposition; it is implied by the combination of Consensus, Weak Continuity and Internal Separability. The Appendix contains the proof of Proposition 2 as well as examples showing that all five axioms are independent.

We conclude this section with two remarks.

First, simple consensual leximin orderings remain arguably too crude. In particular, because they satisfy Strong Symmetry, they violate the following condition.

Permutation Pareto Principle. For all $(x_1, ..., x_n) \in X^N$ and every permutation π on N, $[x_{\pi(i)} P_i x_i$ for all $i \in N] \Rightarrow [(x_{\pi(1)}, ..., x_{\pi(n)}) \mathbf{P} (x_1, ..., x_n)]$.

This axiom is a restricted form of the standard weak Pareto principle. It says that if permuting bundles results in an allocation where every individual prefers her new bundle to the old, this new allocation should be regarded as better. The important point is that the new allocation is generated from the old through a very specific type of exchange: a permutation of consumption bundles. Such a permutation is distributionally neutral in the sense that it preserves any consumption inequalities originally present. Since there is no reason to favor either allocation on distributive grounds, a dash of efficiency should tip the balance in favor of the new allocation.

There is a simple way to generate a social ordering that satisfies this condition along with our two fundamental axioms, Consensus and Dominance Aversion. As before, fix a continuous ordering R on X agreeing with $R_1, ..., R_n$ and let $\mathbf{R}$ be the leximin extension of R to X^N. Next, let $\mathbf{R}'$ be a *standard weakly Paretian* social ordering on X^N in the sense that $[y_i P_i x_i$ for all $i \in N] \Rightarrow [(y_1, ..., y_n) \mathbf{P}' (x_1, ..., x_n)]$. Construct $\mathbf{R}''$ by lexicographic application of $\mathbf{R}$ and $\mathbf{R}'$: $(y_1, ..., y_n) \mathbf{R}'' (x_1, ..., x_n)$ if and only if $(y_1, ..., y_n) \mathbf{P} (x_1, ..., x_n)$ or $[(y_1, ..., y_n) \mathbf{I} (x_1, ..., x_n)$ and $(y_1, ..., y_n) \mathbf{R}' (x_1, ..., x_n)]$.

The social ordering $\mathbf{R}''$ is a consensual leximin ordering. It satisfies Consensus and Dominance Aversion (as well as Weak Continuity) because $\mathbf{R}$ does: see the proof of Proposition 1. To check the Permutation Pareto Principle, note that $(x_{\pi(1)}, ..., x_{\pi(n)}) \mathbf{I} (x_1, ..., x_n)$ for all $(x_1, ..., x_n) \in X^N$ and every permutation π on N. Therefore $(x_{\pi(1)}, ..., x_{\pi(n)}) \mathbf{P}'' (x_1, ..., x_n)$ if and only if $(x_{\pi(1)}, ..., x_{\pi(n)}) \mathbf{P}' (x_1, ..., x_n)$. The claim now follows from the fact that $\mathbf{R}'$ is a standard weakly Paretian social ordering.

Our second remark bears on both Propositions 1 and 2. In either result, the only axiom linking the social ordering of allocations to individual preferences is Consensus. The following weak form of that axiom, found in the literature on multidimensional inequality measurement, abstracts from preferences altogether.

Monotonicity. For all $(x_1, ..., x_n)$, $(y_1, ..., y_n) \in X^N$, $[y_i > x_i$ for all $i \in N] \Rightarrow$ $[(y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)]$.

Replacing Consensus with Monotonicity yields preference-free versions of our earlier results. In particular, we obtain the following variant of Proposition 2. We omit the proof, which is a straightforward modification of the proofs of Propositions 1 and 2.

Proposition 3. *A social ordering* $\mathbf{R}$ *on* X^N *satisfies Monotonicity, Dominance Aversion, Weak Continuity, Strong Symmetry, and Internal Separability if and only if it is the leximin extension of a continuous and strictly monotonic preference ordering* R *on* X.

4 Radical egalitarianism

We turn to a second class of social orderings satisfying Consensus and Dominance Aversion. Our purpose is mainly to illustrate that these two axioms leave us with non-trivial alternatives to the consensual orderings. The orderings defined here constitute a more radical form of resource egalitarianism which is perhaps not very appealing when there are more than two individuals.

Instead of evaluating an allocation on the basis of its worst component according to a preference agreeing with individual preferences, we now pay attention to all bundles that at least one individual finds worst among those composing the allocation. If $i \in N$ and $x \in X$, let $W_i(x) = \{y \in X \mid xR_iy\}$ denote the lower contour set of R_i at bundle x. For each allocation $(x_1, ..., x_n) \in X^N$, let $W(x_1, ..., x_n) = \bigcap_{i,j \in N} W_i(x_j)$ and write $W(y_1, ..., y_n) > W(x_1, ..., x_n)$ if for each $x \in W(x_1, ..., x_n)$ there exists $y \in W(y_1, ..., y_n)$ such that $y > x$. This property implies that $W(y_1, ..., y_n) \supsetneq W(x_1, ..., x_n)$.

Definition 3. A social ordering $\mathbf{R}$ is a *radical Rawlsian* ordering if and only if, for all allocations $(x_1, ..., x_n), (y_1, ..., y_n) \in X^N$, $[W(y_1, ..., y_n) > W(x_1, ..., x_n)] \Rightarrow [(y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)]$.

A word of interpretation is in order. Since the lower contour sets $W_i(x_1), ..., W_i(x_n)$ are nested for each individual i, the set $W(x_1, ..., x_n)$ is just the intersection of the smallest lower contour sets of all individuals. It contains the bundles that all individuals consider at most as good as their least preferred component of the allocation $(x_1, ..., x_n)$. An allocation becomes socially better when this set expands.

A radical Rawlsian ordering can be viewed as the result of the following fictitious two-stage procedure. First, on behalf of society, each individual evaluates allocations using a Rawlsian extension of her own preference: the lower contour set at the worst bundle in an allocation reflects her assessment of the value of that allocation – or, rather, society's tentative assessment based on her individual preference. Second, these assessments are aggregated using the unanimity criterion: the intersection of the individual lower contour sets reflects society's final evaluation of the allocation.

By comparison, the consensual Rawlsian orderings of the previous section result from a procedure where these two stages come in reverse order. The unanimity criterion is applied first to construct a social preference over bundles which agrees with individual preferences. Each allocation is then evaluated using the Rawlsian extension of this aggregate preference.

To illustrate the difference between the two approaches, consider again our two-individual, two-commodity example where R_1, R_2 are the linear preferences represented by $u_1(\alpha,\beta) = 2\alpha + \beta$ and $u_2(\alpha,\beta) = \alpha + 2\beta$. Whenever $\varepsilon > 0$, we have $W((4+\varepsilon, 4+\varepsilon),(4+\varepsilon,4+\varepsilon)) > W((8,2),(2,8))$, hence $((4+\varepsilon,4+\varepsilon),(4+\varepsilon,4+\varepsilon))\ \mathbf{P}\ ((8,2),(2,8))$ for every radical Rawlsian ordering $\mathbf{R}$. By comparison, consider the consensual Rawlsian ordering $\mathbf{R}'$ based on the preference R represented by $u(\alpha,\beta) = \alpha + \beta$ (which agrees with R_1, R_2). According to this social ordering, $((8,2),(2,8))\ \mathbf{P}'\ ((4+\varepsilon,4+\varepsilon),(4+\varepsilon,4+\varepsilon))$ whenever $\varepsilon < 1$. Moreover, for any consensual Rawlsian ordering $\mathbf{R}''$ based on any preference agreeing with R_1, R_2, we have $((8,2),(2,8))\ \mathbf{P}''\ ((4+\varepsilon,4+\varepsilon),(4+\varepsilon,4+\varepsilon))$ if $\varepsilon > 0$ is small enough. That is, the class of consensual Rawlsian orderings is disjoint from the class of radical Rawlsian orderings for the preference profile under consideration.

Because they need not pay attention to the components of an allocation that are not worst according to individual preferences, radical Rawlsian orderings may violate Dominance Aversion. We now describe a subclass of orderings which evaluate allocations by considering the worst bundles first, but also take higher-ranked bundles into consideration in a lexicographic fashion. Given an allocation $(x_1,...,x_n)$, denote by $(x_1^i,...,x_n^i)$ any rearrangement of it from worst to best according to R_i, so that $x_n^i R_i ... R_i x_1^i$. With this notation, $W(x_1,...,x_n) = \bigcap_{i\in N} W_i(x_1^i)$. Next, for any $t = 1,...,n$, define $W_{(t)}(x_1,...,x_n) = \bigcap_{i\in N} W_i(x_t^i)$: this is the set of bundles that everyone finds at most as good as their tth worst bundle among $x_1, ..., x_n$. In particular, $W_{(1)}(x_1, ..., x_n) = W(x_1, ..., x_n)$. The orderings we are about to define evaluate an allocation by looking successively at the sets $W_{(1)}(x_1, ..., x_n)$, ..., $W_{(n)}(x_1, ..., x_n)$. For simplicity, we restrict our attention to preference profiles where $W_i(x)$ is bounded (hence compact in view of our earlier assumption of continuity of preferences) for each $i \in N$ and $x \in X$.

Definition 4. A social ordering $\mathbf{R}$ is a *radical leximin* ordering if and only if it is constructed as follows. Let R be a continuous and strictly monotonic preference ordering over X and let u be a continuous numerical representation of R. Let $\overline{u}_{(t)}(x_1, ..., x_n) = \max \left\{ u(x) \mid x \in W_{(t)}(x_1, ..., x_n) \right\}$ and write $\overline{u}(x_1, ..., x_n) = (\overline{u}_{(1)}(x_1, ..., x_n), ..., \overline{u}_{(n)}(x_1, ..., x_n))$. For all $(x_1, ..., x_n), (y_1, ..., y_n) \in X^N$, let $[(y_1, ..., y_n)\mathbf{R}(x_1, ..., x_n)] \Leftrightarrow [\overline{u}(y_1, ..., y_n) \succsim_{lex} \overline{u}(x_1, ..., x_n)]$, where $\succsim_{lex}$ is the usual leximin ordering on $\mathbb{R}^N$.

It is easy to check that every radical leximin ordering satisfies Consensus and Dominance Aversion. An interesting independent property is the following: if all individuals prefer a certain bundle x to their own consumption bundle at a given allocation, then the egalitarian allocation where everyone gets x is better than the current allocation.

Egalitarian Pareto Principle. For all $(x_1, ..., x_n) \in X^N$, $x \in X$, $[xP_ix_i$ for all $i \in N] \Rightarrow [(x, ..., x)\mathbf{P}(x_1, ..., x_n)]$.

This condition is in the same spirit as the Permutation Pareto Principle: to follow the recommendations of the standard Pareto principle when it does not conflict with egalitarian concerns. By construction, all radical leximin orderings satisfy the Egalitarian Pareto Principle. Our earlier example shows that consensual Rawlsian orderings typically violate it[6].

The following result, proved in the Appendix, is a partial defense of radical Rawlsian orderings.

Proposition 4. *Let $n = 2$. If a social ordering satisfies Full Consensus, Dominance Aversion, Strong Symmetry, the Egalitarian Pareto Principle, and Weak Continuity, then it is a radical Rawlsian ordering.*

[6]In those degenerate cases where all individual preferences coincide, consensual Rawlsian orderings coincide with radical Rawlsian orderings, hence satisfy the axiom.

5 Weaker forms of egalitarianism

The leximin orderings identified in the previous two sections are egalitarian in a strong sense: they satisfy the very demanding axiom of Dominance Aversion. We now return to the Dominance-Reducing Transfer Principle proposed in the introduction and discuss weak forms of egalitarianism.

The *Dominance-Reducing Transfer Principle* is just the restriction of Dominance Aversion to those cases where no resources are lost when transferring commodities. Formally, define the relation $\succ$ on the set of allocations by $(y_1, ..., y_n) \succ (x_1, ..., x_n)$ if and only if there exist $i, j \in N$ and $t \in X$ such that i) $x_i > y_i = x_i - t \geq x_j + t = y_j > x_j$ and ii) $y_k = x_k$ for all $k \in N \setminus \{i, j\}$. Note that the commodity transfer t reduces the dominance between the bundles of individuals i and j but preserves their comparability.

Dominance-Reducing Transfer Principle. For all $(x_1, ..., x_n)$, $(y_1, ..., y_n) \in X^N$, $[(y_1, ..., y_n) \succ (x_1, ..., x_n)] \Rightarrow [(y_1, ..., y_n)\mathbf{R}(x_1, ..., x_n)]$.

A stronger version is natural. Define the relation $\rhd$ by $(y_1, ..., y_n) \rhd (x_1, ..., x_n)$ if and only if there exist $i, j \in N$ and $t \in X$ such that i) $x_i > y_i = x_i - t > x_j$, ii) $x_i > x_j + t = y_j > x_j$, and iii) $y_k = x_k$ for all $k \in N \setminus \{i, j\}$. Here the comparability of the bundles of individuals i and j is not preserved by the commodity transfer t but both individuals still end up poorer than the originally richer individual and richer than the originally poorer one.

Strong Dominance-Reducing Transfer Principle. For all $(x_1, ..., x_n)$, $(y_1, ..., y_n) \in X^N$, $[(y_1, ..., y_n) \rhd (x_1, ..., x_n)] \Rightarrow [(y_1, ..., y_n)\mathbf{R}(x_1, ..., x_n)]$.

In the unidimensional case, this stronger principle follows from the weaker under the standard anonymity condition. The two have far-reaching consequences because $\mathbf{R}$ is required to be transitive: any *sequence* of dominance-reducing transfers is desirable. In particular, denoting by $\overline{\rhd}$ the transitive closure of $\rhd$, the Strong Dominance-Reducing Transfer Principle implies that $(y_1, ..., y_n)\mathbf{R}(x_1, ..., x_n)$ whenever $(y_1, ..., y_n)\overline{\rhd}(x_1, ..., x_n)$. The relation $\overline{\rhd}$ is fully understood in the single-commodity case; it coincides with standard Lorenz dominance when aggregate wealth is fixed. But it differs fundamentally from all existing extensions of Lorenz-dominance in the multidimensional case: a two-individual, two-commodity allocation such as $((3,3),(3,3))$ Lorenz-dominates $((4,2),(2,4))$ according to any of the existing definitions but it is obviously not true that $((3,3),(3,3)) \overline{\triangleright} ((4,2),(2,4))$.

While we do not have a useful characterization of the relation $\overline{\triangleright}$, we do propose a new definition of multidimensional Lorenz dominance which is fairly tightly related to it. Fix an allocation $(x_1,...,x_n) \in X^N$. For any group of individuals $S \subseteq N$, let $|S|$ be the size of S and denote by $x(S) = \sum_{i \in S} x_i$ the aggregate commodity bundle allocated to the members of S. Say that a subset of individuals $S \subseteq N$ is a *poorest group at* $(x_1,...,x_n)$ if no group of equal or larger size is allocated fewer resources than S : for all $T \subseteq N$, $[x(S) > x(T)] \Rightarrow [|S| > |T|]$. For each $k = 1,...,n$, let $\mathcal{P}_k(x_1,...,x_n)$ denote the set of poorest groups of size k at the allocation $(x_1,...,x_n)$.

Definition 5. An allocation $(y_1,...,y_n)$ *bundle-Lorenz-dominates* an allocation $(x_1,...,x_n)$, which we write $(y_1,...,y_n)\mathbf{L}(x_1,...,x_n)$, if and only if, for every $k = 1,...,n$ and every $T \in \mathcal{P}_k(y_1,...,y_n)$ there is some $S \in \mathcal{P}_k(x_1,...,x_n)$ such that $y(T) \geq x(S)$. A social ordering $\mathbf{R}$ satisfies the *bundle-Lorenz-dominance criterion* if $(y_1,...,y_n)\mathbf{R}(x_1,...,x_n)$whenever $(y_1,...,y_n)\mathbf{L}(x_1,...,x_n)$.

In words, $(y_1,...,y_n)$ bundle-Lorenz-dominates $(x_1,...,x_n)$ when every poorest group at $(y_1,...,y_n)$ is allocated at least as much resources as some poorest group of equal size at $(x_1,...,x_n)$. Bundle-Lorenz dominance is clearly transitive. It holds fairly "rarely": in particular, the allocation $(y_1,y_2) = ((3,3),(3,3))$ does *not* bundle-Lorenz-dominate $(x_1,x_2) = ((4,2),(2,4))$ since $\{1\}$ is a poorest group of size 1 at (y_1,y_2) and there is no individual i such that $y_1 \geq x_i$. The bundle-Lorenz-dominance criterion is therefore relatively weak.

When there are only two individuals, it is easy to check that a social ordering satisfies this criterion if and only if it satisfies the Strong Dominance-Reducing Transfer Principle, Strong Symmetry, and the modified version of Monotonicity asking that $(y_1,...,y_n)\mathbf{R}(x_1,...,x_n)$ whenever $y_i \geq x_i$ for all $i \in N$. This equivalence no longer holds when there are more individuals. Yet, in spite of being much weaker than the criteria based on existing definitions of Lorenz dominance, the bundle-Lorenz-dominance criterion does imply the Dominance-Reducing Transfer Principle.

Proposition 5. *Every social ordering satisfying the bundle-Lorenz-dominance criterion satisfies the Strong Dominance-Reducing Transfer Principle.*

The easy proof is in the Appendix. We conclude with an explicit procedure for constructing orderings which satisfy both the Strong Dominance-Reducing Transfer Principle and Consensus. Our purpose is to illustrate that the two conditions are far from implying Dominance Aversion.

Let $\mathcal{R}$ denote the set of strictly monotonic preference orderings on X having a numerical representation in $\mathcal{U} = \left\{u \in \mathbb{R}^X | \ \forall x, y, t \in X, \ [x \leq y] \ \Rightarrow \ [u(x+t) - u(x) \ \leq \ u(y+t) - u(y)]\right\}$. The set $\mathcal{R}$ is very rich: for instance, it contains every strictly monotonic preference ordering R having a twice differentiable representation u such that, for some $\beta > 0$, $\partial_{hk}u/\partial_h u \partial_k u \leq \beta$ for all (possibly identical) commodities h, k. This weak condition does not imply that R is convex.

Pick $R \in \mathcal{R}$, $u \in \mathcal{U}$ representing R, and define $(y_1, ..., y_n)\mathbf{R}(x_1, ..., x_n)$ if and only if $\sum_{i \in N} u(y_i) \geq \sum_{i \in N} u(x_i)$. It is plain that the social ordering $\mathbf{R}$ satisfies the Strong Dominance-Reducing Transfer Principle. To guarantee Consensus, all we need is that R agrees with $R_1, ..., R_n$. This causes no difficulty if at least one individual preference ordering belongs $\mathcal{R}$: just let R coincide with it.

6 Concluding comments

We have formulated preference-sensitive definitions of egalitarianism respecting the principle that dominance between consumption bundles should always be reduced. Two important problems remain unsolved.

1) Each of the simple consensual leximin orderings advocated in Section 3 is built upon a different "social preference" over the commodity space which agrees with individual preferences. What should this social preference be? An answer would likely necessitate a multiprofile framework and conditions specifying how the social ordering of allocations varies in response to changes in individual preferences.

2) A characterization of all social orderings meeting Consensus and the (perhaps Strong) Dominance-Reducing Transfer Principle is still lacking.

7 Appendix: proofs

Proof of Proposition 1. In order to prove the first statement, let $\mathbf{R}$ be a social ordering satisfying Consensus, Dominance Aversion, Intrinsic Dominance, and Weak Continuity. Define the binary relation R on X as follows: for all $x, y \in X$,

$$yRx \Leftrightarrow (y, ..., y)\mathbf{R}(x, ..., x). \tag{1}$$

Since $\mathbf{R}$ is an ordering and satisfies Weak Continuity, R is a continuous ordering on X. Moreover, R agrees with $R_1, ..., R_n$: if yP_ix for all $i \in N$, Consensus implies $(y, ..., y)\mathbf{P}(x, ..., x)$, hence yPx by (1).

Now, fix two allocations $(x_1, ..., x_n)$,$(y_1, ..., y_n)$ such that

$$y_1^R P x_1^R. \tag{2}$$

We claim that $(y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)$. Suppose, by way of contradiction, that

$$(x_1, ..., x_n)\mathbf{R}(y_1, ..., y_n). \tag{3}$$

Without loss of generality, assume that $x_1^R = x_1$. By (2), y_iPx_1 for all $i \in N$. Because R is continuous, there exists a bundle $a > 0$ such that

$$y_iP(x_1 + a) \text{ for all } i \in N. \tag{4}$$

Let $x > x_i + a$ for all $i \in N$ and choose a bundle b such that $0 < b < a$. By Consensus, $(x_1 + b, x, ..., x)\mathbf{P}(x_1, ..., x_n)$, hence, by (3),

$$(x_1 + b, x, ..., x)\mathbf{P}(y_1, ..., y_n).$$

By repeated application of Dominance Aversion, $(x_1 + a, x_1 + a, ..., x_1 + a)\mathbf{R}(x_1 + b, x, ..., x)$, hence,

$$(x_1 + a, x_1 + a, ..., x_1 + a)\mathbf{P}(y_1, ..., y_n). \tag{5}$$

By definition of R, $(y_i, ..., y_i)\mathbf{R}(y_1^R, ..., y_1^R)$ for all $i \in N$. By Intrinsic Dominance, $(y_1, ..., y_n)$ $\mathbf{R}$ $(y_1^R, ..., y_1^R)$. Hence, by (5), $(x_1 + a, x_1 + a, ..., x_1 + a)\mathbf{P}(y_1^R, ..., y_1^R)$, which by definition of R means $(x_1 + a)Py_1^R$, a contradiction to (4).

In order to prove the second statement in Proposition 1, let $\mathbf{R}$ be the leximin extension of some continuous ordering R agreeing with $R_1, ..., R_n$.

To check Consensus, suppose $y_i P_j x_i$ for all $i, j \in N$. Then $y_1^R P_j x_1^R$ for all $j \in N$ and since R agrees with $R_1, ..., R_n$, $y_1^R P x_1^R$. By definition of $\mathbf{R}$, $(y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)$.

To prove Dominance Aversion, suppose $x_i > y_i \geq y_j > x_j$ for some $i, j \in N$ and $y_k = x_k$ for all $k \in N \setminus \{i, j\}$. Because R agrees with $R_1, ..., R_n$, we have $x_i P y_i R y_j P x_j$ and $y_k I x_k$ for all $k \in N \setminus \{i, j\}$. Since $\mathbf{R}$ is the leximin extension of R, it follows that $(y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)$.

To prove Intrinsic Dominance, fix $(x_1, ..., x_n) \in X^N$, $x \in X$, and suppose $(x_i, ..., x_i)$ $\mathbf{R}$ $(x, ..., x)$ for all $i \in N$. By definition of R, $x_i R x$ for all $i \in N$. Hence, by definition of $\mathbf{R}$, $(x_1, ..., x_n)$ $\mathbf{R}$ $(x, ..., x)$.

Weak Continuity of $\mathbf{R}$ follows directly from continuity of R.■

Proof of Proposition 2. We have shown in the proof of Proposition 1 that all simple consensual leximin social orderings satisfy the first three axioms in Proposition 2. It is straightforward to check the last two, Strong Symmetry and Internal Separability.

Conversely, let $\mathbf{R}$ be a social ordering satisfying the five axioms in Proposition 2. We begin by showing that $\mathbf{R}$ satisfies Intrinsic Dominance. Let $(x_1, ..., x_n) \in X^N$ and $x \in X$ be such that $(x_i, ..., x_i)\mathbf{R}(x, ..., x)$ for all $i \in N$. By Weak Continuity, there exist bundles $a_1, ..., a_n \in X$ such that

$$(x_i, ..., x_i)\mathbf{I}(x + a_i, ..., x + a_i) \text{ for all } i \in N. \tag{6}$$

Using (6), repeated applications of Internal Separability yield $(x_n, x_n, ..., x_n)\mathbf{I}(x+a_n, x+a_n, ..., x+a_n) \Rightarrow (x_1, x_n, ..., x_n)\mathbf{I}(x+a_1, x_n, ..., x_n) \Rightarrow (x_1, x_2, x_n, ...x_n)\mathbf{I}(x+a_1, x+a_2, x_n, ..., x_n)$ $\Rightarrow (x_1, ..., x_n)\mathbf{I}(x + a_1, ..., x + a_n)$. By Consensus, $(x + a_1, ..., x + a_n)\mathbf{R}(x, ..., x)$. Hence $(x_1, ..., x_n)\mathbf{R}(x, ..., x)$ by transitivity of $\mathbf{R}$. This establishes Intrinsic Dominance.

Next, define the binary relation R on X as in (1). The proof of Proposition 1 shows that R is an ordering agreeing with $R_1, ..., R_n$ and that $\mathbf{R}$ is a consensual Rawlsian ordering based on R: for all $(x_1, ..., x_n)$, $(y_1, ..., y_n) \in X^N$,

$$y_1^R P x_1^R \Rightarrow (y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n). \tag{7}$$

We claim that, in fact, $\mathbf{R}$ is the leximin extension of R, that is, $\mathbf{R} = \mathbf{R}_{lex}(R)$. To show this, we fix $(x_1, ..., x_n)$, $(y_1, ..., y_n) \in X^N$ and proceed in two steps.

Step 1. We show that $(y_1, ..., y_n)\mathbf{I}_{lex}(R)(x_1, ..., x_n) \Rightarrow (y_1, ..., y_n)\mathbf{I}(x_1, ..., x_n)$.

If $(y_1, ..., y_n)\mathbf{I}_{lex}(R)(x_1, ..., x_n)$, then $y_i^R I x_i^R$ for all $i \in N$. By (1), $(y_i^R, ..., y_i^R)\mathbf{I}(x_i^R, ..., x_i^R)$ for all $i \in N$. Therefore, starting from the fact that $(y_1^R, ..., y_n^R)\mathbf{I}(y_1^R, ..., y_n^R)$ and applying Internal Separability n times, $(y_1^R, ..., y_n^R)$ $\mathbf{I}$ $(x_1^R, y_2^R, ..., y_n^R)$ $\mathbf{I}...\mathbf{I}$ $(x_1^R, ..., x_{n-1}^R, y_n^R)$ $\mathbf{I}$ $(x_1^R, ..., x_n^R)$. Hence, $(y_1^R, ..., y_n^R)\mathbf{I}(x_1^R, ..., x_n^R)$. Invoking Strong Symmetry, $(y_1, ..., y_n)\mathbf{I}(x_1, ..., x_n)$.

Step 2. We show that $(y_1, ..., y_n)\mathbf{P}_{lex}(R)(x_1, ..., x_n) \Rightarrow (y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)$.

If $(y_1, ..., y_n)\mathbf{P}_{lex}(R)(x_1, ..., x_n)$, there exists $j \in N$ such that

$$y_i^R I x_i^R \text{ for all } i < j \text{ and } y_j^R P x_j^R. \tag{8}$$

If $j = 1$, (8) means that $y_1^R P x_1^R$, and (7) implies $(y_1, ..., y_n)\mathbf{P}(x_1, ..., x_n)$.

If $j > 1$, (8) implies that

$$(y_i^R, ..., y_i^R)\mathbf{I}(x_i^R, ..., x_i^R) \text{ for } i = 1, ..., j-1. \tag{9}$$

Choose bundles $z_1, ..., z_{j-1} \in X$ such that

$$z_i P y_j^R \text{ for } i = 1, ..., j-1. \tag{10}$$

Given (9) and since trivially $(z_i, ..., z_i)\mathbf{I}(z_i, ..., z_i)$ for $i = 1, ..., j-1$, applying Internal Separability $j-1$ times yields

$$(y_1^R, ..., y_n^R)\mathbf{R}(x_1^R, ..., x_n^R) \Leftrightarrow (z_1, ..., z_{j-1}, y_j^R, ..., y_n^R)\mathbf{R}(z_1, ..., z_{j-1}, x_j^R, ..., x_n^R). \tag{11}$$

Defining $(a_1, ..., a_n) = (z_1, ..., z_{j-1}, x_j^R, ..., x_n^R)$ and $(b_1, ..., b_n) = (z_1, ..., z_{j-1}, y_j^R, ..., y_n^R)$, (8) and (10) imply that $a_1^R = x_j^R$, $b_1^R = y_j^R$, and $b_1^R P a_1^R$. Therefore, by (7), $(b_1, ..., b_n)\mathbf{P}(a_1, ..., a_n)$, that is, $(z_1, ..., z_{j-1}, y_j^R, ..., y_n^R)$ $\mathbf{P}$ $(z_1, ..., z_{j-1}, x_j^R, ..., x_n^R)$. By (11), $(y_1^R, ..., y_n^R)\mathbf{P}(x_1^R, ..., x_n^R)$. By Strong Symmetry, $(y_1, ..., y_n)\mathbf{I}(y_1^R, ..., y_n^R)$ and $(x_1, ..., x_n)\mathbf{I}(x_1^R, ..., x_n^R)$, hence $(y_1, ..., y_n)$ $\mathbf{P}$ $(x_1, ..., x_n)$.■

The following examples show that the axioms in Proposition 2 are independent.

An ordering violating only Consensus is the universal indifference relation: $(y_1, ..., y_n)$ $\mathbf{R}$ $(x_1, ..., x_n)$ for all $(x_1, ..., x_n), (y_1, ..., y_n) \in X^N$.

An example violating only Dominance Aversion is the following simple consensual *leximax* ordering: let R be a continuous ordering agreeing with $R_1, ..., R_n$ and define $(y_1, ..., y_n)$ $\mathbf{R}$ $(x_1, ..., x_n)$ if and only if either $\left[\text{there is } j \in N \text{ such that } y_j^R P x_j^R \text{ and } y_i^R I x_i^R \text{ for all } i > j\right]$ or $\left[y_i^R I x_i^R \text{ for all } i \in N\right]$.

For an example violating only Weak Continuity, choose a *discontinuous* ordering R agreeing with $R_1, ..., R_n$ and let $\mathbf{R}$ be the leximin extension of R. For instance, R could be a lexicographic refinement of, say, R_1: in the case $X = \mathbb{R}_+^2$ for instance, define $(\alpha', \beta') R(\alpha, \beta)$ if and only if $\left[(\alpha', \beta') P_1(\alpha, \beta)\right]$ or $\left[(\alpha', \beta') I_1(\alpha, \beta) \text{ and } \alpha' \geq \alpha\right]$.

For an example violating only Strong Symmetry, let R be a continuous preference agreeing with $R_1, ..., R_n$ and let $(y_1, ..., y_n)\mathbf{R}(x_1, ..., x_n)$ if and only if $[(y_1, ..., y_n)\mathbf{P}_{lex}(R)(x_1, ..., x_n)]$ or $[(y_1, ..., y_n)\ \mathbf{I}_{lex}(R)\ (x_1, ..., x_n) \text{ and } y_1 R x_1]$.

For an example violating only Internal Separability, choose again a continuous preferences R agreeing with $R_1, ..., R_n$. If $n \geq 3$, let $\mathbf{R}$ be the leximin extension of R. If $n = 2$, however, let $\mathbf{R}$ be the "Rawlsian extension" of R, that is, $(y_1, y_2)\mathbf{R}(x_1, x_2)$ if and only if $y_1^R R x_1^R$.

Proof of Proposition 4. Let $\mathbf{R}$ satisfy Full Consensus, Dominance Aversion, Strong Symmetry, the Egalitarian Pareto Principle, and Weak Continuity. We begin with a fact that does not depend on the restriction $n = 2$.

Step 1. $\mathbf{R}$ satisfies the following condition.

Unjustified-Inequality Aversion. For all $(x_1, ..., x_n)$, $(y_1, ..., y_n) \in X^N$, $[x_i P_k y_i R_k y_j P_k x_j$ for some $i, j \in N$ and all $k \in N$, and $y_k = x_k$ for all $k \in N \setminus \{i, j\}] \Rightarrow [(y_1, ..., y_n)\mathbf{R}(x_1, ..., x_n)]$.

The proof uses only the assumptions that $\mathbf{R}$ satisfies Full Consensus and Dominance Aversion. Let $(x_1, ..., x_n)$, $(y_1, ..., y_n) \in X^N$. Suppose $x_i P_k y_i R_k y_j P_k x_j$ for some $i, j \in N$ and all $k \in N$, and $y_k = x_k$ for all $k \in N \setminus \{i, j\}$. Without loss of generality, let $i = 1, j = 2$. Suppose, contrary to the claim, that

$$(x_1, ..., x_n)\mathbf{P}(y_1, ..., y_n). \tag{12}$$

Since $y_1 R_k y_2$ for all $k \in N$, Full Consensus implies

$$(y_1, y_2, y_3, ..., y_n)\mathbf{R}(y_2, y_2, y_3, ..., y_n). \tag{13}$$

Because $y_2 P_k x_2$ for all $k \in N$ and individual preferences are strictly monotonic, $y_2 > 0$. By continuity of individual preferences, $(y_2 - a) P_k x_2$ for all $k \in N$ and any small enough bundle $a > 0$. By Full Consensus and recalling that $y_k = x_k$ for $k = 3, ..., n$, $(x_1, y_2 - a, y_3, ..., y_n) \mathbf{R} (x_1, ..., x_n)$. Hence, by (12), $(x_1, y_2 - a, y_3, ..., y_n) \mathbf{P} (y_1, ..., y_n)$, which, combined with (13), yields

$$(x_1, y_2 - a, y_3, ..., y_n) \mathbf{P} (y_2, y_2, y_3, ..., y_n). \tag{14}$$

Next, choose $b \in X$ large enough to guarantee that $y_1 + b > x_1 \vee y_2$. By strict monotonicity of preferences, $(y_1 + b) P_k x_1$ for all $k \in N$. By Full Consensus, $(y_1 + b, y_2 - a, y_3, ..., y_n) \mathbf{R} (x_1, y_2 - a, y_3, ..., y_n)$, hence by (14), $(y_1 + b, y_2 - a, y_3, ..., y_n) \mathbf{P} (y_2, y_2, y_3, ..., y_n)$. This contradicts Dominance Aversion since $y_1 + b > y_2 > y_2 - a$.

From now on we restrict attention to the two-individual case, $N = \{1, 2\}$. For any $Y \subseteq X$, let ∂Y be the set of "Pareto-undominated" *bundles* in Y: for all $x \in X$, $x \in \partial Y$ if and only if $x \in Y$ and for all $y \in Y$, $[y R_i x \text{ for } i = 1, 2] \Rightarrow [y I_i x \text{ for } i = 1, 2]$.

Step 2. For all $(x_1, x_2) \in X^{\{1,2\}}$ and all $x \in \partial W(x_1, x_2)$,

$$(x + a, x + a) \mathbf{P} (x_1, x_2) \text{ for all } a > 0. \tag{15}$$

Fix $(x_1, x_2) \in X^{\{1,2\}}$ and $x \in \partial W(x_1, x_2)$. Depending on the preference profile over $\{x_1, x_2\}$, distinguish four cases.

Case 1. Preferences are strict and agree, say, $x_1 P_i x_2$ for $i = 1, 2$.

In this case, $W(x_1, x_2) = W_1(x_2) \cap W_2(x_2)$. Observe that

$$(x, x) \mathbf{I} (x_2, x_2). \tag{16}$$

Indeed, since $x \in W(x_1, x_2)$, we have $x_2 R_i x$ for $i = 1, 2$. Since $x \in \partial W(x_1, x_2)$ and $x_2 \in W(x_1, x_2)$, the definition of $\partial W(x_1, x_2)$ yields $x_2 I_i x$ for $i = 1, 2$. By Full Consensus, (16) follows.

To establish (15), fix $a > 0$. By strict monotonicity of preferences and by Consensus, $(x + a, x + a) \mathbf{P} (x, x)$, hence, by (16), $(x + a, x + a) \mathbf{P} (x_2, x_2)$. By Weak Continuity, it follows that $(x + a, x + a) \mathbf{P} (x_2 + b, x_2 + b)$ for all $b > 0$ small enough. Because preferences are strictly monotonic and continuous, $x_1 P_i (x_2 + b) P_i x_2$ for $i = 1, 2$ and $b > 0$ small enough.

Invoking Unjustified-Inequality Aversion (established in Step 1), $(x_2 + b, x_2 + b)\mathbf{P}(x_1, x_2)$, hence $(x + a, x + a)\mathbf{P}(x_1, x_2)$.

Case 2. Preferences are strict and conflict: $x_1 P_i x_2$ and $x_2 P_j x_1$ for some $i, j \in \{1, 2\}$.

Because of Strong Symmetry, there is no loss of generality in assuming that $x_1 P_2 x_2$ and $x_2 P_1 x_1$. In this case, $W(x_1, x_2) = W_1(x_1) \cap W_2(x_2)$ and thus $x_i R_i x$ for $i = 1, 2$.

We begin by showing that

$$xR_i x_i \text{ for } i = 1, 2. \tag{17}$$

Suppose, by contradiction, that, say, $x_1 P_1 x$. Then

$$x_2 P_1 x_1 P_1 x. \tag{18}$$

If $x_2 P_2 x$, continuity of individual preferences guarantees that for $a > 0$ small enough, $x_i P_i (x+a)$ for $i = 1, 2$, that is, $x + a \in W(x_1, x_2)$. But, by strict monotonicity of preferences, $(x + a)P_i x$ for $i = 1, 2$, contradicting the fact that $x \in \partial W(x_1, x_2)$.

If $x_2 I_2 x$, then both x and x_2 belong to the set $Y = \{y \in X \mid yI_2 x_2 \text{ and } x_2 R_1 y R_1 x\}$. Equation (18) and continuity of preferences imply that there exists some $y \in Y$ such that $yI_1 x_1$ (pick a continuous numerical representation u_1 of R_1 and note that $u_1(Y) = [u_1(x), u_1(x_2)]$ is a closed interval containing $u_1(x_1)$). By definition, $y \in W_1(x_1) \cap W_2(x_2) = W(x_1, x_2)$ and $yP_1 x$ and $yI_2 x$, contradicting again the fact that $x \in \partial W(x_1, x_2)$.

This proves (17). By strict monotonicity of preferences, it follows that $(x + a)P_i x_i$ for $i = 1, 2$ and all $a > 0$. Now (15) follows from the Egalitarian Pareto Principle.

Case 3. One individual is indifferent, say, $x_1 I_1 x_2$.

By Strong Symmetry, we may assume $x_1 P_2 x_2$. So $W(x_1, x_2) = W_1(x_2) \cap W_2(x_2)$. Because R_2 is continuous, there exists $x_1' \in X$ such that $x_1' P_1 x_2$ and $x_1' I_2 x_1$ (for instance, let $x_1' = x_2 + b$: since $x_1 P_2 (x_2 + b)$ when $b = 0$ and $(x_2 + b)P_2 x_1$ when b is large enough, $(x_2 + b)I_2 x_1$ for some $b > 0$).

Observe that $W(x_1', x_2) = W_1(x_2) \cap W_2(x_2) = W(x_1, x_2)$. So $x \in \partial W(x_1', x_2)$. Since $x_1' P_i x_2$ for $i = 1, 2$, we know from Case 2 that $(x + a, x + a)\mathbf{P}(x_1', x_2)$ for all $a > 0$. But $(x_1', x_2)\mathbf{R}(x_1, x_2)$ by Full Consensus, hence (15) follows.

Case 4. Both individuals are indifferent, $x_1 I_i x_2$ for $i = 1, 2$.

In this case, $W(x_1, x_2) = W_1(x_2) \cap W_2(x_2)$ and therefore (16) holds. Hence, using strict monotonicity of preferences and Consensus, $(x + a, x + a)\mathbf{P}(x_2, x_2)$ for all $a > 0$. But since $x_1 I_i x_2$ for $i = 1, 2$, Full Consensus also implies $(x_2, x_2)\mathbf{I}(x_1, x_2)$, hence (15) follows.

Step 3. For all $(x_1, x_2), (y_1, y_2) \in X^{\{1,2\}}$,

$$W(y_1, y_2) > W(x_1, x_2) \Rightarrow y_1^i P_i x_1^i \text{ for } i = 1, 2.$$

Let $W(y_1, y_2) > W(x_1, x_2)$. Contrary to the claim, suppose that, say, $x_1^1 R_1 y_1^1$. Then $W(x_1, x_2) < W(y_1, y_2) \subseteq W_1(y_1^1) \subseteq W_1(x_1^1)$. Therefore $W(x_1, x_2) < W_1(x_1^1)$. On the other hand, $W(x_1, x_2) = W_1(x_1^1) \cap W_2(x_1^2)$. Combining these two facts, $W_2(x_1^2) < W_1(x_1^1)$. This implies that $x_1^1 P_1 x_1^2$, contradicting the definition of x_1^1.

Step 4. For all $(x_1, x_2), (y_1, y_2) \in X^{\{1,2\}}$,

$$W(y_1, y_2) > W(x_1, x_2) \Rightarrow (y_1, y_2)\mathbf{P}(x_1, x_2).$$

Let $W(y_1, y_2) > W(x_1, x_2)$. By Step 3, $y_1^i P_i x_1^i$ for $i = 1, 2$. Let $x \in \partial W(x_1, x_2)$. Since $x \in W_1(x_1^1) \cap W_2(x_1^2)$, we have $y_1^i P_i x$ for $i = 1, 2$. For $a > 0$ small enough, continuity of preferences guarantees $y_1^i P_i(x+a)$ for $i = 1, 2$, hence $y_j P_i(x+a)$ for all $i, j \in \{1, 2\}$. By Consensus, $(y_1, y_2)\mathbf{P}(x + a, x + a)$. By Step 2, $(x + a, x + a)\mathbf{P}(x_1, x_2)$. Therefore $(y_1, y_2)\mathbf{P}(x_1, x_2)$.■

Proof of Proposition 5. We begin with the following remark: for every allocation $(x_1, ..., x_n) \in X^N$, for every $k \in N$ and every $S \subseteq N$ such that $|S| = k$, there exists some $S^* \in \mathcal{P}_k(x_1, ..., x_n)$ such that $x(S) \geq x(S^*)$. The bundle allocated to any group is at least at large as the bundle allocated to some *poorest* group of its size. The proof if obvious because N is finite.

Now, let $\mathbf{R}$ be a social ordering satisfying the bundle-Lorenz-dominance criterion. In order to prove that $\mathbf{R}$ meets the Strong Dominance-Reducing Transfer Principle, fix $(x_1, ..., x_n) \in X^N$, $t \in X$, and $i, j \in N$ such that $x_i > x_i - t > x_j$ and $x_i > x_j + t > x_j$. Define $(y_1, ..., y_n)$ by $y_i = x_i - t$, $y_j = x_j + t$; and $y_k = x_k$ for all $k \in N \backslash \{i, j\}$. We must show that $(y_1, ..., y_n)\mathbf{R}(x_1, ..., x_n)$.

We claim that $(y_1, ..., y_n)\mathbf{L}(x_1, ..., x_n)$. To see this, fix $k \in N$ and $T \in \mathcal{P}_k(y_1, ..., y_n)$. We must find $S \in \mathcal{P}_k(x_1, ..., x_n)$ such that $y(T) \geq x(S)$.

Case 1: $T \cap \{i, j\} \neq \{j\}$ (i.e., either $T \cap \{i, j\} = \{i, j\}$, or $T \cap \{i, j\} = \{i\}$, or $T \cap \{i, j\} = \emptyset$).

Then $y(T) \geq x(T)$. By the remark at the beginning of the proof, there exists $T^* \in \mathcal{P}_k(x_1, ..., x_n)$ such that $x(T) \geq x(T^*)$. Then $y(T) \geq x(T^*)$.

Case 2: $T \cap \{i, j\} = \{j\}$.

Define $S = (T \backslash \{j\}) \cup \{i\}$. Note that $|S| = |T| = k$. Since $y_j = x_j + t > x_i$, we have $y(T) > x(S)$. By the remark at the beginning of the proof, there exists $S^* \in \mathcal{P}_k(x_1, ..., x_n)$ such that $x(S) \geq x(S^*)$. Then $y(T) \geq x(S^*)$.

Since $(y_1, ..., y_n)\mathbf{L}(x_1, ..., x_n)$ and $\mathbf{R}$ satisfies the bundle-Lorenz-dominance criterion, $(y_1, ..., y_n)\mathbf{R}(x_1, ..., x_n)$.■

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