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Emanuela CARDIA[1] and Philippe MICHEL[2]

1 Centre interuniversitaire de recherche en économie quantitative (CIREQ) and Département de sciences économiques, Université de Montréal

2 GREQAM, Université de la Méditerranée and I.U.F.

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CIREQ, Université de Montréal
C.P. 6128, succursale Centre-ville
Montréal (Québec) H3C 3J7
Canada

téléphone : (514) 343-6557
télécopieur : (514) 343-5831
cireq@umontreal.ca
http://www.cireq.umontreal.ca

Résumé

Cet article étudie les transferts intergénérationnels qui sont faits sous deux formes, en biens et en temps, dans le cadre du modèle à générations imbriquées standard. Les transferts en biens - les legs - ont été analysés dans de nombreux travaux, mais les transferts en temps n'ont été que rarement envisagés. Nous explicitons la dynamique d'équilibre en supposant une fonction d'utilité log-linéaire et une fonction de production Cobb-Douglas. Nous montrons qu'il peut y avoir des transferts en temps même dans le cas où les transferts en biens sont nuls en raison de la contrainte de non-négativité de ces transferts. Si l'accumulation du capital augmente avec le degré d'altruisme, l'offre de travail n'est pas nécessairement monotone et elle peut décroître quand les legs sont positifs.

Mots clés : altruisme, transferts en temps, legs

Abstract

This paper uses a standard two-period overlapping generation model to examine the behavior of an economy where both intergenerational transfers of time and bequests are available. While bequests have been examined extensively, time transfers have received little or no attention in the literature. Assuming a log-linear utility function and a Cobb-Douglas production function, we derive an explicit solution for the dynamics and show that altruistic intergenerational time transfers can take place in presence of a binding non-negativity constraint on bequests. We also show that with either type of transfers capital is an increasing function of the intergenerational degree of altruism. However, while with time transfers the labor supply of the young increases with the degree of altruism, with bequests it may decrease.

Keywords : altruism, time transfers, bequests

1 Introduction

In this paper we examine the properties of a standard overlapping generation model where both bequests and intergenerational time transfers are available. The effects of bequests within overlapping generation models have been widely studied. Since the seminal papers of Becker (1965) and Barro (1974), many papers have been devoted to the OLG-model with altruism (see for example Burbidge, 1983, Weil, 1987, Abel, 1987).[1] Almost all of these studies only consider transfers of good (bequests and/or gifts). Only a few studies consider transfers of time, and generally from an empirical point of view, like Ioannides and Kahn (1994). A particular study is that of Cardia and Ng (2002) which calibrates an overlapping generation to allow for both bequests and time transfers. The authors have examined US and Canadian data and found that time transfers are important and in some cases as important as in-vivo monetary transfers.

Despite the empirical importance of time transfers, this type of transfer has not received much attention in the theoretical literature. The aim of this paper is to examine the macroeconomic implications of time transfers versus bequests. The existing literature on dynastic altruism usually assumes an inelastic labor supply. Here we assume that labor supply decisions are endogenous as the young can choose to work at home on a home produced good or on the market place to produce a market good. We find that the labor supply of the young responds differently to bequests and time transfers although both types of transfers increase capital accumulation. An important issue is whether the degree of altruism that is necessary for bequests to be operative and for households to behave altruistically in standard dynastic models

[1] For a simple presentation see chapter 3 in Blanchard and Fisher (1989).

is the same for time transfers to be operative. Our results show that time transfers may take place for degrees of altruism that are not sufficient to generate bequests.[2]

The remainder of the paper is organized as follows. In section II, the model is presented. In section III we study the dynamic of capital accumulation in the cases of operative and non-operative bequests. In section IV we derive the labor supply and the steady state value of capital stock. Their response to higher degrees of altruism is examined. Four cases are considered: no transfers, bequests only, time transfers only and both transfers operative. In section V we present a numerical example and in section VI conclusions are drawn. A technical appendix at the end (section VII) discusses the special case where labor supply is at its maximum.

2 The Model

We use a two-period overlapping generations model. Individuals work when they are young and retire when they are old. We assume that households are one-sided altruistic and maximize their utility which combines their life-cycle utility with the utility of their immediate descendants. Population grows at the constant rate n, N_t is the number of young individuals born at time t and $(1+n)$ is the number of children born to the young. Both a market good and a non-market good enter in the utility function and the old may transfer resources to the young in two ways: via bequests and via transfers in the form of time. Time transfers are modeled by allowing the old to participate in the production of the home good consumed by the young. Altruistic households maximize:

$$v_t = u_t + \gamma v_{t+1} = \sum_{i=0}^{\infty} \gamma^i u_{t+i} \quad (1)$$

[2] Such type of results are also obtained when the utility function depends on variables like the environment (Jouvet and al., 2000) or an inherited taste (De la Croix and Michel, 1998).

where:

$$u_t = \log(c_t) + \log(\varphi_t) + \delta \left[\log(d_{t+1}) + \log(\psi_{t+1}) \right]$$

c_t and d_{t+1} are the consumptions of the market good during youth and old age, respectively, and φ_t and ψ_{t+1} are their consumptions of the non-market good. The parameter $\gamma < 1$ measures the degree of altruism and δ is the subjective discount factor. The home good consumed by the young is produced by using time, λ_t, a market good, a_t, and a Cobb-Douglas technology:

$$\varphi_t = a_t^{\alpha_1} \lambda_t^{\alpha_2} \tag{2}$$

The time input λ_t combines the time spent by the young and the time spent by the old in producing the home good consumed by the young. We assume perfect substitutability between the time of the old and that of the young in the production of the home good of the young:

$$\lambda_t = 1 - l_t + \beta(1 - \mu_t) \tag{3}$$

l_t is the time spent by the young producing the market good, $1 - l_t$ is the time spent on the production of the home good. The old spend μ_t units of time producing their own home good and $1-\mu_t$ units of time to help producing their children's home good. The old are less efficient than the young for values of β smaller than $1/(1+n)$.[3] The home good consumed by the old is produced using their own time μ_{t+1}, a market good, b_{t+1}, and a Cobb-Douglas production function:

$$\psi_{t+1} = b_{t+1}^{\alpha_3} \mu_{t+1}^{\alpha_4} \tag{4}$$

[3] $\beta_1(1 - \mu_t)$ represents the total time transfer, in efficiency units, of the old to the young. The per-child time transfer (in efficiency units) is $\beta(1 - \mu_t) = \frac{\beta_1}{1+n}(1 - \mu_t)$. β therefore measures the per-child efficiency of the transfer of time from the old to the young.

Agents take the real wage w_t and the gross real interest rate R_t as given. Altruistic households maximize (1) subject to the following resource constraints:

$$c_t + a_t + s_t = l_t w_t + x_t, \tag{5}$$

$$d_{t+1} + b_{t+1} + (1+n)x_{t+1} = R_{t+1}s_t, \tag{6}$$

s_t and $l_t w_t$ are the saving and the labor income of the young agent in period t, respectively, x_t is the bequest that he receives in period t from his parents and $(1+n)x_{t+1}$ is the bequest that he is to give to his children. The life–cycle budget constraint is obtained by combining the two resource constraints (equations 5 and 6):

$$R_{t+1}(l_t w_t + x_t - c_t - a_t) = d_{t+1} + b_{t+1} + (1+n)x_{t+1} \tag{7}$$

In each period bequests and time transfers must be non-negative and labor supply and time transfers must be smaller or equal to 1:

$$x_{t+1} \geq 0,\ 0 \leq l_t \leq 1 \text{ and } 0 \leq \mu_{t+1} \leq 1$$

In order to calculate the optimal decisions for all generations one considers the following Lagrangian expression:[4]

$$\sum_{t=o}^{\infty} \gamma^t \Big\{ \log(c_t) + \alpha_1 \log(a_t) + \alpha_2 \log(\lambda_t) + \delta \left[\log(d_{t+1}) + \alpha_3 \log(b_{t+1}) + \alpha_4 \log(\mu_{t+1}) \right]$$
$$+\ q_{t+1}\left[R_{t+1}\left(l_t w_t + x_t - c_t - a_t\right) - d_{t+1} - b_{t+1} - (1+n)x_{t+1}\right]\Big\}$$

where q_t is the Lagrangian multiplier and is the shadow price of bequests. The first order conditions with respect to $c_t, a_t, d_{t+1}, b_{t+1}, l_t, \mu_{t+1}$ and x_{t+1}, are:

$$c_t = \frac{1}{q_{t+1}R_{t+1}} \tag{8}$$

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4 It is useful to notice that for values of $\alpha_1 = \alpha_3 = \alpha_4 = \beta = 0$ the altruistic agent maximizes:

$$v_t = \log(c_t) + \alpha_2 \log(1 - l_t) + \delta \log(d_{t+1}) + \gamma v_{t+1}$$

which corresponds to the standard maximization problem with both a market good and leisure in the utility function.

$$a_t = \frac{\alpha_1}{q_{t+1}R_{t+1}} = \alpha_1 c_t \tag{9}$$

$$d_{t+1} = \frac{\delta}{q_{t+1}} \tag{10}$$

$$b_{t+1} = \frac{\delta\alpha_3}{q_{t+1}} = \alpha_3 d_{t+1} \tag{11}$$

$$-\frac{\alpha_2}{\lambda_t} + q_{t+1}R_{t+1}w_t = -\frac{\alpha_2}{\lambda_t} + \frac{w_t}{c_t} \geq 0, \tag{12}$$

$$= 0 \; if \; l_t < 1 \tag{13}$$

$$\frac{\delta\alpha_4}{\mu_{t+1}} - \gamma\beta\frac{\alpha_2}{\lambda_{t+1}} \geq 0, \tag{14}$$

$$= 0 \; if \; \mu_{t+1} < 1 \tag{15}$$

$$-(1+n)q_{t+1} + \gamma q_{t+2}R_{t+2} \leq 0, \tag{16}$$

$$= 0 \; if \; x_{t+1} > 0 \tag{17}$$

and the transversality condition is (Michel, 1990):

$$\lim_{t\to\infty} \gamma^t q_t x_t = 0 \tag{18}$$

On the production side, competitive firms use the Cobb-Douglas production function to produce the sole market good in the economy. Given that N_t is the size of the young population at time t, total labor force at time t is $L_t = N_t l_t$ and total output is

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha} = AK_t^{\alpha}(N_t l_t)^{1-\alpha}$$

With $k_t = \frac{K_t}{N_t}$, we can define output per capita, y_t, as:

$$y_t \equiv \frac{Y_t}{N_t} = Ak_t^{\alpha}l_t^{1-\alpha}$$

Profit maximization implies that factors are paid the value of their marginal product. Assuming that capital fully depreciates after one period, we have that:

$$w_t = (1-\alpha)Ak_t^{\alpha}l_t^{-\alpha} = (1-\alpha)\frac{y_t}{l_t}, \tag{19}$$

$$R_t = \alpha Ak_t^{(\alpha-1)}l_t^{(1-\alpha)} = \alpha\frac{y_t}{k_t} \tag{20}$$

As is standard of overlapping generations model, capital market equilibrium is summarized by:

$$k_{t+1} = \frac{s_t}{(1+n)}, \tag{21}$$

so that the savings by the young become productive capital the following period. Goods market equilibrium is given by the aggregate resource constraint and can be obtained by using the resource constraints (5) and (6), equations (19) and (20) and the capital accumulation equation (21):

$$y_t = k_{t+1}(1+n) + c_t + a_t + \frac{1}{1+n}(d_t + b_t) \tag{22}$$

3 Study of the Dynamic of Capital Accumulation

In a model with altruism where negative bequests are not allowed, there are two different situations depending on whether the degree of altruism, γ, is sufficient to generate non-negative bequests or not. For low values of γ ($\gamma < \bar{\gamma}$) the economy behaves as if households were selfish because they cannot leave negative bequests. For high values of γ ($\gamma \geq \bar{\gamma}$) households have an infinite dynastic horizon. To study the dynamic of capital accumulation we only need to distinguish between the cases of operative and non-operative bequests as time transfers are a-temporal decisions that do not change the dynamic of the model. In this section we derive the dynamic of capital with and without bequests and the threshold value of γ, $\bar{\gamma}$.

3.1 The Dynamic of Capital for Constrained Bequests ($x_t = 0$)

We consider first the no bequest case. In this case $x_t = x_{t+1} = 0$ and equation (7) becomes the life-cycle budget constraint:

$$l_t w_t = c_t + a_t + \frac{1}{R_{t+1}}\left(d_{t+1} + b_{t+1}\right)$$

Using equations (9), (10) and (11), labor income verifies:

$$l_t w_t = (1+\alpha_1)(1+\Delta)c_t \tag{23}$$

where:

$$\Delta = \frac{(1+\alpha_3)}{(1+\alpha_1)}\delta \tag{24}$$

can be interpreted as the modified discount factor where α_1 and α_3 represent the weights of the market good in the production of the non-market good of the young and old, respectively. The economy's saving is equal to the labor income of the young minus their consumption:

$$s_t = l_t w_t - (1+\alpha_1)c_t = \frac{\Delta}{1+\Delta} l_t w_t$$

Using equations (19) and (21) we can now derive the equation of motion for capital:

$$(1+n)k_{t+1} = \frac{\Delta}{1+\Delta} l_t w_t = \frac{\Delta}{1+\Delta}(1-\alpha)A k_t^{\alpha} l_t^{1-\alpha} \tag{25}$$

The dynamic of capital stock depends on α, Δ and l_t. In the next section we study the labor supply and show that it is constant over time. With non operative bequests the capital accumulation equation is the only dynamic equation of the model. To find the critical value of γ, $\bar{\gamma}$, such that bequests are not operative, we use equations (8), (23) and (25). We have that:

$$\frac{q_{t+1}R_{t+1}}{q_{t+2}R_{t+2}} = \frac{c_{t+1}}{c_t} = \frac{w_{t+1}l_{t+1}}{w_t l_t} = \frac{\Delta w_{t+1} l_{t+1}}{(1+\Delta)(1+n)k_{t+1}}$$

Thus condition (16) for zero bequests is equivalent to:

$$\gamma \leq \frac{(1+n)q_{t+1}}{q_{t+2}R_{t+2}} = \frac{\Delta w_{t+1}l_{t+1}}{(1+\Delta)R_{t+1}k_{t+1}} = \frac{1-\alpha}{\alpha}\frac{\Delta}{1+\Delta} \equiv \bar{\gamma}. \tag{26}$$

When $\alpha_1 = \alpha_3$ the threshold value $\bar{\gamma}$ becomes the standard threshold value found in the literature on dynastic altruism when a Cobb-Douglas production function is assumed (see Weil, 1987). There bequests are not operative if:

$$\gamma \leq \bar{\gamma} = \frac{1-\alpha}{\alpha}\frac{\delta}{1+\delta}$$

3.2 The Dynamic of Capital for Positive Bequests ($x_t > 0$)

We now consider the case of operative bequests. Using equations (17) and (22) we can rewrite the national income identity as follows:

$$y_t = k_{t+1}(1+n) + (1+\alpha_1)\frac{1}{q_{t+1}R_{t+1}} + (1+\alpha_1)\Delta\frac{1}{\gamma q_{t+1}R_{t+1}} \tag{27}$$

Defining z_t as the shadow value of output for the dynasty (see also Michel and Pestieau, 1998):

$$z_t \equiv q_t y_t \tag{28}$$

Using equations (17), (20) and (27):

$$z_t = \frac{\gamma q_{t+1}R_{t+1}}{1+n}y_t = \gamma\alpha z_{t+1} + \frac{1+\alpha_1}{1+n}\{\gamma + \Delta\} \tag{29}$$

This is a first-order differential equation which has an explosive root ($\frac{1}{\gamma\alpha}$). Rearranging equations (6), (10), (11), (20) and (21) it can be shown that:

$$(1+n)x_{t+1} = R_{t+1}s_t - d_{t+1} - b_{t+1} = (1+n)\alpha y_{t+1} - \frac{1+\alpha_1}{q_{t+1}}\Delta. \tag{30}$$

By multiplying both sides of the previous equation by q_{t+1} we can easily see that transversality condition (18) implies a transversality condition on the shadow value of output, z_{t+1} :

$$\lim_{t\to\infty}\gamma^{t+1}q_{t+1}x_{t+1}=0\Leftrightarrow\lim_{t\to\infty}\gamma^{t+1}z_{t+1}=0$$

Therefore the optimality conditions imply that z_t is a constant and that it is equal to:

$$z_t=\frac{1+\alpha_1}{1+n}\frac{\gamma+\Delta}{1-\gamma\alpha}\equiv z(\gamma) \tag{31}$$

As can be seen the shadow value of output, $z(\gamma)$, is an increasing function of the degree of altruism, γ. Since in the case studied here bequests are positive, $x_{t+1}>0$ and $q_{t+1}x_{t+1}>0$. From equation (30) we then have that $q_{t+1}x_{t+1}>0\Leftrightarrow z_{t+1}>\frac{1+\alpha_1}{\alpha(1+n)}\Delta$. But because equation (31) implies that for $z_t>0\Rightarrow\frac{1+\alpha_1}{1+n}\frac{\gamma+\Delta}{1-\gamma\alpha}>0$ we have that, for bequests to be operative, both relations have to be true and γ has to be greater than $\bar{\gamma}$:

$$\gamma>\bar{\gamma}=\frac{1-\alpha}{\alpha}\frac{\Delta}{1+\Delta}. \tag{32}$$

This is the same critical value we found in the previous section except that there bequests were not operative for $\gamma\leq\bar{\gamma}$. To find the equation of motion for capital accumulation we use equations (27) and (29):

$$y_t=\left(k_{t+1}(1+n)+[(1+\alpha_1)\gamma+(1+\alpha_3)\delta]\frac{y_t}{z(\gamma)(1+n)}\right)$$

Substituting out $z(\gamma)$:

$$k_{t+1}(1+n)=\gamma\alpha Ak_t^{\alpha}l_t^{1-\alpha} \tag{33}$$

We use equation (30) to find the equation of motion for bequests:

$$x_{t+1}=\alpha y_{t+1}-\frac{1+\alpha_1}{(1+n)q_{t+1}}\Delta=\frac{\alpha\gamma(1+\Delta)-(1-\alpha)\Delta}{\gamma+\Delta}y_{t+1}$$

As a ratio to output, bequests are always an increasing function of the degree of altruism:

$$\frac{x_{t+1}}{y_{t+1}}=\alpha-\Delta\frac{1-\alpha\gamma}{\gamma+\Delta}$$

4 Study of the Equilibrium

There are 8 possibilities that result from the three inequalities $l \leq 1$, $\mu \leq 1$ and $x \geq 0$ depending on which of these constraints is binding. But for $\mu = 1$ it is necessarily true that $l < 1$ and therefore 6 possibilities remain. We show in this section that the critical value of β for positive intergenerational time transfers is a decreasing function of γ. In the technical appendix we examine the two special cases where $l = 1$. We summarize the different cases on the plane β, γ in Figure 1 below and examine each of the 4 cases with $l < 1$ in a separate subsection.

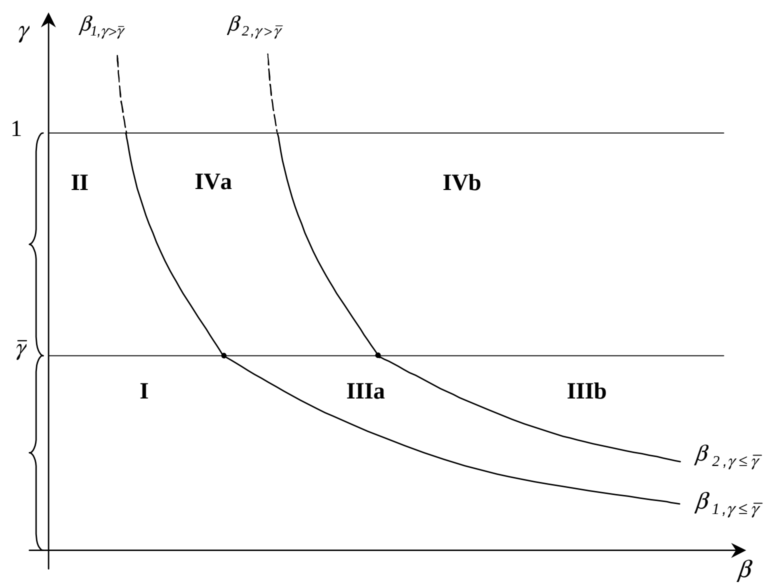

Figure 1

For small values of β and γ (see Figure 1, case I) there are no bequests nor time transfers. This is typically the Diamond model. For larger values of γ ($\gamma > \bar{\gamma}$) and low efficiency of time transfers (i.e. low values of β) there are bequests but no time transfers (case II). For small values of γ (lower than $\bar{\gamma}$) and large values of β (case IIIa) there will be time transfers but not bequests. This case is not typically discussed in the literature on altruistic behavior and its properties are going to be compared to the properties that characterize the more standard version of altruism (case II) and the case of no transfers (case I). The fourth region in Figure 1 (case IVa) is a combination of case II and IIIa as both γ and β are sufficiently high to imply both bequests and time transfers. Regions IIIb and IVb are special cases of regions IIIa and IVa, respectively. They describe how regions IIIa and IVa are modified when the labor supply is equal to 1 and the young only spend time working in the production of the market good. In the sections that follow we examine case I, II, IIIa and IVa in details. Case IIIb and IVb are examined in the technical appendix at the end of the paper. In all cases labor supply and time transfers are constant over time.

4.1 Case I (no bequests $x_t = 0$ and no time transfers $\mu_t = 1$)

First we examine the case in which neither bequests nor transfers of time are operative (see region I in Figure 1). It implies $l_t < 1$. Rearranging equations (3), (13) and (23), we have that:

$$w_t(1 - l_t) = \alpha_2 c_t = \frac{\alpha_2}{(1 + \alpha_1)(1 + \Delta)} w_t l_t$$

which implies that the labor supply of the young is independent of γ and β and is given by:

$$l_t = \frac{1}{1 + \frac{\alpha_2}{(1+\alpha_1)(1+\Delta)}} \equiv l_I. \tag{34}$$

From equations (3) and (14) we can derive the restrictions on β for $\mu_t = 1$:

$$\beta \leq \frac{\delta\alpha_4}{\gamma\alpha_2}\frac{\lambda_t}{\mu_t} = \frac{\delta\alpha_4}{\gamma\alpha_2}(1 - l_I) = \frac{\delta\alpha_4}{\alpha_2 + (1 + \alpha_1)(1 + \Delta)}\frac{1}{\gamma} \equiv \beta_{1,\gamma\leq\bar{\gamma}}$$

which is the curve $\beta_{1,\gamma\leq\bar{\gamma}}$ in Figure 1. Below the curves $\beta_{1,\gamma\leq\bar{\gamma}}$ and $\gamma \leq \bar{\gamma}$ there are no time transfers nor bequests. The critical value of β is a negative function of the degree of altruism. The more are households altruistic, the lower is the critical value for positive time transfers. It is interesting to notice that the critical value of β does not depend on the capital intensity of the economy which, together with Δ, determines $\bar{\gamma}$. This independence results from the fact that l_I itself is independent of the capital intensity of the economy and also from the log-linear utility function. This implies that although economies with low capital intensity are less likely to give bequests than otherwise identical economies with high capital intensity, they are equally likely to make time transfers. Time transfers are driven by the productivity of the old and the young in the non-market good (α_4 and α_2, respectively) and by the relative weight of the utility of the old and the young (δ and γ, respectively). The higher the productivity of the old in producing their own non-market good, the higher the threshold value of β for positive time transfers. With constant labor supply, the dynamic equation for capital accumulation (using equation 25) is:

$$(1+n)k_{t+1} = \frac{\Delta(1-\alpha)}{1+\Delta} A l_I^{1-\alpha} k_t^{\alpha} \tag{35}$$

and the steady state of capital stock, k_I^*, is:

$$k_I^* = \left[\frac{(1-\alpha)\,\Delta}{1+\Delta} \frac{A}{1+n} \right]^{\frac{1}{1-\alpha}} l_I \tag{36}$$

Equations (35) and (36) show that capital accumulation and the steady state value of capital are also not a function of β and γ.

4.2 Case II (positive bequests $x_t > 0$ and no time transfers $\mu_t = 1$)

We now consider the case where the bequests motive is strong enough ($\gamma > \bar{\gamma}$) to generate positive bequests but β is too low to have time transfers (region II, Figure 1).

This case is standard in the literature on dynastic altruism (see Barro, 1974 and Weil, 1987) except that here the labor supply is endogenous. To derive the labor supply of the young we rearrange equations (3), (8), (12), (20) and (26):

$$w_t(1-l_t) = \alpha_2 c_t = \frac{\alpha_2}{q_{t+1}R_{t+1}} = \frac{\alpha_2}{\alpha z(\gamma)} k_{t+1}$$

We substitute out k_{t+1}using equations (19) and (29):

$$w_t(1-l_t) = \frac{\alpha_2 \gamma}{(1+n)z(\gamma)} y_t = \frac{\alpha_2 \gamma}{(1+n)(1-\alpha)z(\gamma)} w_t l_t$$

and obtain:

$$l_t = \frac{1}{1 + \frac{\alpha_2 \gamma}{(1+n)(1-\alpha)z(\gamma)}} \equiv l_{II}(\gamma). \tag{37}$$

It can be shown that for values of $\gamma > \bar{\gamma}$, as γ increases the labor supply first decreases and than increases again to the no-bequest level, l_I:

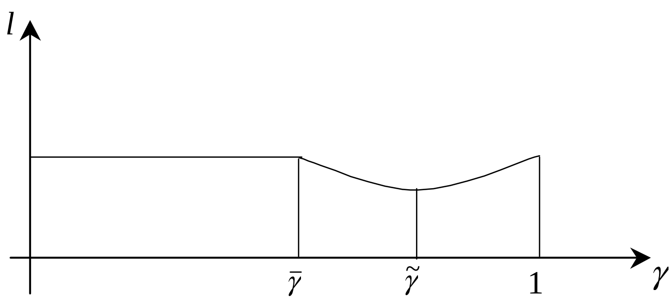

Figure 2

A simple interpretation of this particular form of labor supply can be obtained by using the life-cycle budget constraint at steady state:

$$lw + \left(\frac{R-(1+n)}{R} \right) x = c + a + \frac{1}{R}(d+b)$$

Total households' income is given by the sum of the labor income of the young and the net return on bequests, $(R-(1+n))x/R$. The labor income of the young is augmented by the gross return on the bequest they receive (Rx) and decreased by the bequests they give to their descendent, $(1+n)x$. When there are no positive bequests ($\gamma \leq \bar{\gamma}$) the income of the young is simply their labor income. With operative bequests ($\gamma > \bar{\gamma}$) the steady state value of the gross interest rate is a negative function of the degree of altruism, $R = (1+n)/\gamma$. This is the modified golden rule[5] and is standard in models with operative bequests. Therefore for values of γ greater than $\bar{\gamma}$, the real interest rate is greater than the rate of growth of population and bequests increase the net wealth of the young as at the margin, what the young receive is higher than what they give as bequests. The increase in wealth increases the consumption and the leisure of the young and decreases their labor supply. As γ increases the gap between the real interest rate and the rate of growth of population narrows and the wealth effect becomes less important. Eventually for $\gamma = 1$, $R = 1+n$ (golden rule), and the young receive exactly what they give. In this case, as with $\gamma \leq \bar{\gamma}$, total income is simply the labor income of the young.

From the optimality condition for μ_{t+1} when μ_{t+1} is equal to 1 (equation 15) and $l_{II} < 1$, we have:

$$\delta\alpha_4 \geq \frac{\gamma\beta\alpha_2}{1 - l_{II}(\gamma)}$$

which is equivalent to:

$$\beta \leq \frac{\delta\alpha_4}{\gamma\alpha_2}(1 - l_{II}(\gamma)) = \frac{\delta\alpha_4}{\alpha_2\gamma + (1+n)(1-\alpha)z(\gamma)} \equiv \beta_{1,\gamma>\bar{\gamma}}$$

With values of β below the curve $\beta_{1,\gamma>\bar{\gamma}}$ there are no time transfers. Although the values of β are not high enough to generate time transfers, intergenerational altruism

[5] To obtain the modified golden rule use the capital accumulation equation when bequests are operative (equation 33) and equation (20).

is strong enough to make bequests operative since $\gamma > \bar{\gamma}$.

The equation of motion for capital is derived by substituting equation (37) in equation (33):

$$k_{t+1}(1+n) = \gamma\alpha A k_t^{\alpha} l_{II}(\gamma)^{1-\alpha} = \gamma^{\alpha}\alpha A k_t^{\alpha}(\gamma l_{II}(\gamma))^{1-\alpha}$$

It is clear that since $\gamma l_{II}(\gamma)$ is increasing in γ, capital is, as is standard in the literature on dynastic altruism, an increasing function of γ. The steady state level of capital is:

$$k^*_{II} = \left(\gamma^{\alpha}\alpha\frac{A}{1+n}\right)^{\frac{1}{1-\alpha}}\gamma l_{II}(\gamma) \tag{38}$$

Bequests in the steady state are given by:

$$x^*_{II} = \left[1 - \frac{(1+\alpha_3)\frac{\delta}{\alpha}}{z(\gamma)(1+n)}\right]\frac{(1+n)}{\gamma}k^*_{II} \tag{39}$$

4.3 Case IIIa (no bequests $x_t = 0$ and time transfers $\mu_t < 1$)

This case is specific to our model and it shows that there will be values of γ smaller than $\bar{\gamma}$ for which although there are no bequests there are altruistic transfers in the form of time (region IIIa and IIIb, Figure 1). Here we assume that $l < 1$ and limit our analysis to the region IIIa. We use equation (23) in equation (13) to substitute out consumption:

$$w_t\lambda_t = \alpha_2 c_t = \frac{\alpha_2}{(1+\alpha_1)(1+\Delta)}w_t l_t$$

Using equations (3) and (15) we can derive λ_t as a function of l_t:

$$\lambda_t = (1-l_t) + \beta(1-\mu_t) = (1-l_t) + \beta(1 - \frac{\delta\alpha_4}{\gamma\alpha_2\beta}\lambda_t)$$

Combining the two previous equations, we have that:

$$\lambda_t = \frac{(1+\beta)}{1 + \frac{(1+\alpha_1)(1+\Delta)}{\alpha_2} + \frac{\delta\alpha_4}{\gamma\alpha_2}} \equiv \lambda_{IIIa}$$

and

$$l_t = \frac{(1+\beta)}{1+\frac{\alpha_2+\delta\alpha_4/\gamma}{(1+\alpha_1)(1+\Delta)}} \equiv l_{IIIa} \tag{40}$$

When only time transfers are operative the labor supply of the young is increasing in both γ and β. Therefore time transfers and bequests produce qualitatively different effects on the labor supply of the young. We showed in fact in the previous section that an increase in the degree of altruism does not increase the labor supply of the young above the no-bequest level, l_I and may decrease it. Using equation (3) it can be shown that time transfers from the old to the young, $1-\mu_t$, are also a positive function of β :

$$1-\mu_t = 1-\frac{\delta\alpha_4}{\gamma\alpha_2}\frac{(1+\frac{1}{\beta})}{1+\frac{(1+\alpha_1)(1+\Delta)}{\alpha_2}+\frac{\delta\alpha_4}{\gamma\alpha_2}} \equiv 1-\mu_{IIIa} \tag{41}$$

Using equation (41) and the condition for positive time transfers, i.e. $\mu_t < 1$, we can find the values of β for which there are positive time transfers:

$$\beta > \frac{\delta\alpha_4}{\alpha_2+(1+\alpha_1)(1+\Delta)}\frac{1}{\gamma} \equiv \beta_{1,\gamma\leq\bar{\gamma}}$$

This is the same threshold value as in case I. Here for values of β higher than $\beta_{1,\gamma\leq\bar{\gamma}}$ (regions IIIa and IIIb) intergenerational altruism in not strong enough to generate bequests but β is sufficiently high for households to make altruistic transfers of time. Using equation (41) we can find for which values of β time transfers are positive and also $l_t < 1$:

$$\beta < \left(1+\frac{\delta\alpha_4}{\alpha_2\gamma}\right)\frac{\alpha_2}{(1+\alpha_1)(1+\Delta)} \equiv \beta_{2,\gamma\leq\bar{\gamma}}$$

For values of β smaller than $\beta_{2,\gamma\leq\bar{\gamma}}$, labor supply is inferior to 1 (region IIIa) and for values higher or equal than $\beta_{2,\gamma\leq\bar{\gamma}}$, labor supply is equal to 1 (region IIIb). The threshold value of β is decreasing in γ and increasing in Δ.[6] It does not depend on α,

[6] It is easy to verify that for both $i = 1$ and $i = 2$, $\beta_{i,\gamma\leq\overline{\gamma}}$ coincides with the limit of $\beta_{i,\gamma>\overline{\gamma}}$ for $\gamma = \overline{\gamma}$ when γ converges to $\overline{\gamma}$.

the degree of capital intensity of the economy on which $\bar{\gamma}$ depends upon.

With constant labor supply, the dynamic equation for capital accumulation (using equation 25) and the steady state of capital are:

$$(1+n)k_{t+1} = \frac{\Delta(1-\alpha)}{1+\Delta} A l_{IIIa}^{1-\alpha} k_t^{\alpha}$$

and:

$$k_{IIIa}^{*} = \left[\frac{(1-\alpha)\,\Delta}{(1+\Delta)} \frac{A}{1+n} \right]^{\frac{1}{1-\alpha}} l_{IIIa} \tag{42}$$

As in case II the capital stock is an increasing function of the degree of altruism, γ. In this case it is also an increasing function of the degree of efficiency of the old generation, β.

4.4 Case IVa (positive bequests $x_t > 0$ and time transfers $\mu_t < 1$)

This case incorporates cases II and IIIa. Here γ and β are sufficiently high to have both bequests and time transfers (see regions IVa and IVb in Figure 1). Using equations (19) and (28):

$$\frac{1}{q_t} = \frac{y_t}{z(\gamma)} = \frac{w_t l_t}{(1-\alpha)z(\gamma)}$$

To find the labor supply of the young substitute out y_t and q_t using equations (3), (8), (15) and (17):

$$l_t = \frac{z(\gamma)(1+n)(1-\alpha)(1+\beta)}{(1-\alpha)z(\gamma)(1+n) + \alpha_2\gamma + \delta\alpha_4} = l_{IVa} \tag{43}$$

The equation of motion for capital and its steady state value are derived by substituting equation (43) in equation (31):

$$k_{t+1}(1+n) = \gamma\alpha A_t k_t^{\alpha} {l_{IVa}}^{1-\alpha}$$

$$k_{IVa}^{*} = \left(\gamma\alpha \frac{A}{1+n} \right)^{\frac{1}{1-\alpha}} l_{IVa} \tag{44}$$

It can be easily shown that the capital stock is increasing in γ and β, as in IIIa. It is however not possible to establish analytically the effects of γ on labor supply. The old are transferring time to the young and the young can increase their labor supply to produce the market good (as in case IIIa). This positive effect on the labor supply can offset the initial negative wealth effect (which we discussed for case II). In the next section we use a numerical example and show that for reasonable parameter values labor supply is an increasing function of γ and the negative wealth effect is dominated by the other effect.

Bequests in the steady state are given by:

$$x_{IVa} = \left[1 - \frac{(1+\alpha_1)\frac{\Delta}{\alpha}}{z(\gamma)(1+n)}\right] \frac{(1+n)}{\gamma} k^*_{IVa} \equiv x^*_{IVa} \tag{45}$$

Since k^*_{IV} is increasing in β, bequests are also an increasing function of β. This is an interesting result as it shows that, in the steady state, time transfers do not crowd out bequests. This results is due to the fact that time transfers increase capital accumulation further and that higher capital accumulation increases output and therefore bequests (see results in section 3). Using equations (3), (15) and (44) we find that time transfers are:

$$1 - \mu_{IVa} = 1 - \frac{\delta\alpha_4(1+\beta)/\beta}{(1-\alpha)z(\gamma)(1+n) + \alpha_2\gamma + \delta\alpha_4} \tag{46}$$

Time transfers are a positive function of the degree of altruism and of the degree of efficiency of the old, β.

To find the threshold value of β for which we have positive time transfers we use equation (46). Positive time transfers imply:

$$\beta > \frac{\delta\alpha_4}{\alpha_2\gamma + (1+n)(1-\alpha)z(\gamma)} \equiv \beta_{1,\gamma>\bar{\gamma}}$$

This is the same critical value we found for case II. For values of β higher than $\beta_{1,\gamma>\bar{\gamma}}$ (regions IVa and IVb) there are both time transfers and bequests. We can use (45) to find the values of β such that $l_{IVa} < 1$:

$$\beta < \frac{(\gamma\alpha_2 + \delta\alpha_4)}{(1-\alpha)(1+n)z(\gamma)} \equiv \beta_{2,\gamma>\bar{\gamma}}$$

5 Numerical Example

In this section we use a numerical example to illustrate the role of altruism on the labor supply of the young and on the steady state level of capital stock. We let γ and β vary and assume a subjective discount factor, δ, equal to .5 which, considering one period equivalent to 25 years, implies a yearly subjective discount factor of 0.02. We assume a labor share, $1-\alpha$, equal to .65. We don't have strong priors for the weights of time and market goods and simply assume them to be all equal to .5. Our analytical results suggested that these parameters don't play an important role in determining the effects of γ and β on labor supply and capital accumulation. However because α affects the threshold value of γ for positive bequests but does not affect the threshold value of β for positive time transfers we also consider the case of an economy with higher capital intensity (with $\alpha = .5$). The parameters used in these two scenarios are summarized in table 1.

Figures 3 through 6 assume $\alpha = .35$ and Figures 7 through 10 assume $\alpha = .5$. Case I of no bequests and no time transfers corresponds to the region close to the origin. For low values of γ and β there are no transfers and both labor supply and capital stock are constant.

These figures summarize all the important results we obtained. First, labor supply responds differently to bequests than to time transfers. This can be seen easily by looking at Figures 3 and 7. For low values of β we have the same functional form as in Figure 2. For high values of β such that time transfers are operative, the labor supply of the young becomes a positive function of the degree of altruism. Second, for economies with lower capital intensity the effects on capital accumulation that are due to time transfers can be important as, for given consumers' preferences, a lower α increases the threshold value of γ after which bequests become operative. Since α does not affect the threshold value of β after which time transfers become operative, time transfers can become an important determinant of capital accumulation in low capital intensity economies (see Figure 4). Third, when both bequest and time transfers are included labor supply increases with the degree of altruism. We could not assess analytically the effects of altruism on labor supply when both bequests and time transfers were included. This result was obtained with all the parameter values we tried.

6 Conclusions

This paper has examined the role of intergenerational time transfers in an overlapping generation model where bequests are also included. We have shown that although bequests and time transfers have both positive effects on capital accumulation, they act through different channels. Bequests increase savings and capital accumulation. This capital accumulation does not require the young to work more in the production of the market good. Time transfers instead increase capital accumulation by relaxing the young's time constraint and thus making them work more.[7]

We have also shown that time transfers may take place when intergenerational altruism is not sufficient to generate bequests. The critical level for operative time transfers depends on different variables than the ones we need for operative bequests. In particular, operative bequests depend on the capital intensity of the economy, while time transfers do not. The lower the capital intensity of the economy the higher is the critical value for positive bequests while the critical value for positive time transfers is not affected. This has an interesting implication with intuitive appeal: for less developed economies although the degree of altruism may no be sufficient to generate bequests, there may still be important altruistic intergenerational transfers in the form of time transfers.

7 Technical Appendix

Here we examine two special cases of case III and IV where the labor supply is exactly equal to 1. These two cases are represented in Figure 1 by regions IIIb and IVb, respectively.

[7] In this paper we assumed a log-linear utility function and Cobb-Douglas production functions. We know that the condition of positive operative bequests can be studied in a more general framework and produces similar conclusions, see for example Weil (1987). Cardia and Ng (2002) use numerical simulation methods to understand the steady-state implications of intergenerational transfers of time and of bequests, on child care policies. They assume a CES utility function and CES home production functions and find that time transfers play an important role on the steady-state level of income and capital. The qualitative results are very similar to those found in this paper. There too time transfers encourage labor supply while bequests have an income effect which discourages market work.

7.1 Case IIIb (no bequests $x_t = 0$, time transfers $\mu_t < 1$ and $l_t = 1$)

We first consider the case of no bequests (see region IIIb). In this case $l_t = 1$ implies that $\lambda_t = (1 - l_t) + \beta(1 - \mu_t) = \beta(1 - \mu_t)$. We also have that for time transfers to be positive $\mu_t = \frac{\delta\alpha_4}{\gamma\alpha_2}\lambda_t$ (from equation 15) and therefore:

$$1 - \mu_t = 1 - \frac{\delta\alpha_4}{\delta\alpha_4 + \gamma\alpha_2} \equiv 1 - \mu_{IIIb}$$

In this special case time transfers are an increasing function of the degree of altruism but do not depend on β. It can be easily seen from equation (25) that given that $l_t = 1$ both capital stock and its steady state level are independent of either γ or β. By using equations (12) and (23):

$$\frac{\alpha_2}{\lambda_t} \leq \frac{w_t}{c_t} = (1 + \alpha_1)(1 + \Delta)$$

Substituting the λ_t so obtained in $\lambda_t = \beta(1 - \mu_t)$ and using the previous result, we have that for both time transfers to be positive and the labor supply to be 1 we must have that:

$$\beta \geq \left(1 + \frac{\delta\alpha_4}{\alpha_2\gamma}\right)\frac{\alpha_2}{(1 + \alpha_1)(1 + \Delta)} \equiv \beta_{2,\gamma \leq \bar{\gamma}}$$

7.2 Case IVb (positive bequests $x_t > 0$, time transfers $\mu_t < 1$ and $l_t = 1$)

We now consider the case of positive bequests (see region IVb). In this case it is easy to verify that $l_t = 1$ implies, as in the previous case, that:

$$1 - \mu_t = 1 - \frac{\delta\alpha_4}{\delta\alpha_4 + \gamma\alpha_2} \equiv 1 - \mu_{IVb}$$

Rearranging equations (8), (17), (19) and (28) we have that:

$$\lambda_t \geq \frac{\alpha_2 l_t \gamma}{(1 - \alpha)(1 + n)z(\gamma)}$$

Using the fact that $\lambda_t = \beta(1 - \mu_t)$ and substituting out $\mu_t = \mu_{IVb}$ we find that with $l_t = 1$ for bequests and time transfers to be both operative we must have that:

$$\beta > \frac{(\gamma\alpha_2 + \delta\alpha_4)}{(1-\alpha)(1+n)z(\gamma)} \equiv \beta_{2,\gamma>\bar{\gamma}}.$$

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Table 1

Parameter Values		
	Economy 1	Economy 2
δ	.5	.5
α	.35	.5
α_1	.5	.5
α_2	.5	.5
α_3	.5	.5
α_4	.5	.5

Figure 3
Labor Supply

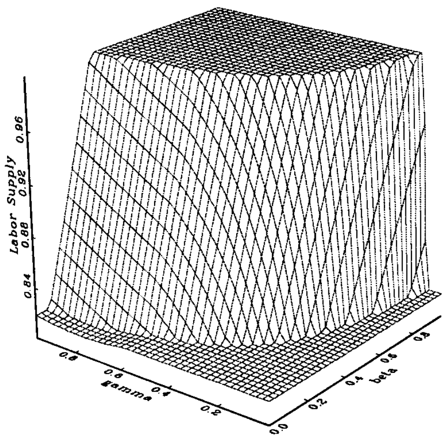

Figure 4
Steady State Capital Stock

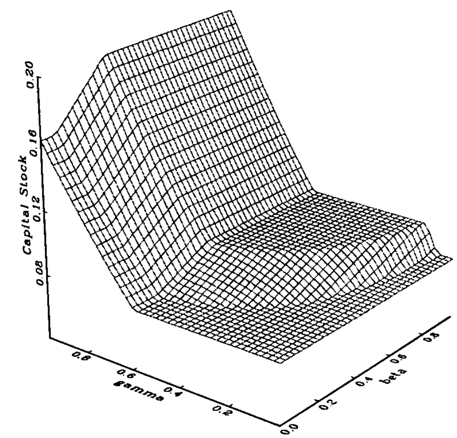

Figure 5
Steady State Bequests

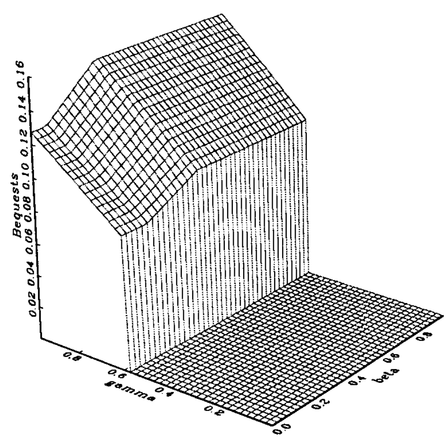

Figure 6
Time Transfers

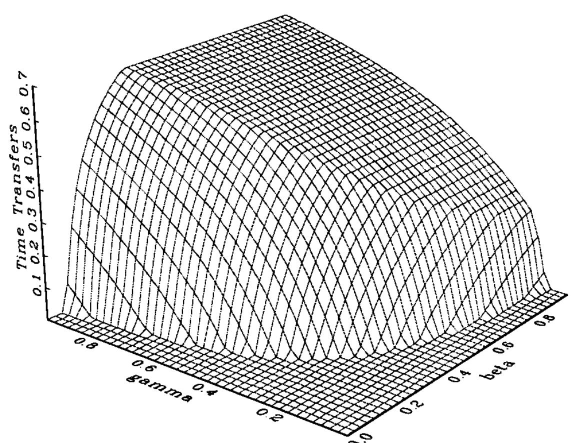

Figure 7
Labor Supply

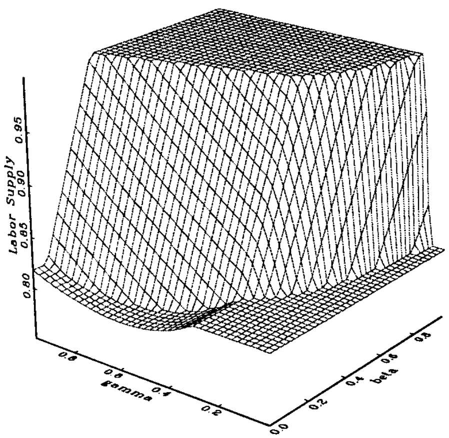

Figure 8
Steady State Capital Stock

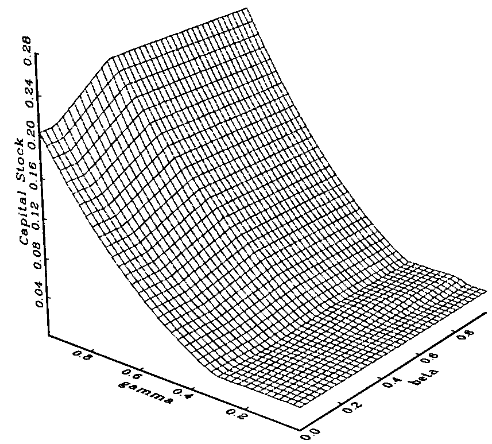

Figure 9
Steady State Bequests

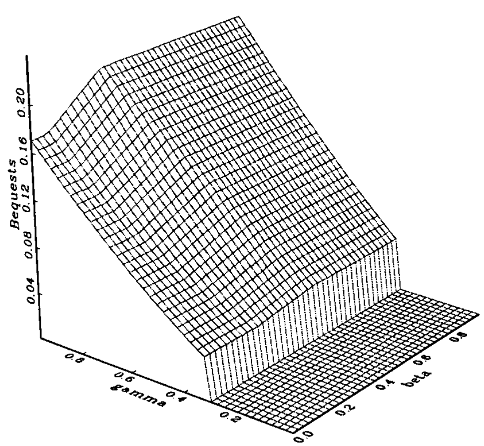

Figure 10
Time Transfers