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Farsight and myopia in a transboundary pollution game*

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Abstract

We study the impact of farsightedness in a transboundary pollution game; i.e. the ability of a country to forecast the relationship between current emissions and future levels of pollution and thus on future damages. We show that when all countries are farsighted their payoffs are larger than when all countries are myopic. However in the case where one myopic country becomes farsighted we show that the welfare impact of farsightedness on that country is ambiguous. Farsightedness may be welfare reducing for the country that acquires it. This is due to the reaction of the other farsighted countries to that country's acquisition of farsight. The country that acquires farsight reduces its emissions while the other farsighted countries extend their emissions. The overall impact on total emissions is ambiguous.

JEL Classification: C73, D90, Q59.
Keywords: myopia, differential games, transboundary pollution.

1 Introduction

In this paper we investigate the impact of myopia in a transboundary pollution game. It is well known that in a transboundary pollution game where emissions that are a by-product of product production and that accumulate into a harmful stock pollutant, the non-cooperative equilibrium typically results in an over-polluted environment. Countries typically ignore the externality imposed on each other. An important feature of transboundary pollution games, is that pollution emissions accumulate and therefore the action at any given moment has a lasting

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impact on the environment. In this paper we seek to study the impact of the ability to forecast the impact of current emissions on future levels of pollution and thus on future damages.

We consider two types of behavior: (i) a country can be myopic and ignores the impact of its current emissions on future emissions and (ii) a country can be farsighted and there can take into account the impact of its emissions on the pollution stock.

In the case of a single decision maker, the impact of farsight on the decision maker's welfare is clearly positive. Indeed, a farsighted country can still choose the path chosen under myopia and therefore the acquisition of farsight will typically allow the country to attain a higher level of utility.

In the case of several decision makers the impact of the acquisition is turns out to be ambiguous. It is in principle possible for a farsighted country to pick the same pollution path she would have chosen under myopia, however it is not necessarily true that the path chosen under myopia constitutes a best-response to the vector of strategies played by the other countries. The acquisition of farsight by a country typically induces that country to reduce its emissions compared to the case where it was myopic. The other countries respond to this reduction in emissions by increasing their emissions. In a multiplayer setup the reaction of other players (countries) is important and as it turns out can negate a goodwill gesture of a myopic country adopting a farsighted behavior (and reducing its emissions compared to the case where it was myopic). The sum of all countries' emissions may increase if one myopic player (country) became farsighted. This result is quite surprising, since myopia is generally associated with careless management of the environment and therefore one would assume that environmental quality unambiguously improves when a myopic country becomes farsighted. For a given total number of countries, the quality of the environment is not a monotonic function of the number of myopic countries. This is true for the short run only; we show that the acquisition of farsight by a country results in a decrease of the steady-state level of pollution. Thus in the long run farsight results in a better quality of the environment. We also examine the change in welfare from the acquisition of farsight and show that in a transboundary pollution game, contrary to a single decision maker problem, it can be negative. This can happen when the value of the damage parameter or when the stock of pollution is large enough. This is a rather pessimistic result since it is precisely in circumstances where pollution causes severe damage or when the stock of pollution is large that one would like all the countries to be farsighted and reduce their emissions. Numerical simulations reveal that, starting from an initial stock of pollution such that the present value of the gains from farsight is zero, the instantaneous welfare path of a myopic country crosses from below the path of instantaneous welfare it would enjoy if it were farsighted. Therefore, the change in instantaneous welfare from the acquisition of farsight is initially negative before turning positive. This suggests that incentive mechanisms, that involve a very small (possibly zero) present of value of transfers, can play an important role in inducing a country to adopt a farsighted behavior and diminishing the number of myopic countries. These incentives would compensate the myopic country for the short-run losses incurred from the acquisition of farsight and can be reimbursed by the country from the gains from farsight that it enjoys in the long run.

To the best of our knowledge this work is the first study of the impact of myopia on the outcome of a pollution game. It was inspired by related studies on the impact of myopia in the area of management and marketing. The issue of myopic pricing has been addressed first in the study of situations where there is a single decision maker. In a model where costs evolve through the effect of learning and demand described by a diffusion process, Robinson and Lakhani (1975) compared the outcomes of static models to their dynamic counterparts. The authors concluded that dynamic strategies are more profitable than myopic ones. Fibich et al. (2003) examined also the issue of dynamic vs. static pricing in dynamic monopolies and oligopolies under price stickiness (à la Ferhstman and Kamien (1987)) where sales could be affected by reference prices. Taboubi and Zaccour (2002) compared price and marketing efforts, at the equilibrium under the scenarios of a myopic and a non-myopic retailer selling the product of a monopolist. They concluded that under a myopic behavior, the retailer reduces the retail price, but exerts less marketing efforts than if she were farsighted. Martín-Herrán et al. (2008) studied competitive emailing by firms where firms can choose to be myopic (by acting as spammers) or farsighted (by avoiding to send spams). Benchekroun et al. (2009) identified the conditions under which a myopic pricing behavior could be a profit enhancing tool in two distribution channels: a bilateral monopoly and a channel with competition at the manufacturing level.

In this paper we investigate the impact of myopia and farsightedness in a pollution game à la Dockner and Long (1993) or Ploeg and Zeeuw (1992). The next section presents the model and gives the Markov-Perfect equilibrium of the differential game where a subset of countries is myopic. The comparison of the case where all countries are myopic to the case where all are farsighted is given in section 3 and section 4 gives the impact on the equilibrium outcomes of having one country changing from a myopic behavior to a farsighted behavior. Our results are summarized in section 5.

2 Model

Consider $N + M$ countries indexed by $l = 1, .., N + M$. Each country produces a single consumption good, the production of which generates emission of a pollutant. The preferences of consumers and the emission-consumption trade-off functions are such that the instantaneous benefits of country l from $E_l \geq 0$, the emission rates of country l, is $AE_l - \frac{1}{2}E_l^2$. The objective of country l is to maximize the discounted sum of utility net of the environmental damage caused by the accumulated stock of pollution, P,

$$\max_{E_l(t)} \int_0^\infty [AE_l(t) - \frac{1}{2}E_l^2(t) - \frac{s}{2}P^2(t)]e^{-rt}\,dt$$

and where $s > 0$ is a damage parameter and $r > 0$ is the discount rate. The stock of pollution P accumulates according to

$$\dot{P}(t) = \sum_{l=1}^{N+M} E_l(t) - kP(t), \quad P(0) = P_0 \geq 0, \tag{1}$$

where $k > 0$ denotes the natural rate of decay.

We consider the case where M countries, indexed by $j = 1,..,M$, are myopic, i.e. when maximizing their utility they are not able to take into account their impact on the stock of pollution, they ignore the accumulation function $\dot{P}$ given by (1). The other N countries, indexed by $i = 1,..,N$, are farsighted, i.e. they take into account (1) when maximizing their utility.

The following vector of strategies where a myopic country chooses:

$$E_j^{M,N}(P) = A$$

and the non-myopic country chooses

$$E_i^{M,N}(P) = \begin{cases} A - \beta_i^{M,N} - \alpha_i^{M,N}P & \text{for} \quad P \leq P_{M,N} \\ 0 & \text{for} \quad P \geq P_{M,N} \end{cases} \tag{2}$$

where

$$\begin{aligned} \alpha_i^{M,N} &= \frac{1}{2(1-2N)}\left(2k + r - \sqrt{(2k+r)^2 + 4s(2N-1)}\right), \\ \beta_i^{M,N} &= \frac{A\alpha_i^{M,N}(M+N)}{k + r + \alpha_i^{M,N}(2N-1)}, \\ \mu_i^{M,N} &= -\frac{A^2 - 2A\beta_i^{M,N}(M+N) + (2N-1)(\beta_i^{M,N})^2}{2r} \end{aligned}$$

and

$$P_{M,N} = \frac{A - \beta_i^{M,N}}{\alpha_i^{M,N}},$$

constitutes a Markov-Perfect Nash equilibrium of the pollution game. This is a straightforward generalization of Dockner and Long (1993) to the case of several players (countries) and where a subset of players are myopic.

A farsighted country i's value function is then

$$V_i^{M,N}(P) = -\frac{1}{2}\alpha_i^{M,N}P^2 - \beta_i^{M,N}P - \mu_i^{M,N},$$

and a myopic country j's value function:

$$V_j^{M,N}(P) = -\frac{1}{2}\alpha_j^{M,N}P^2 - \beta_j^{M,N}P - \mu_j^{M,N}.$$

There exists an asymptotically stable steady-state pollution stock given by:

$$\begin{aligned} P_{ss}^{M,N} &= \frac{A(M+N) - N\beta_i^{M,N}}{k + N\alpha_i^{M,N}} \\ &= \frac{A(M+N)(k+r+\alpha_i^{M,N}(N-1))}{(k+r+\alpha_i^{M,N}(2N-1))(k+\alpha_i^{M,N}N)}. \end{aligned}$$

Note that $\alpha_i^{M,N}$ is positive and therefore $\beta_i^{M,N}$ and $P_{ss}^{M,N}$are positive too.

Some preliminary remarks are worth making. We show in Appendix A that

$$\left(\beta_i^{M-1,N+1} - \beta_i^{M,N}\right) < 0 \text{ and } \left(\alpha_i^{M-1,N+1} - \alpha_i^{M,N}\right) < 0, \tag{3}$$

therefore

$$P_{M,N} < P_{M-1,N+1}. \tag{4}$$

Moreover it can be shown that $P_{M,N}$, the threshold value of the stock of pollution beyond which a farsighted country ceases production, can take negative values. This is illustrated through the case where $M = N = 1$ for which we have

$$P_{11} > 0 \Leftrightarrow s < \underline{s} \equiv (3k+r)(k+r).$$

When $s \geq \underline{s}$, in the case where $M = N = 1$, the equilibrium emission strategies are given by $E_i^{1,1}(P) = 0$ and $E_j^{1,1}(P) = A$. When the damage parameter is large enough the farsighted country emits zero for all stocks of pollution. This does not happen if the myopic country becomes farsighted, i.e. when $M = 0$ and $N = 2$. In that case, since both are farsighted, both restrict their emissions as the stock increases, leaving for any value of s an interval of the stock of pollution where emissions are positive. This is true when $M = 0$ for all $N > 0$: we have $P_{0,N} > 0$. In the case where $M > 1$ a myopic country always emits at level A. When s is large enough the accumulation of the stock at rate A renders any marginal emissions too costly for the farsighted country.

3 Farsight vs. Myopia

In this section we compare countries' payoffs under two scenarios: (i) all countries are farsighted and (ii) all countries are myopic. More precisely, we compare the outcomes of the case where $(M,N) = (N,0)$ to the case where $(M,N) = (0,N)$.

$$\Delta_0(P_0) \equiv V_i^{0,N}(P_0) - V_j^{N,0}(P_0) = \frac{2(X_1P_0^2 + X_2P_0 + X_3)}{(2N-1)(2k+r)(k+r)(r+\Gamma)^2},$$

for all $P_0 > 0$, where:

$$\begin{aligned}
X_1 &= r(k+r)\left[k^2(2k+r)^2 + 2s(2N-1)(3k^2+kr+(2N-1)s)\right.\\
&\quad \left.-k(k(2k+r)+2(2N-1)s)\Gamma\right],\\
X_2 &= 2ANr\left[-k(k+r)(2k+r)^2 - (2N-1)s(2k^2+4kr+r^2-2(2N-1)s\right.\\
&\quad \left.+(k(k+r)(2k+r)+(2N-1)rs)\Gamma\right],\\
X_3 &= A^2N^2\left[r(k+r)(2k+r)^2 - 2s(2N-1)(kr-2(2N-1)s)\right.\\
&\quad \left.-r((k+r)(2k+r)-2(2N-1)s)\Gamma\right],
\end{aligned}$$

with

$$\Gamma = \sqrt{(2k+r)^2+4s(2N-1)}.$$

It can be proved that the discriminant of the quadratic polynomial $X_1P_0^2 + X_2P_0 + X_3$ is always negative, and hence, the sign of this polynomial coincides with the sign of its independent term, X_3. Tedious but straightforward calculations have shown that X_3 is always positive, and therefore,

$$\Delta_0(P_0) > 0.$$

4 Unilateral acquisition of farsight

Suppose we have M myopic countries and N farsighted countries and that one myopic country, say player m, acquires farsight. We determine the impact of this acquisition of farsight on all countries' emission strategies, and on country m's welfare.

4.1 Impact on emissions

4.1.1 Individual countries' emissions

Country m's emission strategy changes from A to $E_i^{M-1,N+1}(P)$ with $A - E_i^{M-1,N+1}(P) > 0$ for all $P \geq 0$. Moreover since $\left(E_i^{M-1,N+1}\right)'(P) < 0$, the larger the stock of pollution the larger the reduction of emissions of country m.

While the emission strategy of the remaining $M-1$ myopic countries is unchanged, clearly the equilibrium emission strategies of the N farsighted countries, change from $E_i^{M,N}(P)$ to $E_i^{M-1,N+1}(P)$.

$$E_i^{M,N}(P) - E_i^{M-1,N+1}(P) = \left(\beta_i^{M-1,N+1} - \beta_i^{M,N}\right) + \left(\alpha_i^{M-1,N+1} - \alpha_i^{M,N}\right)P.$$

From (3) and (4) we have

$$E_i^{M,N}(P) - E_i^{M-1,N+1}(P) < 0$$

for all $P < P_{M-1,N+1}$ and $E_i^{M,N}(P) - E_i^{M-1,N+1}(P) = 0$ for $P > P_{M-1,N+1}$.

Thus, farsighted countries react by increasing their emissions. The larger the stock of pollution, the larger the increase in emissions.

4.1.2 Total emissions

The impact on total emissions is not a priori clear since country m reduces its emissions while the farsighted countries increase their emissions.

We determine the impact of country m's acquisition of farsight on total emissions. Let $E_W^{M,N}(P)$ denote total emissions when there are M myopic countries, N farsighted countries and when the stock of pollution is P. We have:

$$E_W^{M,N}(P) = MA + NE_i^{M,N}(P)$$

and

$$\begin{aligned} E_W^{M,N}(P) - E_W^{M-1,N+1}(P) &= \left(MA + NE_i^{M,N}(P)\right) \\ &- \left((M-1)A + (N+1)E_i^{M-1,N+1}(P)\right) \end{aligned}$$

that is,

$$E_W^{M,N}(P) - E_W^{M-1,N+1}(P) = A - E_i^{M-1,N+1}(P) + N\left(E_i^{M,N}(P) - E_i^{M-1,N+1}(P)\right).$$

While $A - E_i^{M-1,N+1}(P) > 0$ we have $N\left(E_i^{M,N}(P) - E_i^{M-1,N+1}(P)\right) < 0$. The net effect on total emissions is unclear. We show that the direction of the change in total emissions is in fact ambiguous.

This result is quite surprising, since myopia is generally associated with careless management of the environment. In a multiplayer setup the reaction of other players (countries) is important and as it turns out can outweigh a goodwill gesture of a myopic player (country) adopting a farsighted behavior.

We illustrate this possibility through the simple case $M = N = 1$. The same type of qualitative results could be obtained for the general case $M, N > 1$. We first describe the different possible outcomes graphically and then give a more precise statement of the results in Proposition 1. In Appendix B, part D) we show that $2\alpha^{0,2} > \alpha^{1,1}$ and therefore when the slope of the sum of emissions' rules when $M = 0$ and $N = 2$ is steeper than the sum of emissions' rules when $M = N = 1$.

For the purpose of illustration we consider a numerical example where $A = 10, k = 1$ and $r = 1$. We cover different cases that can arise as the value of s changes in Figures 1 to 3. Throughout Figures 1, 2, 3 the solid line corresponds to $E_W^{0,2}(P)$ and the dashed curve to $E_W^{1,1}(P)$.

In Figure 1 (left) $s = 1$: the acquisition of farsight by the myopic results in a decrease of total emissions for all $P \geq 0$.

In Figure 1 (right) $s = 7$, Figure 2 (left) $s = 8$ and Figure 2 (right), $s = 10$: the impact of the acquisition of farsight by the myopic country has an ambiguous impact on total emissions.

In Figure 3, $s = 25$: the acquisition of farsight by the myopic results in a decrease of total emissions for all $P \geq 0$.

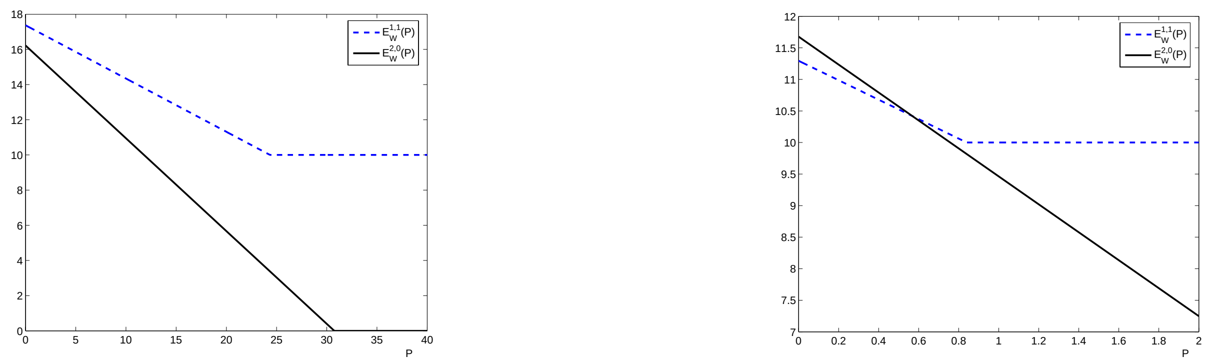

Figure 1: Comparison total emissions: $s = 1$ (left); $s = 7$ (right)

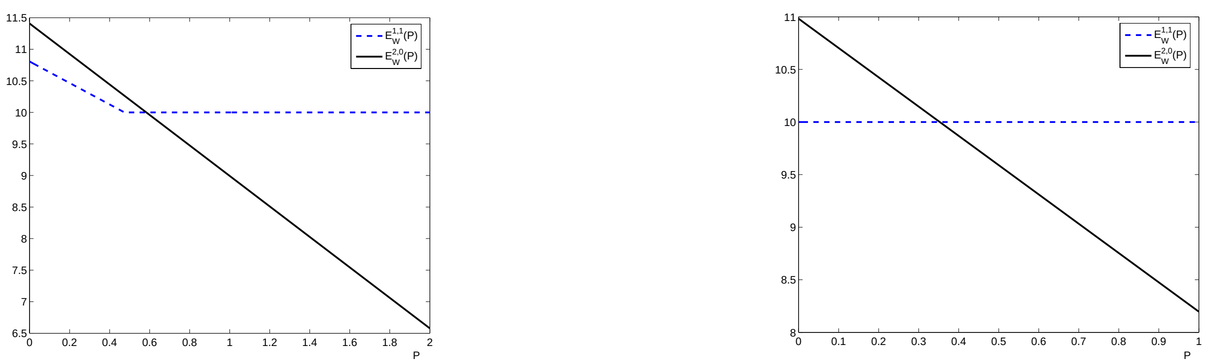

Figure 2: Comparison total emissions: $s = 8$ (left); $s = 10$ (right)

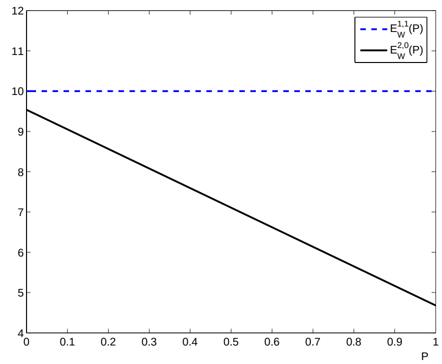

Figure 3: Comparison total emissions: $s = 25$

Proposition 1 *Let* $\underline{\underline{s}} = (3k + r)(k + r)$ *and* $\bar{s} = (5k + 4r)(k + r) > \underline{\underline{s}}$. *There exists* $\hat{s} < \underline{\underline{s}}$ *such that*

(i) for $s \leq \hat{s}$ *we have*

$$E_W^{0,2}(P) < E_W^{1,1}(P) \text{ for all } P > 0$$

(ii) for $\hat{s} < s < \bar{s}$ *we have that there exists a unique* $\tilde{P} > 0$ *such that*

$$E_W^{0,2}(P) > E_W^{1,1}(P) \Leftrightarrow P < \tilde{P}.$$

(iii) for $s \geq \bar{s}$ *we have*

$$E_W^{0,2}(P) < E_W^{1,1}(P) \text{ for all } P \geq 0.$$

Proof. In Appendix B, we prove

in part A) that the threshold $\bar{s}$ is such that $E_W^{0,2}(0) \gtreqless A$ iff $s \lesseqgtr \bar{s}$.

in part B) that the thresholds $\underline{\underline{s}}$ is such that $P_{1,1} \lesseqgtr 0$ iff $s \gtreqless \underline{\underline{s}}$.

in part C), that there exists $\hat{s}(< \underline{\underline{s}})$ such that $E_W^{1,1}(0) - E_W^{0,2}(0) = 2\beta^{0,2} - \beta^{1,1} \lesseqgtr 0$ iff $s \gtreqless \hat{s}$.

In part D) we show that $2\alpha^{0,2} - \alpha^{1,1} > 0$.

If $s \leq \hat{s}$, then, $A < E_W^{0,2}(0) \leq E_W^{1,1}(0)$ with $E_W^{0,2}(0) = E_W^{1,1}(0)$ only for $s = \hat{s}$. This along with $2\alpha^{0,2} - \alpha^{1,1} > 0$ yields (i).

If $\hat{s} < s < \bar{s}$ we distinguish two cases:

a) if $\hat{s} < s < \underline{\underline{s}}$, then $E_W^{0,2}(0) > E_W^{1,1}(0) > A$ and therefore since $2\alpha^{0,2} > \alpha^{1,1} > 0$ and at $P_{0,2}$ we have $E_W^{0,2}(P_{0,2}) = 0 < E_W^{1,1}(P_{0,2}) = A$, there exists a unique $\tilde{P}$ such that

$$E_W^{0,2}(P) > E_W^{1,1}(P) \Leftrightarrow P < \tilde{P}.$$

b) if $\underline{\underline{s}} \leq s < \bar{s}$. The same arguments invoked in a) apply to obtain (ii). The only difference with case a) is that $E_W^{1,1}(P) = E_W^{1,1}(0) = A$ for all $P \geq 0$.

If $\underline{\underline{s}} < \bar{s} \leq s$ then $E_W^{0,2}(0) < A$ and $E_W^{1,1}(P) = 0$ for all P since $P_{1,1} \leq 0$. From the fact that $E_W^{0,2}(\cdot)$ is a strictly decreasing function of P we conclude (iii). ■

Part ii) of Proposition 1 shows an interesting and rather surprising possibility. It states that, holding the total number of countries fixed, the total level of emissions is not a monotonic function of the number of myopic countries.

We note that the threshold $\tilde{P}$ in Proposition 1 ii) can be larger or smaller than $P_{1,1}$. Numerical simulations show that there exists $\check{s}$ is such that $\tilde{P} < P_{1,1}$ iff $s < \check{s}$ with the following two possibilities $\hat{s} < \check{s} < \underline{\underline{s}}$ or $\hat{s} < \underline{\underline{s}} < \check{s}$, depending on the values of k and r. In the latter case for all $s < \underline{\underline{s}}$ we have $\tilde{P} < P_{1,1}$.

While the short-run impact of the acquisition of farsight on total emissions is ambiguous, it turns out that the long-run impact is not.

Proposition 2 *We have*

$$P_{SS}^{0,2} < P_{SS}^{1,1}.$$

Proof.

(i) For $s < \underline{\underline{s}}$

$$\begin{aligned} P_{SS}^{1,1} > P_{SS}^{0,2} \quad &\Leftrightarrow \quad -8As(-8k - 7r + \sqrt{(2k+r)^2 + 12s}) > 0 \\ &\Leftrightarrow \quad -8k - 7r + \sqrt{(2k+r)^2 + 12s} < 0 \\ &\Leftrightarrow \quad -12((5k+4r)(k+r) - s) < 0. \end{aligned}$$

Because where are assuming $s < \underline{\underline{s}} < \bar{s} = (5k+4r)(k+r)$, then, $P_{SS}^{1,1} > P_{SS}^{0,2}$.

(ii) For $\underline{\underline{s}} < s < \bar{s}$

$$\begin{aligned} P_{SS}^{1,1} = \frac{A}{k} > P_{SS}^{0,2} \quad &\Leftrightarrow \quad 4k^2 - 12s + 5kr + k\sqrt{(2k+r)^2 + 12s}) < 0 \\ &\Leftrightarrow \quad -12(k^4 + 3k^3 r + k^2(2r^2 - 9s) - 10krs + 12s^2) < 0 \\ &\Leftrightarrow \quad (s - s_1)(s - s_2) > 0, \end{aligned}$$

where

$$s_{1,2} = \frac{1}{24}k\left(9k + 10r \pm \sqrt{33k^2 + 36kr + 4r^2}\right), \tag{5}$$

$0 < s_1 < s_2$.

Therefore,

$$P_{SS}^{1,1} = \frac{A}{k} > P_{SS}^{0,2} \ \Leftrightarrow \ s < s_1 \text{ or } s > s_2.$$

It can be proved that $\underline{\underline{s}} > s_2$, and then, $s > \underline{\underline{s}}$ implies $s > s_2$, and $P_{SS}^{1,1} = \frac{A}{k} > P_{SS}^{0,2}$.

(iii) For $s > \bar{s}$ it can be shown that $P_{SS}^{0,2} > P_{SS}^{1,1}$. Indeed we have

$$P_{SS}^{1,1} = \frac{A}{k} > P_{SS}^{0,2} \quad \Leftrightarrow \quad s < s_1 \text{ or } s > s_2,$$

where s_1, s_2 are given in (5).

It can be proved that $\bar{s} > s_2$, and then, $s > \bar{s}$ implies $s > s_2$, and $P_{SS}^{1,1} = \frac{A}{k} > P_{SS}^{0,2}$.

■

By means of numerical simulations we have obtained that the result of Proposition 2 holds for all $M, N \geq 1$:

$$P_{ss}^{M,N} > P_{ss}^{M-1,N+1}.$$

Thus the acquisition of farsight by a single country may result in an increase of overall pollution emissions, however it always results in a decrease of the steady-state stock of pollution.

4.2 Impact on the payoffs

We now examine the impact of the acquisition of farsight on different countries' payoff. We can use the payoffs determined in section 2. We evaluate the sign of the following difference of welfare

$$\Delta(P_0) = V_i^{M-1,N+1}(P_0) - V_j^{M,N}(P_0),$$

where we recall that $V_j^{M,N}(P_0)$ denotes the value function of a representative myopic country (player j) when there are M myopic countries and N farsighted countries; and $V_i^{M-1,N+1}(P_0)$ denotes the value function of a representative non-myopic country (player i) when there are $M-1$ myopic countries and $N+1$ farsighted countries. The sign of this difference $\Delta(P_0)$ will typically depend on the model parameters in particular the initial level of the stock of pollution and the damage parameter s.

We have run numerical simulations to compare the difference for two different scenarios concerning the initial value of the stock of pollution. We have fixed different values of the discount rate r, $r \in \{0.1, 0.5, 1\}$ and different values of the regeneration rate k, $k \in \{0.1, \ldots, 0.9\}$ and analyzed the difference $\Delta(P_0)$ as a function of the damage cost parameter s. For the numerical simulations we have assumed $N \in \{1, 2, \ldots, 10\}$.

The main findings are summarized below.

- **Result 1:** The case where $P_0 = 0$.

  For any k, r and N fixed there exists $\tilde{s} > 0$ such that

  $$\Delta(0) = V_i^{M-1,N+1}(0) - V_j^{M,N}(0) < 0 \text{ if } s > \tilde{s}.$$

  The sign of the difference above is independent of the value of M, and then, result 1 remains valid for any value of $M \geq 1$.

  We have ceteris paribus:

  - $\tilde{s}$ decreases as N increases.
  - $\tilde{s}$ increases as k increases.

– $\tilde{s}$ increases r increases.

Under this assumption, for any N, k, r and s such that $V_i^{M-1,N+1}(0) - V_j^{M,N}(0) < 0$, then it is better for a myopic country (belonging to a group of M symmetric myopic countries) to remain myopic than to be in the non-myopic group.

While the level of the initial stock is a priori arbitrary, a particular level of stock $P_0 = P_{ss}^{M,N}$ is a natural candidate to investigate.

- **Result 2:** The case where $P_0 = P_{ss}^{M,N}$

  For any k, r and N fixed there exists $s^+ > 0$ such that

$$\Delta\left(P_{ss}^{M,N}\right) = V_i^{M-1,N+1}(P_{ss}^{M,N}) - V_j^{M,N}(P_{ss}^{M,N}) < 0 \text{ if } s > s^+. \tag{6}$$

  The sign of the difference above is independent of the value of M, and then, result 2 remains valid for any value of $M \geq 1$.

  We have, ceteris paribus:

  – s^+ decreases as N increases.

  – s^+ increases as k increases.

  – s^+ increases r increases.

The main implication of results 1 and 2 is that the unilateral acquisition of farsight can be welfare reducing for the country acquiring farsight. This is explained by the fact a farsighted country pollutes less than when if it were myopic, and the fact that the other (farsighted) countries react by expanding their emissions. The overall impact of farsightedness may well be negative. This disincentive to acquire farsight exists when the damage parameter is large enough.

Remark 1 *From results 1 and 2 we have that the range of the damage parameter for which a myopic country has a positive incentive to acquire farsight is smaller the larger the number of countries N. Under a larger competition it is less likely that a myopic country has an incentive to acquire farsight.*

Remark 2 *The role played by the discount rate is not straightforward. The smaller the discount rate the smaller the range of the damage of parameter under which a myopic country has a positive incentive to acquire farsight. This can be surprising since a more patient country would in principle value the information about the future more and would therefore gain more from farsight. This intuition does not carry over in a game theoretic setting. A smaller discount rate implies that the competitors of the myopic country value the future more, are more mindful of* *the long-run impact of emissions and its damage. This can eliminate the incentive of a myopic country to acquire farsight.*

Moreover the numerical simulations suggest that

$$s^+ > \tilde{s}$$

therefore we have

	$s < \tilde{s} < s^+$	$\tilde{s} < s < s^+$	$\tilde{s} < s^+ < s$
$\Delta(0)$	$+$	$-$	$-$
$\Delta\left(P_{ss}^{M,N}\right)$	$+$	$+$	$-$

If s is large enough all countries have a disincentive to become farsighted.

For $s \in (\tilde{s}, s^+)$ the sign of the gains from farsight depends on P_0. The numerical simulations suggest that, if $\tilde{s} < s < s^+$ there exists a $\bar{P} \in \left(0, P_{ss}^{M,N}\right)$ such that $\Delta\left(\bar{P}\right) = 0$ and $\Delta(P) > 0$ iff $P > \bar{P}$, a myopic country has an incentive to acquire farsight iff $P_0 > \bar{P} \in \left(0, P_{ss}^{M,N}\right)$ moreover, as the stock of pollution converges to the steady state, the incentive to acquire farsight remains positive.

We can also infer from this analysis that for $\bar{P}$ such that $\Delta\left(\bar{P}\right) = 0$ the impact of the acquisition of farsight on the 'instantaneous' welfare of a country is not uniformly positive. This is true from the fact that at the steady state the incentive to acquire farsight is strictly positive. Since the overall impact of farsight is nil, the impact of farsight in the short-run must be negative.

This is illustrated through an example. Consider the case where $N = 10$, $k = 0.1$, $r = 0.1$ we then have $\tilde{s} = 0.00635334$ and $s^+ = 0.0079026$ so we consider for instance $s = \frac{\tilde{s}+s^+}{2}$ we then have, after setting, $A = 1$, for $M = 1$, then $\bar{P} = 18.9502$ and if $M = 2$, then $\bar{P} = 20.6729$.

We consider the time path of the instantaneous welfare, i.e., for the myopic country

$$W_j^{M,N}(t) = \frac{A^2}{2} - (N+M)\frac{s}{2}P_{M,N}^2(t)$$

and for a farsighted country

$$W_i^{M,N}(t) = AE_i^{M,N}\left(P_{M,N}(t)\right) - \frac{1}{2}\left(E_i^{M,N}\left(P_{M,N}(t)\right)\right)^2 - (N+M)\frac{s}{2}P_{M,N}^2(t)$$

for $P_0 = \bar{P}$ and $P_{M,N}(t)$ is the equilibrium path of the stock of pollution when there are M myopic countries and N farsighted countries. We can notice from Figure 4 that the instantaneous welfare path of a myopic country, $W_j^{M,N}(t)$, crosses from below the path of instantaneous welfare it would enjoy if it were farsighted, $W_i^{M-1,N+1}(t)$. Therefore, the change in instantaneous welfare from the acquisition of farsight is initially negative before turning positive. This suggests that incentive mechanisms can play an important role in inducing a country to adopt a farsighted behavior and diminishing the number of myopic countries. These incentives would compensate the myopic country for the short-run losses incurred from the acquisition of farsight and can in principle be reimbursed by the country from the gains from farsight that it enjoys in the long run. A very small net present value of transfers (possibly zero) can be enough to induce a myopic country to acquire farsight; facilitating access to credit can be, for example, an important tool to entice a country to acquire farsight.

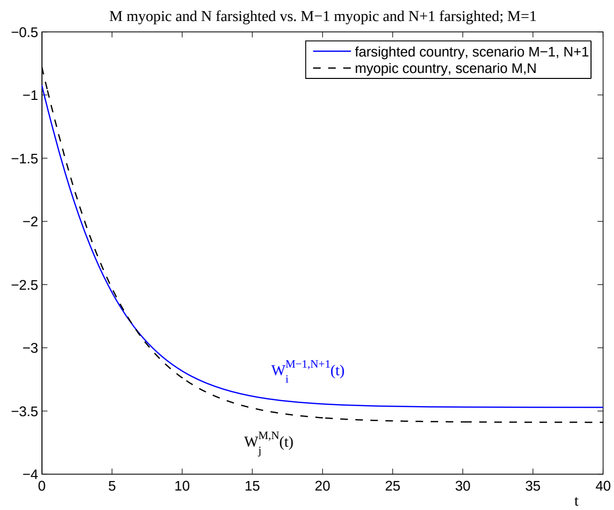

Figure 4: Comparison instantaneous welfare

From the same numerical simulations above we have obtained that the acquisition of farsight by country m results in an increase of all the other countries' payoffs. This is not straightforward because a 'friendly' unilateral action by the country that reduces its emissions (in this case the myopic country acquiring farsight) may be met by an excessive increase in emissions the other farsighted countries that could end up reducing all countries' welfare. All the numerical simulations we carried point to an increase of all the other countries' welfare.

5 Concluding remarks

In this paper we have studied the impact of the ability to forecast the effect of current emissions on future levels of pollution and thus on future damages.

When all countries are farsighted their payoffs are larger than when all countries are myopic. However, we have shown that in the case where one myopic country becomes farsighted we have shown that the welfare impact of farsightedness on that country is ambiguous. This is due to the reaction of the other farsighted countries to that country's acquisition of farsight. The country that acquires farsight reduces its emissions while the other farsighted countries extend their emissions. Global emissions may increase in the short run. In contrast, the sum of all countries' emissions at the steady state is a decreasing function of the number of myopic countries. In the long run farsight results in a better quality of the environment. The analysis of welfare shows that farsightedness may not benefit the country that acquires it when pollution causes severe damage or when the stock of pollution is beyond a certain threshold. Numerical simulations reveal that, starting from an initial stock of pollution such that the present value of the gains from farsight is zero, the change in instantaneous welfare from the acquisition of farsight is initially negative before turning positive. Incentive mechanisms can play an important role in inducing a country to adopt a farsighted behavior and diminishing the number of myopic countries. These incentives would compensate the myopic country for the short-run losses incurred from the acquisition of farsight and can in principle be reimbursed by the country from the gains from farsight that it enjoys in the long run. Access to credit may result in more countries being farsighted.

It would be interesting to analyze the impact of myopia on the implementation of a cooperative solution such as in Petrosjan and Zaccour (2003). For instance, transfers to implement a cooperative outcome, such as optimal reductions in the costs from a transboundary stock pollutant (à la Petrosjan and Zaccour (2003)) can modify countries' incentives to acquire farsight. Although challenging, the design of an allocation over time of transfers that allow the implementation of the first-best solution, while taking into account myopic countries' incentives to acquire farsight promises to be a fruitful line of future research.

Appendix A

$$\alpha_i^{M,N} = f(k,r,s,N) = \frac{1}{2(1-2N)}\left(2k + r - \sqrt{(2k+r)^2 + 4s(2N-1)}\right),$$

can be rewritten as

$$\alpha_i^{M,N} = f(k,r,s,N) = \frac{2s}{2k + r + \sqrt{(2k+r)^2 + 4s(2N-1)}}.$$

$f(k,r,s,N)$ is clearly decreasing in N; the larger N the smaller $f(k,r,s,N)$ and therefore $\alpha_i^{M-1,N+1} - \alpha_i^{M,N} < 0$.

Coefficient

$$\beta_i^{M,N} = \frac{A\alpha_i^{M,N}(M+N)}{k+r+\alpha_i^{M,N}(2N-1)}$$

after some manipulations and once the expression of $\alpha_i^{M,N}$ has been replaced reads:

$$\beta_i^{M,N} = \frac{A(M+N)2s}{(k+r)\left(2k+r+\sqrt{(2k+r)^2+4s(2N-1)}\right)+(2N-1)2s}.$$

With $M+N$ fixed, $\beta_i^{M,N}$ is a decreasing function of N, and therefore

$$\left(\beta_i^{M-1,N+1} - \beta_i^{M,N}\right) < 0.$$

Appendix B

A)

$$\begin{aligned} P_{1,1} = \frac{A-\beta_i^{1,1}}{\alpha_i^{1,1}} > 0 \;\; &\Leftrightarrow \;\; A-\beta_i^{1,1} > 0 \;\Leftrightarrow\; \frac{A\left(4k+3r-\sqrt{(2k+r)^2+4s}\right)}{r+\sqrt{(2k+r)^2+4s}} > 0 \\ &\Leftrightarrow \;\; 4k+3r-\sqrt{(2k+r)^2+4s} > 0 \\ &\Leftrightarrow \;\; (3k+r)(k+r)-s > 0 \\ &\Leftrightarrow \;\; s < \underline{\underline{s}} = (3k+r)(k+r). \end{aligned} \tag{7}$$

B)

$$\begin{aligned} E_W^{0,2}(0) > A \;\; &\Leftrightarrow \;\; E_W^{0,2}(0) = 2E_i^{0,2}(0) > A \;\Leftrightarrow\; 2(A-\beta_i^{0,2}) > A \;\Leftrightarrow\; A - 2\beta_i^{0,2} > 0 \\ &\Leftrightarrow \;\; A - \frac{A8s}{(k+r)(2k+r)+6s+(k+r)\sqrt{(2k+r)^2+12s}} > 0 \\ &\Leftrightarrow \;\; (k+r)(2k+r)-2s+(k+r)\sqrt{(2k+r)^2+12s} > 0. \end{aligned}$$

If $(k+r)(2k+r)-2s > 0$ (i.e. $s < \frac{(k+r)(2k+r)}{2}$), then $E_W^{0,2}(0) > A$.
If $(k+r)(2k+r)-2s < 0$ (i.e. $s > \frac{(k+r)(2k+r)}{2}$), then

$$E_W^{0,2}(0) > A \;\Leftrightarrow\; 4s((5k+4r)(k+r)-s) > 0.$$

It can be easily proved that

$$\frac{(k+r)(2k+r)}{2} < (5k+4r)(k+r).$$

Therefore, we can conclude that

$$E_W^{0,2}(0) > A \;\Leftrightarrow\; s < (5k+4r)(k+r).$$

C) We first show that there exists a unique root $\hat{s}$ to $-\beta_i^{1,1} + 2\beta_i^{0,2} = 0$.

C.1) Let us denote

$$\begin{aligned} h(s) &= -\beta_i^{1,1} + 2\beta_i^{0,2} \\ &= -\frac{4As}{(k+r)\left(2k+r+\sqrt{(2k+r)^2+4s}\right)+2s} \\ &+ \frac{8As}{(k+r)\left(2k+r+\sqrt{(2k+r)^2+12s}\right)+6s}. \end{aligned}$$

Let us note that

$$h(0)=0, \quad h'(0)=\frac{2A}{(k+r)(2k+r)}>0, \quad \lim_{s\to\infty} h(s) = -\frac{2A}{3}<0.$$

Therefore, when s is small enough we have $h>0$, while for s big enough we have $h<0$. We want to prove that function h decreases monotonically and therefore, there exists a unique $\hat{s}$ such that

$$h(s) = -\beta_i^{1,1} + 2\beta_i^{0,2} > 0 \Leftrightarrow s < \hat{s}. \tag{8}$$

After some manipulations we have:

$$\begin{aligned} h(s) &= -\beta_i^{1,1} + 2\beta_i^{0,2} > 0 \\ \Leftrightarrow \quad & g(s) = -\frac{2s}{k+r} + 2k + r - \sqrt{(2k+r)^2+12s} + 2\sqrt{(2k+r)^2+4s} > 0. \end{aligned}$$

If we prove that $g'(s)<0$ for $s>0$, then h decreases monotonically.

$$g'(s) = -\frac{2}{k+r} - \frac{6}{\sqrt{(2k+r)^2+12s}} + \frac{4}{\sqrt{(2k+r)^2+4s}}.$$

$$\lim_{s->0} g'(s) = -2\left[\frac{1}{k+r}+\frac{1}{2k+r}\right]<0; \quad \lim_{s->\infty} g(s) = -\frac{2}{k+r}<0.$$

Let see that the maximum value of $g'(s)$ is negative. The possible extrema of $g'(s)$ are the roots of

$$g''(s)=0.$$

$$g''(s) = -2\left[-18\left((2k+r)^2+12s\right)^{-3/2} + 4\left((2k+r)^2+4s\right)^{-3/2}\right].$$

The unique possible root of $g''(s)=0$ is given by:

$$\breve{s} = (2k+r)^2 \frac{2^{-2/3}-9^{-2/3}}{4\left(3\times 9^{-2/3}-2^{-2/3}\right)}.$$

$$g'(\breve{s}) = -2\left[\frac{1}{k+r}+\frac{3}{(2k+r)\sqrt{1+12a}}-\frac{2}{(2k+r)\sqrt{1+4a}}\right],$$

where

$$a = \frac{2^{-2/3} - 9^{-2/3}}{4\left(3 \times 9^{-2/3} - 2^{-2/3}\right)} \simeq 1.5727$$

Therefore,

$$\begin{aligned} g'(\check{s}) &< -2\left[\frac{1}{2k+r} + \frac{3}{(2k+r)\sqrt{1+12a}} - \frac{2}{(2k+r)\sqrt{1+4a}}\right] \\ &= -\frac{2}{2k+r}\left[1 + \frac{3}{\sqrt{1+12a}} - \frac{2}{\sqrt{1+4a}}\right] \\ &\simeq -\frac{2}{2k+r} 0.9323 < 0. \end{aligned}$$

The value of $g'(s)$ evaluated at its extremum is negative, so it is $g'(s)$ for all $s > 0$.

Let us argue by contradiction and assume that there exists s^* such that $g'(s^*) \geq 0$. Then, necessarily g' possesses at least a maximum point $\underline{\underline{s}}$ and the value of g' at this point is positive. So that, under the hypothesis that there exists s^* such that $g'(s^*) > 0$, there exists s^{**} such that $g''(s^{**}) = 0$ and $g'(s^{**}) \geq 0$. Since we have proved that there is at most one zero ($\check{s}$) of $g''(s) = 0$, necessarily $s^{**} = \check{s}$. But we have computed that $g'(\check{s}) < 0$, in contradiction with $g'(s^{**}) > 0$. So that the hypothesis that there exists s^* such that $g'(s^*) \geq 0$ is false, and $g'(s) < 0$ for all s.

C.2) We now prove that $\hat{s} < \underline{\underline{s}}$.

From (8) we known that there exists $\hat{s}$ such that $E_W^{0,2}(0) = E_W^{1,1}(0) > A$.

For convenience we use the notation $E_{W,\, s=\hat{s}}^{M,N}(0)$ to indicate that the value of $E_W^{M,N}(0)$ is evaluated at $s = \hat{s}$, and we have:

$$E_{W,\, s=\hat{s}}^{0,2}(0) = E_{W,\, s=\hat{s}}^{1,1}(0) > A.$$

Moreover, $P_{1,1} = 0$ when $s = \underline{\underline{s}}$, that is

$$E_{W,\, s=\underline{\underline{s}}}^{1,1}(0) = A.$$

Therefore we have

$$E_{W,\, s=\hat{s}}^{1,1}(0) > E_{W,\, s=\underline{\underline{s}}}^{1,1}(0),$$

or

$$E_{i,\, s=\hat{s}}^{1,1}(0) > E_{i,\, s=\underline{\underline{s}}}^{1,1}(0) = 0.$$

From the fact that $E_W^{1,1}(0)$ (or $E_i^{1,1}(0)$) is a decreasing function of s we have that $\hat{s} < \underline{\underline{s}}$.

D)

$$\begin{aligned} 2\alpha_i^{0,2} - \alpha_i^{1,1} > 0 &\Leftrightarrow 2s\left[\frac{2}{2k+r+\sqrt{(2k+r)^2+12s}} - \frac{1}{2k+r+\sqrt{(2k+r)^2+4s}}\right] > 0 \\ &\Leftrightarrow 2k+r+2\sqrt{(2k+r)^2+4s} - \sqrt{(2k+r)^2+12s} > 0. \end{aligned}$$

Because $2\sqrt{(2k+r)^2+4s} - \sqrt{(2k+r)^2+12s}$ is positive, therefore, $2\alpha_i^{0,2} - \alpha_i^{1,1} > 0$.

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