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IN SEARCH EQUILIBRIUM*

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## Cahier 26-2005

### *SPECIALIZATION AND HUMAN CAPITAL IN SEARCH EQUILIBRIUM*

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# Specialization and Human Capital in Search Equilibrium\*

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## Abstract

This paper constructs a general equilibrium search-matching model with heterogeneous workers. Workers choose whether to invest in general human capital or specific human capital when they enter the labor market. We analyze how unemployment benefits affect the choice of the type of human capital investment.

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# 1 Introduction

For many decades, the human capital approach has been very successful in analyzing various labor market phenomena. Following Ben-Porath (1967), the standard analysis of human capital has largely assumed that human capital is homogenous and examined *how much* human capital is accumulated. In this paper, we analyze instead *what kind of* human capital is accumulated. In particular, we ask the question: In which environment do people accumulate specific human capital and in which environment do people choose to invest in general human capital? By specific human capital we mean that the skills obtained can only be used in particular tasks or occupations, and by general human capital we mean that the skills can be applicable to a broad range of tasks or occupations.

Rosen (1983) pointed out that when the investment cost for human capital does not depend on the subsequent utilization rate, people tend to invest in specialized skills rather than a broad range of skills. To see his point, consider the following model.<sup>1</sup> There are two tasks,  $X$  and  $Y$ , and to perform each task, one has to obtain the corresponding skill. At the beginning of life, each worker has a choice of investing in both skills (general education), or in just one of them (specialized education). Each worker is endowed with one unit of working time. If the worker has invested in only one skill, he will specialize in that skill and will work only on the corresponding task. If the worker has invested in both skills, then the worker can decide how much time to spend working for task  $X$  and how much time to spend for task  $Y$ . Let the time spent for task  $X$  be  $t_X$ . Let the cost of general education be  $c_G$  and the cost of special education be  $c_S$ , with  $c_G > c_S$ . Assume that the wage for task  $X$  is  $w_X$  and the wage for task  $Y$  is  $w_Y$ . Then, the payoff of obtaining general education is

$$W_G = -c_G + \max_{t_X} \{t_X w_X + (1 - t_X) w_Y\},$$

while the payoff of obtaining specialized education in skill  $i$ ,  $i = X, Y$ , is

$$W_i = -c_S + w_i.$$

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<sup>1</sup>The model can be seen as Roy's (1951) model with a human capital decision.

Therefore,  $\max\langle W_X, W_Y \rangle > W_G$ . In particular, when  $w_X = w_Y$ ,  $W_X > W_G$  and  $W_Y > W_G$  always hold, and therefore people prefer to specialize. Rosen’s point is particularly relevant for schooling decisions, since schooling investments tend to occur earlier in life and are not reversible. In reality, however, we observe that a large part of education is general; it is not targeted towards one particular task or occupation. Although there are many reasons why general education can be beneficial, we will consider only one particular reason—it helps people to cope with uncertainty.

This idea is not entirely new. Murphy (1986) made the point that when there is uncertainty, there is a benefit of becoming a generalist. This can easily be seen from the above example. Assume that there is uncertainty in  $w_X$  and  $w_Y$ , which is resolved after the investment has occurred (and the investment is not reversible). In particular, let  $(w_X, w_Y) = (w, 0)$  with probability 1/2 and  $(w_X, w_Y) = (0, w)$  with probability 1/2 (assume that  $w > 0$ ). Then,  $W_G = -c_G + w$ , while  $E[W_X] = E[W_Y] = -c_S + w/2$ . Therefore, when workers are risk neutral and  $c_G - c_S < w/2$ , there is an incentive to invest in general education.

We explore this idea in a search-matching setting. In our model, a worker needs to form a match with another worker and specialize in one task for production. Therefore, search friction (uncertainty regarding whom to match with) creates uncertainty regarding which task one will engage in when the match is formed. Training for general skills broadens the set of skills and helps one to cope with uncertainty. We solve a simple dynamic general equilibrium model and examine under what kind of environment people decide to specialize.<sup>2</sup>

The paper is organized as follows. In Section 2, we set up and solve the model. In Section 3, we present a numerical example. Section 4 concludes.

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<sup>2</sup>Sundaram (2002) is the closest to our paper in spirit. She also constructs a search-matching model to analyze when people choose to specialize. In contrast to our model, her model is more focused on the marriage market application. She assumes that matched people are immediately replaced by unmatched people (the so-called “cloning” assumption). In our model, matched people are later separated with some probability, so that the rate of unemployment is endogenously determined. The consequence of this assumption is discussed in footnote 6.

## 2 Model

The general setup follows Mukoyama and Şahin (2005). A worker  $i$  is characterized by a two-dimensional skill vector,  $(x_i, y_i) \in [0, 1] \times [0, 1]$ .  $x_i$  represents the ability to perform task  $X$ , and  $y_i$  is the ability to perform task  $Y$ . When worker  $i$  and worker  $j$  match and worker  $i$  specializes in task  $X$  and worker  $j$  specializes in task  $Y$ , both receive a flow payoff  $x_i + y_j$ .

There are three types of workers, categorized according to the skill they possess. There are two types of “specialists”:  $X$ -type and  $Y$ -type. An  $X$ -type worker is good at performing task  $X$  and has the talent vector  $(1, 0)$ . Similarly, a  $Y$ -type worker has the talent vector  $(0, 1)$ . There is also a “generalist” type, denoted by  $G$ , whose talent vector is  $(1, 1)$ .

We consider a “perpetual youth” type overlapping-generations model, set in continuous time. A worker faces a Poisson probability  $\eta$  of death at each moment. We normalize the total population to one, therefore  $\eta$  number of people die at each moment. We assume that  $\eta$  number of people are born at each moment, thus the total population remains constant. When workers are born, they choose their skills (education choice). We assume that it is more expensive to accumulate a general skill. We normalize the cost of acquiring only one skill to zero, and assume that the cost of acquiring the general skill is  $c > 0$ . We also assume that a newborn is unemployed. In an equilibrium where some people choose to become specialists and some people choose to become generalists, a newborn has to be indifferent between these choices, thus the following has to hold:

$$U_G - c = U_i \text{ for } i = X, Y, \tag{1}$$

where  $U_i$  is the expected lifetime utility of a type- $i$  unemployed worker.

The Poisson probability of an unemployed worker meeting with another unemployed worker is  $\alpha$ . After meeting, two workers form a match only if both agree to start working together. A match is destroyed with an exogenous probability  $\delta$ . While unemployed, a worker receives  $h$  amount of flow utility in the form of unemployment insurance. Unemployment insurance is financed by a lump-sum tax  $\tau$  on all workers. We assume that the

government always balances the budget, and therefore  $\tau = hu$  holds, where  $u$  is the aggregate unemployment rate. The workers discount the future at the rate  $r$ .

In the following, we consider the steady-state equilibrium of the economy. There are three levels of equilibrium conditions to consider. First is the equilibrium in workers' matching behavior. We call it the "Nash equilibrium" following Burdett and Coles (1999). Given the relative population of each type in the unemployment pool, a worker decides whether or not to accept a match. Second is the equilibrium in the distribution of unemployed workers, given the entire population of each type. Following Burdett and Coles (1999), we call it the "market equilibrium". In the steady-state, it is determined by the condition that the inflow to the employment pool is equal to the outflow from the employment pool for each type. Third is the equilibrium in educational choice. When there is a strictly positive population of each type, (1) has to hold. We call it "education equilibrium". In the next section, we analyze the Nash equilibrium. In the section following, the conditions for the market equilibrium and the education equilibrium are examined.

## 2.1 Nash Equilibrium

In this section, we analyze the matching decisions of the workers. We focus on pure strategies.

### 2.1.1 Generalists

The matching decision of a generalist is trivial. A generalist will be accepted by any type, and he will accept any type. The Bellman equations for a generalist are:

$$(r + \eta)V_G^j = 2 - \tau - \delta(V_G^j - U_G), \quad j \in \{G, X, Y\},$$

$$(r + \eta)U_G = h - \tau + \alpha[\pi_X(V_G^X - U_G) + \pi_Y(V_G^Y - U_G) + \pi_G(V_G^G - U_G)],$$

where  $V_i^j$  is the value of an  $i$ -type employed worker matched with a  $j$ -type worker.  $U_i$  is the value of an  $i$ -type worker.  $\pi_i$  is the fraction of  $i$ -type individuals in the unemployment pool. Therefore,  $\pi_X + \pi_Y + \pi_G = 1$ . From these Bellman equations,  $U_G$  can be solved as  $(r + \eta)U_G = h - \tau + \alpha(2 - h)/(\alpha + r + \eta + \delta)$ .



### 2.1.2 Specialists

In this paper, we focus on the equilibrium that is symmetric between  $X$ -type and  $Y$ -type. In the following, we explicitly write only the  $X$ -type cases when we analyze the specialists.

A specialist has two alternatives: **A**: Accept everyone; **B**: Accept everyone except for his own type. When choice **A** is made, the Bellman equations are:

$$\begin{aligned}(r + \eta)V_X^X &= 1 - \tau - \delta(V_X^X - U_X), \\ (r + \eta)V_X^j &= 2 - \tau - \delta(V_X^j - U_X), \quad j \in \{G, Y\}, \\ (r + \eta)U_X &= h - \tau + \alpha[\pi_X(V_X^X - U_X) + \pi_Y(V_X^Y - U_G) + \pi_G(V_X^G - U_X)].\end{aligned}$$

The  $X$ -type worker selects choice **A** if and only if  $V_X^X - U_X \geq 0$ . From the above Bellman equations, this condition is equivalent to

$$h \leq 1 - \frac{\alpha}{r + \eta + \delta}(1 - \pi_X). \quad (2)$$

When the specialists make choice **i** ( $\mathbf{i} = \mathbf{A}, \mathbf{B}$ ) in a Nash equilibrium, call it the Nash equilibrium **i**. We will use the above condition to determine whether the economy is consistent with Nash equilibrium **A** or Nash equilibrium **B**.

## 2.2 Market Equilibrium and Education Equilibrium

Let the population of  $i$ -type workers ( $i = G, X, Y$ ) be  $n_i$ . Denote the unemployment rate of  $i$ -type workers by  $u_i$ . In the steady-state market equilibrium, the inflow to the employment pool is equal to the outflow from the employment pool. For a  $G$ -type individual, this implies  $\alpha u_G n_G = (\delta + \eta)(n_G - u_G n_G)$ , therefore  $u_G = (\delta + \eta)/(\alpha + \delta + \eta)$  holds.

Suppose that the Nash equilibrium is **A**. We will check (2) to examine the conditions required on the parameters to be consistent with choice **A**. The market equilibrium condition for a specialist is identical to the one for the generalist under choice **A**, therefore,  $u_X = u_G = u = (\delta + \eta)/(\alpha + \delta + \eta)$ . From the definition of  $\pi_X$ ,  $\pi_X = n_X$  holds.

Let us consider the education equilibrium in this case. Calculating the values of the unemployment state for each type, we obtain

$$(r + \eta)U_G = h - \tau + \frac{\alpha}{\alpha + r + \eta + \delta}(2 - h)$$

and

$$(r + \eta)U_X = h - \tau + \frac{\alpha}{\alpha + r + \eta + \delta}(2 - h - \pi_X).$$

Thus, using the above solution for  $\pi_X$ , condition (1) implies

$$\pi_X = n_X = \frac{(r + \eta)(\alpha + r + \eta + \delta)c}{\alpha}. \quad (3)$$

By substituting this into (2), equilibrium **A** exists when

$$h \leq h^* \equiv 1 - \frac{\alpha - (r + \eta)(\alpha + r + \eta + \delta)c}{r + \eta + \delta} \quad (4)$$

is satisfied. Thus, when  $h \in [0, h^*]$ , equilibrium **A** exists, and the values of  $\pi_X$  and  $n_X$  are given by (3).

Now, suppose that the Nash equilibrium is **B**. We will need to check that

$$h > 1 - \frac{\alpha}{r + \eta + \delta}(1 - \pi_X) \quad (5)$$

is satisfied. To this end, we will first obtain  $\pi_X$  as a function of the parameters.

In this equilibrium,  $U_G$  is the same as above, but  $U_X$  is not. Specifically,

$$(r + \eta)U_X = h - \tau + \alpha[\pi_Y(V_X^Y - U_G) + \pi_G(V_X^G - U_X)].$$

Thus  $U_X$  is solved as

$$(r + \eta)U_X = h - \tau + \frac{\alpha(1 - \pi_X)}{\alpha(1 - \pi_X) + r + \eta + \delta}(2 - h).$$

Therefore, the education equilibrium condition (1) results in

$$\frac{\alpha}{\alpha + r + \eta + \delta}(2 - h) - (r + \eta)c = \frac{\alpha(1 - \pi_X)}{\alpha(1 - \pi_X) + r + \eta + \delta}(2 - h). \quad (6)$$

It can be shown that when  $h$  is equal to  $h^*$  in (4),  $\pi_X$  given by (3) also satisfies (6). Moreover, by solving (6) for  $\pi_X$  and plugging it into (5), it turns out that (5) is equivalent to

$$(h - 2)(h - h^*) < 0.$$

The above holds when  $h \in (h^*, 2)$ . Therefore, equilibrium **B** exists when  $h$  is in this range. In equilibrium **B**, specialized workers are more selective than in equilibrium **A**. The result that people are more selective in an economy with higher unemployment compensation ( $h$ ) is a consequence of the fact that the unemployment compensation is acting as a search subsidy.

From the market equilibrium conditions, the unemployment rates in equilibrium **B** are:  $u_G = (\delta + \eta)/(\alpha + \delta + \eta)$  and  $u_X = (\delta + \eta)/(\alpha(1 - \pi_X) + \delta + \eta)$ . Thus,

$$u = 2n_X \frac{\delta + \eta}{\alpha(1 - \pi_X) + \delta + \eta} + (1 - 2n_X) \frac{\delta + \eta}{\alpha + \delta + \eta}, \quad (7)$$

and

$$\pi_X = \frac{u_X n_X}{u_X n_X + u_Y n_Y + u_G n_G} = \frac{(\delta + \eta) n_X}{(\alpha(1 - \pi_X) + \delta + \eta) u}. \quad (8)$$

Given  $\pi_X$  [from (6)], two unknowns,  $u$  and  $n_X$ , are solved by equations (7) and (8).

### 2.3 Results

In this section we characterize the equilibrium given the conditions in the previous section.<sup>3</sup> In particular, we will focus on how human capital investment (represented by the population of each type) and the unemployment rate change as the unemployment insurance policy changes. Let's imagine a situation where  $h^* \in [0, 2)$  and we move  $h$  from zero to 2. When  $h$  is sufficiently low, the economy is in equilibrium **A**. In equilibrium **A**,  $u$ ,  $\pi_X$ , and  $n_X$  do not change as  $h$  changes. When  $h$  exceeds the value  $h^*$ , the equilibrium switches from **A** to **B**. When this switch occurs, there can be jumps in the endogenous variables. Within equilibrium **B**,  $u$ ,  $\pi_X$ , and  $n_X$  all depend on the value of  $h$ .

First, let's concentrate on the equilibrium **B**. Then, the following holds.

**Proposition 1**  $n_X$  is increasing in  $h$  in equilibrium **B**.

<sup>3</sup>The working paper version (Mukoyama and Şahin, 2005) contains a numerical example of the model.

**Proof:**

It can easily be seen from (6) that  $\pi_X$  is increasing in  $h$ . Now we will show that  $u$  increases as  $h$  increases. Suppose not. From (7) and (8),  $u(1 - 2\pi_X) = (1 - 2n_X)(\delta + \eta)/(\alpha + \delta + \eta)$  holds, and therefore  $n_X$  has to go up. The right-hand-side of (7) is increasing in  $n_X$  and  $\pi_X$  and therefore goes up. This contradicts the decrease in  $u$ .

Now, from (8),  $\pi_X(\alpha(1 - \pi_X) + \delta + \eta)u = (\delta + \eta)n_X$  holds, and the left-hand-side is increasing in both  $u$  and  $\pi_X$  (note that  $\pi_X \leq 1/2$ ). Since  $u$  and  $\pi_X$  both increase,  $n_X$  also increases.  $\square$

In our model, general skills are attractive because generalists can find a good match more frequently. As  $h$  increases, people become less desperate to match. Therefore, the attractiveness of obtaining general skill declines, and more people will choose to be specialists. An economy with high  $h$  is characterized by high  $u$ ,  $\pi_X$ , and  $n_X$ . This accords well with the European experience.<sup>4</sup>

Figure 1 plots several indices regarding the generosity of unemployment insurance against the enrollment rate in vocational programs for OECD countries. The left and the center panels use the replacement ratio and the duration index<sup>5</sup> as the indices of generosity of unemployment insurance. Both panels indicate that there is a positive association between the the generosity of unemployment insurance and the enrollment rate in vocational programs. In particular, European countries tend to be located on the north-east region of the panels, compared to Japan and the United States. In the right panel, we constructed a new index by adding up the replacement ratio and the duration index. There is a strong positive association between this unemployment benefit index and the vocational enrollment rate, and the correlation coefficient is approximately 0.5.

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<sup>4</sup>Krueger and Kumar (2004) argue that the emphasis on specialized education in Europe may be helpful in understanding its recent growth performance.

<sup>5</sup>The duration index is defined as  $\alpha[BRR_2/BRR_1] + (1 - \alpha)[BRR_4/BRR_1]$ , where  $BRR_1$  is the benefit replacement ratio received during the first year of unemployment,  $BRR_2$  is the benefit replacement ratio received during the second and third year of unemployment, and  $BRR_4$  is the replacement rate received during the fourth and fifth year of unemployment.  $\alpha$  is set to 0.6.

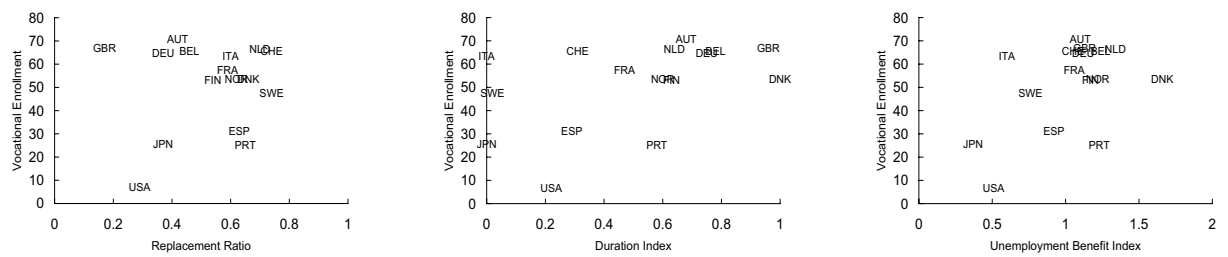


Figure 1: Unemployment Benefit and Vocational Program Enrollment

Source:

Nickell, Nunziata, and Ochel (2005, Table 2, for 1999) for the replacement ratio and duration index.

The unemployment benefit index is based on the authors' calculations (see the main text).

OECD (2001, Figure C2.1) for vocational enrollment rate (%) (except for US, which is taken from Krueger and Kumar, 2004).

Let's now consider the switch from equilibrium **A** to equilibrium **B**. It is already established that this transition occurs at  $h = h^*$ .

**Proposition 2** *At  $h = h^*$ , the Nash equilibrium switches from **A** to **B**. When this occurs,  $\pi_X$  remains the same,  $u$  jumps up, and  $n_X$  jumps down.*

**Proof:**

We have already shown that  $\pi_X$  stays the same when the economy switches from equilibrium **A** to equilibrium **B**. From (7),  $\pi_X > 0$ , and  $n_X > 0$  (which can be seen from (8),  $u > 0$ , and  $\pi_X > 0$ ), it follows that  $u > (\delta + \eta)/(\alpha + \delta + \eta)$ .

Since  $n_X < 1/2$  (which can be seen from (7), (8), and  $\pi_X \neq 1/2$ ), from (7),  $u < (\delta + \eta)/(\alpha(1 - \pi_X) + \delta + \eta)$ . Thus, from (8),  $\pi_X > n_X$  holds. Since  $\pi_X = n_X$  in equilibrium **A** and  $\pi_X$  stays the same at the transition,  $n_X$  has to jump down.  $\square$

The drop of  $n_X$  can be understood intuitively as follows. Suppose that  $n_X$  does not change when the switch occurs. Then,  $\pi_X$  increases, since  $u_G$  is the same but  $u_X$  jumps up. This has a negative impact on the utility of  $X$ -type unemployed workers, since they do not want to meet another  $X$ -type worker. This composition effect makes it unattractive to be a specialist. Thus more people choose to become generalists, and  $n_X$  has to fall. It falls until  $\pi_X$  becomes low enough so that the education equilibrium is restored.

In sum, if we move  $h$  from a very low value to higher values,  $n_X$  behaves non-monotonically. Initially it stays the same while the economy is at equilibrium **A**, jumps down when the economy moves to equilibrium **B**, and gradually increases while at equilibrium **B**. Note that  $u$  and  $\pi_X$  are always (weakly) increasing in  $h$ .

The non-monotonicity of  $n_X$  comes from the fact that the change in  $h$  has two effects on the utility of an unemployed specialist. First, there is the *direct effect*. An increase in  $h$  makes the unemployment state less painful and makes specialization more attractive. Second, there is the *indirect effect*. Changing  $h$  changes the matching behavior. As a result,  $\pi_X$  changes, which affects the utility of the specialist. In the current model, the indirect effect appears

only when the equilibrium switches from **A** to **B**.<sup>6</sup>

### 3 Numerical Example

In this section, we examine the property of the model numerically. Although the purpose of this section is an illustration rather than a quantitative evaluation of the model, we try to set reasonable parameter values.

One period corresponds to one month in our model.  $\alpha$  is set to 0.33 to match the duration of unemployment of 12 weeks. We set  $\delta$  to 0.015 to obtain empirically plausible values for the unemployment rate (between 5% and 10%).  $\eta$  is set to 0.0028 to match the average working life of a worker to 30 years. We use the standard value of  $r = 0.005$ . There is no available guide for setting the values of  $c$  and  $\gamma$ . Here we set  $c = 20$  and  $\gamma = 13$ . We let  $h$  change from 0 to 15(= 2 +  $\gamma$ ) and examine how the economy changes.

Figures 2, 3, and 4 show that the economy is in Nash equilibrium **A** for  $h \leq 1.94$ . In equilibrium **A**,  $u$ ,  $\pi_X$ , and  $n_X$  do not change as  $h$  changes. The economy switches from equilibrium **A** to equilibrium **B** at  $h^* = 1.94$ . When this switch occurs, Figure 2 shows that  $n_X$  jumps down and Figure 3 shows that  $\pi_X$  stays the same. The unemployment rate of specialists,  $u_X$ , also jumps up since specialists become more selective in equilibrium **B** as Figure 4 illustrates. As a result, the aggregate unemployment rate,  $u$ , also jumps up. As we further increase  $h$  within equilibrium **B**,  $n_X$ ,  $u_X$ ,  $u$ , and  $\pi_X$  all increase.

In this example, the decline of  $n_X$  at the switch of the equilibria was relatively small. As the example illustrates, however, when  $h$  becomes substantially larger,  $n_X$  always increases.

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<sup>6</sup>Note that this non-monotonicity of  $n_X$  is driven by the fact that the unemployment rate is endogenous. For example, consider a setup where there is no death or separation, and the workers who are matched are immediately replaced by unemployed workers (“clones”), where these clones determine their own type at the initial stage (like our newborns). This is similar to the situation in Sundaram (2002). In this case, by assumption, it is always the case that  $\pi_X = n_X$ . Thus, the change in  $n_X$  due to a change in  $h$  is the same as the change in  $\pi_X$  from Propositions 1 and 2:  $n_X$  is always (weakly) increasing in  $h$ .

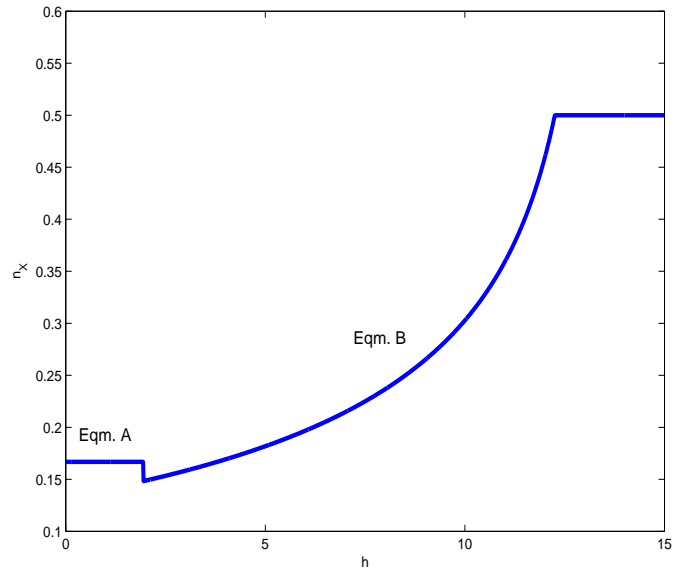


Figure 2:  $n_X$  as a function of  $h$ .

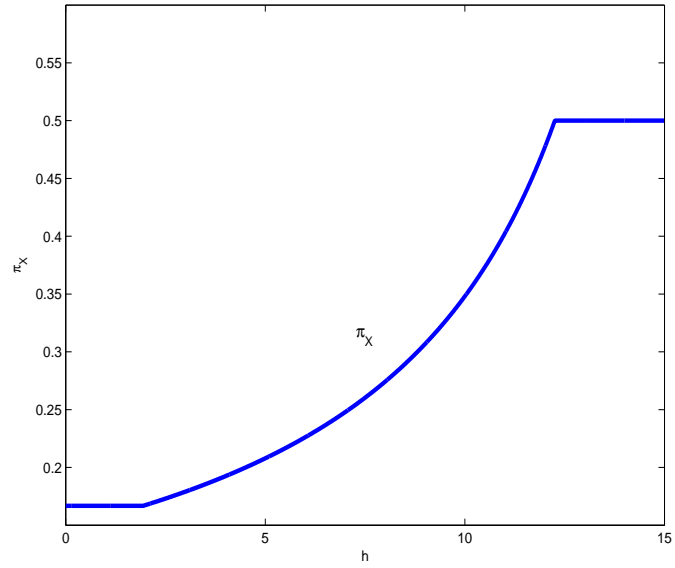


Figure 3:  $\pi_X$  as a function of  $h$ .



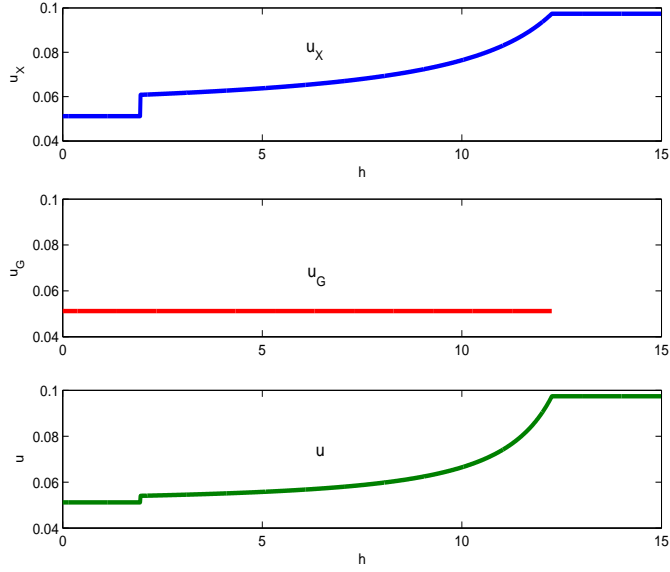


Figure 4:  $u_X$ ,  $u_G$  and  $u$  as a function of  $h$ .

### 3.1 Output

The numerical example allows us to analyze other properties of the model. Here, we examine how aggregate output responds to the change in  $h$ .

First, we calculate the steady-state distribution of the working population. In equilibrium **A**, the probability that an  $i$ -type unemployed worker matches with a  $j$ -type unemployed worker is  $\pi_j$ . Thus, in the steady-state, the population of  $i$ -type employed workers matched with a  $j$ -type is  $\pi_j(1 - u_i)n_i$ , and the population of unemployed  $i$ -type workers is  $u_i n_i$ .

In equilibrium **B**, an  $X$ -type rejects a match with another  $X$ -type. Therefore, conditional on employment, the  $X$ -type is matched with a  $G$ -type with probability  $\pi_G/(\pi_G + \pi_Y)$  and is matched with a  $Y$ -type with probability  $\pi_Y/(\pi_G + \pi_Y)$ . The population of  $X$ -type workers matched with  $G$ -type is therefore  $\pi_G(1 - u_X)n_X/(\pi_G + \pi_Y)$ , and the population of  $X$ -type workers matched with  $Y$ -type workers is  $\pi_Y(1 - u_X)n_X/(\pi_G + \pi_Y)$ .

Figure 5 shows the steady-state aggregate output, net of the education cost, in the above numerical example. It shows that the steady-state output can increase as  $h$  increases. This

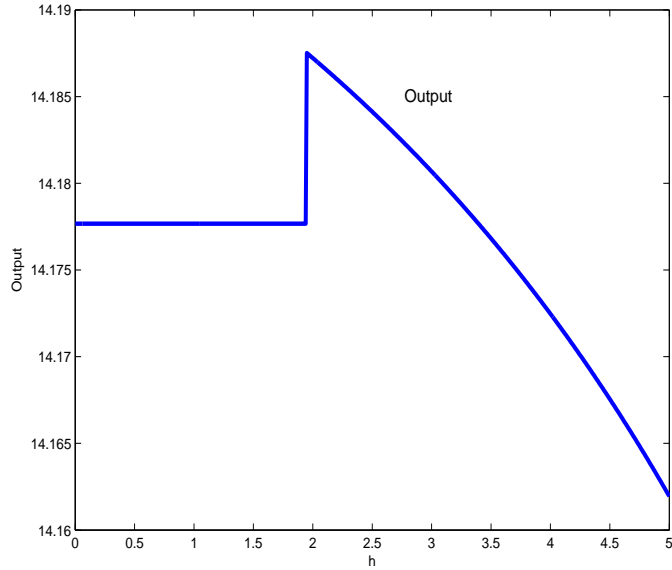


Figure 5: Output as a function of  $h$ .

occurs because specialists become more selective and wait until they find a good match. The same phenomenon is described in Mukoyama and Şahin (2005) in a model without human capital choice. Here, the increase can occur only at the switch between equilibrium **A** and equilibrium **B**, since this is the only point where workers change their matching behavior. In contrast with Mukoyama and Şahin (2005), the decline of output within equilibrium **B** is driven by the human capital choice, rather than the matching decision. There are more specialists, whose unemployment duration is longer, when  $h$  is larger.

## 4 Conclusion

In this paper, we analyzed the human capital choice under search friction. In particular, we focused on the choice between obtaining a general skill or a specialized skill. The unemployment insurance policy affects the workers' choice through two channels. First, it directly affects the relative utility of workers by making unemployment less painful. Second, it indirectly affects utility by changing the workers' matching decision. In particular, the change of the matching decision affects the composition of workers in the unemployment pool.

In the model, when the equilibrium switches from **A** to **B**, only the indirect effect is present, whereas in equilibrium **B** only the direct effect is present. In reality, it is more likely that both effects are at work. The cross-country comparison in Figure 1 seems to suggest that the first effect is stronger. It is an important future research topic to empirically evaluate which effect dominates.

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