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MONETARY POLICY IN SUDDEN STOP-PRONE ECONOMIES*

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Abstract

In a model featuring sudden stops and pecuniary externalities, I show that the ability to use capital controls has radical implications for the conduct of monetary policy. Absent capital controls, following an inflation targeting regime is nearly optimal. However, if the central bank lacks commitment, it will follow a monetary policy that is excessively procyclical and not desirable from an ex ante welfare perspective: it increases overall indebtedness as well as the frequency of financial crisis and reduces social welfare relative to an inflation targeting regime. Access to capital controls can correct this monetary policy bias. With capital controls, relative to an inflation targeting regime, the time-consistent regime reduces both the frequency and magnitude of crises, and increases social welfare. This paper rationalizes the procyclicality of the monetary policy observed in many emerging market economies.

Keywords: Financial crises; monetary policy; capital controls; time consistency; aggregate demand externality; pecuniary externality.

JEL Classifications: E44, E52, F38, F41, G01

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1 Introduction

In the aftermath of a series of financial crises in emerging market countries, also known as sudden stops (Calvo, 1998), there has been sharp debate over the optimal policies for preventing recurrent credit crunches that emphasize the need to adopt an inflation targeting regime as the main monetary framework, and the resort to capital flow management policies. In their recent work, Vegh and Vuletin (2012) documented that, unlike developed economies where the monetary policy is countercyclical, many emerging markets economies have procyclical monetary policies. This empirical finding is somewhat puzzling and raises questions about what is the optimal monetary policy. Can a procyclical monetary policy be optimal? How does the availability of macroprudential policy tools change the optimal monetary policy framework? What are the costs and benefits of adopting inflation targeting in sudden stop-prone economies?

I tackle the issue of the design of optimal monetary policy in emerging market economies using a small open economy model featuring endogenous sudden stops and nominal rigidities. The model builds on the real framework of Mendoza (2002), Bianchi (2011), Korinek (2011), which features domestic agents borrowing from foreign investors subject to a credit constraint that limits borrowing to a fraction of the market value of output. The novel features introduced in this framework to study monetary policy are nominal price rigidities in the non-tradable production sector and flexible exchange rate.

This article presents a normative framework that rationalizes the behavior of the monetary authority in a sudden stop-prone economy, as an optimal time-consistent policy based on the interplay between distortions arising from nominal rigidities and credit frictions. Then, I develop a theoretical explanation of how the optimal uses of capital flow taxes can change the design of the optimal time-consistent policy. Finally, in a calibrated version of the model, I compare the effectiveness of an optimal time-consistent monetary policy to that of a simple inflation targeting rule for reducing the incidence and severity of crises and increasing social welfare.

Two key features of the environment are aggregate demand externality and pecuniary externality. Aggregate demand externality emerges from the 'costly' adjustment of nominal prices and the change in the demand for labor that accompanies the increase in the demands for goods on the part of individual households. Because households fail to internalize this externality, they tend to over-consume, and thus to supply too much labor when the economy is undergoing a boom. Furthermore, with credit constraints depending on market prices, a financial accelerator mechanism is occasionally set in motion: when the constraint binds, demand for tradables falls, the real wage and real exchange rates drop, tightening the credit constraint further. Households also fail to internalize this pecuniary externality. Hence, the model embodies market failures that a government that internalizes these externalities can improve upon.

I start by studying the optimal monetary policy problem of a government that cannot commit to future policies in an economy into which capital flows freely. The optimal time-consistent monetary policy is solved as a Markov perfect equilibrium, and its formulation can easily be related to the standard monetary policy rules developed in the New Keynesian literature (see, e.g., [Woodford, 2003](#); [Farhi and Werning, 2016](#)).

The optimal time-consistent monetary policy derived in this model expresses the path of the inflation rate as a linear function of a measure of the output gap (i.e., the labor wedge) and variables that account for the relevant financial conditions before and during periods when credit constraints bind. This characterization highlights the role of financial frictions in the design of monetary policy and suggests that monetary policy should react to financial conditions. In the absence of credit frictions, a price stability policy is always optimal. This finding is in line with the standard result in the New Keynesian literature suggesting that a policy that offsets distortion stemming from nominal rigidities will be welfare-dominant when nominal rigidities and monopolistic competition are the only causes of friction (see, e.g., [Kollmann, 2002](#) and [Schmitt-Grohé and Uribe, 2007](#)). Credit frictions, however, create policy trade-offs between price stability and financial stability. Indeed, under flexible price allocation, when the credit constraint is not binding households tend to overborrow due to the pecuniary externality, while when it is binding they tend to over-consume goods due to the aggregate demand externality. Hence, with credit frictions, a price stability policy is only optimal in the knife-edge case where the intra- and intertemporal elasticities of substitution are equal.

The manner in which financial conditions influence monetary policy crucially depends on the values of the intra- and intertemporal elasticities of substitution. In the empirically plausible case in which the intratemporal elasticity is greater than the intertemporal elasticity and the former is less than 1, optimal monetary policy involves a form of pro-cyclicality, being expansionary ahead of a potential financial crisis and contractionary when a crisis occurs. On the one hand, ahead of a potential crisis, when borrowing is high but the credit constraint does not bind, generating a boom in the non tradable sector discourages foreign borrowing when goods are substitutes, and thus mitigates the overborrowing problem arising from pecuniary externalities. On the other hand, during a crisis, when the credit constraint binds, a contractionary monetary policy reducing activity in the non-tradable sector serves two purposes: it sustains the value of collateral and it tames the overheating induced by the excessive labor supply of private agents. This characterization helps account for the observation of [Vegh and Vuletin \(2012\)](#) that monetary policy appears to be procyclical in many emerging economies.

This paper also contributes to the literature on macroprudential policies in financially fragile economies. Naturally, the availability of capital flow taxes modifies the features of the optimal

monetary policy. When capital flow taxes are used optimally, the conditions under which a price stability policy is desirable are less restrictive than when they cannot be used. Ahead of potential crises, capital inflow taxes are effective in correcting the pecuniary externality, and thus stabilizing prices is optimal. A policy trade-off between price stability and correcting the externalities only arises during periods when the credit constraint is binding since capital controls are then of little help.

The calibrated version of the model delivers important messages that contribute to the current debate on the dilemma between monetary policy and capital controls. First, in the absence of capital controls, relative to a simple targeting rule that stabilizes the producer price index, hereafter an inflation targeting regime, the gains from adopting a procyclical time-consistent regime are too small. The unconditional probability of crises is higher under the time-consistent regime (12.7 percent versus 5.5 percent under an inflation targeting regime), but crises, when they erupt, are less severe: for instance, total output drops by 21.9 percent (versus 29.7 percent under an inflation targeting regime). Overall, a time-consistent regime is welfare-dominated by an inflation-targeting regime.

The second message is that capital controls are always useful under both an inflation targeting regime and a time-consistent regime. This is because capital controls prevent excessive risk exposure (see [Bianchi, 2011](#); [Jeanne and Korinek, 2012](#); [Bianchi and Mendoza, forthcoming](#)), which reduces the volatility of the external accounts. Finally, the gains associated with the time-consistent monetary policy are more important when capital controls are used optimally. When the government has access to capital controls, a time-consistent regime is more effective for increasing the social welfare, and for reducing both the incidence of crises (1.1 percent versus 1.3 percent under an inflation targeting regime with capital controls) and the severity of crises (total output drops by 17.2 percent versus 17.8 percent under an inflation targeting regime with capital controls).

This paper is related to the literature on the design of monetary policy in economies with financial frictions.¹ [Christiano et al. \(2004\)](#), [Curdia \(2007\)](#) [Gertler et al. \(2007\)](#) and [Braggion et al. \(2009\)](#) study monetary policy in times of crisis in frameworks where financial markets are incomplete and crises are unexpected one-shot events. [Caballero and Krishnamurthy \(2003\)](#), [Aghion et al. \(2004\)](#) and [Benigno et al. \(2011\)](#) examined the design of monetary policy in an environment where economies last two or three periods and in which the credit constraint becomes binding unexpectedly and remains binding afterwards. My analysis differs from these studies in that it allows for crises to alternate with normal times. The dynamics of debt

¹An earlier literature compared the performance of different monetary regimes in an environment with financial market imperfections (see, e.g., [Clarida et al., 2002](#); [Céspedes et al., 2004](#) and [Moron and Winkelried, 2005](#)).

accumulation is also important for the design of monetary policy.

More recent studies have explored monetary policy in dynamic environments featuring both nominal rigidities and financial frictions. In particular, [Fornaro \(2015\)](#) compares the performance of different monetary regimes and shows that a monetary regime that responds to financial conditions in times of crisis delivers higher welfare than an inflation targeting regime. [Ottonello \(2015\)](#) studies exchange rate policy with optimal capital flow taxes in an economy featuring downward nominal wage rigidity and a collateral constraint. As in my model, in his work the optimal exchange-rate policy fully eliminates the inefficiency stemming from nominal rigidities when the intratemporal elasticity of substitution is greater than one. However, unlike in the present paper, his finding relies on the asymmetry of nominal frictions (downward rigidity). In addition, nominal rigidities in this framework introduce forward-looking effects, which are central to time-inconsistency aspects of the optimal policy. I also discuss analytically how the availability of capital controls can change importantly the properties of the optimal time-consistent monetary policy.

In the same vein, using a model with an asymmetric nominal price adjustment and a collateral constraint based on the expected future (resale) value of assets, [Devereux et al. \(2015\)](#) study the optimal conduct of monetary policy and capital controls during a sudden stop. In their paper, capital controls are only used under sudden stops to address the inefficiencies of international capital market. As a consequence, without commitment capital controls are not optimal from an ex-ante social welfare perspective. In contrast, I argue that when capital controls are used to prevent excessive risk exposure and stabilize the external accounts (as in [Bianchi \(2011\)](#)), they become a useful tool that complements monetary policy under both targeting rules and discretion. This paper also complements this literature by providing a framework that explains the differences in the behavior of monetary authorities when the economy is vulnerable to a financial crisis. This is important for a better assessment of the potential benefits and welfare implications from other regulatory measures.

This paper also builds on the quantitative macro-finance literature studying pecuniary externalities and inefficiency resulting from endogenous borrowing constraints (see, e.g., [Lorenzoni, 2008](#)). [Bianchi \(2011\)](#), [Korinek \(2011\)](#) and [Bengui and Bianchi \(2014\)](#) studied how this externality leads to overborrowing and showed to what extent capital control can restore constrained efficiency and reduce vulnerability to financial crises. Focusing on asset prices as a key factor driving debt dynamics and pecuniary externalities, [Bianchi and Mendoza \(forthcoming\)](#) point out the time-inconsistency issues in macroprudential policies originating from the forward-looking nature of asset prices. This paper also can be related to the work of [Acharya and Bengui \(2015\)](#), [Schmitt-Grohé and Uribe \(2016\)](#) and [Farhi and Werning \(2016\)](#), examining

the use of taxes on financial transactions as a tool for managing aggregate demand in the presence of nominal rigidities and constraints on monetary policy. My paper draws upon to both strands of the literature and stands out by analyzing how monetary policy should be designed to simultaneously address both aggregate demand externalities and pecuniary externalities.

The remainder of the paper is organized as follows: Section 2 describes the analytical framework. Section 3 presents the optimal policy analysis. Section 4 conducts the quantitative analysis. Section 5 concludes, and is followed by an extended appendix.

2 Model

Consider a dynamic model of a small open economy. Households in this economy consume two goods (a tradable good and a nontradable good) and can also engage in borrowing with foreign investors. The tradable good can be exchange with the rest of the world and the nontradable good is consumed by domestic agents only. The government (or central bank) in this economy sets the path of the nominal exchange rate as its monetary policy instrument.

2.1 Households

The economy is inhabited by a continuum of mass one of identical households with preferences described by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t) \quad (1)$$

where $\beta \in (0, 1)$ is a subjective discount factor, ℓ_t is labor supply and c_t denotes consumption. The period utility function takes the standard constant relative-risk-aversion form, with a relative-risk-aversion coefficient σ . The consumption good is a composite of tradable consumption c_t^T and nontradable consumption c_t^N , according to an Armington-type CES aggregator:

$$c_t = [a(c_t^T)^{-\eta} + (1 - a)(c_t^N)^{-\eta}]^{-\frac{1}{\eta}}, \quad \eta > -1, a \in (0, 1) \quad (2)$$

The intratemporal elasticity of substitution between tradable and nontradable goods is $\gamma = 1/(\eta + 1)$. Households receive a stochastic endowment of tradable goods y_t^T , profits Φ_t from the ownership of firms producing the nontradable good, and labor income in each period t . Households can trade internationally with foreign investors in one period state non-contingent foreign bonds denominated in units of the foreign currency. The foreign bond pays a stochastic interest rate R_t , determined exogenously in the world market. The tradable endowment and the gross interest rate are the only sources of uncertainty, and the vector of shocks $s_t \equiv (y_t^T, R_t) \subseteq \mathbb{R}_+^2$ is assumed to follow a first-order Markov process. The sequential budget constraint of the

household in terms of the domestic currency is given by:

$$P_t^T c_t^T + P_t^N c_t^N + \frac{\mathcal{E}_t b_{t+1}}{R_t} = P_t^T y_t^T + W_t \ell_t + \Phi_t + \mathcal{E}_t b_t \quad (3)$$

where b_{t+1} denotes bond holdings that households choose at the beginning of time t . \mathcal{E}_t is the nominal exchange rate defined as the price of the foreign currency in terms of the domestic currency. In terms of the domestic currency, P_t^T is the nominal price of the tradable good, P_t^N the nominal price of the nontradable good and W_t the nominal wage rate. The law of one price holds for the tradable good, which implies that $P_t^T = \mathcal{E}_t P_t^{T*}$ where P_t^{T*} denotes the foreign currency price of the tradable good. To simplify the analysis, the foreign-currency price of the tradable good is assumed to be constant and normalized to one. It follows that the domestic currency price of the tradable good is equal to the nominal exchange rate (i.e. $P_t^T = \mathcal{E}_t$).

Households' borrowing capacity is limited by a fraction κ of total income composed of tradable income and nontradable income

$$\frac{\mathcal{E}_t b_{t+1}}{R_t} \geq -\kappa \left[P_t^T y_t^T + W_t \ell_t + \Phi_t \right] \quad (4)$$

This type of borrowing constraint is common in the literature (e.g. [Mendoza, 2002](#); [Bianchi \(2011\)](#) and [Benigno et al., 2013](#)). The standard motivation for this credit constraint is that it can result from institutional features of credit markets or financial-market frictions (such as monitoring costs, bankruptcy risk or imperfections in the judicial system) and captures the willingness of foreign lenders in such an environment to not allow borrowing beyond a certain limit tied to the borrower's income.²

The household's problem is to choose stochastic processes $\{c_t^T, c_t^N, b_{t+1}\}$ to maximize the expected utility (1) subject to (2)-(4), taking as given the sequence of prices, profits, exchange rates, stochastic tradable endowments and interest rates, as well as the initial debt level b_0 . Letting λ_t/P_t^T denotes the multiplier associated with the budget constraint (3) and μ_t/P_t^T the multiplier associated with the credit constraint (4), the household's first-order conditions are:

$$\lambda_t = u_T(t) \quad (5)$$

$$p_t^N = \frac{1-a}{a} \left(\frac{c_t^T}{c_t^N} \right)^{1/\gamma} \quad (6)$$

$$\frac{-u_\ell(t)}{u_N(t)} = \underbrace{\left[1 + \frac{\kappa \mu_t}{u_T(t)} \right]}_{z_t} \frac{w_t}{p_t^N}, \quad (7)$$

²As shown by [Bianchi and Mendoza \(forthcoming\)](#), the credit constraint depends on current prices (rather than future prices) when the possibility of default arises at the end of the current period.

$$\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t \quad (8)$$

$$\mu_t \geq 0, \mu_t [b_{t+1} + \kappa (y_t^T + w_t \ell_t + \phi_t)] = 0 \quad (9)$$

where $p_t^N \equiv P_t^N / \mathcal{E}_t$ denotes the relative price of nontradables in terms of tradables. Similarly, $w_t \equiv W_t / \mathcal{E}_t$ represents the wage in terms of tradables and $\phi_t \equiv \Phi_t / \mathcal{E}_t$ is the firm profits in terms of tradables. The optimality condition (6) equates the marginal rate of substitution between the two goods, the tradable and the nontradable, to their relative price. This condition describes the demand for the nontradable good as a function of their relative price and the level of tradable absorption, and can be re-written as:

$$c_t^N = \left[\frac{a}{1-a} p_t^N \right]^{-\gamma} c_t^T \equiv \alpha (p_t^N) c_t^T.$$

The optimality condition for labor supply (7) equates marginal cost in terms of nontradable consumption from working one additional unit with the marginal benefit, which includes the relative wage w_t / p_t^N and the relaxation effect on the credit constraint. The variable z_t indicates the wage multiplier of an increase in labor supply. In particular, when the credit constraint binds, an increase of one unit of labor relaxes the constraint by $\kappa \mu_t \cdot w_t$, and the wage multiplier of this increase in labor is greater than 1 ($z_t > 1$). The Euler equation for debt (8) states that the current shadow value of wealth equals the expected value of reallocating wealth to the next period plus an additional term that represents the shadow price of relaxing the credit constraint. Thus, conditions (7) and (8) show that the credit constraint introduces two distortions: an intertemporal distortion arising from the presence of a credit constraint and an intratemporal distortion that hinges on the wage income entering the credit constraint.

2.2 Nontradable Sector

Nontradable goods are supplied by firms and following standard practice in the New-Keynesian literature, I introduce nominal rigidities in the nontradable goods market by separating the sector into monopolistically competitive intermediate producers and perfectly competitive retailers. The nontradable final good is produced by competitive firms that combine a continuum of nontradable varieties indexed by $j \in [0, 1]$ using the constant returns to scale CES technology

$$y_t^N = \left(\int_0^1 y_{j,t}^N \frac{\varepsilon-1}{\varepsilon} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon > 1$ is the elasticity of substitution between any two varieties. Each variety $y_{j,t}^N$ is produced by a monopolistically competitive producer using labor $h_{j,t}$ according to a linear

production function $y_{j,t}^N = Ah_{j,t}$. These producers hire labor in a competitive market with wage W_t , but pays $(1 - \tau^h)W_t$ net of a labor subsidy. Cost minimization implies that each firm faces the same real marginal cost (or unitary cost): $mc_t = \frac{1-\tau^h}{A} \frac{W_t}{P_t^N}$.

Price setting The intermediate goods firms face sticky price setting à la Rotemberg (1982). Accordingly, each firm j faces a cost of adjusting prices which, when measured in terms of the final nontradable good, is given by:

$$\frac{\varphi}{2} \left(\frac{P_{j,t}^N}{P_{j,t-1}^N} - 1 \right)^2 y_t^N$$

where φ is an adjustment cost parameter which determines the degree of nominal price rigidity and $P_{j,t}^N$ is the nominal price of variety j . Taking as given the sequence for mc_t, y_t^N and \mathcal{E}_t , the monopolistically competitive firm j optimally chooses the sequence of prices of its own variety, $P_{j,t}^N$, to maximize the stream of its expected discounted profit given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[\left(\frac{P_{j,t}^N}{P_t^N} - mc_t \right) y_{j,t}^N - \frac{\varphi}{2} \left(\frac{P_{j,t}^N}{P_{j,t-1}^N} - 1 \right)^2 y_t^N + T_t \right]$$

where $\Lambda_{i,t+i} \equiv \beta^t u_N(t+i)/u_N(i)$ is the household's stochastic discount factor for the nontradable good between date t and date $t+i$. T_t represents the lump sum tax that firms pay to the government at date t . Each period, the intermediate goods firm faces a demand curve for its product arising from competitive final good firms' production function:

$$y_{j,t}^N = \left(\frac{P_{j,t}^N}{P_t^N} \right)^{-\varepsilon} y_t^N.$$

In a symmetric equilibrium, all firms choose the same price ($P_{j,t}^N = P_t^N$ for all j), and firms' optimal pricing rule is described by the following condition:

$$\pi_t^N (1 + \pi_t^N) = \frac{\varepsilon}{\varphi} \left[mc_t - \frac{\varepsilon - 1}{\varepsilon} \right] + \mathbb{E}_t \Lambda_{t,t+1} \left[\frac{y_{t+1}^N}{y_t^N} \pi_{t+1}^N (1 + \pi_{t+1}^N) \right] \quad (10)$$

where $1 + \pi_t^N \equiv P_t^N / P_{t-1}^N$ is the inflation rate of the nontradable good. Condition (10) is the Rotemberg version of the non-linear New-Keynesian Phillips curve that relates the current inflation to the current deviation of marginal cost from marginal revenue and to the expected future inflation. It states that, given marginal costs, firms expecting higher inflation in the future would already raise prices in the current period to smooth out the necessary price adjustments over the time. According to this condition, firms would optimally set prices to equate the cost

of adjusting prices today to a weighted average of current and future expected deviation of marginal cost from marginal revenue. Therefore, under full price flexibility (i.e. $\varphi = 0$), firms would always set prices to equate marginal revenue to marginal cost. At the other extreme, when prices are fully rigid (i.e. $\varphi \rightarrow \infty$) firms would set prices once and for all to equate an average of current and future expected marginal revenues to an average of current and future expected marginal costs.

2.3 Government

By controlling for the exchange-rate level \mathcal{E}_t , the government influences the relative price of nontradables and is thus able to set the path for the inflation rate in the production sector, which can be seen as representing the government's monetary policy rule in this environment.³ To see more clearly how monetary policy operates, notice that any change in the relative price of nontradables has an expenditure switching effect, and the demand for the nontradable good is thus affected. This in turn requires a change in employment, which necessitates a change in the equilibrium wage through households' labor supply condition (7). Therefore, with its action on the exchange rate, the government affects firms' current marginal costs and thus their price-setting decisions. It then follows that by setting the level of the exchange rate, the government implicitly determines the path of the inflation rate.

The government also sets, once and for all, a constant labor subsidy τ^h which is financed through a lump sum tax on firms such that the government budget is balanced:

$$\tau^h W_t h_t = T_t$$

I assume that this constant labor subsidy is set at the level that would be optimal under flexible prices. This level, given by $\tau^h = 1/\varepsilon$, would fully offset the monopoly distortion in that world.

2.4 Recursive Competitive Equilibrium

Given a constant labor tax τ^h and an exchange rate path $\{\mathcal{E}_t\}_{t=0}^\infty$, a competitive equilibrium is defined by stochastic sequences of allocations $\{c_t^T, c_t^N, b_{t+1}, \ell_t, h_t\}_{t=0}^\infty$ and prices $\{P_t^N, W_t\}_{t=0}^\infty$ such that: (a) agents maximize their lifetime utility (1) subject to the sequence of budget and credit constraints given by (3) and (4) for $t = 0, \dots, \infty$, taking as given $\{P_t^N, W_t\}_{t=0}^\infty$; (b) the markets for labor, nontradable goods and tradable goods clear at each date t . The market

³There exists a nominal interest rate that would implement this policy. This interest rate can be found by introducing into the model a domestic bond that can be traded only among domestic households (its net supply is equal to zero in equilibrium).

clearing condition in the labor market, nontradable goods market and tradable goods market are respectively given by

$$\begin{aligned}
h_t &\equiv \int_0^1 h_{j,t} dj = \ell_t \\
y_t^N &= c_t^N + \frac{\varphi}{2} (\pi_t^N)^2 y_t^N \\
c_t^T + b_{t+1} &= y_t^T + (1+r)b_t
\end{aligned} \tag{11}$$

I now turn to describing a competitive equilibrium in recursive form. The aggregate state vector of the economy is (B, s) where B is the aggregate bond holdings and s is the exogenous shocks realization. The state variables for an household's optimization problem are the individual bond holdings b , and the aggregate state (B, s) . Denoting by $\Gamma(B, s)$ the perceived law of motion for aggregate bonds that households need to form expectations of future prices, and by $w(B, s)$ and $p^N(B, s)$ the respective pricing functions for labor and the nontradable good, the optimization problem of households in recursive form is given by:

$$\begin{aligned}
V(b, B, s) &= \max_{c^T, c^N, \ell, b'} u(c(c^T, c^N), \ell) + \beta \mathbb{E}_{s'|s} V(b', B', s') \\
s.t. \quad c^T + p^N(B, s)c^N + b' &= y^T + w(B, s)\ell + \phi(B, s) + (1+r)b \\
\frac{b'}{R} &\geq -\kappa \left[y^T + w(B, s)\ell + \phi(B, s) \right] \\
B' &= \Gamma(B, s)
\end{aligned}$$

The solution to this problem yields decision rules for individual bond holdings $\hat{b}'(b, B, s)$, labor supply $\hat{\ell}(b, B, s)$, tradable consumption $\hat{c}^T(b, B, s)$ and nontradable consumption $\hat{c}^N(b, B, s)$. In a recursive rational expectations equilibrium, the law of motion for aggregate bonds must coincide with the actual law of motion for aggregate bonds induced by the decision rule for bond holdings, and given by $\hat{b}'(B, B, s)$. The firm j 's optimal pricing rule satisfies

$$(1 + \pi_j^N) \pi_j^N = \frac{\varepsilon - 1}{\varphi} \left[\frac{1}{A} \frac{w(B, s)}{p^N(B, s)} - 1 \right] + \mathbb{E}_{s'|s} \Lambda \left[\frac{\hat{h}'(B', s')}{\hat{h}_j} (1 + \pi^N(B', s')) \pi^N(B', s') \right] \tag{12}$$

In a symmetric equilibrium, the decision rules satisfy $\hat{\pi}_j^N(B, s) = \hat{\pi}^N(B, s)$ and $\hat{h}_j(B, s) = \hat{h}(B, s)$ for all firms. A recursive rational expectations equilibrium is defined below.

Definition 1 (Recursive Competitive Equilibrium). *For a given government's policy rule $\pi_t^N(B, s)$, a recursive competitive equilibrium is defined by pricing functions $\{w(B, s), p^N(B, s)\}$, a perceived law of motion for aggregate bond holdings $\Gamma(B, s)$, firms' policies $\{\hat{\pi}^N(B, s), \hat{h}(B, s)\}$, and households' decision rules $\{\hat{b}'(b, B, s), \hat{c}^T(b, B, s), \hat{c}^N(b, B, s), \hat{\ell}(b, B, s)\}$ with associated value function $V(b, B, s)$ such that:*

1. $\{\hat{b}'(b, B, s), \hat{c}^T(b, B, s), \hat{c}^N(b, B, s), \hat{\ell}(b, B, s)\}$ and $V(b, B, s)$ solve households' recursive optimization problem, taking as given $w(B, s)$, $p^N(B, s)$ and $\Gamma(B, s)$.
2. $\{\hat{\pi}^N(B, s), \hat{h}(B, s)\}$ satisfies (12), taking as given $w(B, s)$, $p^N(B, s)$ and $\pi^N(B, s)$.
3. The perceived law of motion for aggregate bonds and the government's policy rule are consistent with the actual law of motion for individual bonds and actual inflation policy, respectively: $\Gamma(B, s) = \hat{b}'(B, B, s)$ and $\pi(B, s) = \hat{\pi}(B, s)$.
4. The labor market and the tradable good market clear

$$\begin{aligned}\hat{h}(B, s) &= \hat{\ell}(B, B, s) \\ \hat{c}^T(B, B, s) + \Gamma(B, s) &= y^T + (1+r)B\end{aligned}$$

3 Optimal Policy Analysis

This section discusses the trade-off between macroeconomic stabilization and financial stabilization that a policymaker faces in the model economy presented in the previous section and formally characterizes the optimal time-consistent monetary policy.

I assume that the government (or central bank) lacks the ability to commit to future policies, and chooses its policy instruments subject to the credit constraint and all others competitive equilibrium conditions. Since the set of restrictions on the optimal policy includes forward-looking constraints, namely the Euler equation for households (4) and an intertemporal pricing rule for firms (10), the optimal policy setup amounts to a dynamic game between successive governments. Thus, following [Kein et al. \(2008\)](#) and [Bianchi and Mendoza \(forthcoming\)](#), I focus on Markov-stationary policy rules that are expressed as functions of the payoff-relevant state variables (B, s) . A Markov-perfect equilibrium is a fixed point in the policy rule chosen by the government in any given period, taking as given the policy rule that represent the future governments' decisions. The key property of this fixed point is that the policy rule of the current government matches the policy rules of future governments that the current government takes as given to solve its optimization problem.

For the optimal policy analysis of this section, I focus on additively separable preferences, that is preferences satisfying $u_{c,\ell} = 0$.⁴ This specification of preferences is common in analytical

⁴For the purpose of the quantitative analysis of Section 4, I follow the growing macro-finance literature (for example [Bianchi and Mendoza, forthcoming](#); [Fornaro, 2015](#)) in specifying households' preferences following [Greenwood et al. \(1988\)](#), where utility is defined in terms of the excess of consumption over the disutility of labor, $u(c-g(\ell))$, with $g(\ell)$ being twice-continuously differentiable, strictly increasing and convex. All the results are robust to more general specification of preferences, especially for preferences specified following [Greenwood et al. \(1988\)](#) where $u_{c,\ell} \leq 0$. Section 4 provides more details.

studies of monetary policy open economy models (see for example [Benigno and Benigno, 2003](#) and [Gali and Monacelli, 2005](#)).

3.1 Government's Optimization Problem

Problem The formulation of the government optimal policy's problem is standard, with the government setting its policy to maximize the agents' welfare subject to the resource, credit and implementability constraints. Unlike private agents, the government internalizes the general equilibrium effects of borrowing decisions on market prices. To simplify the description of the government's optimal policy problem, I introduce the concept of a labor wedge, defined as the gap between firms' marginal product of labor in the nontradable production sector and households' marginal rate of substitution between leisure and nontradable consumption:

$$\omega_t \equiv 1 + \frac{1}{A} \frac{u_\ell(t)}{u_N(t)} \quad (13)$$

The labor wedge is zero at efficient allocation. Let $\mathcal{M}(b, s) = (1 + \hat{\pi}^N(b, s))\hat{\pi}^N(b, s)$ be the monetary policy rule of future governments that the current government takes as given, and $\{\mathcal{C}(b, s), \mathcal{L}(b, s), \mathcal{B}(b, s), \mathcal{V}(b, s)\}$ the equilibrium functions that return the values of the corresponding variables under that policy rule. Taking these functions as given, the government's time-consistent problem in recursive form is:

$$\mathcal{V}(b, s) = \max_{c^T, c^N, \ell, b', p^N, \pi^N, \mu} u [c(c^T, c^N), \ell] + \beta \mathbb{E}_{s'|s} \mathcal{V}(b', s') \quad (14)$$

$$s.t. \quad c^N = \alpha(p^N)c^T \quad (15)$$

$$c^N = \left[1 - \frac{\varphi}{2}(\pi^N)^2\right] A\ell \quad (16)$$

$$c^T = y^T + b - \frac{b'}{R} \quad (17)$$

$$\frac{b'}{R} \geq -\kappa(p^N A\ell + y^T) \quad (18)$$

$$\mu = u_T(c, \ell) - \beta R \mathbb{E}_{s'|s} u_T(\mathcal{C}(b', s'), \mathcal{L}(b', s')) \quad (19)$$

$$\mu \times [b' + \kappa(p^N A\ell + y^T)] = 0, \quad \mu \geq 0 \quad (20)$$

$$0 = \varphi \pi^N (1 + \pi^N) + (\varepsilon - 1)[1 - z^{-1}(1 - \omega)] - \varphi \ell^{-1} \mathbb{E}_{s'|s} \Lambda [\mathcal{L}(b', s') \mathcal{M}(b', s')] \quad (21)$$

where the variables z_t and ω_t are respectively defined in (7) and (13). (15) is an intratemporal implementability constraint, (16) is the resource constraint for the non-tradable good, (17) is the resource constraint for the tradable good, (18) is the credit constraint, (19)-(20) are the intertemporal implementability constraints associated with households' borrowing choices, and

(21) is the intertemporal implementability constraint associated with firms' pricing decision (i.e., the non-linear New Keynesian Philipps Curve). I denote the multiplier on (17), (18), (19) by $\lambda^* \geq 0$, $\mu^* \geq 0$ and $v \geq 0$. Note that λ^* and μ^* differ from λ and μ .

Aggregate demand externality Factoring the nontradable good's demand (15) and resource constraint (16) into the period utility function, an indirect utility function can be defined as $V(t) \equiv u \left[c(c^T, \alpha(p^N)c^T), (1 - \frac{\varphi}{2}(\pi^N)^2)^{-1} \alpha(p^N)c^T \right]$. The social marginal value of the tradable good in equilibrium is given by:

$$V_T(t) = \left[1 + \left(\frac{p_t^N y_t^N}{c_t^T} \right) \left(\omega_t - \frac{\varphi}{2}(\pi_t^N)^2 \right) \right] u_T(t) \quad (22)$$

There is thus a wedge between households' private marginal value of the tradable good and this social marginal value in equilibrium, due to the presence of an aggregate demand externality. This wedge is proportional to the labor wedge net of the cost of inflation, weighted by the relative expenditure share of the nontradable good relative to the tradable good.⁵ To understand the source of this wedge, consider a marginal increase in households' tradable good consumption. Households only value this increase according to their private marginal utility $u_T(t)$. But as they increase their consumption of tradables, households also demand more nontradables. And since price adjustment is costly, this larger demand translates into a partial price adjustment as well as into more nontradable good production. These price and production adjustments have non-internalized welfare ramifications. The relevant nontradable output multiplier is given by the relative expenditure share of the nontradable to the tradable good $p_t^N y_t^N / c_t^T$, and the price and employment adjustment detailed above create a net benefit of $\omega_t - (\varphi/2)(\pi_t^N)^2$. Therefore, the non-internalized benefit is precisely $(p_t^N y_t^N / c_t^T)(\omega_t - (\varphi/2)(\pi_t^N)^2)$, which has a marginal utility $u_T(t)$.

Pecuniary externality The second externality present in this environment is a pecuniary externality originating from the credit constraint. To describe this externality, I compare the government's and households' bond choices in the absence of nominal rigidities (i.e. for $\varphi = 0$).⁶ The government's optimal decision for bonds in sequential form can be described by the two

⁵This result is in line with [Farhi and Werning \(2016\)](#). In a similar environment where firms sets their prices once and for all, they highlight the existence of an aggregate demand externality and show that the wedge between the social and private marginal values equals the *weighted labor wedge*; that is, the labor wedge weighted by the relative expenditure share of nontradable goods relative to tradable goods.

⁶When prices are flexible ($\varphi = 0$), there is no role for the monetary policy due to the dichotomy between nominal and real variables (see [Lucas, 1972](#) and [Caplin and Spulber, 1987](#)).

equations:⁷

$$\lambda_t^* = u_T(t) + \gamma^{-1} \kappa \frac{p_t^N y_t^N}{c_t^T} \mu_t^* \quad (23)$$

$$\lambda_t^* = \beta R_t \mathbb{E}_t \lambda_{t+1}^* + \mu_t^* \quad (24)$$

The difference between government solutions relative to the one based on households can be seen by comparing (24) with the corresponding households' optimality conditions (5). The government's shadow value of tradable consumption λ_t^* is equal to the marginal utility of the tradable consumption, $u_T(t)$, plus an additional term that represents the relaxation effect on the credit constraint induced by the rise in the relative price of nontradables associated with a marginally higher tradable consumption. The credit constraint relaxation term is absent from the private condition (5). As emphasized in the normative macro-finance literature (see Korinek, 2011 and Bianchi, 2011), this wedge between social and private valuations of tradable consumption, when the credit constraint is binding, generates overborrowing at a time when it is not binding. Indeed, when the credit constraint is not currently binding, households equate the benefits $u_T(t)$ of an additional unit of borrowing to its private costs $\beta(1+r)\mathbb{E}_t u_T(t+1)$. However, the government has a higher marginal cost given by $\beta R_t \mathbb{E}_t [u_T(t+1) + \mu_{t+1}^* \Theta_{t+1}]$ with $\Theta_t = (1/\gamma)\kappa p_t^N y_t^N / c_t^T$. The additional cost, $R_t \Theta_{t+1}$, incurred by the government represents how a one unit increase in borrowing at date t tightens the ability to borrow at date $t+1$, which has a marginal utility cost of μ_{t+1}^* .

3.2 Time-Consistent Monetary Policy in Absence of Credit Frictions

I start the normative analysis by considering a case in which the economy features unconstrained access to the international credit market. This economy can be regarded as a financially robust economy in which either there is no credit constraint, or the credit coefficient κ is large enough that the credit constraint never binds.⁸ In this case, the only constraint faced by agents on their borrowing is their natural borrowing limit. The allocation of the competitive equilibrium in this economy is the same as the one described in Section 2, but with $\mu_t = 0$ for all t . This case will serve as a benchmark.

The optimal policy solves (14) subject to (15)-(17), (19) with equality $\mu_t = 0$ and (21). The optimal policy problem amounts to choosing a path for the inflation rate of the nontradable good to maximize private agents' welfare. The following lemma describes the solution to this

⁷Because if the government decisions for bonds coincide with households decisions for bonds, the implementability constraint (19) would be always satisfied (that is, the associated multiplier would be equal to zero), the multiplier on this constraint is set to zero in order to discuss the presence of the externality.

⁸Absence of a credit constraint or higher values of κ can be interpreted as representing a highly-developed financial system in which households are able to borrow against a large share of their total income.

problem.

Proposition 1 (Optimal Monetary Policy without Credit Frictions). *With no credit frictions, a price stability policy (i.e. $\pi_t^N = 0$ for all t) is the optimal monetary policy. It perfectly stabilizes the economy (i.e. $\omega_t = 0$), and the allocation satisfies (6), (11), $-u_\ell(t) = Au_N(t)$ along with $c_t^N = y_t^N = Al_t$.*

Proof. See Appendix A.1

This result can be intuited from the observation that when there are no financial market imperfections (no credit frictions), the only distortions in this economy arise from price stickiness. Hence, as is standard in the New Keynesian literature, the optimal monetary policy eliminates the inefficiency stemming from sticky prices by making price adjustments unnecessary and production supply determined. Specifically, when the economy experiences a negative tradable endowment shock, private agents feel poor and want to reduce their demand for both goods. Firms in turn aim to adjust their price downward in response to the resulting decrease in the demand for nontradables. The optimal policy thus features an exchange rate depreciation that generates expenditure switching toward the nontradable good and renders an adjustment of the price of the nontradable good unnecessary. Because the labor tax is assumed to be set optimally to completely offset monopolistic distortions, the resulting allocation is efficient.

3.3 Time-Consistent Monetary Policy without Capital Flow Taxes

I now turn to analyzing the optimal policy in an economy featuring limited access to the international credit market. The optimal policy under free capital mobility solves (14) subject to (15)-(21). The two aforementioned externalities create policy trade-offs in the presence of credit frictions, and a price stability policy is generally not optimal. The proposition below characterizes the optimal time-consistent monetary policy in this case.

Proposition 2 (Optimal Monetary Policy without Capital Flow Taxes). *In the presence of credit frictions, the path of the inflation rate under the optimal time-consistent monetary policy satisfies:*

$$\varphi \Delta_t \pi_t^N = \omega_t + (1 - \gamma^{-1}) \frac{\kappa}{u_T(t)} \mu_t^* + (\sigma - \gamma^{-1}) \frac{c_T(t)}{c_t y_t^N} c_t^N v_t$$

where Δ_t is the sum of three terms.⁹ As a result, price stability is always optimal in the knife-edge case where $\gamma = \sigma = 1$.

Proof. See Appendix A.2

⁹ $\Delta_t = \Delta_{o,t} + \mathbb{E}_t[\Lambda_{t,t+1} \ell_{t+1} (1 + \pi_{t+1}^N)] \vartheta_t \pi_{t+1}^N + (1/2) \pi_t^N$ with $\Delta_{o,t} > 0$. Appendix A.2 provides further details.

This proposition states that the optimal path of the inflation rate is a linear equation that is a function of the labor wedge, Lagrange multipliers on the credit constraint (18) and households' Euler (19), with coefficients given by substitution elasticities. The basic idea of this proposition is as follows. There are distortions arising from price stickiness (output loss and aggregate demand externalities) and from credit frictions (the pecuniary and the aggregate demand externalities). A price stability policy eliminates the inefficiency stemming from price stickiness at the costs of making private agents overborrow ex ante, and causing them to overconsume the tradable good ex post. Hence, the government has an incentive to deviate from solely granting price stability. The effectiveness of the monetary policy in correcting the pecuniary externality when the constraint is not binding depends on how an increase in the tradable consumption affects households' private marginal utility of tradables (that is, their cost of borrowing), which is captured by the difference between the intratemporal and the intertemporal elasticity of substitution. Likewise, the effectiveness of the monetary policy in correcting the demand externality when the constraint binds depends on how an increase in the tradable consumption affects the value of the collateral, which is captured by the intratemporal elasticity of substitution. The optimal time-consistent monetary policy thus strikes a compromise between price stability and financial stability.

In the discussion that follows, a boom is defined as a situation in which the labor wedge in the economy is lower than its level under flexible price allocation, in line with the general class of monetary policy open economy models.¹⁰ The remarks below provide further details on the characterization of the optimal policy.

Remark 1 (Prudential Monetary Policy). *When the credit constraint does not currently bind, a price stability policy is optimal if and only if $\gamma = 1/\sigma$ and it stabilizes the economy. Further, the current government optimally deviates from price stability policy to generate a boom if $\gamma > 1/\sigma$; and a recession if $\gamma < 1/\sigma$.*

This remark adds further insights and describes how monetary policy is used as a prudential tool. Because under a price stability policy private agents overborrow ex ante, the government does not use monetary policy solely to make any adjustments in the prices of nontradables unnecessary, but also to address the pecuniary externality. When the intratemporal elasticity of substitution is greater than the intertemporal elasticity of substitution (i.e., when $\gamma > 1/\sigma$), an expansionary monetary policy generating a boom in the nontradable sector reduces households' marginal propensity to consume the tradable good and addresses overborrowing. Likewise,

¹⁰In this class of models (see, among others, [Gali and Monacelli, 2005](#) and [Schmitt-Grohé and Uribe, 2007](#)), there is a boom when the level of output is above its natural level, which is defined as the output level under flexible price allocation.

when the intratemporal elasticity of substitution is less than the intertemporal elasticity of substitution (i.e. when $\gamma < 1/\sigma$), a contractionary monetary policy generates a recession in the nontradable sector and reduces households' marginal propensity to consume the tradable good, thus also addressing overborrowing.

In the knife-edge case where $\gamma = 1/\sigma$, the intratemporal and the intertemporal effects of the monetary policy stance on tradable consumption cancel each other out, and generating either a boom or a bust in the nontradable sector is impotent to address overborrowing. Therefore, in this case, the government only focuses on price stability and replicates flexible price allocation.

Remark 2 (Ex Post Policy). *When the credit constraint binds, the current government optimally stabilizes prices which generates a boom in the economy when $\gamma \geq 1$. When $\gamma < 1$, the current government optimally deviates from a price stability policy and generates a recession.*

When the credit constraint binds, the social marginal value of tradable consumption is $V_T(t) = [1 + (p_t^N c_t^N / c_t^T) u_T(t)]$ with $\omega_t = -\kappa \mu_t / u_T(t) < 0$, and households over-consume the tradable good. The government then faces a trade-off between price stability and the relaxation of the credit constraint to address the aggregate demand externality. When the intratemporal elasticity of substitution is greater than 1 ($\gamma \geq 1$), the quantity effect dominates the price effect ($\partial p_t^N y_t^N / \partial y_t^N < 0$). Thus, in order to relax the constraint, the government would need to create a boom, which would amplify the effect of the aggregate demand externality. Therefore, the current government has no incentive to deviate from a price stability policy. In contrast, when the intratemporal elasticity of substitution is less than 1 ($\gamma < 1$), the price effect dominates the quantity effect. The current government then finds it optimal to deviate from a price stability policy to create a recession in order to sustain the value of the collateral, which in turn mitigates the aggregate demand externality.

Discussion An important result of this section is that in an economy featuring an endogenous sudden mechanism; that is, in the presence of credit frictions, a price stability policy is optimal under special conditions, $\gamma = 1/\sigma \geq 1$. While monetary policy is countercyclical in almost all developed countries, [Vegh and Vuletin \(2012\)](#) document that it is procyclical in 51 percent of developing countries (24 countries out of 47) in their sample for the period 1960-2009.¹¹ This section highlights conditions under which procyclical monetary policy can be optimal in an economy in which financial markets are imperfect, capital flows freely and the government lacks commitment. [Remarks 1](#) and [2](#) together show that the optimal time-consistent monetary policy is procyclical when the intratemporal elasticity is greater than the

¹¹The procyclicality of the monetary policy is measured by the correlation between the cyclical components of short-term interest rates and real GDP.

intertemporal elasticity and the former is less than 1 ($1/\sigma < \gamma < 1$). The basic idea here is that lowering the degree of nominal rigidity is not always welfare increasing due to the tradeoff between price and financial stability documented in this environment. During crisis periods (that is periods in which the constraint binds) a contractionary monetary policy slackens the credit constraint when $\gamma < 1$. In these circumstances, at the margin there are gains from moving away from a price stability policy to relax the credit constraint and restrain capital outflows. Furthermore, in advance of crisis periods (that is periods in which the credit constraint does not bind), a procyclical monetary policy induces private agents to accumulate a desirable level of precautionary savings. Once again, at the margin the gain from this policy is higher than the cost of inflation.

3.4 Time-Consistent Monetary Policy with Capital Flow Taxes

I now consider the possibility of the government having an additional policy instrument consisting in capital flow taxes and look at how the availability of this additional tool affects the optimal design of monetary policy. The budget constraint of households is:

$$c_t^T + p_t^N c_t^N + \frac{b_{t+1}}{R_t(1 + \tau_t^b)} = y_t^T + w_t \ell_t + \phi_t + b_t + \mathcal{T}_t$$

where \mathcal{T}_t represents a lump-sum transfer that households receive from the government. The households' Euler equation for bonds becomes:

$$u_T(t) = \beta R_t(1 + \tau_t^b) \mathbb{E}_t u_T(t+1) + \mu_t \tag{25}$$

Given quantities, the tax on debt can be used to back out households' Euler equation. It turns out that allowing the government to use capital taxes is equivalent to assuming that it controls the credit operations of households and rebates back the proceeds of the transactions in a lump-sum fashion. Hence, the government's optimal policy problem amounts to choosing the path for the inflation rate and making debt choices for households, while allowing them to choose their labor supply and their allocation of consumption between tradable goods and nontradable goods in a competitive way. The government's optimization problem thus reduces to solving (14) subject to (15)-(18) and (21).¹² The following proposition characterizes the optimal monetary policy in an economy with credit frictions when capital flow taxes are used optimally.

Proposition 3 (Optimal Policy under Availability of Capital Flow Taxes). *When capital flow*

¹²Appendix A.3 shows that any allocation that satisfies (15)-(18) and (21) also satisfies the general equilibrium.

taxes are available, the path of the inflation rate under the optimal time-consistent monetary policy satisfies:

$$\varphi \Delta_t \pi_t^N = \omega_t + (1 - \gamma^{-1}) \frac{\kappa}{u_T(t)} \mu_t^*,$$

Proof. See Appendix [A.3](#)

This proposition provides insight into how the availability of capital flow taxes changes the optimal monetary policy. When the credit constraint binds, the design of the optimal monetary policy is quite similar to that in an economy where capital flow taxes cannot be used. In that case, the optimal time-consistent monetary policy is a compromise between two objectives: correcting nominal rigidities (macroeconomic stability) and relaxing the credit constraint to correct aggregate demand externalities (financial stability). The government optimally stabilizes prices to create a boom in the nontradable sector if the intratemporal elasticity of substitution is less than 1 ($\gamma < 1$), and deviates from a price stability policy to create a recession if $\gamma > 1$. However, monetary policy is qualitatively different from the case where capital flow taxes are not available when the credit constraint does not currently bind. In this case, capital flow taxes appear to be the preferred tool for correcting the pecuniary externality.

The proposition also shows that the conditions under which a price stability policy is optimal in the absence of capital flow taxes are a subset of those when capital flow taxes are available and used optimally. In the latter, the optimal time-consistent monetary policy is a price stability policy when $\gamma \geq 1$, while in the former a price stability policy is optimal only under the more restrictive aforementioned conditions ($\gamma = 1/\sigma \geq 1$).

4 Quantitative Analysis

This section evaluates the quantitative implications of the model. I solve numerically for the problem of the government, under both free capital mobility and availability of capital flow taxes, using global non-linear methods. I also compute numerically the competitive equilibrium in which the monetary policy is characterized by a price stability policy.

4.1 Calibration

The model is calibrated to annual data from Argentina. Preferences in this section are specified following [Greenwood et al. \(1988\)](#) where the argument of $u(\cdot)$ is the composite commodity $c_t - g(\ell_t)$ and $g(\ell_t)$ is a function that measures the disutility of the labor supply. This formulation of preferences allows international real business cycle models to explain observed business cycle facts, and delivers realistic/empirically plausible, dynamics for employment in emerging

economies. The functional forms for preferences are:

$$u(c - g(\ell)) = \frac{(c - g(\ell))^{1-\sigma}}{1-\sigma}, \text{ and}$$

$$g(\ell) = \chi \frac{\ell^{1+\theta}}{1+\theta},$$

The international interest rate is set to 4 percent, a standard value for the world risk-free interest rate in the real business cycle literature for small open economies. The coefficient of relative risk aversion is set to $\sigma = 2$, also a standard value. In line with evidence by [Kimball and Shapiro \(2008\)](#), the Frisch elasticity of labor supply $1/\theta$ is set to 1. The labor disutility parameter χ is set so that mean employment in the nontradable sector is equal to 1, which requires $\chi = 0.69$. The value of the total factor productivity (TFP) in the nontradable sector is normalized to 1. The elasticity of substitution among differentiated intermediate goods ε is set to 7.66, corresponding to a 15% net markup that is in the range found by [Diewert and Fox \(2008\)](#). I also set the Rotemberg price adjustment cost parameter to ensure that nominal prices are sticky for three quarters on average, which requires $\varphi = 62$.¹³ The process for the exogenous driving forces $s_t = (y_t^T, R_t)$ is taken to be a first-order bivariate autoregressive process

$$\begin{bmatrix} \ln(y_t^T) \\ \ln\left(\frac{R_t}{\bar{R}}\right) \end{bmatrix} = \rho_s \begin{bmatrix} \ln(y_{t-1}^T) \\ \ln\left(\frac{R_{t-1}}{\bar{R}}\right) \end{bmatrix} + \begin{bmatrix} \epsilon_t^T \\ \epsilon_t^R \end{bmatrix} \quad \text{where} \quad [\epsilon_t^T, \epsilon_t^R] \sim i.i.d. N(0, \Sigma_\epsilon^2)$$

I estimate this process by OLS with the risk-free rate and the cyclical component of tradable GDP from the World Development Indicators for the 1965-2014 period. The risk-free rate is measured by a U.S. real interest rate (Treasury-bill rate, deflated with expected U.S. CPI inflation). The tradable endowment is measured with the cyclical component of value added in agriculture and manufacturing. The OLS estimates of ρ_s and Σ_ϵ are respectively

$$\hat{\rho}_s = \begin{bmatrix} 0.6663 & -0.5238 \\ 0.0842 & 0.7861 \end{bmatrix}, \quad \hat{\Sigma}_\epsilon = \begin{bmatrix} 0.0028889 & -0.0001182 \\ -0.0001182 & 0.0001648 \end{bmatrix} \quad \text{and} \quad \bar{R} = 1.0219$$

The vector of shocks is discretized into a first-order Markov process, with seventeen points, using the quadrature-based procedure of [Tauchen and Hussey \(1991\)](#). The mean of the endowment is set to 1 without loss of generality.

¹³The parameter φ is set to ensure that the first-order approximation of the New-Keynesian Phillips curve (NKPC), equation (10) in the Rotemberg model, is equivalent to the one in a Calvo model where firms keep their price unchanged with a probability δ . The log-linear version of the NKPC is: $\hat{\pi}_t^N = \beta \hat{\pi}_{t+1}^N + \bar{\kappa} \hat{y}_t^N$, where \hat{y}_t^N represents the output gap, and $\bar{\kappa} = (\varepsilon - 1)/\varphi$ in the Rotemberg model and $\bar{\kappa} = (1 - \delta)(1 - \beta\delta)/\delta$ in the Calvo model.

Table 1: Calibration

Description	Parameter-Value	Source/Target
Risk aversion	$\sigma = 2$	Standard value
Frisch elasticity parameter	$\theta = 1$	Kimball and Shapiro (2008)
Elasticity of substitution	$\gamma = 0.83$	Conservative value
Monopoly power	$\varepsilon = 7.66$	15% net markup
Adjustment cost parameter	$\varphi = 62$	Three quarter of price stickiness
Discount factor	$\beta = 0.905$	Average NFA-GDP ratio = -29%
Weight on tradables in CES	$a = 0.315$	Share of tradable output = 32%
Collateral coefficient	$\kappa = 0.319$	Frequency of crisis = 5.5%
Labor disutility coefficient	$\chi = 0.686$	Mean labor = 1
TFP in nontradable sector	$A = 1$	Normalization

The parameters $\{\gamma, \beta, a, \kappa\}$ are calibrated following the baseline calibration of [Bianchi \(2011\)](#). The calibration strategy consists in choosing values for the parameters so that the model economy in which the monetary policy is characterized by a price stability policy matches some key aspects of the Argentina data. The intratemporal elasticity of substitution between tradable and nontradable goods, γ , is set to a conservative value of 0.83. The three parameters $\{\beta, a, \kappa\}$ are respectively set so that the long-run moments of the competitive equilibrium match the following three historical moments of the data: (i) an average net foreign asset position to GDP of -29 percent, (ii) a share of tradable goods in production of 32 percent and (iii) an observed frequency of 5.5 percent of "Sudden Stops". Sudden Stops are defined as events in which the credit constraint binds, and this leads to an increase in net capital outflows that exceeds one standard deviation. This approach leads to $\beta = 0.905$, $a = 0.315$ and $\kappa = 0.319$.

4.2 Policy Functions

I start by analyzing the policy functions under different monetary regimes. I consider an *inflation targeting regime* in which the policy rule is set to stabilize the producer price index by offsetting all of the distortions originating from nominal rigidities, and there is no tax on capital. This regime captures the price stability objective of central banks. I also consider *time-consistent regimes* with and without capital flow taxes, under which the monetary policy is characterized by the optimal time-consistent monetary policy rules derived in section [3.3](#).

Figure [1](#) plots the decision rules as a function of the current holdings of bonds for a negative one-standard deviation shock. The presence of the endogenous borrowing constraint generates a kink in the policy function, and the bond decision rule is non-monotonic. I distinguish two different regions in each panel below focusing on the change in the slope of the borrowing decision rule under an inflation targeting regime, which occurs at the point where the credit

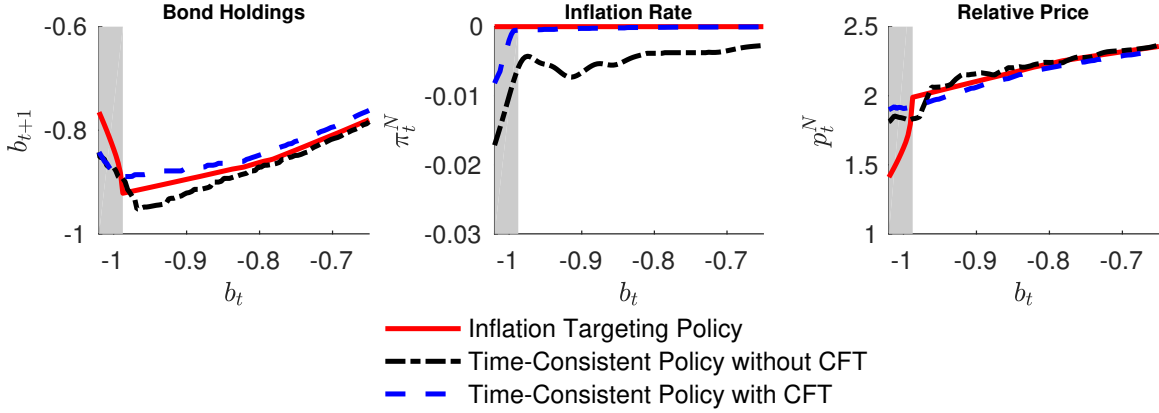


Figure 1: Decision rules for negative one-standard-deviation shocks

constraint binds. The solid red line corresponds to the bond decision rule under an inflation targeting regime. The dashed blue line and the dashed-dotted black line corresponds to the bond decision rule under a time-consistent regime, with and without capital flow taxes, respectively.

The constrained region (shaded region) represents the situations in which the current debt level is sufficiently high such that the credit constraint binds under an inflation targeting regime. In this region, under an inflation targeting regime, by the market clearing condition for the tradable good (11) for a given choice of next-period level of debt, an increase in the current level of debt would imply a decrease in tradable consumption. Notice that because the intratemporal elasticity of substitution is less than 1 ($\gamma < 1$), the price effect dominates the quantity effect. Thus, the decrease of the relative price of nontradables dominates the increase in the demand of nontradables that accompanies the decrease in tradable consumption, which means that the next-period level of debt must be decreased further to satisfy the credit constraint.

Under both time-consistent regimes, the government uses its policy instrument (here, the exchange rate) to increase the relative price of nontradables and sustain the value of the collateral, which means that households can borrow more. Moreover, the increase in the relative price of nontradables implies a lower marginal cost. Thus, for a given future government policy, monopolistically competitive firms reduce prices of the nontradable good by equation (10), which in turn generates a lower level of inflation when compared to its level under the inflation targeting regime.

In the unconstrained region, which corresponds to the state-space where the credit constraint does not bind, under a monetary regime that replicates flexible price allocation (an inflation targeting regime), households do not accumulate sufficient precautionary savings, due to the presence of the pecuniary externality. Under time-consistent regimes, the government also uses its policy instruments to increase the cost of borrowing and prevent a larger drop in households' borrowing ability if the credit constraint becomes binding in the next period. Without capital

flow taxes (CFT), given future policies lowering the exchange rate level to increase the relative price of nontradables has two effects: an intertemporal effect that reduces the cost of borrowing since it is relatively cheaper to purchase debt, and an intratemporal effect that increases the cost of borrowing since the nontradable good is relatively more expensive. As shown in Proposition 2, since the intertemporal effect dominates ($\sigma > 1/\gamma$), the optimal policy is expansionary and thus generates a higher level of debt. Furthermore, a higher relative price of nontradables implies that firms face lower marginal costs and then adjust their nominal prices downward according to their optimal pricing decision given by equation (10). The inflation rate under an optimal time-consistent regime without capital flow taxes thus turns out to be lower than under an inflation targeting regime. Under the time-consistent regime with capital flow taxes, the government uses taxes to control for households' credit operations and the monetary policy focuses only on stabilizing prices, as implied by Proposition 3. Thus, households accumulate uniformly lower levels of debt, which in turn help contain the rise in the relative price of nontradables under a flexible price allocation and prevent a larger drop in households' borrowing ability if the credit constraint becomes binding in the next period.

4.3 Monetary Regimes and the Dynamics of Financial Crises

This section analyzes the costs and gains associated with the adoption of an inflation targeting regime as opposed to a time-consistent regime, in an economy in which capital flows are free.

4.3.1 Economic Behavior During Crises

To describe the effectiveness of a time-consistent regime in reducing the severity of crises, I construct an event analysis using simulated data and analyze the dynamics of the economy during financial crises. A financial crisis is defined as a period in which the credit constraint is binding, and in which the current account is one standard deviation above its mean in the ergodic distribution corresponding to the economy under each monetary regime.

The construction of the event analysis follows the procedure proposed by Bianchi and Mendoza (forthcoming). First, the model economy under an inflation targeting regime is simulated for 500,000 periods. After dropping the first 1,000 periods and identifying all of the crisis events under an inflation targeting regime, I construct five-year event windows centered in the period in which the crisis takes place. Then at each period, I compute averages for each simulated variable across the event windows in each year $t - 2$ to $t + 2$, and produce the economic dynamic under an inflation targeting regime. An initial value for bonds and a five-year sequence of tradable realizations is determined by calculating the median initial debt at $t - 3$, and the median tradable shock across a cross-section of crisis events. Finally, I feed this sequence of shocks

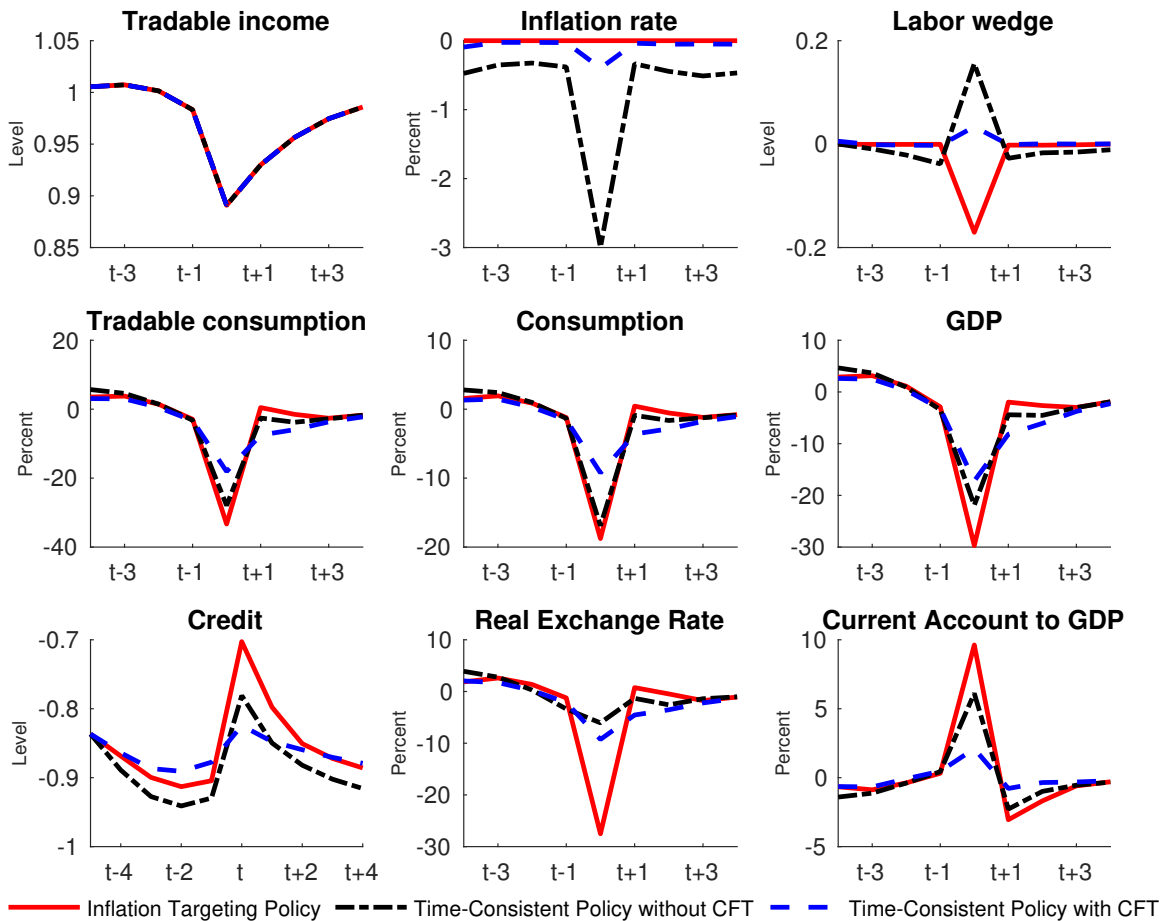


Figure 2: Comparison of crises dynamics

Note: The real exchange rate, tradable consumption, consumption and GDP are expressed in percentage deviations from averages in the ergodic distribution.

and initial value of bonds into the decision rules of the model economy under time-consistent regimes and compute the corresponding endogenous variables.¹⁴ The model's predictions during financial crises for both monetary regimes is depicted by Figure 2.

The top middle panel shows that, under the time-consistent regime without capital flow taxes, there is a negative inflation rate in the run-up to crises. The reduction of the inflation level (-3.0 percent) is more important in the year of the crisis. In this way, the government allows for more credit access, and the economy under this monetary regime features more debt than the economy under an inflation targeting regime during crises but also in the years before and after the occurrence of the crises (see the bottom right panel). As a result, consumption of the tradable good falls by a much smaller percentage during crises than it does the under the inflation targeting regime (-28.1 vs. -33.3 percent). This relatively large fall of tradable

¹⁴This procedure ensures that the dynamics under each model economy are simulated using the same initial state and the same sequence of shocks.

consumption under the inflation targeting regime arises because the binding credit constraint forces households to reduce their next period level of debt, as captured by the sharp reversal of the current account-to-GDP ratio. Under the time-consistent regime, the government generates a boom ahead of financial crises and a recession during crises, as measured by the negative labor wedge, to sustain the value of the collateral. As a consequence, the fall in the real exchange rate is smaller during crises (-6.0 vs. -27.5 percent under the inflation targeting regime), which in turn mitigates the drop-in output and absorption during crises. The middle panel of Figure 2 shows that, in contrast to a time-consistent regime, decline in the total output and consumption is larger under inflation targeting: total output and consumption drop by 21.9 and 18.0 percent, respectively, under the time-consistent regime (vs. 29.7 percent for total output and 18.8 percent for consumption under the inflation targeting regime).

4.3.2 Economic Behavior ahead of Potential Crises

I now turn to analyzing the impact of monetary policy on debt accumulation and the frequency of crises in sudden stop-prone economies. In the absence of capital flow taxes, because households in an economy under an inflation targeting regime fail to accumulate sufficient precautionary savings due to the pecuniary externality, the government optimally deviates from a price stability policy to sustain the value of the collateral ahead of a potential financial crisis. Therefore, as shown in figure 3, the economy under the time-consistent regime is likely to have higher debt than the economy under an inflation targeting regime. Formally, there is a 7.6 percent chance that households in the economy under the time-consistent regime carry an amount of debt larger than -1.0, which corresponds to the maximum amount of debt that households

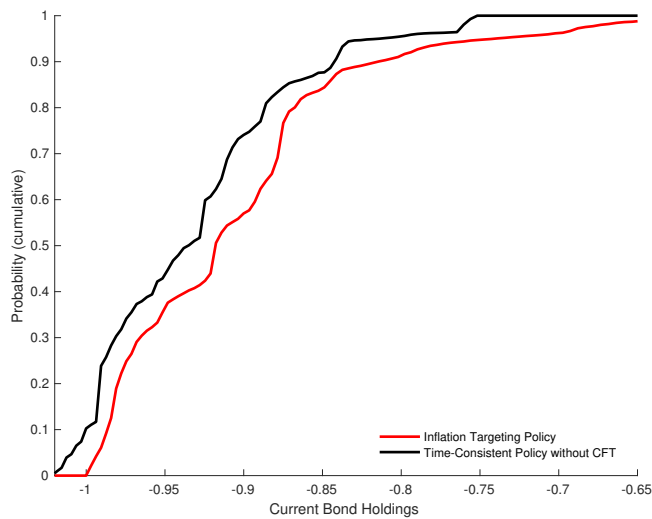


Figure 3: Distribution of bond holdings.

in the economy under an inflation targeting regime can hold. It is then apparent that the

long-run probability of financial crises is 12.7 percent under the time-consistent regime versus 5.5 percent under an inflation targeting regime.

4.3.3 Monetary Policy and Capital Controls

The use of capital controls in the form of capital flow taxes along with a time-consistent monetary policy appears to be very effective in reducing the magnitude of crises as shown in Figure 2. Ahead of potential financial crises, taxes are used to diminish households' debt and restrain the boom in tradable consumption. This in turn prevents a larger drop in households' borrowing ability during crises, as captured by the smaller reversal of the current account-to-GDP ratio. The decline in tradable consumption, aggregate consumption and total output are thus smaller than their decline under both the time-consistent regime without capital flow taxes and the inflation targeting regime. Moreover, since the government uses taxes on debt to encourage households to accumulate sufficient precautionary savings, the economy under the time-consistent regime is less vulnerable to financial crises.

Table 2: Probability and severity of crises

	Inflation Targeting		Time-Consistent	
	no CFT	with CFT	no CFT	with CFT
Probability of crises	5.5	1.3	12.7	1.1
Current-Account to GDP	9.6	2.4	6.2	2.1
Real exchange rate depreciation	27.5	11.4	6.1	8.3
Output				
average	-29.7	-17.8	-21.9	-17.2
$\mathbb{P}(\hat{y} < -20\%)$	87.7	24.4	80.3	14.8
Consumption				
average	-18.8	-9.6	-18.0	-9.2
$\mathbb{P}(\hat{c} < -15\%)$	83.3	8.1	78.9	7.2

Note: Consumption, output and real exchange rate are expressed in percentage deviations from averages in the corresponding ergodic distribution.

Table 2 highlights the importance of capital flow taxes in reducing the long-run frequency of crises regardless of the monetary regime. The long-run probability of crises is 1.1 percent under a time-consistent regime with capital flow taxes, and 1.3 percent under an inflation targeting regime with capital flow taxes (vs. 5.5 percent under an inflation targeting regime, and 12.7 percent under a time-consistent regime). Table 2 also points out the role of capital flow taxes in reducing the severity of crises. Because monetary policy is less effective in correcting the pecuniary externality, the drop in capital inflows is more pronounced when capital flows taxes are not available. The probability that the decline in total output will exceed 20 percent is

80.3 percent under a time-consistent regime, and 14.8 percent under a time-consistent regime with capital flow taxes (87.7 percent under inflation targeting vs. 24.4 percent under inflation targeting with capital flow taxes). Further, the probability that the consumption will drop by more than 15 percent is 78.9 percent under a time-consistent regime, and 7.2 percent under a time-consistent regime with capital flow taxes (83.3 percent under inflation targeting vs. 8.1 percent under inflation targeting with capital flow taxes). These results from Table 2 suggests that monetary policy should be supplemented with capital flows taxes.

Another important result is that the design of the monetary policy also affects the optimal tax rate on debt. It is apparent that the optimal tax rate under the time-consistent regime is higher than under the inflation targeting regime. On average, tax on debt is about 6.1 percent under a time-consistent regime and 5.0 percent under an inflation targeting regime. This finding can be inferred from the fact that under a time-consistent regime, the government uses its monetary policy to allow for more credit access during crises. Further, because future choices of bond holdings affect current optimal choices, the government needs to raise (relatively) more taxes in order to cause households to accumulate a socially desirable level of debt. This additional result suggests that monetary policy and macroprudential policy are complementary.

4.4 Long-run moments

The table below depicts unconditional second moments computed using the ergodic distribution for the economy under each monetary regime considered. In line with the macro-finance literature, this model, which incorporates an occasionally binding credit constraints, accounts for some key regularities of the business cycles of emerging countries: the variability in GDP is higher than the variability in consumption and the strong procyclicality of capital flows. Furthermore, Table 3 points toward strong effects of monetary regimes on the volatility of the macroeconomic indicators, especially GDP, consumption, unemployment, and real exchange rate.

The first result from Table 3 enhances the existence of a trade-off between the variability in GDP and employment. Indeed, compared to the economy under a time-consistent regime, the economy under an inflation targeting regime features lower business cycle variability in employment and consumption, pointing to the importance of price stability policy in absorbing shocks and stabilizing employment. By stimulating employment and reducing the variability in the relative price of nontradables during episodes of financial crises, a time-consistent regime lowers the volatility of the real exchange rate, the current account-to-GDP ratio, and the trade balance-to-GDP ratio. Table 3 further highlights the strong role of capital flow taxes in stabilizing the economy regardless the monetary regime considered. As discussed by [Bianchi \(2011\)](#)

Table 3: Second Moments

	Inflation Targeting		Time-Consistent		Data
	no CFT	with CFT	no CFT	with CFT	
Standard Deviations					
Consumption	5.4	5.2	7.3	5.7	6.2
Employment	2.3	2.8	5.3	3.4	2.9
Real Exchange Rate	8.5	6.7	6.5	5.9	8.2
Current Account-GDP	2.9	1.3	2.0	1.2	3.0
Trade Balance-GDP	3.1	1.4	2.1	1.2	2.8
Correlation with GDP					
Consumption	0.93	0.98	0.88	0.96	0.88
Employment	0.80	0.99	0.69	0.95	0.74
Current Account-GDP	-0.65	-0.54	-0.59	-0.54	-0.63

Notes: Data are annual from WDI and Global Financial Data (GFD). Data period covers 1965-2014.

and [Fornaro \(2015\)](#), there are two main reasons for this to happen: the probability of crises is much larger in an economy where capital flow taxes are not available, and the externalities interfere with households' desire to smooth consumption over time. Clearly, when there is a bad shock and the credit constraint binds, households are forced to reduce their debt, which in turn generates a countercyclical current account balance-to-GDP ratio. Thus, the correlation of the current account-to-GDP ratio with the GDP is key to explaining the role played by capital flow taxes in smoothing consumption. From [table 3](#), the lowest countercyclicality of the current account-to-GDP ratio is obtained when capital flow taxes are used optimally. The consumption smoothing therefore works better under a monetary regime with capital flow taxes.

4.5 Welfare Effects

The welfare implications of a monetary regime for each initial state, denoted $\lambda(b, s)$, is defined as the percent variation in the lifetime consumption stream that equalizes the expected utility of an household living in the economy under an inflation targeting regime (IT) to the expected utility of an household living in an economy under the alternative monetary regime (GP) considered. Formally, for each initial state (b, s) , the welfare implications of the alternative monetary regime are computed as follows:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{IT} (1 + \lambda) - g(\ell_t^{IT})) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{GP} - g(\ell_t^{GP}))$$

where c_t^s and ℓ_t^s denote consumption and labor supply allocations under the monetary regime $s \in \{IT, GP\}$. Figure 4 depicts the welfare gains of moving away from an inflation targeting regime to an alternative monetary regime as a function of the current level of bond holdings, and for negative, one standard deviation shocks.

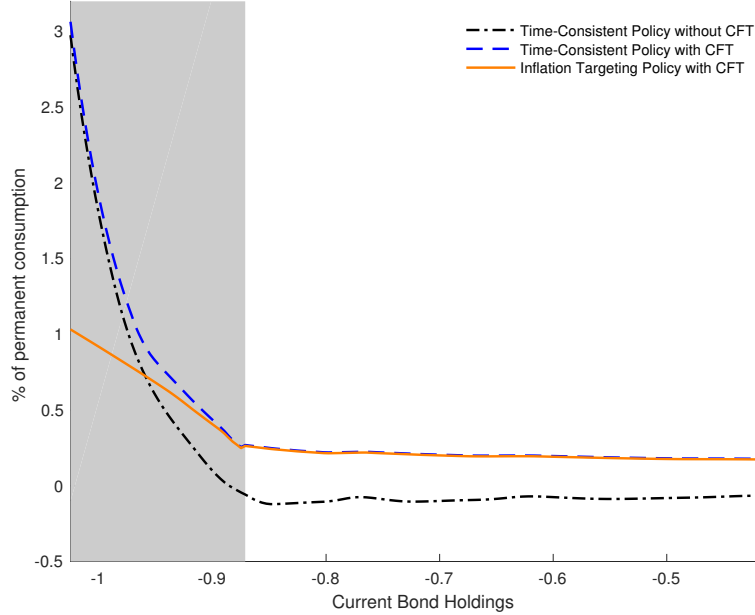


Figure 4: Welfare gains/costs of moving away from inflation targeting regime.

First, in the unconstrained region, the monetary regimes with capital flows taxes deliver larger welfare gains. This is because with capital flow taxes used by the government to address the pecuniary externality, households living in an economy under one of the monetary regimes with capital flow taxes act in a more precautionary way and the welfare increases. Furthermore, Figure 4 shows that there are welfare losses from adopting a time-consistent regime without capital flow taxes. The rationale is that under a time-consistent regime, the government selects the best action given the current situation, which does not result in the social objective function being maximized. Rather, by relying on the inflation targeting policy rules, economic performance is improved (Kydlund and Prescott, 1977). Secondly, in the region where the credit constraint binds, comparing the welfare effects under the time-consistent regime without capital flow taxes and under the inflation targeting regime with capital flow taxes, it is apparent that monetary policy is more effective in correcting the externalities than capital flow taxes. In this region, the time-consistent monetary policy sustains the value of the collateral, allows for more credit access and thus significantly improve the welfare. It is also important to notice that there are benefits from using capital controls under the inflation targeting regime, in contrast to the previous studies (e.g. Bianchi, 2011 and Ottonello, 2015) where the welfare gain from using capital flow taxes under a policy that replicates the flexible price allocation arises only

in relation to how future allocations will differ. The reason is that, in this environment, when the credit constraint binds, capital flow taxes are used to offset the intratemporal distortion in the labor supply decision and stabilize the economy.

I also calculate the average welfare gain $\bar{\lambda}$ as the average $\lambda(b, s)$ computed with the ergodic distribution in the economy under the inflation targeting regime. Because the time-consistent monetary policy without capital flow taxes only delivers welfare gains in the constrained region, and the economy under the inflation targeting regime without capital flow taxes spends less than 16 percent of the time in this region, on average there is a welfare cost of adopting the time-consistent regime rather than the inflation targeting regime, which corresponds to 0.04 percent of permanent consumption. Another key result of this welfare analysis indicates the importance of capital flows taxes in changing the desirable property of the time-consistent regime. The time-consistent regime with capital flow taxes delivers the largest welfare gain (0.27 percent of permanent consumption). In comparison, the welfare gain from using capital flow taxes under the inflation targeting regime is 0.21 percent of permanent consumption.

5 Conclusion

In a dynamic stochastic general equilibrium model of an open economy featuring nominal rigidities and credit constraint that limits borrowing to a fraction of borrowers' current income, I study the optimal time-consistent monetary policy. Theoretically, the optimal time-consistent monetary policy is expressed in close form as a linear equation that is a function of the labor wedge and variables that account for financial conditions before and during periods in which the credit constraint binds. A price stability policy is not generally optimal. In particular, when the intratemporal elasticity is greater than the intertemporal elasticity and the former is less than one, the optimal time-consistent monetary policy is procyclical. Conditions under which a price stability policy is optimal are less restrictive when capital flow taxes are available. Quantitatively, relative to the outcome of a simple inflation targeting regime, financial crises are more likely to occur but will be less severe when they do under a monetary regime that implements the time-consistent policy. Moreover, the quantitative analysis suggests that there is much to be gained when monetary policy and macroprudential regulation such as capital controls are conducted jointly. Capital controls are found to be more effective in correcting the externality stemming from credit frictions, which in turn reduce excessive risk exposure of the economy, and result in higher social welfare.

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Appendix

A Proofs

The government's time-consistent problem in recursive form is rewritten here for convenience:

$$\mathcal{V}(b, s) = \max_{c^T, \ell, b', p^N, \pi^N, \mu} u [c (c^T, \alpha(p^N)c^T), \ell] + \beta \mathbb{E}_{s'|s} \mathcal{V}(b', s') \quad (\text{A.1})$$

$$\alpha(p^N)c^T = \left[1 - \frac{\varphi}{2}(\pi^N)^2\right] Al \quad (\text{A.2})$$

$$c^T = y^T + b - \frac{b'}{R} \quad (\text{A.3})$$

$$\frac{b'}{R} \geq -\kappa (p^N Al + y^T) \quad (\text{A.4})$$

$$\mu = u_T(c, \ell) - \beta R \mathbb{E}_{s'|s} u_T(\mathcal{C}(b', s'), \mathcal{L}(b', s')) \quad (\text{A.5})$$

$$\mu \times [b' + \kappa (p^N Al + y^T)] = 0 \quad (\text{A.6})$$

$$\mu \geq 0 \quad (\text{A.7})$$

$$0 = \varphi \pi^N (1 + \pi^N) + (\varepsilon - 1) [1 - z^{-1}(1 - \omega)] - \varphi \ell^{-1} \mathbb{E}_{s'|s} \Lambda [\mathcal{L}(b', s') \mathcal{M}(b', s')] \quad (\text{A.8})$$

Let $\lambda^* \geq 0$ and $\mu^* \geq 0$ be the multiplier on the resource constraint for tradables and the credit constraint respectively; $\iota \geq 0$, $v \geq 0$, δ , $\chi \geq 0$ and ξ be the multiplier on (A.2), (A.5)-(A.7), (A.8). I define an auxiliary variable: $\psi \equiv (\varepsilon - 1) [1 - z^{-1}(1 - \omega)] - \varphi \ell^{-1} \mathbb{E}_{s'|s} \Lambda [\mathcal{L}(b', s') \mathcal{M}(b', s')]$.

A.1 Proof of Proposition 1

In the absence of credit frictions, the government's optimisation problem reduces to solving (A.1) subject to (A.2), (A.3), (A.5) with equality $\mu_t = 0$ and (A.8). The proof proceeds by analyzing a relaxed problem where the government is not subject to (A.5) and then showing that this condition is satisfied. Abstracting from the implementability constraint (A.5), the government's optimality conditions, after eliminating the multiplier ι in the key equations, are:

$$c_t^T :: \quad \lambda_t^* = \mathcal{V}_T(t) + \xi_t \left(\psi_{T,t} + \frac{\ell_t}{c_t^T} \psi_{\ell,t} \right) \quad (\text{A.9})$$

$$b_{t+1} :: \quad \lambda_t^* = \beta R_t \mathbb{E}_t \lambda_{t+1}^* + \xi_t \psi_{b',t} \quad (\text{A.10})$$

$$p_t^N :: \quad c_t^N u_N(t) \left(\omega_t - \frac{\varphi}{2} (\pi_t^N)^2 \right) = \xi_t (\gamma^{-1} p_t^N \psi_{p,t} - \ell_t \psi_{\ell,t}) \quad (\text{A.11})$$

$$\pi_t^N :: \quad \xi_t = \frac{\varphi \ell_t y_t^N}{1 + 2\pi_t^N} \pi_t^N \quad (\text{A.12})$$

Consider now a price stability policy, $\pi_t^N = 0$ for all t . Under this policy, by equation (A.12), it follows that $\xi_t = 0$. Substituting into (A.11) implies that $\omega_t = 0$, and into (A.9) implies that $\lambda_t^* = u_T(t)$. Using the latter together with (A.10) leads to:

$$u_T(t) = \beta R_t \mathbb{E}_t u_T(t+1).$$

The implementability constraint (A.5) is then satisfied. Therefore, a price stability policy is optimal and it stabilizes the economy (i.e. $\omega_t = 0$).

A.2 Proof of Proposition 2

To characterize the optimal time-consistent monetary policy, I solve for the government's optimization problem (A.1) given that future path of the inflation rate \mathcal{M} are chosen by future government with which are associated with policies $\{\mathcal{C}(b, s), \mathcal{L}(b, s), \mathcal{B}(b, s), \mathcal{V}(b, s)\}$.

The optimality conditions of the government problem (A.1), after eliminating the multiplier ι in the key equations, are given by:

$$c_t^T :: \quad \lambda_t^* = \mathcal{V}_T(t) + \kappa \frac{p_t^N y_t^N}{c_t^T} \tilde{\mu}_t^* - (1 + p_t^N \alpha(p_t^N)) u_{TT}(t) v_t + \xi_t \left(\psi_{T,t} + \frac{\ell_t}{c_t^T} \psi_{\ell,t} \right) \quad (\text{A.13})$$

$$b_{t+1} :: \quad \lambda_t^* = \beta R_t \mathbb{E}_t \lambda_{t+1}^* + \tilde{\mu}_t^* + \beta R_t \mathbb{E}_t \frac{\partial u_T(t+1)}{\partial b_{t+1}} v_t + \xi_t \psi_{b,t} \quad (\text{A.14})$$

$$\mu_t :: \quad v_t + \delta_t [b_{t+1} + \kappa (p_t^N y_t^N + y_t^T)] + \chi_t + \xi_t (\varepsilon - 1) (1 - \omega_t) z_t^{-2} \frac{\kappa}{u_T(t)} = 0 \quad (\text{A.15})$$

$$p_t^N :: \quad y_t^N u_N(t) \left(\omega_t - \frac{\varphi}{2} (\pi_t^N)^2 \right) + (1 - \gamma^{-1}) \tilde{\mu}_t^* \kappa p_t^N y_t^N \\ + (\sigma - \gamma^{-1}) \frac{c_T(t)}{c_t} c_t^N u_N(t) v_t = \xi_t (\gamma^{-1} p_t^N \psi_{p,t} - \ell_t \psi_{\ell,t}) \quad (\text{A.16})$$

$$\pi_t^N :: \quad \xi_t = \frac{\varphi \iota_t y_t^N}{1 + 2\pi_t^N} \pi_t^N \quad (\text{A.17})$$

These expressions are obtained by assuming that the policies and value functions are differentiable, with $\tilde{\mu}_t^* = \mu_t^* + \delta_t \mu_t$ that represents the government's effective shadow value on the credit constraint. It can also be shown that $\tilde{\mu}_t^* = \mu_t^*$ for all t . To clearly see this, notice first that when the constraint does not binds (i.e. $\mu_t^* = 0$), the implementability constraint (A.6) is always satisfied. Thus, setting δ_t to zero is optimal and it follows that $\tilde{\mu}_t^* = \mu_t^* = 0$. When the constraint binds, the implementability constraint (A.6) implies that $\mu_t = 0$ and again $\tilde{\mu}_t^* = \mu_t^*$.

Combining (A.16) with (A.17) and rearranging the expression yields

$$\varphi \Delta_t u_N(t) \pi_t^N = u_N(t) \omega_t + (1 - \gamma^{-1}) \kappa p_t^N \mu_t^* + (\sigma - \gamma^{-1}) \frac{c_T(t)}{c_t y_t^N} c_t^N u_N(t) v_t \quad (\text{A.18})$$

To simplify the notation, I define $\tilde{\iota}_t \equiv \iota_t / (1 + 2\pi_t^N)u_N(t)$, the auxiliary variable Δ_t is then given by $\Delta_t = \tilde{\iota}_t(\gamma^{-1}p_t^N\psi_{p,t} - \ell_t\psi_{\ell,t}) + (1/2)\pi_t^N$, which can be rewritten as:

$$\Delta_t = \Delta_{o,t} + \left(\frac{-c_t^N u_{NN}(t)}{u_N(t)} - 1 \right) \frac{\tilde{\iota}_t}{\ell_t} \mathbb{E}_t[\Lambda \ell_{t+1}(1 + \pi_{t+1}^N)] \pi_{t+1}^N + \frac{1}{2} \pi_t^N$$

where

$$\Delta_{o,t} = (\varepsilon - 1) \frac{-z_t^{-1} \tilde{\iota}_t u_{\ell}(t)}{A u_N(t)} \left[\frac{\gamma^{-1} \kappa p_t^N \mu_t - c_t^N u_{NN}(t)}{z_t u_N(t)} + \ell_t \frac{u_{\ell\ell}(t)}{u_{\ell}(t)} \right] > 0$$

The equilibrium under the optimal time-consistent monetary policy can be characterized by sequences $\{c_t^T, c_t^N, \ell_t, b_{t+1}, \mu_t, \pi_t^N, p_t^N\}_{t=0}^{\infty}$ that satisfy (4)-(8), (10), (11) along with the complementary slackness condition and (A.18).

Because the optimal time-consistent monetary policy is characterized by a fixed point in the policy rules chosen by the government, to characterize conditions under which a price stability policy is optimal in this environment, I start by assuming that future governments choose their policy instrument to stabilize prices (i.e. $\pi_{t+1}^N = 0$). Then, I characterize conditions under which the current government stabilizes prices. Finally, I show whether the current government deviates from a price stability policy to create a boom or a recession in the economy.

Let's assume that $\pi_{t+1}^N = 0$. From the implementability constraint (A.5) that combines (7) and (10), it follows that:

$$\omega_t = -\varphi \frac{z_t}{\varepsilon - 1} (1 + \pi_t^N) \pi_t^N - \kappa \frac{\mu_t}{u_T(t)} \quad (\text{A.19})$$

Substituting into (A.18) implies that:

$$\begin{aligned} \varphi \left[\Delta_{o,t} + \left(z_t \frac{1 + \pi_t^N}{\varepsilon - 1} + \frac{\pi_t^N}{2} \right) \right] \pi_t^N \\ = \frac{\kappa}{u_T(t)} \left((1 - \gamma^{-1}) \mu_t^* - \mu_t \right) + (\sigma - \gamma^{-1}) \frac{c_T(t)}{c_t y_t^N} c_t^N v_t \end{aligned} \quad (\text{A.20})$$

When the credit constraint does not bind (i.e. $\mu_t^* = 0$), it follows from the complementary slackness condition (A.6) that $\mu_t = 0$ and $\tilde{\mu}_t^* = 0$. Thus, (A.20) becomes:

$$\varphi \left[\Delta_{o,t} + \left(\frac{1 + \pi_t^N}{\varepsilon - 1} + \frac{\pi_t^N}{2} \right) \right] \pi_t^N = (\sigma - \gamma^{-1}) \frac{c_T(t)}{c_t y_t^N} c_t^N v_t$$

Therefore, the current government also stabilizes prices, i.e. $\pi_t^N = 0$, if and only if $\gamma = 1/\sigma$. This policy stabilizes the economy, $\omega_t = 0$, by equation (A.19). Moreover for $\gamma > 1/\sigma$, the current government deviate from a price stability policy, $\pi_t^N > 0$. This in turn implies by (A.19) that the economy experiences a boom, $\omega_t < 0$. Conversely, for $\gamma < 1/\sigma$, the current government deviate from a price stability policy, $\pi_t^N < 0$, and the economy experiences a recession, $\omega_t > 0$.

When the credit constraint binds (i.e. $\mu_t^* \neq 0$), the optimality condition (A.15) becomes:

$$v_t = -\varphi(\varepsilon - 1)\tilde{l}_t\kappa\frac{p_t^N y_t^N - u_\ell(t)}{z_t^2} \frac{-u_\ell(t)}{u_N(t)} \pi_t^N. \quad (\text{A.21})$$

For $\gamma \geq 1$, set $\mu_t = (1 - \gamma^{-1})\tilde{\mu}_t^* \geq 0$. Substituting it along (A.21) into (A.20) implies that:

$$\varphi \left[\Delta_{o,t} + (\gamma^{-1} - \sigma)(\varepsilon - 1)\tilde{l}_t z_t^{-2} \frac{\kappa u_\ell(t)}{c_t u'(t)} + \left(\frac{1 + \pi_t^N}{\varepsilon - 1} + \frac{\pi_t^N}{2} \right) \right] \pi_t^N = 0$$

Thus, the current government optimally stabilizes prices ($\pi_t^N = 0$) which implies that $v_t = 0$.

Further, it follows from (A.19), that this policy generates a recession, $\omega_t = -\kappa\mu_t/u_T(t) \leq 0$.

For $\gamma < 1$, set $\mu_t = \max \left\{ 0, (\sigma - \gamma^{-1}) \frac{c_T(t)}{c_t y_t^N} c_t^N u_N(t) v_t \right\}$. Substituting it along (A.21) into (A.20):

$$\varphi \left[\bar{\Delta}_t + \left(\frac{1 + \pi_t^N}{\varepsilon - 1} + \frac{\pi_t^N}{2} \right) \right] \pi_t^N = \frac{\kappa}{u_T(t)} (1 - \gamma^{-1}) \mu_t^*$$

where $\bar{\Delta}_t = \Delta_{o,t} - \min \left\{ 0, (\sigma - \gamma^{-1}) \frac{c_T(t)}{c_t y_t^N} c_t^N v_t \right\} > 0$. Thus, the current government do not find optimal to stabilize prices, $\pi_t^N < 0$, which also implies that $v_t > 0$ and $\mu_t \geq 0$. This policy generates smaller boom than under a price stability policy by equation (A.19).

A.3 Proof of Proposition 3

The proof proceeds by first showing that any allocation $\{c_t^T, \ell_t, b_{t+1}, p_t^N, \pi_t^N\}$ that satisfy (16)-(18) and (21) also satisfy the general equilibrium, and then describing the optimal monetary policy and optimal tax rate.

Consider any allocation $\{c_t^T, \ell_t, b_{t+1}, p_t^N, \pi_t^N\}$ that satisfy (16)-(18) and (21). Then, set $c_t^N = \left[1 - \varphi(\pi_t^N)^2 \right] A \ell_t$ and $\mu_t = 0$ to satisfy (15) and (20) respectively. Choose $\tau_t^b = 1 - \beta R_t \frac{\mathbb{E}_t u_T(t+1)}{u(t)}$. By definition, this makes (25) hold also. The government's optimisation problem then reduces to

$$\mathcal{V}(b, s) = \max_{c^T, \ell, b', p^N, \pi^N} u \left[c(c^T, \alpha(p^N)c^T), \ell \right] + \beta \mathbb{E}_{s'|s} \mathcal{V}(b', s')$$

$$\alpha(p^N)c^T = \left[1 - \frac{\varphi}{2}(\pi^N)^2 \right] A \ell$$

$$c^T = y^T + b - \frac{b'}{R}$$

$$\frac{b'}{R} \geq -\kappa(p^N A \ell + y^T)$$

$$\varphi \pi^N (1 + \pi^N) + (\varepsilon - 1)\omega - \varphi \ell^{-1} \mathbb{E}_{s'|s} \Lambda [\mathcal{L}(b', s') \mathcal{M}(b', s')] = 0$$

Again $\iota \geq 0$, $\lambda^* \geq 0$ are the multiplier on the resource constraint for the nontradable good and the tradable good respectively, $\mu^* \geq 0$ is the multiplier on the credit constraint, $\xi \geq 0$ is the multiplier the nontradable good pricing implementability constraint. The optimality conditions of the government's problem, when capital flow taxes are available, are given by:

$$c_t^T :: \quad \lambda_t^* = \mathcal{V}_T(t) + \kappa \frac{p_t^N y_t^N}{c_t^T} \mu_t^* + \xi_t \left(\psi_{T,t} + \frac{\ell_t}{c_t^T} \psi_{\ell,t} \right) \quad (\text{A.22})$$

$$b_{t+1} :: \quad \lambda_t^* = \beta R_t \mathbb{E}_t \lambda_{t+1}^* + \mu_t^* + \xi_t \psi_{b',t} \quad (\text{A.23})$$

$$p_t^N :: \quad y_t^N u_N(t) \left(\omega_t - \frac{\varphi}{2} (\pi_t^N)^2 \right) + (1 - \gamma^{-1}) \kappa p_t^N y_t^N \mu_t^* = \xi_t (\gamma^{-1} p_t^N \psi_{p,t} - \ell_t \psi_{\ell,t}) \quad (\text{A.24})$$

$$\pi_t^N :: \quad \xi_t = \frac{\varphi \iota y_t^N}{1 + 2\pi_t^N} \pi_t^N \quad (\text{A.25})$$

Combining (A.24) with (A.25) and rearranging the expression yields

$$\varphi \Delta_t u_N(t) \pi_t^N = u_N(t) \omega_t + (1 - \gamma^{-1}) \kappa p_t^N \mu_t^* \quad (\text{A.26})$$

This complete the proof that when taxes are available and used optimally, the path for the inflation rate when the government cannot commit to future policies satisfies (A.26) as in Proposition 3. Define now the tax as:

$$\tau_t^b = \frac{\beta R_t \mathbb{E}_t \left\{ \Theta_{t+1} \mu_{t+1}^* + \xi_{t+1} \left(\psi_{T,t+1} + \frac{c_{t+1}^N}{c_{t+1}^T} \psi_{N,t+1} \right) \right\} - \Theta_t \mu_t^* - \xi_t \left(\psi_{T,t} + \frac{c_t^N}{c_t^T} \psi_{N,t} \right) + \xi_t \psi_{b',t}}{\beta R_t \mathbb{E}_t u_T(t+1)}$$

where $\Theta_t = \gamma^{-1} \kappa p_t^N y_t^N \mu_t^* / c_t^T$. The proof for the optimal tax rate then consists in showing that households' Euler equation for bond holds. Combining (A.22) and (A.24) yields:

$$\lambda_t^* = u_T(t) + \Theta_t \mu_t^* + \xi_t \left(\psi_{T,t} - \frac{1}{c_t^T} \gamma^{-1} p_t^N \psi_{p,t} \right).$$

Substituting the expression of λ_t^* into (A.23) and using $\gamma^{-1} p_t^N \psi_{p,t} = c_t^N \psi_{N,t}$, it follows that households' Euler equation for bond holds: $u_T(t) = \beta R_t (1 + \tau_t^b) \mathbb{E}_t u_T(t+1)$.

B Numerical Solution Method (Algorithm)

B.1 For Competitive Equilibrium under a Price Stability Policy

This algorithm is build on Bianchi (2011)'s algorithm that incorporates the occasionally binding endogenous constraint, modified to account for the nominal rigidities. Formally, the computation of the competitive equilibrium operates directly on the first-order conditions and requires solving for functions $\{\mathcal{B}(b, s), \mathcal{L}(b, s), \mathcal{C}^T(b, s), \mathcal{P}^N(b, s), \mu(b, s)\}$ such that:

$$\mathcal{C}^T(b, s) + \frac{\mathcal{B}(b, s)}{R} = y^T + b \quad (\text{B.1})$$

$$\alpha (\mathcal{P}^N(b, s)) \mathcal{C}^T(b, s) = A \mathcal{L}(b, s) \quad (\text{B.2})$$

$$\frac{\mathcal{B}(b, s)}{R} \geq -\kappa (A \mathcal{P}^N(b, s) \mathcal{L}(b, s) + y^T) \quad (\text{B.3})$$

$$\begin{aligned} u_T(c(b, s) - g(\mathcal{L}(b, s))) \\ = \beta R \mathbb{E}_{s'|s} \{u_T(c(\mathcal{B}(b, s), s') - g(\mathcal{L}(\mathcal{B}(b, s), s)))\} + \mu(b, s) \end{aligned} \quad (\text{B.4})$$

$$u_N(c(b, s) - g(\mathcal{L}(b, s))) + \frac{1}{A} u_\ell(c(b, s) - g(\mathcal{L}(b, s))) = -\kappa \mathcal{P}^N(b, s) \mu(b, s) \quad (\text{B.5})$$

where $c(b, s) \equiv c(\mathcal{C}^T(b, s), A \mathcal{L}(b, s))$. The steps for the algorithm are the following:

1. Generate discrete grids $G_b = \{b_1, b_2, \dots, b_M\}$ for the bond position and $G_s = \{s_1, s_2, \dots, s_N\}$ for the shock state space, and choose an interpolation scheme for evaluating the functions outside the grid of bonds. The piecewise linear approximation is used to interpolate the functions and the grid for bonds contains 200 points.
2. Conjecture $\mathcal{B}_h(b, s), \mathcal{L}_h(b, s), \mathcal{C}_h^T(b, s), \mathcal{P}_h^N(b, s), \mu_h(b, s)$ at time $H, \forall b \in G_b$ and $\forall s \in G_s$.
3. Set $i = 1$
4. Solve for the values of $\mathcal{B}_{h-i}(b, s), \mathcal{L}_{h-i}(b, s), \mathcal{C}_{h-i}^T(b, s), \mathcal{P}_{h-i}^N(b, s), \mu_{h-i}(b, s)$ at time $h - i$ using (B.1)-(B.5) and $\mathcal{B}_{h-i+1}(b, s), \mathcal{L}_{h-i+1}(b, s), \mathcal{C}_{h-i+1}^T(b, s), \forall b \in G_b$ and $\forall s \in G_s$:
 - (a) First, assume that the credit constraint (B.3) is not binding. Set $\mu_{h-i}(b, s) = 0$ and using (B.4), (B.5) and a root finding algorithm solve for $\mathcal{C}_{h-i}^T(b, s)$ and $\mathcal{L}_{h-i}(b, s)$. Solve for $\mathcal{B}_{h-i}(b, s)$ and $\mathcal{P}_{h-i}^N(b, s)$ using (B.1) and (B.2).
 - (b) Check whether $\frac{\mathcal{B}_{h-i}(b, s)}{R} \geq -\kappa (A \mathcal{P}_{h-i}^N(b, s) \mathcal{L}_{h-i}(b, s) + y^T)$ holds. If the credit constraint is satisfied, move to the next grid point.
 - (c) Otherwise, using (B.1), (B.3), (B.4), (B.5) and a root finding algorithm solve for $\mu_{h-i}(b, s), \mathcal{B}_{h-i}(b, s), \mathcal{C}_{h-i}^T(b, s)$ and $\mathcal{L}_{h-i}(b, s)$ and using (B.2) solve for $\mathcal{P}_{h-i}^N(b, s)$.

5. Convergence. The competitive equilibrium is found if $\|\sup_{B,s} x_{h-i}(b,s) - x_{h-i+1}(b,s) < \epsilon\|$ for $x \in \{\mathcal{B}, \mathcal{L}, \mathcal{C}^T\}$. Otherwise, set $x_{h-i}(b,s) = x_{h-i+1}(b,s)$, $i \approx i + 1$ and go to step 4.

B.2 For Optimal Time-Consistent Monetary Policy

The solution method proposed here uses a nested fixed point algorithm to solve for optimal time-consistent monetary policy and is related to the literature using Markov perfect equilibria (e.g. [Kein et al. \(2008\)](#) and [Bianchi and Mendoza \(forthcoming\)](#)). In the inner loop, using the Bellman equation and value function iteration, solve for value function and policy functions taking as given future policies. Formally, given functions $\{\mathcal{C}^T(b,s), \mathcal{P}^N(b,s), \mathcal{B}(b,s), \mathcal{L}(b,s), \mathcal{M}(b,s)\}$, the Bellman equation is given by:

$$\mathcal{V}(b,s) = \max_{c^T, \ell, b', p^N, \pi^N, \mu} u [c(c^T, \alpha(p^N)c^T) - g(\ell)] + \beta \mathbb{E}_{s'|s} \mathcal{V}(b', s') \quad (\text{B.6})$$

$$s.t. \quad \alpha(p^N)c^T = \left[1 - \frac{\varphi}{2}(\pi^N)^2\right] A\ell \quad (\text{B.7})$$

$$c^T = y^T + b - \frac{b'}{R} \quad (\text{B.8})$$

$$\frac{b'}{R} \geq -\kappa(p^N A\ell + y^T) \quad (\text{B.9})$$

$$\mu = u_T(c, \ell) - \beta R \mathbb{E}_{s'|s} u_T(c(\mathcal{C}^T(b', s'), \mathcal{P}^N(b', s')) - g(\mathcal{L}(b', s'))) \quad (\text{B.10})$$

$$\mu \times [b' + \kappa(p^N A\ell + y^T)] = 0 \quad (\text{B.11})$$

$$\varphi \pi^N (1 + \pi^N) - (\epsilon - 1)[z^{-1}(1 - \omega) - 1] - \varphi \ell^{-1} \mathbb{E}_{s'|s} \Lambda[\mathcal{L}(b', s') \mathcal{M}(b', s')] = 0 \quad (\text{B.12})$$

Given the solution to the Bellman equation, update future policies as the outer loop. The steps for the algorithm are the following:

1. Generate discrete grids $G_b = \{b_1, b_2, \dots, b_M\}$ for the bond position and $G_s = \{s_1, s_2, \dots, s_N\}$ for the shock state space, and choose an interpolation scheme for evaluating the functions outside the grid of bonds. The piecewise linear approximation is used to interpolate the functions and the grid for bonds contains 200 points.
2. Guess policy functions $\mathcal{B}, \mathcal{C}^T, \mathcal{P}^N, \mathcal{M}$ at time H , $\forall b \in G_b$ and $\forall s \in G_s$.
3. For given $\mathcal{L}, \mathcal{C}^T, \mathcal{P}^N, \mathcal{M}$ solve the recursive problem using value function iteration to find the value function and policy functions:
 - (a) First, assume that the credit constraint (B.9) is not binding. Set $\mu = 0$ – (B.11) is thus satisfied – and solve the optimization problem (B.6) subject to (B.7), (B.8), (B.10), (B.12) using a Newton type algorithm and check whether (B.9) holds.

- (b) Second, assume that the credit constraint (B.9) is binding – (B.11) is thus satisfied. Solve the optimization problem (B.6) subject to (B.7)-(B.10), (B.12) using a Newton type algorithm.
- (c) Compare the solutions in (a) and (b). The optimal choices in each state is the best solution. Denote $\{\nu^i\}_i$, with $i \in \{b', \ell, c^T, p^N, \pi^N\}$, the associated policy functions.
4. Evaluate convergence. Compute the sup distance between $\mathcal{B}, \mathcal{C}^T, \mathcal{P}^N, \mathcal{M}$ and $\{\nu^i\}$, with $i \in \{b', c^T, p^N, \pi^N\}$. If the sup distance is not smaller enough (higher than $\epsilon = 1e - 7$), update $\mathcal{B}, \mathcal{C}^T, \mathcal{P}^N, \mathcal{M}$ and solve again the recursive problem.

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