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On the determination of the return to capital when there is no exogenous ownership in firms

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Résumé

Cet article est consacré à l'étude de la détermination du rendement du capital lorsqu'il n'existe pas de propriété exogène des entreprises. Hahn et Solow ont proposé une notion d'équilibre bien adaptée à ce cas. Nous étudions cette notion dans un cadre d'équilibre partiel (le taux de rendement du capital est donné).

Nous présentons une notion de capital compatible ainsi que celle de capital parfaitement compatible avec un taux de rendement donné. Un stock de capital est compatible avec un rendement donné si le ratio des recettes nettes sur ce capital est égal au taux de rendement. Les stocks de capital parfaitement compatibles avec un taux de rendement sont les stocks de capital compatibles qui maximisent les « profits purs », i.e., la différence entre le profit et le produit du capital par son taux de rendement.

Nos hypothèses incluent mais ne sont pas limitées à celles faites dans le cas standard (dans lequel les ensembles de production vérifient la propriété de rendements d'échelles constants). Nous proposons plusieurs exemples de ces notions de capital compatibles.

Les résultats de cet article pourront être utilisés dans l'étude des économies ouvertes ou bien dans des analyses d'équilibre général (en faisant l'hypothèse que les entreprises sont identiques).

Mots clés: équilibre de Hahn et Solow, capital compatible.

Abstract

This paper addresses the issue of the determination of return to capital when there is no exogenous ownership in firms. Hahn and Solow have proposed a notion of equilibrium for this setting and we consider this notion from a partial equilibrium point of view (i.e. under the assumption that the rate of return to capital is given).

We present a study of the notion of compatible and perfectly compatible capital stocks with a given rate of return. A capital stock is compatible with a given rate of return if it yields a gross operating surplus per unit of capital equal to this rate of return. Perfectly compatible capital stocks are capitals which are compatible and which maximize "pure profit", i.e. the difference between profit and the product of the rate of return times the capital stocks. Our assumptions encompass but are not reduced to the standard case where production sets exhibit constant returns to scales. Several examples of independent interest are presented. The results of this paper could be useful in the study of small open economies, or in a general equilibrium setting (with the usual assumption that firms are symmetric).

Key words: Hahn and Solow equilibrium, compatible capital stock.

1. Introduction

In the Arrow-Debreu model of an economy (see Arrow and Debreu (1954)), agents are exogenously given the ownership of firms. Hence, potential pure profits are given to agents according to their shareholdings in firms.

Now, consider a different economy where there are pre-existing firms (which are identified with different production sets), but where there is no exogenously given ownership in firms. Also, assume that inputs are owned by agents and that a particular input, which we call capital, endows its owners with the right to receive firms profits (proportionnally to the total capital). Hence the capital is not only an input but it is also an asset. Since there is no given ownership in firms, this seems to be as sensible way to share firms' profits[1] .

This kind of economy have been used and studied from two different points of view. The first one is that of the theory of general equilibrium with incomplete markets. It originates in a paper by Diamond (1967) [2] and is carefully exposed in the book of Magill and Quinzii (1996). In their section 31, pages 356-357, these authors study what they call a partnership equilibrium. An agent becomes a partner in a venture by contributing to a share of the capital cost which gives him the right to the corresponding share of a random income stream. The key difficulty in this context is for partners to decide how the venture is to be operated.

The second view point is that of dynamic macroeconomic theory, and was presented by Hahn and Solow (1995) (pages 70-71), who write: "...we take it to be characteristic of capitalist firms that their profits go to the suppliers of capital. We assume, therefore, that savings ... are used to buy shares in the gross operating surplus of firms...".

Hahn and Solow have also proposed an equilibrium notion which is useful when ownership in firms is endogenous. This notion relies on the rate of return to capital, that is, the ratio of the gross operating surplus of firms to the capital. It requires that in equilibrium, all firms using capital must provide the same (average) rate of return (otherwise, there would exist profitable trade-offs). Notice that marginal rates of returns are not necessary equalized, except in special, but standard cases. Also, there is no price per se of capital (though it is used as input).

The idea underlying this notion of equilibrium is that firms are more or less passive with regard to the demand of capital. Capital is provided by savers who are in fact only interested in the (average) returns of capital. This is not in constrast with the real world, where agents seem to take average rates of return as given (but of course, arbitrage eliminates differences of *average* rates of return).

The two view points are not directly comparable since they apply to different set ups. However, when there is no uncertainty, a partnership equilibrium "reduces" to a Hahn and Solow equilibrium (every active venture yields the same rate of return).

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[1] Pure profits arise in several interesting settings, e.g. imperfect competition, when there are non constant returns to scales, or when firms received some transfers, like free pollution permits.

[2] We thank J. Drèze for this reference.

In the spirit of Hahn and Solow, we shall be interested in what we called capitals compatible with a given rate of return (we shall always assume that there is no uncertainty). A capital stock is compatible with a given rate of return if its yields a gross operating surplus per unit of capital equal to this rate of return. We also introduce the notion of perfectly compatible capital, namely, capital stocks that maximize "pure profit", i.e. the difference between profit and the product of the rate of return times the capital stocks. We propose to study the properties of compatible and perfectly compatible capital stocks when the rate of return if given. We hope that the results of this paper will be useful in studies of small open economy and in general equilibrium analysis (under the standard assumption that firms are symetric).

The organization of the paper is as follows. In the next section, we study the standard case where profit per unit of capital is constant (this case arises with a standard neoclassical technology, when firms do no receive any transfers [4]). We show that either all (positive) capital stocks are perfectly compatible, or none of them are. We also provide a characterization of perfectly compatible capital stocks. In section 3, we pay more attention to the case where the average profit per capital is a monotonic function. Several examples - of independent interest - are presented (which illustrate the cases of gross operating surplus per unit of capital that are decreasing or increasing functions of capital). Section 4 adresses the case where the profit per capital is a non-monotonic function of capital. Section 5 contains some concluding remarks.

2. A Notion of capital stock compatible with a given return

2.1 Definitions

We consider a production set $\mathcal{Y} \subset \mathbb{R}^{n+1}$ that includes the capital as input (as usual, if $y \in \mathcal{Y}$, negative components are interpreted as inputs while positive components are interpreted as outputs). Given the prices or prices functions[5] of the other goods $X \in \mathbb{R}^n$, we note $G(-K, X)$ the receipts corresponding to the vector $(-K, X)$ in $\mathcal{Y}$ when the capital stock K is given. We define the profit feasible with the capital stock K, $\Pi(K)$, as the supremum of the receipts that are feasible with K, i.e.,

$$\Pi(K) = \sup\{G(-K, X); (-K, X) \in \mathcal{Y}\}$$

If $\mathcal{Y}$ is closed and G is continuous, then $\Pi(.)$ is lower semi continuous. If for any compact subset of $]0, \infty[$, the supremum is realized at some value X_K which belongs to some compact subset of $\mathbb{R}^n$, then $\Pi(.)$ is continuous (see Berge (1997)).

We shall assume:

[4] Magill and Quinzii favors the assumption of constant returns to scale, which is perfectly sensible in their framework.

[5] Prices functions appear when the firm is non-competitive (see below, example (3.2)). For a competitive firm $G(-K, X)$ is equal to $P.X$, where P is a price vector.

H1. $\Pi(.)$ *is finite and continuous on the set* $\mathbb{R}_{++}$ *of positive real numbers. Moreover, it is positive for some capital stock:* $\exists K_0 > 0$, $\Pi(K_0) > 0$.

Definition 1. Let $R > 0$. *A capital stock* $K_R > 0$ *is compatible with the return* R *if the average feasible profit with* K_R *is equal to* R: $\pi_A(K_R) = \frac{\pi(K_R)}{K_R} = R$.

Given a rate of return R, we call *pure profit* the difference $\Pi(K) - RK$. Note that pure profit is nil at K_R when K_R is compatible with R.

Definition 2. Let $R > 0$. *A compatible capital stock* K_R *is perfectly compatible with* R *if it is compatible with* R *and maximizes the pur profit:* $\max_{K>0} \Pi(K) - RK = \Pi(K_R) - RK_R = 0$

This simply means that R is a "super-gradient" of Π at K_R: for all positive K, $\Pi(K) - \Pi(K_R) \geq R(K - K_R)$. When $\Pi(.)$ is differentiable this implies that the marginal productivity is equal to the average productivity.

A perfectly compatible capital stock is solution of the standard problem faced by firm that gets its capital in a competitive market (with price R). Given this price the firm zero-maximizes the pure profit at the capital K_R.

2.2 The standard case

The standard case occurs for a competitive firm with constant returns to scale. If $\mathcal{Y}$ is a closed cone and $G(.)$ is linear-homogenous, then for all positive K setting $Z = \frac{1}{K}X$, one has:

$$\begin{aligned} (-K, X) &= K(-1, Z) \\ (-K, X) \in \mathcal{Y} &\Leftrightarrow (-1, Z) \in \mathcal{Y} \\ G(-K, X) &= KG(-1, Z) \\ \Pi(K) &= K \sup\{G(-1, Z); (-1, Z) \in \mathcal{Y}\} \end{aligned}$$

Assuming that the supremum $R_0 = \sup\{G(-1, Z); (-1, Z) \in \mathcal{Y}\}$ is positive and finite, $\Pi(K) = R_0 K$ is a linear function of K. Then *any* positive capital stock is perfectly compatible with R_0. When $R \neq R_0$, no positive capital stock is compatible with R. Hence, in this standard case *any compatible capital stock is perfectly compatible.*

2.3 Existence of compatible capital stocks

If $\Pi(K)$ is non-linear, i.e. $\Pi_A(K) = \Pi(K)/K$ is non-constant, then there exists a whole interval of values of the return R for which there exist at least one compatible capital stock compatible with R.

Proposition 1. *Assume H1. Let* R *be a positive number such that:*

$$\inf_{K>0} \Pi_A(K) < R < \sup_{K>0} \Pi_A(K) \tag{1}$$

Then, there exists at least one positive capital stock which is compatible with R. Any such capital stock is not perfectly compatible with R.

Proof. By continuity, the function $\Pi_A(K) = \Pi(K)/K$ takes any value intermediate R between the infimum and the supremum of its range $\Pi_A(\mathbb{R}_{++})$. For any such a solution K_R of $\Pi_A(K_R) = R$, there exists K_1 such that $\Pi_A(K_1) > R$ and this implies that:

$$\sup_{K>0} \Pi(K) - RK \geq \Pi(K_1) - RK_1 = K_1(\Pi_A(K_1) - R) > 0$$

This proves the proposition. Q.E.D.

2.4 Existence of perfectly compatible capital stocks

Proposition 2. *A positive capital stock K^* is perfectly compatible with $R^* > 0$ if and only if the supremum of $\Pi_A(\mathbb{R}_{++})$ is realized at K^* and is equal to R^*, i.e.,*

$$R^* = \max_{K>0} \Pi_A(K) = \Pi_A(K^*)$$

Proof. If $K^* > 0$ is perfectly compatible with R^*, then, for any positive K we have:

$$\Pi_A(K) - R^* = \frac{1}{K}(\Pi(K) - R^*K) \leq 0 = \frac{1}{K^*}(\Pi(K^*) - R^*K) = \Pi_A(K^*) - R^*$$

Hence, $\Pi_A(K) \leq \Pi_A(K^*)$.

Conversely, if for any positive capital stock K,

$$\Pi_A(K) \leq R^* = \Pi_A(K^*)$$

then we have

$$\Pi(K) - R^*K = K(\Pi_A(K) - R^*) \leq 0 = K^*(\Pi_A(K^*) - R^*) = \Pi(K^*) - R^*K^*$$

This proves the proposition. Q.E.D.

Remark 1. If K^* is perfectly compatible with R^*, then for any $R > R^*$, and any $K > 0$, we have $\Pi(K) < RK$. This stems from: $R > \sup_{K>0} \Pi_A(K) \geq \Pi(K)/K$ for all positive K.

Proposition 3. *Assume H1. Assume also that the supremum of $\Pi_A(.)$ is not the limit sup of this function when K tends to 0 or to $+\infty$. Then there exists a perfectly compatible stock with $R^* = \sup_{K>0} \Pi_A(K)$.*

Note that the limits of $\Pi_A(.)$ when K tends to 0 or $+\infty$ may not exist. Then the limit assumption in the proposition writes: $\limsup_{K\to+\infty} \Pi_A(K) < \sup_{K>0} \Pi_A(K)$, and $\limsup_{K\to 0} \Pi_A(K) < \sup_{K>0} \Pi_A(K)$. This implies that the two lim sup are finites.

Proof. Let R_0 be smaller that $\sup_{K>0} \Pi_A(K)$ and larger that $\limsup_{K\to 0} \Pi_A(K)$ and $\limsup_{K\to+\infty} \Pi_A(K)$. Then there exist K_1 and K_2, $0 < K_1 < K_2$ such that:

$\sup_{0<K<K_1} \Pi_A(K) < R_0$ and $\sup_{K>K_2} \Pi_A(K) < R_0$. Under H1, $\Pi_A(.)$ is continuous and realizes its maximum (which is larger than R_0) in $[K_1, K_2]$, say at point K_0^*. Defining, $R^* \equiv \Pi_A(K_0^*) = \sup_{K>0} \Pi_A(K)$, it follows that K_0^* is perfectly compatible with R^*. Q.E.D.

3. Monotonic feasible average profit

In this section we analyze compatible and perfectly compatible capital stocks under the assumption that the average profit is monotonic. In subsections 3.2. and 3.3 we shall present economic example illustrating decreasing and increasing average profit functions.

A sufficient condition for the average profit function to be decreasing is: strict concavity of $\Pi(.)$ on $\mathbb{R}_+$ and $\Pi(0) \geq 0$. Indeed, in this case, $\triangle(K) \equiv \frac{\Pi(K)-\Pi(0)}{K}$ is decreasing and $\Pi_A = \triangle + \frac{\Pi(0)}{K}$ is also clearly decreasing.

3.1 Existence and uniqueness of compatible capital stocks

Proposition 4. *Assume H1. If the function Π_A is strictly monotonic on $\mathbb{R}_{++}$ (either increasing or decreasing), then there exits a unique positive capital stock compatible with $R > 0$ if and only if R satisfies (1). Moreover, there is no capital stock perfectly compatible with R.*

Proof. In the case of a decreasing (increasing) function Π_A, the extrema of $\Pi_A(K)$ are its limits when K goes to 0 and ∞ and are not realized by any positive K. Thus, strict monotonicity implies that the solution $K_R > 0$ of $\Pi_A(K) = R$ is unique and satisfies (1). Since there exists no finite positive $\tilde{K}_1$ satisfying $\Pi_A(\tilde{K}_1) = \sup_{K>0} \Pi_A(K)$, there is no perfectly compatible capital stock (see Proposition 2). Q.E.D.

Remark 2. The notion of perfectly compatible stock can be extended by taking the limit of $\Pi_A(.)$ when K goes to 0 when this limit is finite. If Π_A is decreasing, the supremum of $\Pi_A(\mathbb{R}_{++})$ is equal to its limit when K goes to 0 and this limit $\Pi_A(0_+)$ is necessarily non-negative (finite or infinite). If $R_0^* = \Pi_A(0_+)$ is finite, we have the following property:

$$\forall K > 0, \Pi(K) - R_0^* K = K(\Pi_A(K) - R_0^*) \leq 0 = 0(\Pi_A(0_+) - R_0^*)$$

i.e. the maximum of pure profit with R_0^* (and with any $R \geq R_0^*$) is equal to 0 and is "realized" with a zero capital stock. In this case, our definition can be extended as follows: the zero capital stock is said to be *perfectly compatible in the limit* with R_0^*. Notice that in the standard basic case, $\Pi_A(K) = R_0$ for all positive K, the "demand" for capital includes $K = 0$ for all $R \geq R_0$. Such extension may be useful for defining /considering a firm making no pure profit and using zero capital as input.

3.2 An example with a non-competitive firm

Assume that a firm produces a good according to a linear-homogenous production function $Y = F(K, L)$, $F : \mathbb{R}^2_{++} \to \mathbb{R}_{++}$. Assume also that there is an inverse demand curve $p(Y)$, $p : \mathbb{R}_{++} \to \mathbb{R}_+$, which is decreasing with respect to Y.

For all positive K and l, we may define $\phi(K,l) = p(KF(1,l))F(1,l) - wl$ where $l = L/K$ is the labor to capital ratio.

The maximum of the (net receipts) is defined by:

$$\Pi(K) = \sup_{l>0} K\phi(K,l) = K\Pi_A(K)$$

We shall assume that for all $K > 0$, the supremum of $\phi(K,l)$ with respect to l is realized at $l_K > 0$: $\Pi_A(K) = \phi(K, l_K)$.

Let us show that Π_A is decreasing. Let $0 < K_1 < K_2$. There exists l_i such that: $\Pi_A(K_i) = \phi(K_i, l_i)$, $i = 1, 2$. For $l = l_2$, $\phi(K, l_2)$ is decreasing with respect to K. So, $\Pi_A(K_2) = \phi(K_2, l_2) < \phi(K_1, l_2) \leq \Pi_A(K_1)$.

Remark: The existence of a competitive firm which uses capital and labor as inputs may determinate the rate R. Assume indeed that there exists a competitive which produces a good (taken as numéraire) with a standard neo-classical technology ($Y_0 = F^0(K,L)$). Its average profit is determined by the wage w from the factor prices frontier and we have:

$$\Pi_A^0 = \max_l F^0(1,l) - wl \equiv R_0$$

The existence of an equilibrium with these two firms implies that there exists K such that: $\Pi_A^*(K) = R_0$ (see the standard case considered above).

3.3 Examples with competitive firms

3.3.1. Pollution permits

We consider a firm with a production function $F : \mathbb{R}_+^3 \to \mathbb{R}_+$, that uses three inputs, capital (K), labor (L) and emissions of pollutants (E). We assume that $F(.)$ is linear-homogenous. There is a market for pollution permits and it is assumed that the firm is endowed with an amount of $\overline{E}$ permits (see Montgomery (1972), Ono (2002)). Given a positive wage rate w and a positive price q of a permit, the net receipts write: $F(K,L,E) - wL - q(E - \overline{E})$. Let K be a positive capital stock. Define $l = L/K$ and $e = E/K$. We assume that $S \equiv \sup_{l>0,e>0} F(1,l,e) - wl - qe$ is finite. Then: $\Pi(K) = SK + q\overline{E}$, and $\Pi_A(K) = S + q\frac{\overline{E}}{K}$ is decreasing with respect to K.

3.3.2. Fixed costs

Now we consider a firm that is with a linear-homogenous production function $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ that uses capital (K) and labor (L). We assume that in order to produce, the firm must bear a fixed cost μ. The net receipts write: $pF(K,L) - wL - \mu$. Consider $K > 0$ and assume that $S \equiv \sup_{l \geq 0} pF(1,l) - wl$ is finite. Then, $\Pi(K) = KS - \mu$ and $\Pi_A(K) = S - \frac{\mu}{K}$. In this case, $\Pi_A(K)$ is increasing with respect to K.

4. The non-monotonic case

This section investigates the case where the function $\Pi_A(.)$ is no more monotonic (nor constant as in the standard case in section 2). In the preceding section we have seen that whenever the average profit is monotonic there exists a unique capital stock compatible with a given return R (it there is any). There are no perfectly compatible capital stock. Assuming non-monotonicity of the average profit changes yields different results.First we show that if there is a capital stock compatible with R, there is largest capital stock compatible with this R. Second, we provide a sufficient condition for the existence of a perfectly compatible stock with a given return.

4.1. Some properties of compatible capital stocks

Let us start with the following result:

Proposition 5. *Assume H1. Let a positive rate of return R be given. Then there is a largest capital stock compatible with R if there exists a positive capital stock K_0 and* $\limsup_{K\to+\infty}\Pi_A(K) < R \leq \Pi_A(K_0)$.

Proof. Suppose that $R > \limsup_{K\to+\infty}\Pi_A(K)$. Then there is a number K_1 such that: for all $K \geq K_1$, $\Pi_A(K) < R$. This proves that the set of capital stocks compatible with R is upper-bounded. By assumption, there is a capital stock such that $\Pi_A(K_0) \geq R$. Then, under H1, there exists a compatible capital stock. Let $0 < \overline{K}_R$ be the supremum of the capital stocks compatible with R. Suppose that $\overline{K}_R$ is *not* compatible with R. By definition, $\overline{K}_R$ is an accumulation point of the set of compatible stocks. Hence, there is a sequence $(K_n)_n$ of compatible stocks that converges to $\overline{K}_R$. For all n, one has: $\Pi_A(K_n) = R$. Under H1, $\Pi_A(.)$ is continuous and one has: $\Pi_A(\overline{K}_R) = R$. Hence $\overline{K}_R$ is compatible, contradicting our assumption. Hence, $\overline{K}_R$ is indeed compatible. Q.E.D.

Remark 4. Note that when $\Pi(.)$ is non-decreasing with respect to K, Π achieves its maximum on the set of compatible point at point $\overline{K}_R$ (indeed, for all compatible capital stocks, $K \leq \overline{K}_R \Rightarrow \Pi(K) \leq \Pi(\overline{K}_R)$). This gives the maximum of profit under the constraint that the capital yields an average return equal to R. Of course the solution of this problem is unique when $\Pi(.)$ is increasing.

Proposition 6. *Assume H1. Assume also that there is $K_0^* > 0$ such that $\nu_0 = \Pi(K_0^*) - R_0K_0^* = \max_{K\geq 0}\Pi(K) - R_0K > 0$. Then for all R such that $R_0 < R \leq R_0 + \frac{\nu_0}{K_0^*}$, there exists a largest capital stock compatible with R.*

Proof. Indeed, let R be such that $R_0 < R \leq R_0 + \frac{\nu_0}{K_0^*}$. One has: $\Pi(K_0^*) - RK_0^* \geq \Pi(K_0^*) - R_0K_0^* - \nu_0 = 0$. Notice that for all $K > 0$, $\Pi(K) - R_0K \leq \Pi(K_0^*) - RK_0^* = \nu_0$. Hence:

$$\Pi_A(K) \leq R_0 + \frac{\nu_0}{K}$$

It follows that: $\limsup_{K+\infty} \Pi_A(K) \leq R_0$. Hence one can applies Proposition 4 to get the result. Q.E.D.

4.2. Another example with fixed costs

We consider the case of a firm which is price taker and whose technology is described by a Cobb-Douglas production function: $(K, L) \in \mathbb{R}_+^2 \mapsto AK^\alpha L^\beta \in \mathbb{R}_+$, where $\alpha > 0$ and $\beta > 0$, $\alpha + \beta < 1$, $A > 0$. Assuming a fixed cost μ, the receipt function writes: $pAK^\alpha L^\beta - wL - \mu$ (w being equal to the wage rate). The profit maximizing choice of L, K being given, is such that: $p\beta AK^\alpha L^{\beta-1} = w$. The profit function may be written: $\Pi(K) = BK^\lambda - \mu$ where: $\lambda = \frac{\alpha}{1-\beta} < 1$ (for $\lambda \geq 1$, $\Pi_A(K)$ is increasing). Clearly, the average profit $\Pi_A(K) = BK^{\lambda-1} - \mu K^{-1}$ is non-monotonic. We have: $\frac{d\Pi_A}{dK} = K^2[\mu - (1-\lambda)BK^\lambda]$. We have $\frac{d\Pi_A}{dK} > (<) 0$ if $K^\lambda < (>) \frac{\mu}{(1-\lambda)B}$.

When there is free entry, it is possible to find a perfectly compatible capital stock which is $K^\lambda = \frac{\mu}{(1-\lambda)B}$.

5. Conclusion

In this paper, we have studied the notion of compatible and perfectly compatible capital stocks. In the standard case, profit per unit of capital is constant and either all (positive) capital stocks are perfectly compatible, or none of them are. We have characterized perfectly compatible capital stocks. We have also paid attention to the case where the average profit per capital is a monotonic and non-monotonic function of capital (all these cases have been illustrated with examples).

We have limited our study to a partial equilibrium setting. The notion of general equilibrium à la Solow and Hahn is generally difficult to analyze unless one considers the symetric equilibrium assuming that firms use the same technologies. This assumption is often made in macroeconomics in order to ease the analysis involving non-competitive equilibria (see, e.g., Hahn and Solow, page 78, The symetric case). Indeed, in the symetric equilibrium, equality of average returns across firms is achieved when firms use the same amount of capital. In this context, the results of this paper could be useful since analyzing a symetric equilibrium boilds dow to the study of compatible capital stocks for a single firm.

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