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Infinite-horizon choice functions*

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Abstract

We analyze infinite-horizon choice functions within the setting of a simple linear technology. Time consistency and efficiency are characterized by stationary consumption and inheritance functions, as well as a transversality condition. In addition, we consider the equity axioms Suppes-Sen, Pigou-Dalton, and resource monotonicity. We show that Suppes-Sen and Pigou-Dalton imply that the consumption and inheritance functions are monotone with respect to time—thus justifying sustainability—while resource monotonicity implies that the consumption and inheritance functions are monotone with respect to the resource. Examples illustrate the characterization results.

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1 Introduction

The literature on ranking infinite consumption (or utility) streams has produced a number of negative results in the form of the incompatibility of seemingly mild axioms. For example, following Koopmans (1960), Diamond (1965) establishes that anonymity is incompatible with the strong Pareto principle. Finite anonymity weakens anonymity by restricting the application of the standard anonymity requirement to situations where utility streams differ in at most a finite number of components. Diamond (1965) goes on to show that strong Pareto, finite anonymity and a continuity requirement are incompatible if the social relation is required to be transitive and complete. Hara, Shinotsuka, Suzumura and Xu (2005) adapt the well-known strict transfer principle due to Pigou (1912) and Dalton (1920) to the infinite-horizon context. They show that this principle is incompatible with strong Pareto and continuity even if the social preference is merely required to be acyclical. Basu and Mitra (2003) show that strong Pareto, finite anonymity and representability by a real-valued function are incompatible. Epstein (1986) establishes the incompatibility of a set of standard axioms and a substitution property requiring the possibility to improve upon any given constant stream by means of a stream with lower initial consumption.

The main purpose of this paper is to suggest an alternative approach that may provide a promising way to address issues involving intergenerational allocation problems with an infinite horizon. Instead of searching for a *ranking* of infinite streams, we examine a choice-theoretic model where a *choice function* is used to select a consumption stream from each set of feasible streams. Because our focus is on the choice-theoretic aspect of the model, we deliberately consider a simple setting where there is a single resource and a linear and stationary technology with positive renewal. This implies that the feasibility of a consumption stream is determined by the initial amount of the resource available, and the choice function assigns a consumption stream (the chosen consumption stream, given the feasibility constraint) to each possible initial amount.

We begin with an analysis of two fundamental properties whose versions formulated for orderings have been used extensively in the literature, namely, *efficiency* and *time consistency.* We provide characterizations of all infinite-horizon choice functions satisfying either of the two axioms and, moreover, identify all choice functions with both properties. We then consider *equity* properties that are choice-theoretic versions of the Suppes-Sen principle, the Pigou-Dalton transfer principle and resource monotonicity (see Asheim, Mitra and Tungodden, 2006; Bossert, Sprumont and Suzumura, 2006; Hara, Shinotsuka, Suzumura and Xu, 2005, for equity properties imposed on rankings of infinite streams). Again, classes of infinite-horizon choice functions possessing one of these properties are characterized, and further axiomatizations are obtained by adding efficiency or time consistency.

The results we obtain are promising. Unlike in the case of orderings of infinite utility streams, impossibilities can be avoided and rich classes of infinite-horizon choice functions satisfying several desirable properties do exist. In particular, our choice-theoretic version of the Suppes-Sen principle imposes full anonymity rather than merely finite anonymity and our choice functions may be continuous in the initial endowment. Moreover, it turns out that the notion of *sustainability,* which has played a major role in the literature on intergenerational resource allocation, is closely linked to the Suppes-Sen and Pigou-Dalton principles. Our conclusion from these results is that the choice-theoretic approach to intergenerational resource allocation provides an interesting and viable alternative to the models based on establishing orderings of infinite utility streams, and we propose to explore this approach further.

Section 2 contains some basic definitions and a first well-known observation characterizing sets of feasible consumption streams. In Section 3, we examine the fundamental axioms of efficiency and time consistency. We characterize all efficient infinite-horizon choice functions, all time-consistent infinite-horizon choice functions, and the class of choice functions satisfying both requirements. Section 4 deals with the equity axioms à la Suppes-Sen, Pigou-Dalton and resource monotonicity. We characterize all infinite-horizon choice functions satisfying: (i) Suppes-Sen; (ii) efficiency and Pigou-Dalton; (iii) time consistency and Suppes-Sen; (iv) efficiency, time consistency and Pigou-Dalton; (v) efficiency, time consistency and resource monotonicity. As a by-product of our analysis, we show that the conjunction of efficiency and Pigou-Dalton is equivalent to Suppes-Sen. Section 5 provides some examples and Section 6 concludes.

2 Preliminaries

Let $\mathbb{R}_+$ and $\mathbb{R}_{++}$ denote the set of all non-negative real numbers and the set of all positive real numbers, respectively. Analogously, $\mathbb{Z}_+$ and $\mathbb{Z}_{++}$ denote the set of all non-negative integers and the set of all positive integers, respectively.

Define the set $\mathcal{Y} = \mathbb{R}_+^{\mathbb{Z}_+}$ to be the set of all sequences $y = (y_0, y_1, \ldots, y_t, \ldots)$. We interpret y as a consumption stream, where y_t is the amount of a single resource consumed in period $t \in \mathbb{Z}_+$. Time is measured relative to the present: period t is the t^{th} period after today. The initial amount of the resource is $x \in \mathbb{R}_+$. We assume a linear and stationary technology, entailing that in each period, the resource is renewed at the fixed positive rate $r \in \mathbb{R}_{++}$.

We use the following notation for inequalities in $\mathcal{Y}$. For all $y, z \in \mathcal{Y}$, $y \geq z$ if and only if $y_t \geq z_t$ for all $t \in \mathbb{Z}_+$, and $y > z$ if and only if $y \geq z$ and $y \neq z$.

For $x \in \mathbb{R}_+$ and $y \in \mathcal{Y}$, the sequence of resource stocks

$$k(x, y) = (k_0(x, y), k_1(x, y), \ldots, k_t(x, y), \ldots) \in \mathbb{R}^{\mathbb{Z}_+}$$

generated by x and y is defined by $k_0(x, y) = x$ and

$$k_t(x, y) = (1 + r)(k_{t-1}(x, y) - y_{t-1})$$

for all $t \in \mathbb{Z}_{++}$. For $x \in \mathbb{R}_+$, the set of x-feasible consumption streams is

$$\mathcal{S}(x) = \{y \in \mathcal{Y} \mid y_t \in [0, k_t(x, y)] \text{ for all } t \in \mathbb{Z}_+\}.$$

It is immediate that the set of x-feasible consumption streams can equivalently be expressed as in the following lemma; see, for instance, Epstein (1986) who made this observation in his analysis of the linear model in an intertemporal social choice setting.

Lemma 1 *For all $x \in \mathbb{R}_+$,*

$$\mathcal{S}(x) = \left\{ y \in \mathcal{Y} \;\middle|\; \sum_{t=0}^{\infty} \frac{y_t}{(1 + r)^t} \leq x \right\}.$$

3 Efficient and time-consistent choice

An *infinite-horizon choice function* is a mapping $C \colon \mathbb{R}_+ \to \mathcal{Y}$ such that $C(x) \in \mathcal{S}(x)$ for all $x \in \mathbb{R}_+$. This function assigns a consumption stream to any given initial amount of a single resource available in the economy. Note that consumption streams are undated: whether the choice takes place today or tomorrow makes no difference if the same initial endowment is present. This time-independence feature of a choice function ensures that the choice of a starting period is irrelevant. For all $t \in \mathbb{Z}_+$, we write $C_t(x)$ for the t^{th} component of the sequence $C(x)$.

The first fundamental property of an infinite-horizon choice function is the familiar *efficiency* axiom. It requires that no x-feasible consumption stream Pareto dominates the chosen consumption stream with initial stock x.

Efficiency. For all $x \in \mathbb{R}_+$ and for all $y \in \mathcal{Y}$,

$$y > C(x) \quad \Rightarrow \quad y \notin \mathcal{S}(x).$$

Given Lemma 1, it is straightforward to characterize the class of efficient choice functions. We omit the immediate proof of the following lemma stating the relevant result.

Lemma 2 *An infinite-horizon choice function C satisfies efficiency if and only if*

$$\sum_{t=0}^{\infty} \frac{C_t(x)}{(1+r)^t} = x \quad \textit{for all } x \in \mathbb{R}_+. \tag{C1}$$

Time consistency prevents deviations from chosen consumption streams as time progresses. Thus, for any $x \in \mathbb{R}_+$ and for any $t, \tau \in \mathbb{Z}_+$, the consumption $C_{t+\tau}(x)$ in period $t+\tau$ for the initial endowment x should be the same as the consumption $C_\tau(k_t(x, C(x)))$ in period τ for the initial endowment $k_t(x, C(x))$.

Time consistency. For all $x \in \mathbb{R}_+$ and for all $t, \tau \in \mathbb{Z}_+$,

$$C_{t+\tau}(x) = C_\tau(k_t(x, C(x))).$$

We now characterize all infinite-horizon choice functions satisfying time consistency. In order to express this class of choice functions, we use a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ that indicates, for each initial level of the resource, the amount of the resource that is available in the next period after the present consumption has taken place. Hence, we may refer to g as the *inheritance function.* Consequently, $g(x)/(1+r)$ is the bequest that is left behind, and $x - (g(x)/(1+r))$ is the present consumption. Hence, we may refer to the mapping $x \mapsto x - (g(x)/(1+r))$ as the *consumption function.*

For any function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$, let the function $g^0\colon \mathbb{R}_+ \to \mathbb{R}_+$ be defined by $g^0(x) = x$ for all $x \in \mathbb{R}_+$ and, for all $t \in \mathbb{Z}_{++}$, define the function $g^t\colon \mathbb{R}_+ \to \mathbb{R}_+$ by letting $g^t(x) = g(g^{t-1}(x))$ for all $x \in \mathbb{R}_+$. As will become clear once our characterization of time consistency is stated, the functions g^t have a natural interpretation: they identify the amount of the resource available in period t as a function of the initial endowment x only. Because all these functions are determined once a function g is chosen, it is sufficient to specify, for any initial endowment, the amount of the resource remaining at the beginning of period one.

The following lemma characterizes all time-consistent choice functions.

Lemma 3 *An infinite-horizon choice function C satisfies time consistency if and only if there exists a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that*

$$g(x) \leq x(1+r) \quad \textit{for all } x \in \mathbb{R}_+ \tag{G1}$$

and

$$C_t(x) = g^t(x) - \frac{g^{t+1}(x)}{1+r} \quad \textit{for all } t \in \mathbb{Z}_+ \textit{ and for all } x \in \mathbb{R}_+. \tag{CG}$$

Proof. 'If.' Let C be an infinite-horizon choice function and suppose there exists a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that (G1) and (CG) are satisfied. Let $x \in \mathbb{R}_+$ and $t \in \mathbb{Z}_+$. By (G1), it follows that

$$g^{t+1}(x) = g(g^t(x)) \leq g^t(x)(1+r)$$

and, together with (CG), that

$$C_t(x) = g^t(x) - \frac{g^{t+1}(x)}{1+r} \geq 0.$$

Using (CG) and the definition of $k(x, y)$, we obtain

$$k_t(x, C(x)) = g^t(x). \tag{1}$$

Because g is non-negative-valued, (CG) and (1) together imply

$$C_t(x) = g^t(x) - \frac{g^{t+1}(x)}{1+r} = k_t(x, C(x)) - \frac{g^{t+1}(x)}{1+r} \leq k_t(x, C(x)).$$

Hence, $C(x) \in \mathcal{S}(x)$ and C is a well-defined infinite-horizon choice function.

To establish time consistency, let $x \in \mathbb{R}_+$ and $t, \tau \in \mathbb{Z}_+$. By (CG),

$$C_{t+\tau}(x) = g^{t+\tau}(x) - \frac{g^{t+\tau+1}(x)}{1+r}. \tag{2}$$

By (1) and (CG),

$$C_\tau(k_t(x, C(x))) = C_\tau(g^t(x)) = g^\tau(g^t(x)) - \frac{g^{\tau+1}(g^t(x))}{1+r} = g^{t+\tau}(x) - \frac{g^{t+\tau+1}(x)}{1+r}$$

which, together with (2), proves that C is time consistent.

'Only if.' Suppose C is an infinite-horizon choice function that satisfies time consistency. Define the function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ by letting

$$g(x) = (1+r)(x - C_0(x)) \tag{3}$$

for all $x \in \mathbb{R}_+$. By feasibility, $C_0(x) \in [0, x]$, and the definition of g immediately implies $g(x) \in [0, x(1+r)]$ for all $x \in \mathbb{R}_+$, establishing that g indeed maps into $\mathbb{R}_+$ and that (G1) is satisfied.

It remains to be shown that (CG) is satisfied. We proceed by induction. Solving (3) for $C_0(x)$, we obtain

$$C_0(x) = x - \frac{g(x)}{1+r} = g^0(x) - \frac{g^1(x)}{1+r}. \tag{4}$$

Now suppose

$$C_t(x) = g^t(x) - \frac{g^{t+1}(x)}{1+r} \tag{5}$$

for some $t \in \mathbb{Z}_+$. By definition, $k_1(x, C(x)) = (1+r)(x - C_0(x)) = g(x)$. Thus, using time consistency and (5), we obtain

$$C_{t+1}(x) = C_t(k_1(x, C(x))) = C_t(g(x)) = g^t(g(x)) - \frac{g^{t+1}(g(x))}{1+r} = g^{t+1}(x) - \frac{g^{t+2}(x)}{1+r}$$

which completes the proof. ■

We now characterize all infinite-horizon choice functions satisfying both efficiency and time consistency.

Theorem 1 *An infinite-horizon choice function C satisfies efficiency and time consistency if and only if there exists a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that (CG), (G1) and*

$$\lim_{t\to\infty} \frac{g^t(x)}{(1+r)^t} = 0 \quad \textit{for all } x \in \mathbb{R}_+ \tag{G2}$$

are satisfied.

Proof. 'If.' Let C be an infinite-horizon choice function and suppose there exists a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that (CG), (G1) and (G2) are satisfied. Then, by Lemma 3, C is a well-defined infinite-horizon choice function that satisfies time consistency. Furthermore,

$$\sum_{t=0}^{\infty} \frac{C_t(x)}{(1+r)^t} = x - \lim_{t\to\infty} \frac{g^t(x)}{(1+r)^t}.$$

By invoking Lemma 2, (G2) implies that C satisfies efficiency.

'Only if.' Suppose C is an infinite-horizon choice function that satisfies efficiency and time consistency. Then, by Lemma 3, there exists a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that (CG) and (G1) are satisfied. By invoking Lemma 2, efficiency and (CG) imply that

$$x = \sum_{t=0}^{\infty} \frac{C_t(x)}{(1+r)^t} = x - \lim_{t\to\infty} \frac{g^t(x)}{(1+r)^t}.$$

Hence, g satisfies (G2). ■

Condition (G2) is of course a capital value transversality condition, which has been used to characterize efficient capital accumulation at least since Malinvaud (1953).

The properties (G1) and (G2) of a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ are independent, as is straightforward to verify. That (CG) must be satisfied is a consequence of the time-consistency requirement, and (G1) ensures that this is done without violating the resource constraints. Property (G2) is required for the efficiency axiom.

4 Imposing equity axioms

We now examine the consequences of imposing certain equity axioms, in addition to efficiency and time consistency.

The first of the equity axioms that we consider—*Suppes-Sen*—requires that no x-feasible consumption stream has a permutation which Pareto dominates the chosen consumption stream with initial stock x. The term 'permutation' signifies a bijective mapping π of $\mathbb{Z}_+$ onto itself. The Suppes-Sen axiom is a straightforward adaptation of the Suppes-Sen principle for orderings (cf. Suppes, 1966; Sen, 1970) to the present infinite-horizon choice-theoretic setting.

Suppes-Sen. For all $x \in \mathbb{R}_+$ and for all $y, y' \in \mathcal{Y}$, if y' is a permutation of y, then

$$y' > C(x) \quad \Rightarrow \quad y \notin \mathcal{S}(x).$$

Clearly, the Suppes-Sen axiom implies efficiency. Note that we do not restrict the scope of the axiom to finite permutations (that is, permutations π with the property that there is a $t \in \mathbb{Z}_+$ such that $\pi(\tau) = \tau$ for all $\tau \geq t$). In contrast to the Suppes-Sen axiom formulated for orderings of infinite utility streams, allowing for infinite permutations does not lead to an impossibility in the choice-theoretic setting, given our technological environment. This is established by combining our next result, which characterizes all choice functions satisfying the Suppes-Sen principle, with the fact that, for any initial resource stock, there exists a non-empty set of efficient and non-decreasing streams.

Lemma 4 *An infinite-horizon choice function C satisfies Suppes-Sen if and only if (C1) and*

$$C_t(x) \leq C_{t+1}(x) \quad \textit{for all } x \in \mathbb{R}_+ \textit{ and for all } t \in \mathbb{Z}_+ \tag{C2}$$

are satisfied.

Proof. 'If.' Assume (C1) and (C2) are satisfied. Since the sequence $\langle 1/(1+r)^t\rangle_{t\in\mathbb{Z}_+}$ is decreasing and the sequence $\langle C_t(x)\rangle_{t\in\mathbb{Z}_+}$ is non-decreasing, if y is a permutation of $C(x)$, then

$$\sum_{t=0}^{\infty}\frac{y_t}{(1+r)^t}\geq x.$$

Hence, for all $y,y'\in\mathcal{Y}$ such that y' is a permutation of y, $y'>C(x)$ implies

$$\sum_{t=0}^{\infty}\frac{y_t}{(1+r)^t}>x.$$

By Lemma 1, $y\notin\mathcal{S}(x)$. Thus, C satisfies Suppes-Sen.

'Only if.' Let $x\in\mathbb{R}_+$. Suppose first that $\sum_{t=0}^{\infty}C_t(x)/(1+r)^t<x$. Then by Lemma 1, there exists $y\in\mathcal{S}(x)$ such that $y>C(x)$. Thus, there is an x-feasible consumption stream which Pareto-dominates the chosen consumption stream with initial stock x, entailing that C does not satisfy Suppes-Sen. Together with feasibility, this contradiction implies that we must have $\sum_{t=0}^{\infty}C_t(x)/(1+r)^t=x$. By way of contradiction, suppose there exists $\tau\in\mathbb{Z}_+$ such that $C_\tau(x)>C_{\tau+1}(x)$. Construct $y\in\mathcal{Y}$ as follows:

$$y_t=\begin{cases}C_t(x) & \text{if}\quad t\notin\{\tau,\tau+1\},\\ C_{\tau+1}(x) & \text{if}\quad t=\tau,\\ C_\tau(x)+r(C_\tau(x)-C_{\tau+1}(x)) & \text{if}\quad t=\tau+1.\end{cases}$$

Then

$$\begin{aligned}\sum_{t=0}^{\infty}\frac{y_t}{(1+r)^t}&=\sum_{t\notin\{\tau,\tau+1\}}\frac{C_t(x)}{(1+r)^t}+\frac{1}{(1+r)^\tau}\left(C_{\tau+1}(x)+\frac{C_\tau(x)+r(C_\tau(x)-C_{\tau+1}(x))}{1+r}\right)\\&=\sum_{t=0}^{\infty}\frac{C_t(x)}{(1+r)^t}=x,\end{aligned}$$

implying by Lemma 1 that $y\in\mathcal{S}(x)$. Construct $y'\in\mathcal{Y}$ from y by permuting y_τ and $y_{\tau+1}$. Since $r(C_\tau(x)-C_{\tau+1}(x))>0$, we have that $y'>C(x)$. Thus, there is an x-feasible consumption stream with a permutation which Pareto-dominates the chosen consumption stream with initial stock x, entailing that C does not satisfy Suppes-Sen. ∎

As is apparent from the proof, the Suppes-Sen principle as stated in the lemma can be replaced with its finite counterpart, restricting its conclusion to finite permutations. In our setting, the two properties are equivalent and we chose to use the general version in order to illustrate that, unlike the model based on orderings of infinite streams, our approach does not lead to an impossibility when infinite permutations are permitted.

The observation that the Suppes-Sen axiom can allow for infinite permutations without leading to an impossibility in the choice-theoretic setting is robust with respect to modifications in our technological assumptions. To see this, consider the technological assumptions of *immediate productivity* and *eventual productivity*, as defined by Asheim, Buchholz and Tungodden (2001, p. 259). The assumption of immediate productivity states that if $y \in \mathcal{Y}$ with $y_\tau > y_{\tau+1}$ for some $\tau \in \mathbb{Z}_+$ is feasible, then $y' \in \mathcal{Y}$ constructed by

$$y'_t = \begin{cases} y_t & \text{if} \quad t \notin \{\tau, \tau+1\}, \\ y_{\tau+1} & \text{if} \quad t = \tau, \\ y_\tau & \text{if} \quad t = \tau+1 \end{cases}$$

is feasible and *inefficient*. The assumption of eventual productivity states that, for any initial resource stock(s) and time, there exists an efficient and equally-distributed stream. The class of technologies that satisfy the assumptions of immediate productivity and eventual productivity includes the simple linear and stationary technologies that we consider throughout this paper. However, this class is far wider than this, as illustrated by Asheim, Buchholz and Tungodden (2001, Examples 1–3).

In a technology satisfying eventual productivity, the choice function assigning to any initial resource stock(s) and time the efficient and equally-distributed stream is an efficient, time consistent choice function satisfying even the infinite permutation Suppes-Sen axiom. Hence, provided that the assumption of eventual productivity is satisfied, the Suppes-Sen axiom can allow for infinite permutations without leading to an impossibility in the choice-theoretic setting. If we add immediate productivity, we obtain a generalization of Lemma 4: An infinite-horizon choice function satisfies Suppes-Sen if and only if, for any initial resource stock(s) and time, the chosen stream is efficient and non-decreasing. Also the latter result allows for the version of Suppes-Sen axiom that includes infinite permutations, although it continues to hold if the axiom is replaced by its finite permutation counterpart.

The second of the equity axioms—*Pigou-Dalton*—requires that no x-feasible consumption stream can be generated from the chosen consumption stream with initial stock x through a transfer of consumption from a better-off to a worse-off generation. The axiom is a straightforward adaptation of the Pigou-Dalton transfer principle (cf. Pigou, 1912; Dalton, 1920) for social welfare orderings to the present choice-theoretic setting.

Pigou-Dalton. For all $x \in \mathbb{R}_+$ and for all $y, y' \in \mathcal{Y}$, if there exist $\varepsilon \in \mathbb{R}_{++}$ and $\tau, \tau' \in \mathbb{Z}_+$ such that $y_\tau = y'_\tau - \varepsilon \geq y'_{\tau'} + \varepsilon = y_{\tau'}$ and $y_t = y'_t$ for all $t \in \mathbb{Z}_+ \setminus \{\tau, \tau'\}$, then

$$y' = C(x) \quad \Rightarrow \quad y \notin \mathcal{S}(x).$$

Unlike the Suppes-Sen principle, Pigou-Dalton does not imply efficiency. However, it rules out all violations of efficiency that do not involve equally-distributed streams. As will become clear in the proof of the following theorem, efficiency could therefore be replaced with a weaker axiom that applies to equal distributions only. We chose to keep the standard efficiency axiom for clarity and ease of exposition.

We now characterize all infinite-horizon choice functions satisfying efficiency and the Pigou-Dalton principle. Interestingly, this is the same class as the one identified in the previous lemma.

Lemma 5 *An infinite-horizon choice function C satisfies efficiency and Pigou-Dalton if and only if (C1) and (C2) are satisfied.*

Proof. 'If.' Assume (C1) and (C2) are satisfied. By Lemma 2, C satisfies efficiency. Since the sequence $\langle 1/(1+r)^t \rangle_{t\in\mathbb{Z}_+}$ is decreasing and the sequence $\langle C_t(x)\rangle_{t\in\mathbb{Z}_+}$ is non-decreasing, if $y_\tau = C_\tau(x) - \varepsilon \geq C_{\tau'}(x) + \varepsilon = y_{\tau'}$ for some $\varepsilon \in \mathbb{R}_{++}$ and $y_t = C_t(x)$ for all $t \in \mathbb{Z}_+ \setminus \{\tau, \tau'\}$, then

$$\sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} > x.$$

By Lemma 1, $y \notin \mathcal{S}(x)$. Thus, C satisfies Pigou-Dalton.

'Only if.' Suppose $\sum_{t=0}^{\infty} C_t(x)/(1+r)^t < x$. Then by Lemma 1, there exists $y \in \mathcal{S}(x)$ such that $y > C(x)$. Thus, there is an x-feasible consumption stream which Pareto dominates the chosen consumption stream with initial stock x, entailing that C does not satisfy efficiency. Therefore, using feasibility, we must have $\sum_{t=0}^{\infty} C_t(x)/(1+r)^t = x$.

Now suppose there exists $\tau \in \mathbb{Z}_+$ such that $C_\tau(x) > C_{\tau+1}(x)$. Construct $y \in \mathcal{Y}$ as follows:

$$y_t = \begin{cases} C_t(x) & \text{if} \quad t \notin \{\tau, \tau+1\}, \\ C_\tau(x) - \varepsilon & \text{if} \quad t = \tau, \\ C_{\tau+1}(x) + \varepsilon & \text{if} \quad t = \tau+1, \end{cases}$$

where $0 < \varepsilon \leq (C_\tau(x) - C_{\tau+1}(x))/2$, so that $y_\tau = C_\tau(x) - \varepsilon \geq C_{\tau+1}(x) + \varepsilon = y_{\tau+1}$. Then

$$\begin{aligned} \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} &= \sum_{t\notin\{\tau,\tau+1\}} \frac{C_t(x)}{(1+r)^t} + \frac{1}{(1+r)^\tau}\left(C_\tau(x) - \varepsilon + \frac{C_{\tau+1}(x)+\varepsilon}{1+r}\right) \\ &= \sum_{t=0}^{\infty} \frac{C_t(x)}{(1+r)^t} - \frac{r\varepsilon}{(1+r)^{\tau+1}} < x, \end{aligned}$$

implying by Lemma 1 that $y \in \mathcal{S}(x)$. Thus, an x-feasible consumption stream can be generated from the chosen consumption stream with initial stock x through a transfer of consumption from a better-off to a worse-off generation, entailing that C does not satisfy Pigou-Dalton. ■

The following corollary is an immediate consequence of the previous two lemmas.

Corollary 1 *An infinite-horizon choice function C satisfies Suppes-Sen if and only if C satisfies efficiency and Pigou-Dalton.*

The following theorem identifies all choice functions satisfying time consistency in addition to Suppes-Sen (or, equivalently, in addition to efficiency and Pigou-Dalton).

Theorem 2 *An infinite-horizon choice function C satisfies time consistency and Suppes-Sen (or efficiency, time consistency and Pigou-Dalton) if and only if there exists a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that (CG), (G1), (G2),*

$$x \leq g(x) \quad \textit{for all } x \in \mathbb{R}_+ \tag{G3}$$

and

$$x - \frac{g(x)}{1+r} \leq g(x) - \frac{g^2(x)}{1+r} \quad \textit{for all } x \in \mathbb{R}_+ \tag{G4}$$

are satisfied.

Proof. 'If.' Suppose there exists a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that (CG), (G1), (G2), (G3) and (G4) are satisfied. By Theorem 1, C satisfies time consistency and efficiency. Thus, by Lemma 2, (C1) is satisfied. By (CG) and (G4), it follows that

$$\begin{aligned} C_t(x) &= g^t(x) - \frac{g^{t+1}(x)}{1+r} = g^t(x) - \frac{g(g^t(x))}{1+r} \\ &\leq g(g^t(x)) - \frac{g^2(g^t(x))}{1+r} = g^{t+1}(x) - \frac{g^{t+2}(x)}{1+r} = C_{t+1}(x) \end{aligned}$$

for all $x \in \mathbb{R}_+$ and for all $t \in \mathbb{Z}_+$. Hence, by Lemma 4, C satisfies Suppes-Sen.

'Only if.' Assume that C satisfies time consistency and Suppes-Sen. By Lemma 4, (C1) and (C2) are satisfied and, by Lemma 2, C satisfies efficiency. By Theorem 1, there exists a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ satisfying (CG), (G1) and (G2).

To show (G3), suppose there exists $x \in \mathbb{R}_+$ such that $x > g(x)$. By (CG) and (G2), it follows that

$$\sum_{t=0}^{\infty} \frac{C_t(x)}{(1+r)^t} = x > g(x) = \sum_{t=0}^{\infty} \frac{C_{t+1}(x)}{(1+r)^t},$$

contradicting (C2).

To show (G4), suppose there exists $x \in \mathbb{R}_+$ such that

$$x - \frac{g(x)}{1+r} > g(x) - \frac{g^2(x)}{1+r}.$$

By (CG),

$$C_0(x) \;=\; x - \frac{g(x)}{1+r} \;>\; g(x) - \frac{g^2(x)}{1+r} \;=\; C_1(x),$$

again contradicting (C2). ■

Condition (G3) ensures sustainable development in the sense that the current consumption can potentially be shared by all future generations. In the context of a stationary technology with only one resource (or capital good), this requires that the resource stock is maintained from the current period to the next, which is just what condition (G3) entails. Condition (G4) complements (G3) by requiring that the potential for sharing present consumption with future generations actually materializes. Hence, Theorem 2 means that both the Suppes-Sen axiom and the Pigou-Dalton axiom can be used to justify sustainability in the present choice-theoretic setting.

Theorem 2 thereby echoes similar results when infinite-horizon social choice is analyzed through social welfare relations.

- In particular, Asheim, Buchholz and Tungodden (2001) show how the Suppes-Sen principle for social welfare relations can be used to rule out unsustainable consumption streams as maximal elements under technological conditions satisfied by the simple linear model considered here. Given such technological assumptions, this result also implies that social welfare relations like those considered in Asheim and Tungodden (2004), Basu and Mitra (2006), and Bossert, Sprumont and Suzumura (2006), which all satisfy the Suppes-Sen principle, yield sustainable consumption streams as maximal elements as long as maximal elements exist.

- Asheim (1991) shows in a similar way how the Pigou-Dalton principle for social welfare relations can be used to rule out unsustainable consumption streams.

Another equity axiom that appears to be natural in this context is *resource monotonicity*. It requires that no one should be worse off as a consequence of an increase in the initial level of the resource. See Thomson (2006) for a discussion of resource monotonicity in a variety of economic models and further references. Formulated for infinite-horizon choice functions, the axiom is defined as follows.

Resource monotonicity. For all $x, x' \in \mathbb{R}_+$,

$$x > x' \;\Rightarrow\; C(x) \geq C(x').$$

Adding resource monotonicity to efficiency and time consistency leads to the choice functions characterized in the following theorem.

Theorem 3 *An infinite-horizon choice function C satisfies efficiency, time consistency and resource monotonicity if and only if there exists a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that (CG), (G1), (G2),*

$$g \quad \text{is non-decreasing in } x \tag{G5}$$

and

$$x \mapsto x - \frac{g(x)}{1+r} \quad \text{is non-decreasing in } x \tag{G6}$$

are satisfied.

Proof. 'If.' Assume that there exists a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that (CG), (G1), (G2), (G5) and (G6) are satisfied. By Theorem 1, C satisfies efficiency and time consistency. Let $x > x'$. By (G5), we have that

$$g^t(x) \geq g^t(x')$$

for all $t \in \mathbb{Z}_+$. Consequently, since (CG) and (G6) are satisfied, it follows that

$$\begin{aligned} C_t(x) \;&=\; g^t(x) - \frac{g^{t+1}(x)}{1+r} \;=\; g^t(x) - \frac{g(g^t(x))}{1+r} \\ &\geq\; g^t(x') - \frac{g(g^t(x'))}{1+r} \;=\; g^t(x') - \frac{g^{t+1}(x')}{1+r} \;=\; C_t(x') \end{aligned}$$

for all $t \in \mathbb{Z}_+$. Hence, C satisfies resource monotonicity.

'Only if.' Assume that C satisfies time consistency, efficiency and resource monotonicity. By Theorem 1, there exists a function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that (CG), (G1) and (G2) are satisfied.

To show (G5), suppose there exist $x, x' \in \mathbb{R}_+$ such that $x > x'$, but $g(x) < g(x')$. By (CG) and (G2), it follows that

$$\sum_{t=1}^{\infty} \frac{C_t(x)}{(1+r)^{t-1}} \;=\; g(x) \;<\; g(x') \;=\; \sum_{t=1}^{\infty} \frac{C_t(x')}{(1+r)^{t-1}},$$

contradicting resource monotonicity.

To show (G6), suppose there exist $x, x' \in \mathbb{R}_+$ such that $x > x'$, but

$$x - \frac{g(x)}{1+r} < x' - \frac{g(x')}{1+r}.$$

By (CG),

$$C_0(x) \;=\; x - \frac{g(x)}{1+r} \;<\; x' - \frac{g(x')}{1+r} \;=\; C_0(x'),$$

again contradicting resource monotonicity. ■

Note that the proof of (G5) relies on efficiency, whereas (G6) is established without using this axiom.

It follows from Theorems 2 and 3 that the classes of choice functions characterized in Theorem 1 can be narrowed down considerably by adding equity axioms. However, Suppes-Sen or Pigou-Dalton, on the one hand, and resource monotonicity, on the other hand, do so in different ways.

- By Theorem 2, Suppes-Sen or efficiency and Pigou-Dalton in combination with time consistency imply that, for given $x \in \mathbb{R}_+$, $g^t(x)$ and $g^t(x) - (g^{t+1}(x)/(1+r))$ are monotone with respect to t, while

- by Theorem 3, resource monotonicity in combination with efficiency and time consistency implies that $g^t(x)$ and $g^t(x) - (g^{t+1}(x)/(1+r))$ are monotone with respect to x for given $t \in \mathbb{Z}_+$.

5 Examples

To ensure that the choice functions in the examples of this section are well-defined it is important that the renewal rate r is positive, as we have assumed throughout. Consider first the *steady-state* example, where consumption is equalized across generations.

Example 1. The infinite-horizon choice function C^1 of this example corresponds to the case in which the function g is the identity mapping, defined by $g(x) = x$ for all $x \in \mathbb{R}_+$. This implies $g^t(x) = x$ for all $x \in \mathbb{R}_+$ and for all $t \in \mathbb{Z}_+$. (G1) and (G2) are satisfied because

$$g(x) = x \leq x(1+r)$$

and

$$\lim_{t\to\infty} \frac{g^t(x)}{(1+r)^t} = \lim_{t\to\infty} \frac{x}{(1+r)^t} = 0$$

for all $x \in \mathbb{R}_+$. According to (CG),

$$C_t^1(x) = g^t(x) - \frac{g^{t+1}(x)}{1+r} = x - \frac{x}{1+r} = \frac{xr}{1+r} \tag{6}$$

for all $x \in \mathbb{R}_+$ and for all $t \in \mathbb{Z}_+$, that is, every generation consumes the same amount.

In addition to satisfying time consistency and efficiency, the infinite-horizon choice function C^1 is characterized by a g-function for which the conditions of (G3) and (G4) hold with equality. By Theorem 2 this entails that C^1 satisfies both Suppes-Sen and Pigou-Dalton. Furthermore, both $g(x)$ and $x - (g(x)/(1+r))$ are non-decreasing in x. Hence, by Theorem 3, the choice function satisfies resource monotonicity, as can easily be verified directly from (6).

A generalization of the choice function C^1 of Example 1 is obtained by letting g be a linear function such that both $g(x)$ and $x - (g(x)/(1+r))$ are non-decreasing in x, so that resource monotonicity is satisfied.

Example 2. The infinite-horizon choice function $C^{2,a}$ of this example is obtained by letting $g(x) = ax$ for all $x \in \mathbb{R}_+$, where $a \in [0, 1+r]$ is a parameter. Obviously, the steady-state case is obtained for $a = 1$. It follows that $g^t(x) = a^t x$ for all $x \in \mathbb{R}_+$ and for all $t \in \mathbb{Z}_+$. Clearly, (G1) is satisfied because

$$g(x) = ax \leq x \leq x(1+r)$$

for all $x \in \mathbb{R}_+$. (G2) is satisfied if and only if $a < 1 + r$ because

$$\lim_{t\to\infty} \frac{g^t(x)}{(1+r)^t} = \lim_{t\to\infty} \frac{a^t x}{(1+r)^t} = \lim_{t\to\infty} \left(\frac{a}{1+r}\right)^t x = 0.$$

Hence, the case where $a = 1 + r$ illustrates how (G2) can be violated by excessive accumulation of the resource.

Substituting into (CG), it follows that

$$C_t^{2,a}(x) = g^t(x) - \frac{g^{t+1}(x)}{1+r} = a^t x - \frac{a^{t+1}x}{1+r} = \frac{a^t(1+r-a)x}{1+r} \tag{7}$$

for all $x \in \mathbb{R}_+$ and for all $t \in \mathbb{Z}_+$.

In addition to satisfying efficiency and time consistency for $a < 1 + r$, the infinite-horizon choice function $C^{2,a}$ is characterized by a g-function for which the conditions of (G3) and (G4) hold if and only if $a \geq 1$. By Theorem 2 this entails that $C^{2,a}$ satisfies efficiency, time consistency, Suppes-Sen and Pigou-Dalton if and only if $a \in [1, 1+r)$. If

$a \in (1, 1+r)$, then consumption is increasing in t, and the consumption of generations t such that

$$t > \frac{\ln(r) - \ln(1+r-a)}{\ln(a)}$$

is higher than that of the steady-state, at the expense of earlier generations. Moreover, the consumption of generation t approaches infinity as t approaches infinity.

Both $g(x)$ and $x - (g(x)/(1+r))$ are non-decreasing in x for any $a \in [0, 1+r]$. Hence, by Theorem 3, the choice function satisfies time consistency, efficiency and resource monotonicity if and only if $a \in [0, 1+r)$, as can easily be verified directly from (7). Therefore, $C^{2,a}$ satisfies resource monotonicity, but not Suppes-Sen and Pigou-Dalton, if and only if $a \in [0, 1)$. If $a \in (0, 1)$, then consumption is decreasing in t, and the consumption of generations t such that

$$t < \frac{\ln(r) - \ln(1+r-a)}{\ln(a)}$$

is higher than that of the steady-state, at the expense of later generations. Moreover, the consumption of generation t approaches zero as t approaches infinity.

Example 2 shows, in the case where $a < 1$, that $g^t(x)$ and $g^t(x) - (g^{t+1}(x)/(1+r))$ can be non-decreasing with respect to x, without $g^t(x)$ and $g^t(x) - (g^{t+1}(x)/(1+r))$ being non-decreasing with respect to t. In particular, a choice function can satisfy resource monotonicity without satisfying Suppes-Sen and Pigou-Dalton. In the following pair of examples, we show that a choice function can satisfy Suppes-Sen and Pigou-Dalton without satisfying resource monotonicity.

Example 3. The infinite-horizon choice function C^3 of this example is obtained by setting $r = 1$, so that $1 + r = 2$, and by letting g be given by:

$$g(x) = \begin{cases} \frac{3}{2}x & \text{if} \quad 0 \leq x \leq 1, \\ \frac{4}{3}x & \text{if} \quad x > 1\,. \end{cases}$$

Clearly, (G1) is satisfied. Also, $x \leq g(x)$ for all $x \in \mathbb{R}_+$ so that (G3) is satisfied, and $x - g(x)/2$ is an increasing function of x so that (G6) is satisfied. By combining these observations we obtain that $x - g(x)/2 \leq g(x) - g^2(x)/2$ for all $x \in \mathbb{R}_+$ so that (G4) is satisfied. Furthermore, if $x \in \mathbb{R}_{++}$, then C^3 behaves as $C^{2,a}$ with $a \in (0, 1+r)$ when t goes to infinity, implying that (G2) is satisfied. If $x = 0$, then (G2) is trivially satisfied. Hence, it follows from Theorem 2 that the infinite-horizon choice function C^3 satisfies efficiency, time consistency, Suppes-Sen and Pigou-Dalton. However,

$$g(1) \;=\; \tfrac{3}{2} \;>\; \tfrac{17}{12} \;=\; g\big(\tfrac{17}{16}\big).$$

Hence, (G5) does not hold, and it follows from Theorem 3 that C^3 does not satisfy resource monotonicity.

Example 4. The infinite-horizon choice function C^4 of this example is obtained by setting $r = 1$, so that $1 + r = 2$, and by letting g be given by:

$$g(x) \;=\; \begin{cases} \tfrac{4}{3}x & \text{if} \quad 0 \le x \le 1, \\ \tfrac{3}{2}x & \text{if} \quad x > 1. \end{cases}$$

Clearly, (G1) is satisfied. Also, $x \le g(x)$ for all $x \in \mathbb{R}_+$ so that (G3) is satisfied, and $g(x)$ is an increasing function of x so that (G5) is satisfied. Furthermore, if $x \in \mathbb{R}_{++}$, then C^4 behaves as $C^{2,a}$ with $a \in (0, 1 + r)$ when t goes to infinity, implying that (G2) is satisfied. If $x = 0$, then (G2) is trivially satisfied. To verify that (G3) is satisfied, note that

$$\begin{aligned}
x - \tfrac{g(x)}{2} \;&=\; \big(1 - \tfrac{2}{3}\big)x \;=\; \tfrac{1}{3}x \;\le\; \tfrac{4}{9}x \;=\; \big(\tfrac{4}{3} - \tfrac{8}{9}\big)x \;=\; g(x) - \tfrac{g^2(x)}{2} && \text{if } 0 \le x \le \tfrac{3}{4}, \\
x - \tfrac{g(x)}{2} \;&=\; \big(1 - \tfrac{2}{3}\big)x \;=\; \tfrac{1}{3}x \;=\; \tfrac{1}{3}x \;=\; \big(\tfrac{4}{3} - 1\big)x \;=\; g(x) - \tfrac{g^2(x)}{2} && \text{if } \tfrac{4}{3} < x \le 1, \\
x - \tfrac{g(x)}{2} \;&=\; \big(1 - \tfrac{3}{4}\big)x \;=\; \tfrac{1}{4}x \;\le\; \tfrac{3}{8}x \;=\; \big(\tfrac{3}{2} - \tfrac{9}{8}\big)x \;=\; g(x) - \tfrac{g^2(x)}{2} && \text{if } x > 1.
\end{aligned}$$

Hence, it follows from Theorem 2 that the infinite-horizon choice function C^4 satisfies efficiency, time consistency, Suppes-Sen and Pigou-Dalton. However,

$$1 - \tfrac{g(1)}{2} \;=\; 1 - \tfrac{2}{3} \;=\; \tfrac{1}{3} \;>\; \tfrac{5}{18} \;=\; \tfrac{10}{9} - \tfrac{5}{6} \;=\; \tfrac{10}{9} - \tfrac{g(10/9)}{2}.$$

Hence, (G6) does not hold, and it follows from Theorem 3 that C^4 does not satisfy resource monotonicity.

Examples 2, 3 and 4 show that the conditions characterizing Suppes-Sen and Pigou-Dalton—namely that $g^t(x)$ and $g^t(x) - g^{t+1}(x)/(1 + r)$ are monotone with respect to t—are independent of the conditions characterizing resource monotonicity—namely that $g^t(x)$ and $g^t(x) - g^{t+1}(x)/(1 + r)$ are monotone with respect to x.

We conclude with an example showing that condition (G5) is not necessary for an infinite-horizon choice function to satisfy time consistency and resource monotonicity, as long as efficiency is not imposed.

Example 5. The infinite-horizon choice function C^5 of this example is obtained by setting $r = 1$, so that $1 + r = 2$, and by letting g be given by:

$$g(x) \;=\; \begin{cases} 2x & \text{if} \quad 0 \leq x \leq 1, \\ 2(x - \frac{1}{2}) & \text{if} \quad x > 1. \end{cases}$$

Clearly (G1) is satisfied, while condition (G5) is not satisfied, since

$$g(1) \;=\; 2 \;>\; \tfrac{3}{2} \;=\; g\big(\tfrac{5}{4}\big).$$

Resource monotonicity still holds since, by substituting into (CG), it follows that

$$C^5(x) \;=\; \begin{cases} (0, 0, \dots) & \text{if} \quad x = 0, \\ \big(\underbrace{0, \dots, 0}_{n+1 \text{ times}}, \frac{1}{2}, \frac{1}{2}, \dots\big) & \text{if} \quad x \in \big((\frac{1}{2})^{n+1}, (\frac{1}{2})^n\big] \quad \text{for } n \in \mathbb{Z}_+, \\ \big(\frac{1}{2}, \frac{1}{2}, \dots\big) & \text{if} \quad x > 1. \end{cases}$$

It is straightforward to verify that C^5 does not satisfy efficiency; in particular, increasing the initial resource stock beyond x does not lead to increased consumption for any generation, provided that $x > 1$.

Examples 1 and 2 provide infinite-horizon choice functions that are continuous in the initial endowment, even though there are no continuous orderings satisfying strong Pareto and finite anonymity that rationalize them. This observation serves to further underline the gains that are possible from adopting a choice-theoretic approach.

6 Concluding remarks

We conclude this paper with some thoughts on possible directions where the approach of this paper might be taken in future work. An issue that suggests itself naturally when considering a choice function is its *rationalizability* by a relation defined on the objects of choice—in our case, infinite consumption streams. The rationalizability of choice functions with arbitrary domains has been examined thoroughly in contributions such as Richter (1966) and Hansson (1968) and, more recently, Bossert, Sprumont and Suzumura (2005) and Bossert and Suzumura (2005). While the generality of the results obtained in these papers allows for their application in our intergenerational setting, it might be possible to obtain new observations due to the specific structure of the domain considered here. Note that the existence of a rationalizing ordering does *not* conflict with the impossibility results established for such orderings in the earlier literature: the existence of a rationalization of an infinite-horizon choice function satisfying requirements such as Suppes-Sen does not imply that the choice function is rationalizable by an ordering that possesses properties such as the Suppes-Sen principle formulated for binary relations.

An interesting difference emerges when the technology parameter r is equal to zero instead of positive, as we have assumed throughout the paper. In that case, Suppes-Sen and the conjunction of efficiency and Pigou-Dalton no longer are equivalent—in fact, their implications are strikingly different. If $r = 0$, then the Pigou-Dalton principle rules out the choice of any unequal stream. Thereby the principle becomes incompatible with efficiency because, for any finite initial endowment x, the only possible equal choice is zero consumption in every period, which clearly violates efficiency if x is positive. On the other hand, Suppes-Sen reduces to efficiency because no stream that is not dominated according to the efficiency criterion is dominated by a permutation of any feasible stream.

As mentioned earlier, we made the conscious choice to work with a simple model in order to emphasize the novel aspect of the paper—the choice-theoretic approach in an infinite-horizon setting. It might turn out to be of interest to explore possible generalizations in future work.

References

Asheim, G.B. (1991), Unjust intergenerational allocations. *Journal of Economic Theory* **54**, 350–371.

Asheim, G.B., Buchholz, W., and Tungodden, B. (2001), Justifying sustainability. *Journal of Environmental Economics and Management* **41**, 252–268.

Asheim, G.B., Mitra, T., and Tungodden, B. (2006), A new equity condition for infinite utility streams and the possibility of being Paretian. In: J. Roemer and K. Suzumura (eds.), *Intergenerational Equity and Sustainability*, Palgrave, Amsterdam, forthcoming.

Asheim, G.B. and Tungodden, B. (2004), Resolving distributional conflicts between generations. *Economic Theory* **24**, 221–230.

Basu, K. and Mitra, T. (2003), Aggregating infinite utility streams with intergenerational equity: the impossibility of being Paretian. *Econometrica* **71**, 1557–1563.

Basu, K. and Mitra, T. (2006), Utilitarianism for infinite utility streams: A new welfare criterion and its axiomatic characterization. *Journal of Economic Theory,* forthcoming.

Bossert, W., Sprumont, Y., and Suzumura, K. (2005), Consistent rationalizability. *Economica* **72**, 185–200.

Bossert, W., Sprumont, Y., and Suzumura, K. (2006), Ordering infinite utility streams. *Journal of Economic Theory,* forthcoming.

Bossert, W. and Suzumura, K. (2005), Rational choice on general domains. Mimeo, Institute of Economic Research, Hitotsubashi University.

Dalton, H. (1920), The measurement of the inequality of incomes. *Economic Journal* **30**, 348–361.

Diamond, P. (1965), The evaluation of infinite utility streams. *Econometrica* **33**, 170–177.

Epstein, L.G. (1986), Intergenerational preference orderings. *Social Choice and Welfare* **3**, 151–160.

Hansson, B. (1968), Choice structures and preference relations. *Synthese* **18**, 443–458.

Hara, C., Shinotsuka, T., Suzumura, K., and Xu, Y. (2005), On the possibility of continuous, Paretian and egalitarian evaluation of infinite utility streams. Working Paper, Project on Intergenerational Equity, Institute of Economic Research, Hitotsubashi University.

Koopmans, T.C. (1960), Stationary ordinal utility and impatience. *Econometrica* **28**, 287–309.

Malinvaud, E. (1953), Capital accumulation and efficient allocation of resources. *Econometrica* **21**, 233–268.

Pigou, A. (1912), *Wealth and Welfare.* Macmillan, London.

Richter, M.K. (1966), Revealed preference theory. *Econometrica* **34**, 635–645.

Sen, A.K. (1970), *Collective Choice and Social Welfare.* Oliver and Boyd, Edinburgh.

Suppes, P. (1966), Some formal models of grading principles. *Synthese* **6**, 284–306.

Thomson, W. (2006), Fair allocation rules. In: K. Arrow, A. Sen and K. Suzumura (eds.), *Handbook of Social Choice and Welfare, Vol. 2,* Elsevier, Amsterdam, forthcoming.