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Substitution between biofuels and fossil fuels: is there a Green Paradox?*

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Substitution between biofuels and fossil fuels: is there a Green Paradox?

Abstract: We show that (i) subsidies for renewable energy policies with the intention of encouraging substitution away from fossil fuels may accentuate climate change damages by hastening fossil fuel extraction, and that (ii) the opposite result holds under some specified conditions. We focus on the case of subsidies for renewable resources produced under increasing marginal costs, and assume that both the renewable resources and the fossil fuels are currently in use. Such subsidies have a direct effect and an indirect effect working in opposite directions. The direct effect is the reduction in demand for fossil fuels at any given price. The indirect effect is the reduction in the current equilibrium price for fossil fuels, which tends to increase the amount of fossil fuels demanded. Whether the sum of the two effects will actually result in an earlier or later date of exhaustion of the stock of fossil fuels depends on the curvature of the demand curve for energy and of the supply curve for the renewable substitute.

JEL-Classification: Q54, Q42, Q30

Keywords: Biofuels Subsidies, The Green Paradox, Climate Change.

1 Introduction

The need to reduce CO_2 emissions to combat climate change is widely acknowledged. According to standard economic theory the first best measure would be a carbon tax, and, as Hoel (2011) succintly stated, "at any time, the optimal price of carbon emissions should be equal to the present value of all future climate costs caused by present emissions, often called the social cost of carbon." In the real world, however, policy makers are faced with many constraints on the choice of policy instruments and their levels, and consequently first best policies cannot be implemented. Governments, in particular, face political opposition to the introduction of efficient carbon taxes.

A more politically acceptable time path of carbon taxes would be of the "ramp" variety: a government starts with a low tax rate, but commits to raise the tax to efficient levels into the future. As has been demonstrated (e.g. Sinn, 2008a,b), such time paths of carbon tax may induce profit-maximizing fossil resource owners to hasten the extraction of their resources, thus increasing carbon emissions in the near term.[1] Such an outcome has been termed a "Green Paradox" (Sinn, 2008a,b). Sinn's message is that in designing policies one must take into account supply responses by owners of fossil fuels.

To date, much of the existing literature on biofuel subsidies uses static analysis. Many authors have found, in the static context, mechanisms for increased carbon emissions (a Green Paradox outcome) with biofuel subsidies. It has been pointed out that the production process of biofuels is not "green" because it involves the use of many inputs with high carbon contents. Moreover, because the first generation of biofuels displace land for food production it may increase food prices. Our paper complements the existing literature with a dynamic mechanism. The contribution is a much richer understanding of the Green Paradox, delimiting cases when it occurs and when it does not, and shedding light on the inter-temporal consequences of biofuels subsidies.

[1] Long and Sinn (1985) offered an analysis of how extractive firms respond to an anticipated time path of varying tax rate, but they did not explicitly mention the effect on CO_2 concentration levels.

Sinclair (1992) was among the first economists to explicitly consider climate change policies that would reduce the extraction of fossil fuels, stressing that "the key decision of those lucky enough to own oil-wells is not so much how much to produce as when to extract it." Thus, as explained by Hoel (2011), if extraction cost is constant and carbon tax increases at a rate higher than the interest rate, emissions in the near term will be higher than without a carbon tax.

Several authors have explored the implications of a subsidy on a renewable substitute that can be produced under constant marginal costs (Hoel 2008, Strand 2007, Gerlagh 2011). In that setting, assuming a constant cost of extraction of fossil fuels, the Green Paradox outcome is *inevitable*.The intuition behind this inevitability of the Green Paradox is that subsidies on the backstop technology leads to a lower peak price for fossil fuels, whose fixed stock must be exhausted before the renewable is produced.[2] This implies that the whole time path of fossil fuel price is shifted downward, resulting in greater consumption of fossil fuels earlier on.[3] It is important to emphasize that in such models fuel consumption has two distinct phases: in the first phase, only the exhaustible resource is used, and in the second phase, only the renewable substitute is used. By construction, there is *no* phase where both resources are consumed simultaneously. The key assumption which is responsible for the absence of simultaneous use is that the renewable substitute is produced at *constant* unit cost.

While models without a phase of simultaneous consumption are useful devices to illustrate principles, it is also important to consider situations where there is a phase in which exhaustible resources and renewable substitutes are simultaneously consumed in varying proportions. Such situations occur when the renewable substitute is produced under *increasing* marginal cost. This corresponds to the case of first generation biofuels based on conven-

[2]The situation is different if exhaustion is economic rather than physical. Given the global warming externalities, Hoel and Kverndokk (1996) show that under economic exhaustion, both fossil fuels and the backstop technology substitute will be consumed indefinitely after the production of the substitute begins.

[3]See Dasgupta and Heal (1979). Pearce and Turner (1990) provide a good exposition. Earlier models of substitute production include Heal (1976) and Hoel (1978,1983), which, though not dealing with CO_2 emissions, contain all the ingredients from which one can deduce a Green Paradox result.

tional crops such as sugar cane, corn and vegetable oils (soy, canola, etc.). This is because the gradual expansion biofuels output, at least for first generation biofuels, necessitates the increasing use of less productive land with inevitable diminishing returns.[4]

Using a framework where *both* types of fuels are simultaneously consumed in the first phase phase, this paper finds conditions under which a Green Paradox outcome will *not* occur, as well as conditions under which it will occur. We focus on the effect of a biofuel subsidy on the time path of extraction of the fossil fuel resources. A striking result of our paper is that in the case of a linear demand for energy, together with (i) a zero extraction cost for fossil fuels and (ii) an upward-sloping linear marginal cost of biofuels, a biofuel subsidy will have *no* effect on the amount of fossil fuels extracted and consumed at each point of time, even though the subsidy does result in a lowering of the price of both fuels at each point of time, and in a higher quantity of energy demanded at each point of time. Under the stated assumptions it is shown that that the increase in quantity of fuel demanded is exactly matched by an increased output of biofuels, leaving the extraction rate unchanged. Thus, the time at which the stock of fossil fuels is exhausted is unchanged, *irrespective* of the rate of subsidy. This result stands in constrast to the inevitable Green Paradox in the model of Hoel (2008) where the supply curve of the renewable resource is horizontal. We stress that the Green Paradox does not arise in our model in the presence of a linear and increasing marginal cost of the renewable substitute to fossil fuels.

A second and important result of our paper is that when the assumption of zero extraction cost of the fossil fuel is replaced by the assumption of a positive and constant marginal extraction cost, a biofuel subsidy will result in a longer time over which the fossil fuel stock is exhausted. In this second case the Green Paradox does not arise. Indeed, there is no paradox because the subsidy works as intended because it extends the extraction period.

A third setting to investigate the possibility of a Green Paradox is when the marginal

[4] While it is true that to completely replace fossil fuels, humanity must make use of the whole range of alternative clean energy sources, including biofuels, geo-thermal heat, wind, solar, etc., it is convenient to use the expression "biofuels" as a catch-all term.

cost of biofuels increases at an increasing, or a decreasing rate as biofuels supply increases. In particular, we consider the case where the marginal cost of biofuel production is strictly increasing and strictly concave. In this situation along the equilibrium price path, as the price of energy rises gradually along the optimal extraction path of fossil fuels, biofuel output will increase over time, but the rate of the supply increase (per dollar increase in energy price) is *greater* when the price is higher. Consequently, fossil fuel firms, in anticipation of this greater expansion of the substitute in the later stage, respond by increasing their extraction at an earlier date. In this case, a Green Paradox outcome results.

In summary there a multiple cases when the Green Paradox does, and does not hold. The purpose of our analysis is to provide necessary and sufficient conditions for the Green Paradox to hold. We decompose the effect of a biofuel subsidy and show, for the first time, a direct effect of the subsidy (which is "pro-Green") and an indirect effect (which is "anti-Green"). Further, we show that a Green Paradox occurs *if and only if* the indirect effect outweights the direct effect.

2 Related literature

There is a large literature that connects the dynamic analysis of non-renewable fossil resources with climate change damages. Sinclair (1992, 1994) pointed out that climate change policies must aim at delaying the extraction of oil, and argued that a carbon tax must decline over time to encourage owners of fossil fuels to defer extraction. This result, however, depends in part on the assumptions that damages appear multiplicatively in the production function, and that capital and oil are substitutable inputs in a Cobb-Douglas production function.[5]

Recently, Groth and Schou (2007) confirm Sinclair's declining tax result using a similar model, but allowing endogenous growth. Ulph and Ulph (1994), specifying damages as an additive term in the social welfare function, and assuming exponential decay of pollution,

[5] Heal (1985) and Sinn (2008a,b) also model damages from GHGs emissions as a negative externality in production. Most papers however specify damages as an additive term in the social welfare function.

find that the optimal carbon tax time profile has an inverted U shape. Hoel and Kverndokk (1996) specify a model of economic exhaustion that includes rising extraction costs. They show that carbon tax peaks before the peak in atmospheric carbon. Consistent with Ulph and Ulph (1994), they find that in the long run the carbon tax approaches zero. They also consider the case where there is a backstop technology that produces a substitute at constant cost.[6] Farzin and Tahvonen (1996), by assuming that the decay rate of pollution is non-linear in its stock, show that the optimal carbon tax can take a variety of shapes. Similarly, Tahvonen (1997) obtains eleven different tax regimes, depending on initial sizes of the stock of CO_2 concentration and the stock of fossil fuels.

The key point of the Green Paradox literature is that the optimal carbon tax cannot be implemented given the political economy that exists in most countries. In particular, governments adopt policies that are conceived as second-best measures, but may cause greater environmental damages, if owners of fossil fuel stocks hasten their extraction to avoid future carbon taxes. Models that depict this adverse response to anticipation of taxes or substitute production include Sinn (2008a,b), Hoel (2008), Di Maria et al. (2008), Gerlagh and Liski (2008), Smulders et al. (2009), and Eichner and Pethig (2010).[7]Strand (2007) shows that a technological agreement that makes carbon dedundant in the future may increase current emissions. Hoel (2008) assumes that carbon resources remain cheaper than the substitute and analyses the situation where different countries have climate policies of different ambition levels. He shows that, in the absence of an efficient global climate agreement, climate costs may increase as a consequence of improved technology of substitute production.

Concurrent to our work, a number of authors have investigated the possibility of a Green Paradox in the context of policies that facilitate the availability of a substitute for fossil fuels. Hoel (2010, 2011) uses a two-period model where firms invest in capacity of producing

[6]Hoel and Kverndokk (1996, section 4) consider an alternative specification of damages, where the environmental damage is a function of the rate of change in the atmospheric stock of carbon.

[7]As pointed out in Hoel (2008), prior to 2008, "there is little work making the link between climate policies and exhaustible resources when policies are non-optimal or international agreements are incomplete". He mentioned a few exceptions: Bohm (1994), Hoel (1994) in a static framework.

a substitute. A number of key parameters are considered in his model (e.g. a parameter to capture how rapidly extraction costs increase with increasing total extraction, and a parameter affecting the time profile of the returns to investments in the substitute). Whether an investment subsidy results in greater environmental damages (a Green Paradox) depends on the relationship among these parameters.

Gerlagh (2011) also considers a model where extraction is at a constant cost, and a backstop technology can produce unlimited amount of a renewable substitute, at a constant cost per unit. He defines a Weak Green Paradox as an increase in the current emissions in response to an improvement in the backstop technology, and a Strong Green Paradox when the net present value of damages increases as a result of an improvement in the backstop technology.[8] He shows that both the Weak and the Strong Green Paradox arise in this benchmark model. Assuming linear demand, he finds that increasing extraction costs counteract the Strong Green Paradox, while an imperfect energy substitute may reduce the likelihood of both the Weak and the Strong Green Paradox. Ploeg and Withagen (2010) focus on the case where marginal extraction costs of the exhaustible resource depends on the existing stock. They assume that the substitute is available in unlimited supply at a constant marginal cost. After characterizing the social optimum, they turn to the case where first-best policies are not feasible, and show that the Green Paradox prevails if the cost of backstop decreases, provided that the backstop remains expensive such that the non-renewable resource stock is eventually exhausted. By contrast, if the backstop becomes so cheap that physical exhaustion will not take place, the Green Paradox no longer holds.

Our paper emphasises the facts that biofuels are already available, but the expansion of biofuel output is possible only with increasing costs. There is a large literature on the costs and benefits of developing biofuels as a means to reduce green house gas emissions, but most authors focus on static analysis of energy substitution and do not take into account the intertemporal response of fossil fuel producers (e.g. Hill et al. 2006, Steenblik, 2007, Koplow,

[8] His specification of net present value of damages is rather non-standard: it depends on a shadow price of emissions which is not derived from the model.

2007, Lapan and Moschini, 2009, Bandyopadhyay et al. 2009, Moschini et al. 2010).[9]

Our model is consistent with the current state of play in terms of biofuel production. The main producing countries for transport biofuels are the U.S., Brazil and the EU. Brazil and the U.S. combined produced 55 and 35 percent, respectively, of the world's ethanol production in 2009 while the EU produced 60 percent of the total biodiesel output. The main stimulus to the use of biofuels are policies that encourage the substitution from fossil fuels, especially for road transportation. Government mandates for blending biofuels into vehicle fuels have also been enacted in at least 17 countries, and many states and provinces within these countries. Typical mandates require blending 10–15 percent ethanol with gasoline or blending 2–5 percent biodiesel with diesel fuel. Recent targets have encouraged higher levels of biofuel use in various countries (UNEP, 2009, page 15-16).

The range of policies that have stimulated biofuel demand by setting targets and blending quotas has been aided by supporting mechanisms, such as subsidies and tax exemptions. In the US, the total biofuels support encompasses the total value of all government supports to the biofuels industry, including consumption mandates, tax credits, import barriers, investment subsidies and general support to the sector such as public research investment. A report by Koplow (2007, pp. 29, 31) for the Global Subsidies Initiative indicates that the total support estimates for the US alone, in 2008, was between \$9.2 and 11.07 billion.

3 A general model of simulataneous use of fossil fuels and biofuels

Most of the literature on the Green Paradox that involves renewable substitutes is based on the assumption that the substitute is adopted only when the use of fossil fuels comes to an end. Thus a typical model displays two phases. In the first phase, only fossil fuels are consumed. In the second phase, the renewable substitute completely displaces the use of

[9] Chakravorty et al. (2009) provide a comprehensive survey. A dynamic model is developed in Bahel et al. (2011) but their focus is on the evolution of food prices, not on CO_2 emissions.

fossil fuels. These models are based on the assumption that the substitute is available in unlimited supply, at a constant unit cost.

In this paper, we focus on the case where *both* types of fuels are supplied in the market. This is made possible by postulating an upward sloping supply curve for biofuels.We allow this positive slope to be constant for all output levels, or to be increasing or decreasing (while still remaining positive) with output. Our model for the cost of extraction of the fossil fuels allows for the possibility that as the stock dwindles, the extraction cost rises. The special case of stock-independent marginal cost is also admitted.

On the demand side, we assume that fossil fuels and biofuels are perfect substitutes: one unit of energy can be obtained from using one unit of fossil fuels, or alternatively one unit of biofuels. We denote by P_t as the consumer's price of fuels at time t, and by E_t^d the quantity of fuel demanded. The demand function is

$$E_t^d = D_E(P_t)$$

where $D'_E(P_t) < 0$.

We assume that biofuel firms are price takers. Suppose that for each unit of biofuel sold, the biofuel producers receive $(1+v)P_t$ dollars, where v is the ad valorem subsidy rate. (We may interpret v as a production subsidy). Let $z = 1 + v$. We call z the "subsidy factor." Let B_t be the quantity of biofuels supplied at time t. The supply curve of biofuels is upward sloping

$$B_t = S(zP_t)$$

with $S'(zP) > 0$. We allow the second derivative $S''(zP)$ to be negative or positive. The special case $S''(zP) = 0$ implies that the supply curve is linear.

Concerning the extraction of fossil fuels, we assume that the fossil fuels sector consists of n identical producers, each having an initial stock R_{i0}. Let q_{it} denote producer i's rate of extraction at time t, and R_{it} denote his remaning stock. Then

$$\dot{R}_{it} = -q_{it}.$$

Omitting the subscripts for simplicity, we write the extraction cost function for each firm as $C(q, R)$. We assume that (i) marginal extraction cost is non-negative, $C_q \geq 0$, and non-decreasing in extraction rate, $C_{qq} \geq 0$; (ii) the extraction cost is non-increasing in the size of the reserves, $C_R \leq 0$; (iii) as the stock dwindles, the marginal extraction cost rises (or remain constant), $C_{qR} \leq 0$.[10]

$$C_q \geq 0, \quad C_{qq} \geq 0, \quad C_R \leq 0, \quad C_{qR} \leq 0. \tag{1}$$

Let Q_t be the aggregate extraction of fossil fuels at time t :

$$Q_t \equiv \sum_{i=1}^{n} q_{it}$$

By assumption fossil fuels and biofuels are perfect substitutes. Flow equilibrium in the fuel market, thus, requires that total supply (Q_t plus the output of biofuel producers, B_t) equals the quantity of fuels demanded, E_t:

$$Q_t + B_t = E_t \tag{2}$$

This condition implies that

$$Q_t = D_E(P_t) - S(zP_t) \tag{3}$$

Let us define

$$D(P_t; z) \equiv D_E(P_t) - S(zP_t) \tag{4}$$

The function $D(P_t; z)$, defined as the difference between the demand for fuels and the supply of biofuels, is called the demand for fossil fuels. For any given $z > 0$, there is a unique price $\overline{P}$ (which depends on z) at which $D(\overline{P}; z) = 0$. We call $\overline{P}$ the "choke price" for fossil fuels, where

$$D_E(\overline{P}) - S(z\overline{P}) = 0$$

When the price reaches $\overline{P}$, the entire market demand for fuels is met by biofuel producers. A higher z implies a lower choke price:

$$\frac{d\overline{P}}{dz} = \frac{S'(z\overline{P})\overline{P}}{D'_E(\overline{P}) - S'(z\overline{P})z} < 0 \tag{5}$$

[10] See e.g. Farzin (1992).

An increase in z also has an effect on the demand for fossil fuels: at any price below the choke price, a higher z implies a smaller quantity demanded (for fossil fuels). Figure 1 illustrates the effect of an increase in the subsidy factor z, from z_1 to $z_2 > z_1$. We assume in the Figure that $S(0) = 0$, but our mathematical analysis does not require this assumption.

PLEASE PLACE FIGURE 1 HERE

A higher biofuel production-subsidy implies that at any given consumer's price P, a greater quantity of biofuels is supplied. This is represented by a clockwise rotation of the biofuel supply curve in Figure 1. This rotation implies that the demand curve for fossil fuels, $D(P_t; z)$, is rotated anti-clockwise, as is clear from equation (4).In Figure 1, this rotation is from EM_1 to EM_2. The old choke price $\overline{P}(z_1)$ is depicted by point M_1 and the new choke price $\overline{P}(z_2)$ is depicted by point M_2.

Figure 1 appears to indicate that a higher rate of biofuel subsidy will result in lower emissions, but this can only be true at a given price. For example, in Figure 1, at the given P_0, the *direct effect* of a biofuel production subsidy is to increase biofuel output from point A to point B, implying that the quantity of fossil fuels demanded falls from Y_0 to Y'.However, taking into account the full response of extractive firms, the equilibrium price P_0 would also change as a result of the increase in the subsidy rate. Thus, the extraction path of fossil fuels must in general be adjusted, and so must P_0 and all P_t, in order to ensure that: (i) the flow equilibrium condition (2) is satisfied at all points of time; (ii) the intertemporal equilibrium condition, known as the Hotelling Rule, is satisfied along a positive extraction path; and (iii) the stock equilibrium condition holds such that the cumulative extraction up to the exhaustion time T should equal the initial stock. Consequently, the initial price must fall, for instance, from P_0 to, say, P_0' in Figure 1.

As the initial price falls from P_0 to P_0', the output of biofuel moves along the biofuel supply curve $S(z_2P)$ from point B to point N, and thus the quantity demanded for fossil fuels rises. We call this the *indirect effect* of the biofuel subsidy. Figure 1 depicts a very special case where the fall in initial price is just sufficient to restore the initial quantity of fossil fuel demanded back to Y_0 (i.e. in this case the subsidy has no effect on the extraction rate of fossil fuels, because the direct effect and the indirect effect cancel each other out).

In what follows, we show how the equibrium paths of price and quantity would adjust to a permanent increase in the subsidy factor z. We consider two cases.

Case I (stock-independent extraction cost). In this case, $C_R = 0$ for all R.

Case II (stock-dependent extraction cost). In this case, $C_{qR} < 0$.

In Case I, it is convenient to denote the marginal extraction cost by $c(q)$. Since $c(q)$ is assumed to be non-decreasing, marginal extraction cost is lowest when $q = 0$. We assume that $c(0)$ is lower than the choke price $\overline{P}(z)$, so that the firm will find it optimal to (eventually) exhaust its stock.

In Case II, $C_q(0,0)$ is called the "marginal cost of extracting the last drop of oil." We distiguish two sub-cases. In sub-case II-A, $C_q(0,0) \leq \overline{P}(z)$, which implies that eventual exhaustion is optimal. In sub-case II-B, $C_q(0,0) > \overline{P}(z)$, the marginal cost of extracting the last drop of oil is high, and physical exhaustion will not take place. In sub-case II-B there exists a positive reserve level R_L such that $C_q(0, R_L) = \overline{P}(z)$, and the firm will abandon its stock when the reserve level R reaches R_L. This case is economic exhaustion rather than physical exhaustion.

We now consider the optimization problem of the representative extractive firm. We assume it has perfect foresight of the price path, which it takes as given. It chooses the extraction path q_t, the terminal time T and the terminal stock level R_T to maximize

$$\int_0^T e^{-rt}(q_t P_t - C(q_t, R_t))dt$$

where $r > 0$ is the rate of interest. The necessary conditions yield the Hotelling equation

$$\frac{d}{dt}\left[P_t - C_q(q_t, R_t)\right] = r(P_t - C_q) + C_R \text{ for } 0 \leq t \leq T \tag{6}$$

with $q_T = 0$, and the transversality condition takes the form

$$P_T - C_q(0, R_T) \geq 0, \quad R_T \geq 0, \quad \left[P_T - C_q(0, R_T)\right] R_T = 0 \tag{7}$$

The transversality condition (7) is intuitively plausible. If it is optimal to leave some stock R_T unexploited, it must be the case that at the terminal time T the marginal cost of extraction equals the choke price. By contrast, if resource exhaustion is optimal, the terminal marginal extraction cost must generally be lower than the terminal price.

Given that all firms are identical, we focus on the symmetric equilibrium where

$$nq_t = Q_t = D(P_t; z)$$

Differentiation with respect to time yields

$$\dot{Q}_t = D_P(P_t; z)\dot{P}_t$$

Substituting these two equations into the Hotelling Rule (6) gives us the evolution of the equilibrium price path:

$$\dot{P} = r\left[P - C_q\left(\frac{D(P;z)}{n}, R\right)\right] + C_R + C_{qq}\left(\frac{D(P;z)}{n}, R\right)\frac{D_P(P;z)\dot{P}}{n} - C_{qR}\left(\frac{D(P;z)}{n}, R\right)\frac{D(P;z)}{n} \tag{8}$$

For given initial stock size R_0 and given z, this differential equation, together with $\dot{R} = -\frac{D(P;z)}{n}$ and condition (7), can be solved to determine the equilibrium paths of price, output, and the equilibrium terminal time T at which

$$P_T = \overline{P}(z) \tag{9}$$

At the level of generality displayed by equation (8), it is not possible to determine the effect of an increase in the biofuel subsidy factor z on the extraction path and the exhaustion time T. In the next section, we consider the case of stock-independent marginal cost, in which equation (8) reduces to a simpler form.

4 The case of stock-independent marginal extraction cost

Consider the special case where the marginal extraction cost is stock-independent and is a constant $c \geq 0$.Then $C(q, R) = cq$, and $C_q = c$. Assume $c < \overline{P}$, so that it is profitable to extract and to eventually exhaust the resource stock, i.e. $R_T = 0$. Condition (8) reduces to

$$\dot{P} = r(P - c).$$

This equation and the boundary condition (9) imply the following Hotelling price path for all $t \in [0, T]$:

$$P_t = c + (\overline{P}(z) - c)e^{r(t-T)} \equiv \phi(\overline{P}(z), t, T). \tag{10}$$

This equation shows that the price at time t is functionally related to the exhaustion time T as well as the choke price $\overline{P}$. Note that ϕ is decreasing in T and increasing in $\overline{P}$.

From equation (10) and the differential equation $\dot{R}_t = -D(P_t; z)/n$ we obtain the stock equilibrium condition that accumulated extraction (from time zero to the exhaustion time) equals the initial reserves:

$$\int_0^T D(c + (\overline{P}(z) - c)e^{r(t-T)}; z)dt = -\int_0^T n\dot{R}(t)dt = nR_0 \tag{11}$$

Given R_0 and z, this equation uniquely determines the equilibrium exhaustion time T. If an increase in z brings the exhaustion time T closer to the present, we says that we obtain a *Green Paradox.* It is clear that if T is brought closer to the present, the present value of the flow of damages will be greater, even though we have not explicitly modelled the relationship between the time path of damages and the time path of cumulative emissions.[11]

What are the forces that determine the net effect of an increase in the biofuel subsidy factor z on the time of exhaustion T? We know from equation (5) that an increase in z will lower the fossil fuel choke price $\overline{P}$. At the same time, an increase in z rotates the demand curve for fossil fuel in the anti-clockwise direction, as shown in Figure 1. Does an increase in z generate, on average, greater demand for fossil fuels over the time interval T? To answer this question, it is useful to decompose the change in demand into a direct effect and an indirect effect.

Direct Effect: The anti-clockwise rotation of the demand curve for fossil fuels implies that any given price P_t, the quantity demanded is smaller than before. This direct effect is

[11] For the importance of cumulative emissions, see Allen et al. (2009). For additional considerations such as the feedback effects of temperature increases on GHGs emissions, see Winter (2011), among others.

capture by the term $D_z < 0$. The direct effect is "pro-Green": an increase in biofuel subsidy reduces demand for fossil fuels, at any given price P_t.

Indirect Effect: The increase in z (biofuel subsidy) lowers the fossil fuel choke price $\overline{P}$. This implies that, holding T constant, the price P_t must fall, see equation (10). A fall in P_t increases the quantity demanded. The indirect effect, captured by the term $\left(\frac{\partial D}{\partial P_t}\right)\left(\frac{\partial P_t}{\partial z}\right)$, is positive, i.e. it is "anti-Green".

The total effect on fossil fuels consumption at any time t is the sum of the direct effect and the indirect effect at that time. In general, the total effect can be positive at some points of time and negative at some other points of time. Therefore, to find the effect of biofuel subsidy, one has to compute the cumulative total effect, over the interval $[0, T]$. If this cumulative total effect is positive, it means that the exhaustion time must be brought closer to the present, which is a Green Paradox outcome. A more formal analysis follows.

To find the net effect of a change in z on the exhaustion time T, let us write

$$G(T, z) = \int_0^T D(c + (\overline{P}(z) - c)e^{r(t-T)}; z)dt - nR_0 = 0$$

Then

$$\frac{dT}{dz} = -\frac{G_z}{G_T} = \frac{-\int_0^T \left[D_P \frac{\partial P_t}{\partial z} + D_z\right] dt}{D(\overline{P}(z); z) + \int_0^T \left[D_P \frac{\partial P_t}{\partial T}\right] dt} \tag{12}$$

where the denominator is positive because $D(\overline{P}(z); z) = 0$, $D_P < 0$ and $\partial P_t / \partial T < 0$ from equation (10). The integrand in the numerator is ambiguous in sign:

$$D_z + \left(D_P \frac{\partial P_t}{\partial z}\right) \lesseqgtr 0 \tag{13}$$

The comparative static expression (12) depends on the integral of the expression (13). Even though in general the sign of (13) is uncertain, the sign of its integral may be determinate or at least calculated numerically.

Lemma 1: *An increase in the subsidy factor z will bring the resource-exhaustion date T closer to the present (i.e. will lower T) if and only if the cumulative indirect effect dominates the cumulative direct effect*

$$\int_0^T D_P \frac{\partial P_t}{\partial z} dt > \int_0^T (-D_z)\, dt \tag{14}$$

Since in general we cannot determine the sign of dT/dz, let us consider some special cases.

4.1 Linear demand for energy and linear biofuel-supply function

Assume that both the demand function for energy $D_E(P)$ and the biofuel supply function are linear:

$$D_E(P) = a - bP, \text{ and } S(zP) = \beta z P - g$$

where $a > 0$, $b > 0$, $\beta > 0$ and $g \geq 0$. Then the (residual) demand function for fossil fuel is

$$D(P; z) = (a + g) - bP - \beta z P, \; a > 0, b > 0, \beta > 0$$

From equation (10)

$$P_t = c + (\overline{P}(z) - c)e^{r(t-T)} = c\left[1 - e^{r(t-T)}\right] + \overline{P}(z)e^{r(t-T)}$$

The direct effect is then

$$D_z(P_t; z) = -\beta P_t = -\beta\overline{P}(z)e^{r(t-T)} - \beta c\left[1 - e^{r(t-T)}\right] < 0 \tag{15}$$

To calculate the indirect effect, note that

$$\overline{P}(z) = \frac{(a+g)}{b+\beta z} \implies \frac{d\overline{P}}{dz} = -\frac{(a+g)\beta}{(a+g+\beta z)^2} < 0.$$

and

$$\partial P_t/\partial z = (d\overline{P}/dz)e^{r(t-T)} = -\frac{a\beta}{(b+\beta z)^2}e^{r(t-T)} = -\frac{\beta\overline{P}(z)e^{r(t-T)}}{b+\beta z} < 0$$

Then the indirect effect is

$$D_P\frac{\partial P_t}{\partial z} = [(b+\beta z)]\left[\frac{\beta\overline{P}(z)e^{r(t-T)}}{b+\beta z}\right] = \beta\overline{P}(z)e^{r(t-T)} > 0 \tag{16}$$

Comparing the direct effect (15) and the (16), we see that they cancel each other out if $c = 0$. If $c > 0$, then the direct effect dominates the indirect effect, and there is no Green Paradox. Thus, making use of Lemma 1, we can state the following result.

Proposition 1: *Assume that the demand function for fuels and the supply function of biofuels are both linear. Then under perfectly competitive extraction, an increase in the subsidy for biofuel producers will generally delay the exhaustion date, i.e.* $dT/dz \geq 0$. *In particular,*

(i) Assume extraction costs are zero ($c = 0$). *Then an increase in the biofuel subsidy factor* z *has no effect on the date of exhaustion of the resource stock.The cumulative extraction path under the biofuel subsidy is identical to the cumulative extraction path without the biofuel subsidy.(The direct effect and the indirect effect of an increase in* z *cancel each other out.)*

(ii) Assume extraction costs are positive and smaller than the choke price ($\overline{P} > c > 0$). *Then an increase in the biofuel subsidy factor* z *leads to a delay the date of exhaustion of the resource stock. The cumulative extraction path with the biofuel subsidy is uniformly lower than the cumulative extraction path without the biofuel subsidy.*

Remark 1. The intuition behind Proposition 1 is as follows. A permanent increase in biofuel subsidies at time $t = 0$ creates a direct effect: increased biofuel production at any given price (a clockwise rotation of the biofuel supply cuve, see Figure 1). This increased supply of biofuels implies that if the price path for fuels were unchanged, less fossil fuels would be demanded at each point in time. Hence, if the price path were unchanged, the fossil fuel stock would not be exhausted by the time the price reaches the new (and lower) choke price, M_2 in Figure 1. But from our assumption that $c < M_2$ non-exhaustion cannot be an equilibrium. So the price at all t must fall to stimulate demand for fuels (and hence for fossil fuels) sufficiently to clear the market. The fall in P_0 tends to reduce the amount of biofuels supplied (an induced effect, a movement along the already rotated biofuel supply curve, from point B to point N in Figure 1). At the lower price, the demand for energy is higher than in the no-subsidy scenario, and while some of this demand is met by biofuel production, the remaining demand is met by fossil fuel extraction. When $c = 0$ and the demand and supply are linear, the fall in the initial price (from P_0 to P_0') is just sufficient for the flow demand for fossil fuels to be restored to the level that prevails in the no-subsidy scenario (point Y_0 in Figure 1). When $c > 0$, the fall in initial price is smaller (above the point P_0' depicted in Figure 1), so the amount of fossil fuels demanded at time $t = 0$ will be smaller than Y_0 in Figure 1.

Remark 2. We note the possibility that the direct and indirect effects perfectly offset each other. Such outcomes are also possible in different contexts, such as, in macroeconomics, with a logarithmic utility function an increase in the interest rate has no impact on savings because the income effect and the substitution effect cancel each other out.

Remark 3. Another interpretation of part (i) of Proposition 1 is as follows. Assume that the demand for an exhaustible resource is linear, $Q = a - (b + \beta z) P \equiv a - \widetilde{b}P$, and that extraction cost is zero. A rotation of the linear demand curve (keeping the horizontal intercept a unchanged, while changing $\widetilde{b}$) will leave the equilibrium extraction path and the exhaustion time unchanged, but the equilibrium price path will be changed. Figure 2 shows the time paths of price for different values of $\widetilde{b}$, under the assumption that $c = 0$. It is important to note that while superficially the time path of price in Figure 2 looks like the standard textbook figures which treat the case of a substitute that is available at constant cost and that is supplied only after extraction of fossil fuels has ceased, the underlying story is not the same because, in our case, both biofuels and fossil fuels are consumed simultaneously at each point of time, and the increased demand for fuels associated with the lower price path is satisfied by the expansion of biofuel output. This explains why in Figure 2 it is possible that the stock is exhausted at the same time T even though the price path is lowered as the result of a subsidy.

PLEASE PLACE FIGURE 2 HERE

4.2 Isoelastic biofuel-supply function and modified-isoelastic energy demand function

We now consider the special case where the biofuel supply function is isoelastic

$$S(zP) = \beta(zP)^{\mu} \text{ where } \beta > 0 \text{ and } \mu > 0 \tag{17}$$

(where μ is the elasticity of supply) and the demand function for energy is of the modified isoelastic form[12]

$$D_E(P) = (P + \delta)^{-\varepsilon} \text{ where } \delta \geq 0 \text{ and } \varepsilon > 0 \tag{18}$$

The choke price $\overline{P}$ is the solution of the equation

$$(\overline{P} + \delta)^{-\varepsilon} - \beta(z\overline{P})^{\mu} = 0 \tag{19}$$

Then an increase in biofuel subsidy will lower the choke price for fossil fuels,

$$\frac{d\overline{P}}{dz} = -\frac{\beta\mu z^{\mu-1}(\overline{P})^{\mu}}{\varepsilon(\overline{P} + \delta)^{-\varepsilon-1} + \mu\beta z^{\mu}(\overline{P})^{\mu-1}} < 0.$$

From equation (12), we know that a Green Paradox outcome occurs if and only if $\int_0^T \left[D_P \frac{\partial P_t}{\partial z} + D_z\right] dt > 0$. Recalling equation (10), we obtain the indirect effect

$$D_P \frac{\partial P_t}{\partial z} = e^{r(t-T)} \left\{ \frac{\beta\mu z^{\mu-1}(\overline{P})^{\mu} \left[\varepsilon(P_t + \delta)^{-\varepsilon-1} + \mu\beta z^{\mu}(P_t)^{\mu-1}\right]}{\varepsilon(\overline{P} + \delta)^{-\varepsilon-1} + \mu\beta z^{\mu}(\overline{P})^{\mu-1}} \right\} > 0$$

Proposition 2: *Under the demand and supply specification (17) and (18), a Green Paradox occurs if and only if parameter values are such that the cumulative indirect effect dominates the cumulative direct effect, i.e.,*

$$\int_0^T \left\{ e^{r(t-T)} \left[\frac{\varepsilon(P_t + \delta)^{-\varepsilon-1} + \mu\beta z^{\mu}(P_t)^{\mu-1}}{\varepsilon(\overline{P} + \delta)^{-\varepsilon-1} + \mu\beta z^{\mu}(\overline{P})^{\mu-1}} \right] - \left(\frac{P_t}{\overline{P}}\right)^{\mu} \right\} dt > 0 \tag{20}$$

where P_t is given by (10) and $\overline{P}$ is given by (19).This condition is satisfied under plausible specifications of parameter values.

Proof: Using Lemma 1 and our specification of demand and supply as in (17) and (18), condition (14) is satisfied if and only if

$$\int_0^T \left\{ e^{r(t-T)} \left[\frac{\beta\mu z^{\mu-1}(\overline{P})^{\mu} \left(\varepsilon(P_t + \delta)^{-\varepsilon-1} + \mu\beta z^{\mu}(P_t)^{\mu-1}\right)}{\varepsilon(\overline{P} + \delta)^{-\varepsilon-1} + \mu\beta z^{\mu}(\overline{P})^{\mu-1}} \right] - \beta\mu z^{\mu-1}(P_t)^{\mu} \right\} dt > 0$$

[12] A possible interpretation of the energy demand function (18) is that it is a derived demand function. For example, suppose a manufactured good is produced under constant returns to scale using two inputs, energy and labor, in fixed proportions, normalized at unity. Then the price of a unit of the manufatured good is $P + \delta$ where P is the price of energy and δ is the wage rate. Under this interpretation, ε is the price elasticity of demand for the manufactured good.

This condition is equivalent to (20).

Numerical examples

While it is not possible to determine the sign of (20) analytically, we can compute it when parameter values are specified. First, $\overline{P}(z)$ is computed. Then P_t is computed from (10). Next, we integrate (20) numerically. Finally, we compute (12).

In our base-line scenario, the parameters are:

$$\varepsilon = \delta = \beta = \mu = 1, \qquad r = 0.05, \qquad c = 0 \tag{21}$$

This means that the elasticity of supply of biofuel is unity, and the (residual) demand for fossil fuels is strictly convex, with the price elasticity of demand ranging from infinity (at the choke price $\overline{P}$) to zero (when the price approaches zero). Let the reserve size R_0 be large enough so that under this base-line scenario, the exhaustion time is $T = 100$ years. Then we find that, starting at the initial subsidy factor $z = 1$, an increase in z leads to an earlier exhaustion time:

$$\frac{dT}{dz} = -2.09$$

This result is in sharp contrast to the case where the residual demand is linear, where under zero extraction cost an increase in z will cause a fall in the intial price P_0 just sufficiently to keep the quantity demanded q_0 (on the rotated demand curve) unchanged. With a strictly convex demand function, the initial price falls more sharply, increasing the quantity demanded q_0. Figure 3 shows the Green Paradox in this case.

PLEASE PLACE FIGURE 3 HERE.

The above computation shows that, given our specification (21), the Weak Green Paradox holds if the reserve size is such that $T = 100$. Is this result sensitive to the size of the reserve? Let us vary the reserve size so that T varies from 50 years to 200 years. We find that dT/dz remains negative, and not far different from -2.09. Table 1 below reports the value dT/dz for various reserve sizes (and hence various T).

We also consider different initial subsidy levels keeping $\varepsilon = \delta = \beta = \mu = 1$, $r = 0.05$, $c = 0$, $T = 100$. Table 2 reports the results and shows there is a Green Paradox for this set of parameter values, for a wide range of values of the initial subsidy factor. Next, keeping $\beta = \delta = \mu = 1$, $r = 0.05$, $c = 0$, $T = 100$, we consider different values for ε, the price elasticity of demand for the manufactured good. The results are reported in Table 3. Again, for a wide range of possible values of this elasticity, the Green Paradox holds. We find that the greater is the elasticity ε, the stronger is the effect of an increase in the subsidy on the lowering of the exhaustion time. Finally, consider different values for supply elasticity of biofuels, μ. We find that the Green Paradox holds for a wide range of μ, as reported in Table 4. As shown in this Table 4, *with the exception of the first column where the supply elasticity of biofuels is small,* an increase in the subsidy rate will hasten the exhaustion of fossil fuels.

The intuition behind the sensitivity to the supply elasticity μ can be explained by noting that when the price elasticity of supply of biofuels is high (μ large) the rising price along the Hotelling path induces very strong biofuel supply expansion. Hence, the demand for oil primarily decreases in the future rather than earlier. Consequently, oil owners have to sell more sooner and it is this outcome that works in favor of the Green Paradox.

5 The case of stock-dependent marginal extraction cost

We now turn to an investigation of the possibility of a Green Paradox outcome in the case where extraction costs are stock-dependent, as specified in (1) with $C_R < 0$, meaning that the extraction cost rises as the remaining stock falls. There are two possible scenarios. In the first scenario, the marginal cost of extracting the 'last drop of oil', although high, is still below the choke price for fossil fuels, and therefore all the fossil fuel stock will eventually be exhausted. In the second scenario, the last drop of oil is prohibitively expensive to extract, and therefore firms will abandon their deposits without exhausting them.[13] We investigate

[13] Some authors have therefore modeled the "resource exhaustion" in the sense of an "economic abandonment" of the deposit after the profitable part has been exploited (see for example Karp, 1984).

the possibility of the Green Paradox in both cases.

It will be convenient to work with an explicit form of the function $C(q, R)$. Following Karp (1984), we postulate that

$$C(q_t, R_t) = [c_0 + c_1 (R_0 - R_t)]\, q_t, \qquad c_0 \geq 0, c_1 \geq 0, R_0 - R_t \geq 0.$$

Then $c_0 + (R_0 - R_t)c_1$ is the marginal cost of extraction at time t. At time $t = 0$, the marginal extraction cost is at its lowest value, c_0. As extraction proceeds, the remaining stock R_t falls, and the marginal extraction cost rises. As R_t falls to 0, the marginal extraction cost rises to $c_0 + c_1 R_0$. We call $c_0 + c_1 R_0$ the marginal cost of extracting the last drop of oil. The parameter c_1 represents the sensitivity of marginal cost to the remaining stock.

Under this specification, equation (8) reduces to

$$\dot{P} = r(P - (c_0 + c_1(R_0 - R))$$

As before, the demand for fossil fuels falls to zero at the "fossil fuel choke price" $\overline{P}$ defined by $D_E(\overline{P}) - S(z\overline{P}) = 0$. Since the representative oil firm's marginal cost is $c_0 + c_1 (R_0 - R_t)$, it is clear that:[14]

(i) if the marginal extraction cost of the last drop of oil is low, such that $c_0 + c_1 R_0 \leq \overline{P}(z)$, then the representative firm will eventually exhaust all its stock,

(ii) if the marginal extraction cost of the last drop of oil is high, such that $c_0 + c_1 R_0 > \overline{P}(z)$, the representative firm will abandon its deposit when the reserve level falls to some positive level R_L defined by

$$\overline{c} - c_1 \left(\overline{R} - R_L\right) = \overline{P}(z),\; R_L > 0 \tag{22}$$

Let us consider the case where the marginal extraction cost of the last drop of oil is low, such that $c_0 + c_1 R_0 \leq \overline{P}(z)$. Then the equilibrium paths of price and quantity are obtained from solving the following system of equations

$$\dot{P} = r(P - (c_0 + c_1(R_0 - R))$$

[14] See Karp (1984).

$$\dot{R} = -D(P; z)$$

subject to the boundary conditions $R(0) = R_0$, $P_T = \overline{P}(z)$ and $R_T = 0$.

The case of linear demand for fossil fuels

In this case, we can show that the Green Paradox does not hold. This is stated as Proposition 3.

Proposition 3: *If the fossil fuel demand function $D(P; z)$ is linear in P, and the entire stock R_0 is exhausted at some time T, then*

(i) an increase in the subsidy factor z will delay the exhaustion date,

(ii) higher sensitivity of extraction cost with respect to the remaining stock results in a later exhaustion date.

Proof: See the Appendix.

As a numerical example, let $a = 3, b = \beta = z = 1$ and $\overline{R} = 80$. We find that the stock $\overline{R}$ will be exhausted in 370 years. If the subsidy factor is $z = 1.1$, we find that T increases to 410 years. In this special case, the Green Paradox does not hold, at least in the long run.

The case of non-linear demand

We consider the more general case of non-linear demand and suppose that $D(P, z) = (P + \delta)^{-\varepsilon} - \beta(zP)^{\mu}$. Then $\overline{P}(z)$ is the solution of

$$(P + \delta)^{-\varepsilon} - \beta(zP)^{\mu} = 0$$

The system to be analyzed is as follows:

$$\dot{P}_t = r\left[P_t - (c_0 + c_1(R_0 - R_t)\right]$$

$$\dot{R}_t = -(P + \delta)^{-\varepsilon} + \beta(zP)^{\mu}$$

subject to three boundary conditions: R_0 given, $R_T = 0$, $P_T = \overline{P}(z)$. Unlike linear demand case, we cannot obtain an analytical solution.

Consider an example. Let $\varepsilon = \mu = \beta = \delta = 1$. Then $\overline{P}$ is the solution of

$$\frac{1}{(P+1)} - zP = 0$$

At $z = 1$, the fossil fuel choke price is 0.618. Assuming that $c_0 = 0.005$, $c_1 = 0.0001$ and $\overline{R} = 80$, we can solve for the current oil price $P(0) = 0.017$, and the exhaustion date $T \simeq 106.9$ years. These cost and oil reserve parameters imply that the current extraction cost/price ratio is 29%. Until the exhaustion date, costs in this case increases slightly from 0.005 to 0.013, but the fuel price increases faster from 0.017 to 0.618, and thus the relevant cost/price ratio at the exhaustion date is just 2%.[15]

Does a small increase in z lead to an increase or a decrease in T? The answer depends on various parameters, particularly cost sensitivity or c_1. Our numerical simulations show that if extraction costs increase faster as the reserve depletes (a large value of c_1), then, starting from $z = 1$, a small increase in the subsidy will make oil reserves last longer (a larger T). On the other hand, if c_1 is small enough, then an increase in the subsidy may lead to earlier exhaustion. We illustrate this in Table 5 where the subsidy z is increased from 1 to 1.2 ($\Delta z = 0.2$). Table 5 illustrates it is possible to find a large range of cases where fossil fuel reserves are exhausted faster, and the Green Paradox holds.

The findings in Table 5, however, do not imply the Green Paradox is a general result. Indeed, if condition (22) holds, we can show that with linear demand and a marginal extraction costs that increases linearly with accumulated extraction, and without technological change, fossil fuel deposits are abandoned before exhaustion and a subsidy for biofuels production results in a smaller overall consumption of fossil fuels. In that case, we say that the Green Paradox does not hold in the long run. However, the long run is very far away (in fact, abandonment takes place at time infinity). Thus, even when the Green Paradox does not hold even in the long run, our results indicate that it can plausibly hold in the short or

[15] For fossil fuel producers to abandon extraction before reserves are exhausted, the subsidy must satisfy:

$$c_0 + c_1 R_0 = 0.013 > \overline{P}(z) \text{ or } z > 153.8.$$

medium term.

We emphasize that the time period over which the Green Paradox holds is critically important. In terms of climate change and avoiding the severe impacts of climate change, what happens to cumulative emissions over the next 30 years interval is critical. It is this period of time that is likely to be the important period in terms of biofuels subsidies as we might reasonably assume that new, carbon neutral energy technologies will become more widespread beyond 30 years. Thus, in the time periods that matter we have real cause for concern that biofuel subsidies may actually encourage larger GHG emissions from fossil fuel combustion and increase the likelihood of the more severe impacts of climate change.[16]

Another relevant issue is technological change that affects extraction costs and the net price path on non-renewables. Technological change would seem to be especially important in terms of stock-dependent extraction costs. Positive technological change should offset the effect of stock-dependent extraction, and may do so for a long time. The overall impact of technological change would depend on the relative changes of z over time, on the extent to which technological changes affect the extraction costs and the cost of biofuel production, and the nature of stock dependent costs.

6 Extension 1: Green Paradox under cartel extraction

What happens if oil is supplied by a cartel that behaves like a monopolist? Assume that the monopolist takes the subsidy factor z as a given constant. How does an increase in z affect the extraction path? As before, the residual demand function facing the oil producer is $D(P;z) \equiv D_E(P) - S(zP)$. The choke price for fossil fuels is $\overline{P}$, where $D_E(\overline{P}) - S_B(z\overline{P}) = 0$, and $\overline{P}$ is decreasing in z. For simplicity we only consider the case of constant marginal cost of extraction, c.We assume that the monopolist's extraction matches the demand for fossil fuels, $q_t = D(P_t;z)$. The monopolist's optimization problem consists of choosing a time path

[16] If one takes into account the possible feedback effect (such as additional emissions from the permafrost biomass that could result from increased GHGs concentrations in the medium run) the steeper path of cumulative fossil fuel extraction in the medium run can result in greater irreversible damages even if long run total extraction is lower due to abandonment of reserves. See Winter (2011) on this point.

of price $P_t \in \left[0, \overline{P}\right]$ and a terminal date T to maximize the present value of its stream of discounted profit:

$$\max_{T,P_t} \int_0^T e^{-rt} \left[(P_t - c)D(P_t; z)\right] dt$$

subject to the constraint

$$\dot{R}_t = -D(P_t; z), \qquad R_T \geq 0,\, R_0 \text{ given.}$$

Denote the elasticity of the demand for fossil fuels by $\theta_t(P_t; z)$, defined by

$$\theta_t(P_t; z) \equiv -\frac{P_t}{D} \left(\frac{\partial D}{\partial P_t} \right) > 0$$

Assumption A: *The elasticity of demand for fossil fuels is (i) greater than unity for sufficiently high P, and (ii) non-decreasing in P :*

$$\frac{\partial \theta_t(P_t; z)}{\partial P_t} \geq 0$$

Let ψ_t denote the current-value shadow price of the stock R_t and let H_t denote the current-value Hamiltonian. Then

$$H_t = (P_t - c)D(P_t; z) - \psi_t D(P_t; z)$$

The optimality conditions for the monopolist are[17]

$$P_t \left[1 - \frac{1}{\theta_t} \right] - c - \psi_t = 0 \Leftrightarrow \frac{\partial H_t}{\partial P_t} = 0$$

$$\dot{\psi}_t = r\psi_t$$

$$\psi_T \geq 0, \qquad R_T \geq 0, \qquad \psi_T R_T = 0$$

$$H_T = \left[P_T - c - \psi_T\right] D(P_T; z) = 0$$

One can show that conditions imply that $\psi_T = P_T - c > 0$, $R_T = 0$, $P_T = \overline{P}$, $D(P_T; z) = 0$, and $\theta_T = \infty$. In particular, we obtain the Hotelling Rule for the monopolist: the present value of marginal profit is the same for all $t \in [0, T]$:

$$\left[\left(1 - \frac{1}{\theta_t(P_t; z)} \right) P_t - c \right] = \left(\overline{P} - c \right) e^{r(t-T)} \tag{23}$$

[17] As is well known, the monopolist always restricts supply so that $\theta_t > 1$.

Equation (23) implicitly defines the monopolist's optimal price P_t^m as a function of $\overline{P}$, z, t and T :

$$P_t^m = \phi^m(\overline{P}(z), t, T; z) \tag{24}$$

Note that the function ϕ^m in (24) is of the same nature as the function ϕ in (10) but they do not have the same functional form. Furthermore, the function ϕ^m depends on z both via $\theta_t(P_t; z)$ and via $\overline{P}$.

We make use of (24) to determine the monopolist's optimal exhaustion time T. It is the value of T such that total accumulated extraction equals the initial reserve level R_0.

$$\int_0^T D_E(\phi^m(\overline{P}(z), t, T; z); z)dt = R_0 \tag{25}$$

Lemma 2: *An increase in the subsidy factor z will bring the monopolist's resource-exhaustion date T closer to the present if and only if the indirect effect outweighs the direct effect, i.e.,*

$$\int_0^T \left[\frac{\partial D}{\partial P_t}\left(\frac{\partial P_t}{\partial z}\right) + \frac{\partial D}{\partial z}\right] dt > 0 \tag{26}$$

where

$$\frac{\partial P_t}{\partial z} \equiv \frac{\partial \phi^m}{\partial \overline{P}}\frac{d\overline{P}}{dz} + \frac{\partial \phi^m}{\partial z}.$$

6.1 Cartel extraction under linear residual demand

Let $D_E(P) = a - bP$ and $S = \beta z P - g$, where $a > 0$, $b > 0$, $\beta > 0$ and $g \geq 0$. Then

$$\phi^m(\overline{P}(z), t, T; z) = \frac{1}{2}\left[\left(\overline{P} - c\right) e^{r(t-T)} + c + \frac{a}{b+z}\right]$$

and

$$D_P\left(\frac{\partial \phi^m}{\partial \overline{P}}\frac{d\overline{P}}{dz} + \frac{\partial \phi^m}{\partial z}\right) + D_z = -\frac{c}{2}\left(1 - e^{-r(T-t)}\right) \leq 0$$

Proposition 4: *Assume that the demand function for fuel and the supply function of biofuel are both linear. Then, under monopoly extraction,*

(i) if extraction costs are zero ($c = 0$), an increase in the biofuel subsidy factor z will have no effect on the date of exhaustion of the resource stock R_0;

(ii) if extraction costs are positive ($c > 0$), an increase in the biofuel subsidy factor z will delay the date of exhaustion of the resource stock R_0.

6.2 Cartel extraction under non-linear residual demand

In this subsection, we assume the functional forms (18) and (17). Let $\mu = 1$. Then $D = (P+\delta)^{-\varepsilon} - \beta z P$, and the elasticity of demand for fossil fuels is

$$\theta_t = \frac{\left[\varepsilon (P_t+\delta)^{-\varepsilon-1} + \beta z\right] P_t}{(P_t+\delta)^{-\varepsilon} - \beta z P_t} > 0 \text{ for } P_t < \overline{P}$$

Note that if $\beta z > 0$ and $\varepsilon \geq 1$ then $\theta_t > 1$ because

$$\theta_t > \frac{\varepsilon P_t}{P_t + \delta} \geq \varepsilon$$

It can be verified that Assumption A is satisfied. Equation (23) becomes

$$\left\{ P_t - \frac{1}{\theta_t} P_t - c \right\} - \left(\overline{P} - c\right) e^{r(t-T)} = 0 \tag{27}$$

Denote the right-hand side of equation (27) by $F(P_t, \overline{P}, t, T, z)$. Equation (27) yields the implicit function $P_t = \phi^m(\overline{P}(z), t, T; z)$. Then

$$\frac{\partial \phi^m}{\partial \overline{P}} = -\frac{\partial F / \partial \overline{P}}{\partial F / \partial P_t} = \frac{e^{r(t-T)}}{\left(1 - \frac{1}{\theta_t}\right) - P_t \frac{d}{dP_t}\left(\frac{1}{\theta_t}\right)} \equiv \frac{e^{r(t-T)}}{\Omega_t} > 0$$

where $\Omega_t > 0$ because Assumption A is satisfied. Furthermore,

$$\frac{\partial \phi^m}{\partial z} = -\frac{\partial F / \partial z}{\partial F / \partial P_t}$$

where

$$\partial F / \partial z = -\frac{P_t}{\theta_t^2} \left(\frac{\partial \theta_t}{\partial z} \right) < 0$$

and

$$\frac{\partial \theta_t}{\partial z} = \frac{\beta P_t}{(D)^2} \left[D + P_t \varepsilon (P+\delta)^{-\varepsilon-1} + z P_t \right] > 0$$

(a higher biofuel subsidy increases the elasticity of demand for fossil fuels at any given price). Consequently,

$$\frac{\partial \phi^m}{\partial z} = -\frac{\frac{\beta P_t^2}{(\theta_t D)^2} \left[D + P_t \varepsilon (P+\delta)^{-\varepsilon-1} + z P_t \right]}{\Omega_t} < 0$$

We can now compute the crucial expression in Lemma 2, condition (26):

$$\frac{\partial D_f}{\partial P_t}\left(\frac{\partial \phi^m}{\partial \overline{P}}\frac{d\overline{P}}{dz}+\frac{\partial \phi^m}{\partial z}\right)+\frac{\partial D}{\partial z}$$

$$=\frac{\varepsilon\left(P_t+\delta\right)^{-\varepsilon-1}+\beta z}{\Omega_t}\left(\frac{\overline{P}\beta e^{r(t-T)}}{\varepsilon(\overline{P}+\delta)^{-\varepsilon-1}+\beta z}+\frac{\beta P_t^2}{\left(\theta_t D\right)^2}\left[D+P_t\varepsilon(P+\delta)^{-\varepsilon-1}+zP_t\right]\right)-\beta P_t$$

Using this expression, the integral (26) can be positive or negative, depending on parameter values.

Proposition 5: *Under non-linear demand, a Green Paradox can arise when fossil fuels are supplied by a cartel.*

7 Extension 2: Green Paradox in a two-country model

An important policy issue is whether well-meaning environmental policies of advanced industrialized countries is likely to lead to a Green Paradox because emerging market economies are not willing to reduce their GHG emissions. Earlier works on this important topics include Bohm (1993) and Hoel (1994). (For a recent discussion of carbon leakage in the context of trade, see Copeland and Taylor, 2005, and the references therein). However, these authors did not address the relationship between non-optimal biofuel policies and carbon leakage effects in a dynamic setting, a connection which we now take up. As a metaphor, let us consider the case where there are two countries with different energy policies. For simplicity we do not consider game theoretic or food policy issues (Bandyopadhyay et al. 2009). Let us say the "home country" is the U.S. and the "foreign country" is China. There is no biofuel production in China. Assume that U.S. biofuels are not exported to China (e.g. because of high transport costs or other barriers to trade, or because the U.S. production subsidies are only given to domestically earmarked consumption).

Let P_t be the world price of fossil fuels. Assume that China's inverse demand function for fuels is

$$P_t = A - \frac{1}{M^c}E_t^c$$

where E_t^c is the quantity demanded in China, i.e., China's demand for energy is $D_E^c(P_t) = M^c(A - P_t)$. Similarly, the US inverse demand function for fuels is

$$P_t = A - \frac{1}{M^u} E_t^u$$

i.e. , U.S.'s demand for energy is $D_E^u(P_t) = M^u(A - P_t)$. The ratio M^c/M^u is a measure of China's market size relative to the U.S.'s market size. By an appropriate choice of units, we set $M^u = 1$ in what follows.

When the price is P_t, U.S. biofuel producers earn zP_t for each unit they sell domestically, where z is the subsidy factor. The biofuel supply function is $S_B = \beta (zP)^\mu$ and the U.S.'s residual demand for fossil fuels is

$$D^u(P; z) = D_E^u(P_t) - \beta (zP)^\mu = A - P - \beta (zP)^\mu \text{ since } M^u = 1.$$

Let $\overline{P}_u$ be the solution of

$$A - \overline{P}_u - \beta \left(z\overline{P}_u\right)^\mu = 0 \tag{28}$$

Note that

$$\frac{d\overline{P}_u}{dz} = -\frac{z^{\mu-1}\left(\overline{P}_u\right)^\mu}{\frac{1}{\beta\mu} + z^\mu \left(\overline{P}_u\right)^{\mu-1}} < 0 \tag{29}$$

Given a positive constant subsidy factor $z > 0$, the world equilibrium consists of two phases. In the first phase, fossil fuels are consumed in both the U.S. and China. This phase ends at an endogenously determined time T^u, when P_t reaches the value $\overline{P}_u$. In the second phase, fossil fuels are only used in China, and U.S. energy demand is completely satisfied by biofuel production.[18] The second phase ends at time T^w, when the world price of fossil fuels reaches the choke price A and China's demand for fossil fuels becomes zero.

Our task is to find out how the subsidy rate z influences the two critical times T^u and T^w, and how it influences the equilibrium price path of fossil fuels, and hence the rate of emissions of CO_2 at each point of time.

[18] In this section, we use the word biofuels only as a metaphore, representing all other renewable energy sources.

Let $D^w(t)$ be the world (US and China) demand for fossil fuels during phase I, i.e. $D^w(t) = D^u(P_t; z) + D^c_E(P_t)$, for $t < T^u$. For $t \geq T^u$, let $D^c_E(t)$ be China's demand for fossil fuels in phase II. Let R_0 be the initial stock of fossil fuels. Equilibrium requires that the total use of fossil fuels equals its stock:

$$\int_0^{T^u} D^w(t)dt + \int_{T^u}^{T^w} D^c_E(t)dt = R_0 \tag{30}$$

We begin by evaluating the second integral on the left-hand side of (30). Assume zero extraction cost and perfect competition. Hotelling's Rule gives us,

$$P_t e^{-rt} = P(T_u)e^{-rT^u} = P(T^w)e^{-rT^w}$$

Given that $P(T^w) = A$ and $P(T_u) = \overline{P}_u$,

$$T^w - T^u = \frac{1}{r}(\ln A - \ln \overline{P}_u(z)) \equiv J(z)$$

This indicates that the length of Phase II is an increasing function of the subsidy factor z.

$$\frac{dJ}{dz} = -\frac{1}{r\overline{P}_u}\left(\frac{d\overline{P}_u}{dz}\right) > 0$$

China's oil consumption during Phase II is

$$\int_{T^u}^{T^w} D^c_E(P_t)dt = M^c \int_{T^u}^{T^w} (A - P_t)dt = M^c \int_{T^u}^{T^w} (A - A^{-r(T^w - t)})dt$$

$$= AM^c \left\{ T^w - T^u - \frac{1}{r}\left[1 - e^{-r(T^w - T^u)}\right] \right\}$$

$$= AM^c \left\{ J(z) - \frac{1}{r}\left[1 - e^{-rJ(z)}\right] \right\} \equiv K(z)$$

It is easy to verify that $K(z)$ is an increasing function: the higher is the biofuel subsidy factor in the U.S., the greater is China's total fossil fuel consumption in Phase II:

$$\frac{dK}{dz} = AM^c(1 - e^{-rJ(z)})\frac{dJ}{dz} > 0. \tag{31}$$

The inequality (31) is our first result concerning"carbon leakage" in a two-country world.

Let us turn to Phase I, in which fossil demands are positive in both countries. The total accumulated consumption of fossil fuels in Phase I must equal $\overline{R} - K(z)$:

$$\int_0^{T^u} [D^u(P_t; z) + D_E^c(P_t)]\, dt = \overline{R} - K(z)$$

where

$$P_t = \overline{P}_u e^{-r(T^u - t)} \equiv \phi(\overline{P}_u(z), t, T^u)$$

Define

$$G(T^u, z, \overline{R}) \equiv \int_0^{T^u} [D^u(\phi; z) + D_E^c(\phi)]\, dt - \overline{R} + K(z)$$

The effect of an increase in the subsidy factor z on the time T^u at which the U.S.'s demand for fossil fuels vanishes is given by

$$\frac{dT^u}{dz} = -\frac{\partial G/\partial z}{\partial G/\partial T^u}.$$

To determine the sign of this expression, we note that

$$\partial G/\partial T^u = D_E^c(\overline{P}_u) + \int_0^{T^u} \left[\frac{\partial D^u}{\partial P_t} + \frac{\partial D_E^c}{\partial P_t}\right] \frac{\partial \phi}{\partial T^u} dt > 0$$

and

$$\partial G/\partial z = \frac{dK}{dz} + \int_0^{T^u} \left[\left(\frac{\partial D^u}{\partial P_t} + \frac{\partial D_E^c}{\partial P_t}\right) \frac{\partial \phi}{\partial \overline{P}_u} \frac{d\overline{P}_u}{dz} + \frac{\partial D^u}{\partial z}\right] dt \tag{32}$$

Using Hotelling's Rule, the integrand in (32) can be simplified to

$$z^{\mu-1}\mu\beta e^{r(t-T)} \left[\frac{M_c + \mu\beta z^\mu \left[(P_t)^{\mu-1} - \left(\overline{P}_u\right)^{\mu-1}\right]}{1 + \mu\beta z^\mu \left(\overline{P}_u\right)^{\mu-1}}\right] \;, t \leq T^u$$

which is positive if M_c is big. Thus, if China's demand is significant enough, an increase in biofuel subsidy in the U.S. will lead to an earlier beginning of Phase II.

What about the effect of an increase in z on the date where all fosil fuel stocks are exhausted, T^w? We must compute

$$\frac{dT^w}{dz} = \frac{dT^u}{dz} + \frac{dJ}{dz}$$

This expression is negative if and only if

$$(\partial G/\partial T^u)\,\frac{dJ}{dz} < \frac{dK}{dz} + \int_0^{T^u} \left[\left(\frac{\partial D^u}{\partial P_t} + \frac{\partial D^c_E}{\partial P_t}\right)\frac{\partial \phi}{\partial \overline{P}_u}\frac{d\overline{P}_u}{dz} + \frac{\partial D^u}{\partial z}\right] dt$$

i.e. if and only if

$$-\int_0^{T^u} \left[\left(\frac{\partial D^u}{\partial P_t} + \frac{\partial D^c_E}{\partial P_t}\right)\frac{\partial \phi}{\partial \overline{P}_u}\frac{d\overline{P}_u}{dz} + \frac{\partial D^u}{\partial z}\right] dt + \frac{\partial D^u}{\partial z}dt - \left[\frac{dJ}{dz}\right]\left[AM^c(1-e^{-rJ(z)}) - (\partial G/\partial T^u)\right] < 0$$

We conclude that if the U.S.'s choke price is highly responsive to increases in biofuel subsidies, i.e. if $\frac{d\overline{P}_u}{dz}$ is sufficiently large in absolute value, then an increase in biofuel subsidy will bring the resource exhaustion date closer to the present.

Proposition 6: *An increase in the subsidy factor z will lengthen Phase II and may shorten Phase I. The exhaustion date will be brought closer to the present if the U.S.'s choke price is highly responsive to increases in biofuel subsidies.*

8 Concluding Remarks

We examine several possible cases under which a Green Paradox may arise from a policy of biofuel subsidies whereby the supply-side response by fossil fuel producers more than offsets any gains from direct substitution to biofuels. Whether the Green Paradox holds or not depends on demand and supply elasticities, expected changes in subsidies, technological change in fossil fuel extraction and how extraction costs respond to changes in remaining reserves. We find that a Green Paradox is not a general result, but under plausible assumptions it may exist.

A key implication of our theoretical work is the need to test empirically whether a Green Paradox exists. Equally as important, our work shows the importance of considering: (1) the inter-temporal effects of biofuel subsidies on fossil fuel production; (2) the trade effects of biofuel subsidies even for countries that neither produce nor consume biofuels; and (3) the industry structure (competitive or cartel) of fossil fuel production.

The implications of our findings on energy policy are several. First, and foremost, is the importance and necessity of analyzing the impacts of biofuels policies in a dynamic context.

Second, domestic policies of large biofuel producing nations must be evaluated in a welfare context that includes the effects on both biofuel and fossil fuel consumption globally. Third, we provide an additional reason why biofuel subsidies are a sub-optimal policy and, in some cases, may even generate perverse outcomes that are contrary to the stated objectives which supposedly justify their use.

Further development of our dynamic models is necessary to take into account game-theoretic issues, the effects of technological change on extraction costs, the impacts of GHG atmospheric concentrations and the rates of decay, and the direct GHG reductions from biofuel-fossil fuel substitution at a given price. Nevertheless, our findings are sufficiently well developed to require, at the very least, that policy makers carefully evaluate the dynamic and supply-side effects of biofuel subsidies on the extraction rate of fossil fuels.

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APPENDIX I

Exhaustion with stock-dependent cost and linear demand

With the linear demand $D(P_t; z) = a-(b+\beta z)P_t$, the fossil fuel choke price is $\overline{P}(z) = \frac{a}{b+\beta z}$. Define

$$Y_t = R_0 - R_t$$

Assume that the marginal cost of extracting the last drop of oil is lower than the fossil fuel choke price:

$$\overline{R} < \frac{a}{c_1(b+\beta z)} - \frac{c_0}{c_1} \equiv \widetilde{Y}(z)$$

Consider the system

$$\dot{P}_t = r\left[P_t - (c_0 + c_1 Y_t)\right], \qquad r > 0, c_0 > 0, c_1 > 0$$

$$\dot{Y}_t = D(P_t; z) = a - (b+\beta z)P_t, \qquad a > 0, b > 0, \beta > 0, z > 0$$

subject to three boundary conditions: $Y_0 = 0$, $Y_T = \overline{R}$, $P_T = \overline{P}(z)$.

Write the system of differential equations in matrix form:

$$\begin{bmatrix} \dot{P} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} P \\ R \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

where $a_{11} = r, a_{12} = -rc_1, a_{21} = -(b+\beta z), a_{22} = 0, b_1 = -rc_0$ and $b_2 = a$.In simpler notation,

$$\dot{\mathbf{w}} = \mathbf{A}\mathbf{w} + \mathbf{b}$$

where $\det \mathbf{A} = a_{11}a_{22} - a_{12}a_{21} = -rc_1(b+\beta z) < 0$

$$\mathbf{A}^{-1} = \frac{1}{-rc_1(b+\beta z)}\begin{bmatrix} 0 & rc_1 \\ b+\beta z & r \end{bmatrix}$$

Define $\widetilde{\mathbf{w}}$ by

$$\widetilde{\mathbf{w}} = -\mathbf{A}^{-1}\mathbf{b}$$

Then

$$\widetilde{\mathbf{w}} = \begin{bmatrix} \frac{a}{b+\beta z} \\ \frac{a}{(b+\beta z)c_1} - \frac{c_0}{c_1} \end{bmatrix} \equiv \begin{bmatrix} \widetilde{w}_1 \\ \widetilde{w}_2 \end{bmatrix}$$

Now let us define $\mathbf{x}$ by

$$\mathbf{x} \equiv \mathbf{w} - \widetilde{\mathbf{w}}$$

Let $\rho_1 < 0$ and $\rho_2 > 0$ be the characteristic roots,

$$\begin{aligned} \rho_{1,2} &= \frac{a_{11}+a_{22} \pm \sqrt{(a_{11}+a_{22})^2 - 4\left(a_{11}a_{22} - a_{12}a_{21}\right)}}{2} \\ &= \frac{1}{2}\left(r \pm \sqrt{\Delta}\right) \text{ where } \Delta \equiv r^2 + 4rc_1(b+\beta z) \end{aligned}$$

Then the general solution is

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} h_1 \\ -\left(a_{12}\right)^{-1}(a_{11}-\rho_1)h_1 \end{bmatrix} \exp(\rho_1 t) + \begin{bmatrix} h_2 \\ -\left(a_{12}\right)^{-1}(a_{11}-\rho_2)h_2 \end{bmatrix} \exp(\rho_2 t) \tag{33}$$

where h_1 and h_2 are constants (to be determined using boundary conditions).

Define

$$v_1 \equiv -\left(a_{12}\right)^{-1}\left(a_{11}-\rho_1\right) = \frac{\rho_1 - r}{-rc_1} \tag{34}$$

$$v_2 \equiv -\left(a_{12}\right)^{-1}\left(a_{11}-\rho_2\right) = \frac{\rho_2 - r}{-rc_1} \tag{35}$$

Setting $t = T$ in the matrix equation (33), and noting that have $x_1(T) = P_T - \widetilde{w}_1 = 0$ and $x_2(T) = Y_T - \widetilde{w}_2 = \overline{R} - \widetilde{w}_2 \equiv \overline{R} - \widetilde{Y}(z)$, we get two equations:

$$0 = h_1 \exp(\rho_1 T) + h_2 \exp(\rho_2 T) \tag{36}$$

$$\overline{R} - \widetilde{Y}(z) = v_1 h_1 \exp(\rho_1 T) + v_2 h_2 \exp(\rho_2 T) \tag{37}$$

Equation (36) gives

$$h_1 = -h_2 \exp\left[\left(\rho_2 - \rho_1\right) T\right] \tag{38}$$

Substituting into equation (37) to get

$$\overline{R} - \widetilde{Y}(z) = -v_1 h_2 \exp(\rho_2 T) + v_2 h_2 \exp(\rho_2 T)$$

Therefore

$$h_2 = \frac{\overline{R} - \widetilde{Y}(z)}{(v_2 - v_1)\exp(\rho_2 T)} \tag{39}$$

Setting $t = 0$ in the matrix equation (33), we get two equations

$$P_0 - \overline{P}(z) = h_1 + h_2 \tag{40}$$

$$Y_0 - \widetilde{Y}(z) = v_1 h_1 + v_2 h_2 \tag{41}$$

Since $Y_0 = 0$, substituting (38) and (39) into (41) we get

$$-\widetilde{Y}(z) = \frac{\overline{R} - \widetilde{Y}(z)}{(v_2 - v_1)\exp(\rho_2 T)} \left[v_2 - v_1 \exp\left[(\rho_2 - \rho_1) T\right]\right]$$

or

$$\frac{(v_2 - v_1)\exp(\rho_2 T)}{v_2 - v_1 \exp\left[(\rho_2 - \rho_1) T\right]} = \frac{\widetilde{Y}(z) - \overline{R}}{\widetilde{Y}(z)} \tag{42}$$

Now, using (34), (35) and (42)

$$\frac{(\rho_2 - \rho_1)\exp(\rho_2 T)}{-\rho_1 + \rho_2 \exp\left[(\rho_2 - \rho_1) T\right]} = \frac{\widetilde{Y}(z) - \overline{R}}{\widetilde{Y}(z)}$$

Let

$$\phi(T, z) \equiv \frac{(\rho_2 - \rho_1)}{-\rho_1 \exp\left[-\rho_2 T\right] + \rho_2 \exp\left[-\rho_1 T\right]}$$

The function $\phi(T, z)$ has the following properties: $\phi(0, z) = 1, \partial\phi(T, z)/\partial T < 0$ for all $T > 0$, and

$$\lim_{T \to \infty} \phi(T, z) = 0$$

It follows that the equation

$$\phi(T, z) = \frac{\widetilde{Y}(z) - \overline{R}}{\widetilde{Y}(z)}$$

has a unique solution $T > 0$ (which depends on z).

Numerical examples

Suppose $\overline{R} = 80, r = 0.05, a = 3, b = \beta = z = 1, c_0 = 1$.Let $c_1 = 0.001$. Then

$$\widetilde{Y}(z) = \frac{a}{(b+\beta z)\, c_1} - \frac{c_0}{c_1} = \frac{1}{c_1}\left(\frac{a}{b+\beta z} - c_0\right) = \frac{0.5}{c_1} = 500$$

$$\rho_1 = -4.950\,975 \times 10^{-4}$$

$$\rho_2 = 5.049\,509\,8 \times 10^{-2}$$

The equation

$$\frac{(\rho_2 - \rho_1)}{-\rho_1 \exp(-\rho_2 T) + \rho_2 \exp(-\rho_1 T)} = \frac{\widetilde{Y}(z) - \overline{R}}{\widetilde{Y}(z)} = 0.84$$

yields the unique solution $T = 370$.

Now let z increase to 1.1. Then T increases from 370 years to 410 years.

If c_1 is higher, $c_1 = 0.002$, while $z = 1$ and $\overline{R} = 80$. It will take 394 years to exhaust $\overline{R}$.

Now let z increase to 1.1. Then T increases from 394 years to 475 years.

APPENDIX 2

Proof of Proposition 3

From the assumption that $\frac{a}{b+\beta z} > c_0 + c_1\overline{R}$, let us define

$$\widetilde{Y}(z) \equiv \frac{a}{c_1(b+\beta z)} - \frac{c_0}{c_1} > \overline{R}$$

In Appendix 1, we showed that the exhaustion date T is the unique positive solution of the following equation

$$\frac{(\rho_2 - \rho_1)}{\rho_2 e^{-\rho_1 T} - \rho_1 e^{-\rho_2 T}} = \left(\frac{\widetilde{Y}(z) - \overline{R}}{\widetilde{Y}(z)}\right)$$

where ρ_1 and ρ_2 are functions of z :

$$\rho_2 = \frac{1}{2}\left(r + \sqrt{r^2 + 4rc_1(b+\beta z)}\right) > 0$$

$$\rho_1 = \frac{1}{2}\left(r - \sqrt{r^2 + 4rc_1(b+\beta z)}\right) < 0$$

Define

$$F(T,z) = \frac{(\rho_2 - \rho_1)}{\rho_2 e^{-\rho_1 T} - \rho_1 e^{-\rho_2 T}}$$

and

$$G(T,z) \equiv F(T,z) - \left(\frac{\widetilde{Y}(z) - \ \overline{R}}{\widetilde{Y}(z)} \right)$$

To prove part (i), note that the effect on T of an increase in the subsidy z is

$$\frac{dT}{dz} = -\frac{\frac{\partial G}{\partial z}}{\frac{\partial G}{\partial T}}$$

We can show that

$$\frac{\partial G}{\partial T} = -\frac{\rho_1 \rho_2 (\rho_2 - \rho_1)^2 e^{\rho_2 T}}{\left\{ \rho_2 e^{(\rho_2 - \rho_{1)T}} - \rho_1 \right\}^2} \left[e^{\rho_2 T} - e^{(2\rho_2 - \rho_1)} \right] < 0$$

and

$$\frac{\partial G}{\partial z} = \frac{\partial F}{\partial z} - \ \overline{R} \left[\widetilde{Y}(z) \right]^{-2} \frac{d\widetilde{Y}}{dz} > 0$$

since $\frac{\partial F}{\partial z} > 0$.

To prove part (ii), we use $\frac{dT}{dc_1} = -\left(\frac{\partial G}{\partial c_1} \right) / \left(\frac{\partial G}{\partial T} \right)$ and show that $\left(\frac{\partial G}{\partial c_1} \right) > 0.$■

T	50	75	**100**	150	200
$\frac{dT}{dz}$	-1.75	-2.04	$\mathbf{-2.09}$	-2.110	-2.113

Table 1: Effect of a Biofuel Subsidy on Exhaustion Date for Different Fossil Fuel Reserve Sizes

z	0.5	0.75	**1**	1.5	2
$\frac{dT}{dz}$	-6.6	-3.43	$\mathbf{-2.09}$	-0.99	-0.56

Table 2: Effect of a Biofuel Subsidy on Exhaustion Date for Different Initial Subsidy Levels

ε	0.1	0.5	**1**	2	3
$\frac{dT}{dz}$	-0.35	-1.34	$\mathbf{-2.09}$	-2.89	-3.32

Table 3: Effect of a Biofuel Subsidy on Exhaustion Date for Different Demand Elasticities of the Manufactured Good

μ	0.5	0.75	**1**	1.5	2
$\frac{dT}{dz}$	**1.45**	-2.07	$\mathbf{-2.09}$	-3.88	-3.75

Table 4: Effect of a Biofuel Subsidy on Exhaustion Date for Different Supply Elasticities of Biofuel

Cost sensitivity parameter	Marginal extraction cost of the 'last drop'	Choke price of fuel when z=1.2/z=1	ΔT	$\Delta T/\Delta z$	Paradox/ No Paradox
0	0.005	0.54/0.618	-0.270	-1.35	Paradox
0.0001	0.013	0.54/0.618	-0.176	-0.88	Paradox
0.0002	0.021	0.54/0.618	-0.077	-0.39	Paradox
0.0003	0.029	0.54/0.618	0.025	+0.13	No Paradox
0.0005	0.045	0.54/0.618	0.244	+1.22	No Paradox

Table 5: Parameter Values for the Weak Green Paradox

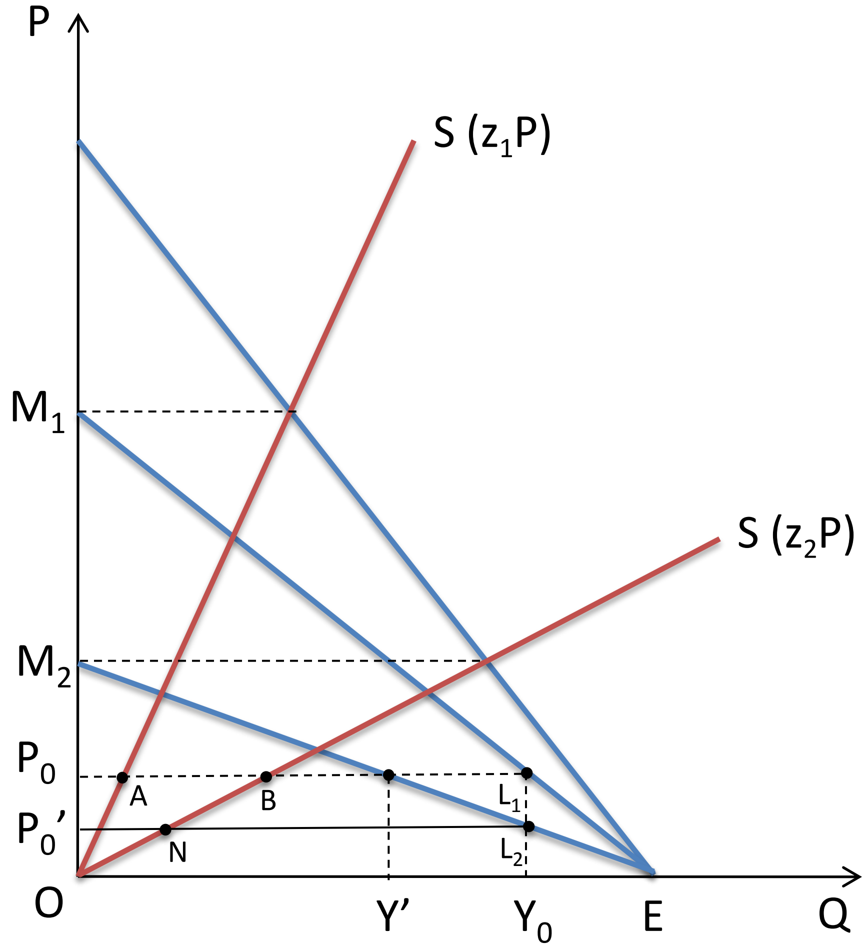

Figure 1. Direct and indirect effects of a biofuel subsidy

Figure 2 – Special case: no impact on exhaustion time and no impact on extraction path

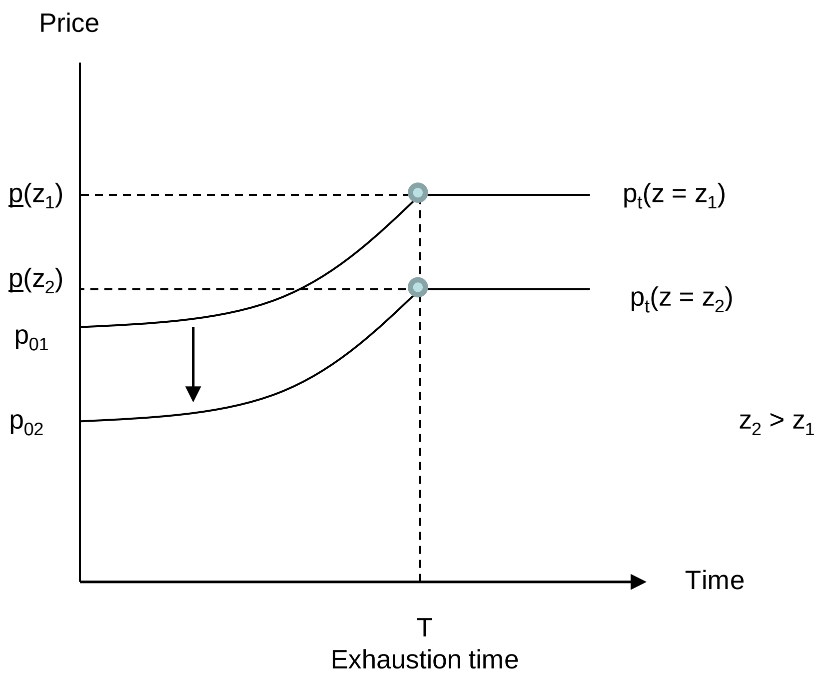

Figure 3 – Green paradox: higher biofuel subsidy causes faster exhaustion