Centre interuniversitaire de recherche
en économie quantitative

CIREQ

Cahier 06-2014

Network Effects, Aftermarkets and the Coase Conjecture : A Dynamic Markovian Approach

Didier LAUSSEL,
Ngo Van LONG and
Joana RESENDE

Cahier 06-2014

Network Effects, Aftermarkets and the Coase Conjecture : A Dynamic Markovian Approach

Didier LAUSSEL, Ngo Van LONG and Joana RESENDE

CIREQ, Université de Montréal
C.P. 6128, succursale Centre-ville
Montréal (Québec) H3C 3J7
Canada
Téléphone : (514) 343-6557
Télécopieur : (514) 343-7221
cireq@umontreal.ca
http://www.cireqmontreal.com

Dépôt légal - Bibliothèque nationale du Canada, 2014, ISSN 0821-4441
Dépôt légal - Bibliothèque et Archives nationales du Québec, 2014
ISBN-13 : 978-2-89382-664-6

Network Effects, Aftermarkets and the Coase Conjecture: a Dynamic Markovian Approach

Didier Laussel*
Ngo Van Long†
Joana Resende‡

May 16, 2014

*Aix-Marseille University (Aix-Marseille School of Economics), CNRS & EHESS. Email: didier.laussel@univ-amu.fr.

†CIREQ, and Department of Economics, McGill University, Montreal H3A 2T7, Canada. Email: ngo.long@mcgill.ca.

‡ Ce.fup, Economics Department, University of Porto. Email: jresende@fep.up.pt.

Network Effects, Aftermarkets and the Coase Conjecture: a Dynamic Markovian Approach

Abstract: This paper analyses the dynamic problem faced by a monopolist firm that produces a durable good (in the primary market) and also participates in the market for complementary goods and services (the aftermarket). Considering the possibility of network effects in both markets, we investigate the Markov Perfect Equilibrium of the dynamic game played by the monopolist and the forward-looking consumers. We characterize the evolution of the monopolist's equilibrium network and the equilibrium price trajectories. We show that the Coase Conjecture remains valid if there are only primary network effects, while it fails when aftermarket network effects are present. We also find that the properties of the Markov Perfect Equilibrium vary drastically with the intensity of aftermarket network effects.

JEL-Classification: L12, L14

Keywords: durable good; network externalities; aftermarkets, Coase Conjecture

1 Introduction

In this paper we study the dynamic aspects of the problem faced by a monopolist who is involved both in the primary market (in the production and sale of the durable good) and in the aftermarket, where she sells complementary goods and services (CGS), possibly in competition with rival CGS producers. As is well known, the value delivered by a durable good often depends on the subsequent consumption of CGS. Examples of this phenomena include hardware and software, wireless services and phone calls, printers and ink cartridges, cars and repairing services, and so on.[1]

We investigate the monopolist's equilibrium time path of production and pricing of the durable good, in the presence of network effects. These effects arise when the benefits derived from a good are increasing in the total number of consumers buying that good.[2] In the literature, it is common to distinguish between direct network effects (DNE) and indirect network effects (INE).[3] DNE are said to exist if the usefulness of a good directly depends on the size of its network (e.g. communication networks), whereas INE arise when the benefits of an increase in the number of users operate indirectly (e.g. "through improved opportunities to trade with another side of the market," as in Farrell and Klemperer, 2007).

In this paper, since the monopolist participates in two distinct markets (the primary market and the aftermarket), we also need to distinguish between primary market network effects (PMNE) and aftermarket network effects (AMNE).[4] Concerning the primary market, we focus on direct PMNE, which take place when the usefulness of the durable good is directly increasing in the size of the consumer base of the monopolist. As an example, computer users tend to benefit when many others use the same operating system (OS) because this makes

[1]See Shapiro (1995) for a more detailed characterization of aftermarkets.

[2]See the seminal papers by Rohlfs (1974), Katz and Shapiro (1985), Grilo *et al.* (2001), or, more recently, Amir and Lazzati (2011) and Griva and Vettas (2011). Network effects often lead to supermodular games (see Amir, 2005, for a survey).

[3]For a more detailed discussion on the diversity of sources of network effects, see, for example, the seminal paper by Katz and Shapiro (1994).

[4]For a duopoly model with both PMNE and AMNE see Laussel and Resende (2013).

it easier for them to find a computer that they know how to use.[5]

Aftermarket network effects arise when the value of CGS is increasing with the size of the consumer base for the durable good. Our paper allows for both direct and indirect AMNE.[6] For instance, the value of applications made available for a certain smartphone/tablet model associated with a given OS may increase with the network of users of such OS in two ways. First, consumers may obtain direct benefits (which increase with network size) from the exclusive applications (CGS) which the OS offers: they can chat, play or share files with more users (direct AMNE). Second, both the number and the quality of the applications made available for a given OS tend to increase with the size of its installed consumer base (indirect AMNE).

The present paper sheds light on the process of expansion of the network of a monopolist who produces a durable good and is also involved in the aftermarket. In particular, we study the extent to which the Coase conjecture, initially developed for a durable good market without network effects, may be applicable to the case where both PMNE and AMNE exist.

Assuming that consumers have rational expectations, Coase (1972) argued that, in continuous time, given her inability to make commitment about her future prices and outputs, a monopolist that produces and sells a durable good will lose all her monopoly power: in equilibrium, her price must be equal to her constant marginal cost, and she must serve all her customers immediately in one go. This conjecture has been subsequently proved rigorously. In particular, using a model with heterogeneous valuations where each consumer buys at most one unit, Bulow (1982) confirms this result for the "No Gap" case, defined as the situation in which the constant marginal cost is higher than willingness to pay of the (unserved) consumer with the lowest valuation. There are a number of exceptions. (See, for example, Kahn, 1987, Karp, 1996b). Specifically, it has been shown that when the durable

[5] As Cabral (2011) put it, "If I use Windows OS then, when I travel, it is more likely I will find a computer that I can use (both in terms of knowing how to use it and in terms of being able to run files and programs I carry with me)."

[6] In Appendix A, we provide a detailed analysis of aftermarkets in which direct or indirect network effects may arise.

good is subject to *stationary* network externalities, the Coase conjecture may fail (Mason, 2000).[7] In contrast, our paper assumes *non-stationary* network externalities in both markets.

In studying whether the Coase conjecture applies to durable good monopoly with network effects, it is useful to make the following distinction. The Coase conjecture is said to hold in the weak sense if the monopolist's present value of the stream of future profits is zero, even though there is intertemporal price discrimination. It is said to hold in the strong sense if the equilibrium displays both (i) zero profit, and (ii) immediate supply to all customers in one go (i.e. there is no intertemporal price discrimination).

In our paper, the dynamics of the interaction between the economic agents is analyzed in the context of a continuous time dynamic game played by the monopolist and forward-looking consumers with heterogeneous valuations. Each consumer, correctly forecasting future prices and network expansions, must determine whether, given his/her type, it is advantageous to buy the durable good and, if so, when to buy it. Understanding their optimal timing is crucial for the analysis of Coasian dynamics. We assume a continuum of infinitely lived consumers, ranked in order of their stand-alone valuation of the durable good. Each consumer demands at most one unit of the durable good, whose value depends on its intrinsic characteristics, the primary network effects (which we assume to be *non-stationary* direct network effects), and the value of the subsequent CGS purchases[8].

The monopolist's problem consists in choosing a time path for the durable good's output and its price so as to maximize the present value of the discounted stream of total profits that she earns in both the primary market and the CGS market. We assume that the monopolist cannot make any commitment about future prices and output.

We focus on the Markov Perfect Equilibrium (MPE) of the game. Hence, all the payoff-relevant information at any given time t is conveyed by the current size of the monopolist's network in the primary market, which constitutes the state variable of the game. The mo-

[7] The network effects are said to be stationary if the customer who purchases the good at time t never derives any benefits from expansion of the network after time t.

[8] The value of CGS depends on their intrinsic benefits as well as on non-stationary aftermarket network effects (which can be direct or indirect).

nopolist uses a Markovian production strategy for the durable good, and the consumers' expectations about the evolution of prices are a Markovian price function, such that, in equilibrium, (i) given consumers' price expectations, the monopolist's Markovian strategy maximizes the present value of its stream of future profits, starting at all possible (date, state) pairs, and (ii) given the monopolist's strategy, the Markovian price function representing consumers' expectations about the evolution of the equipment price is consistent with rational expectations.

Our methodological approach is similar to the one used by Laussel and Long (2012) to study the problem of vertical disintegration, and by Hilli et al. (2013), who study whether ownership dynamics may lead to pure managerial firms.

Our main finding is that the dynamics in the durable good market crucially depend both on the strength of the non-stationary AMNE. In contrast with Mason (2000), who found that stationary PMNE invalidate the strong form of the Coase conjecture, we find that the existence of non-stationary PMNE alone does not affect qualitatively the results obtained in models without network effects, such as Bulow (1982). Absent aftermarket network effects, the monopolist always sells in one go.

Interestingly, when AMNE are present, the picture changes dramatically in two different respects. First, when AMNE are sufficiently weak (so that the No Gap case arises), the monopolist's equilibrium strategy is to sell the durable good gradually, never covering the market completely. This gradual evolution of the network size means that the strong form of the Coase conjecture fails. In this case, the Coase conjecture holds only in its weak sense, i.e., the firm's initial value is equal to zero. In the primary market, the monopolist finds it optimal to sell the durable good at a price below its marginal production cost. She makes positive profits in the aftermarket.

Second, when AMNE are not weak, then we have the "Gap Case," which encompasses two different subcases: the Large Gap subcase, and the Small Gap subcase, according to the intensity of the AMNE. The standard result, that the market is fully covered in one go, obtains only in the Large Gap subcase, in which the AMNE are strong.

In the Small Gap subcase, in which the AMNE are intermediate, a new type of equilibrium emerges, in which the monopolist's equilibrium strategy depends on its installed base of consumers. If the latter is below a critical threshold $\widetilde{D}$, the monopolist's optimal strategy is to sell the durable good gradually, with price falling and below marginal production cost; these feature are similar to the No Gap case. In contrast, if the monopolist's installed base is above the critical threshold $\widetilde{D}$, her optimal strategy consists in selling to all the remaining consumers in one go, at a price equal to the willingness to pay of the lowest-valuation consumer. Finally, if the monopolist's installed base is at the critical threshold $\widetilde{D}$, her equilibrium behavior is a mixed strategy: selling to all remaining consumers in one go with probability λ, not selling with probability $1-\lambda$.

Our model is connected to two strands of literature: the literature on durable good monopolies, with the Coasian Conjecture as a main theme, and the literature on dynamic monopoly pricing in network industries, which also touches on the Coasian Conjecture.[9]

Starting with the seminal work of Coase (1972), a vast literature has studied the optimal monopoly pricing of durable goods. See, for example, Stokey (1981), Bond and Samuelson (1984, 1987), Kahn (1986), Gul et al. (1986), Ausubel and Deneckere (1987, 1989), Karp (1996a,b), among others. Kahn (1986) found that if marginal cost is increasing, the monopolist will not cover the market in one go, because she wants to take advantage of cost smoothing. Karp (1996b) found that depreciation erodes the Coase conjecture. A paper which is close to ours in spirit, though not in applications, nor in modelling, is Küh and Padilla (1996), where, in contrast to our formulation, each consumer buys many units of the durable goods.

In Kühn and Padilla (1996), the monopolist sells both a durable and a non-durable good to a representative consumer (who buys many units of each good). She has linear-quadratic

[9] More recently, there was a boost on the literature on dynamic pricing in network industries dealing with duopoly markets. Some works on this subject include Doganoglu (2003), Laussel et al. (2004), Mitchell and Skrzypacz (2006), Markovich (2008), Markovich and Moenius (2009), Chen et al. (2009), Cabral (2011), Laussel and Resende (2013), among others.

preferences over the two goods, which may be complements or substitutes. There are no network effects. They showed that, in this framework, the strong version of the Coase Conjecture fails: the monopolist finds it optimal to sell the durable good gradually. At the formal level, the non-durable good market in their model plays a similar role to the aftermarket in our paper, and the violation of the Coase Conjecture rests on a formally identical feature: the convexity of instantaneous equilibrium profits in the non-durable good market with respect to the stock of the durable good.[10]There are however substantial differences between the two papers. In our paper, there is a continuum of consumers' types instead of a representative consumer; buying one unit of durable is a necessary condition for consuming CGS in the aftermarket; and the stock of durables enters the instantaneous equilibrium aftermarket payoff functions via network effects rather than complementarity/substitutability in consumption.

In the literature on monopoly pricing in network industries, it has often be argued that the Coase Conjecture may fail when the durable good is subject to stationary direct network effects, where "stationarity" signifies that the network effects that benefit a given consumers depend only on past sales, i.e., consumers do not benefit from further expansion of the network after they have bought the durable good. Assuming stationary network effects, several authors have shown that access-pricing strategies may be time-consistent, and that, in some circumstances, a low introductory price is necessary to reach a critical mass of users and launch the market.[11] However, almost all of these papers (Xie and Sirbu, 1995, Fudenberg and Tirole, 2000, Gabszewicz and Garcia, 2007, 2008) avoid important issues of Coasian dynamics by assuming that consumers may buy the good only when they are young, i.e., consumers do not optimize over the date of purchase. An exception is Bensaid

[10]Kühn and Padilla provide a condition on the second-order derivatives of the "rental rates" under which this result is valid for more general utility functions. This condition, which bears on third-order derivatives of the utility function, is not very intuitive, and there are reasonable examples of utility functions that violate it.

[11]In contrast, Cabral et al. (1999) have shown that if network effects are non-stationary, *"incomplete information about demand or asymmetric information about costs is necessary for introductory pricing to occur in equilibrium when consumers are small"*.

and Lesne (1996) who allow consumers to choose the date of purchase, in a discrete time model. Considering only the "Gap case" and restricting attention to stationary network effects, they show that (i) the price of the good may increase through time, and (ii) prices and profits are bounded below. Under the same assumptions (stationary network effects, and optimization with respect to purchase date), but using a continuous time model and considering only the "No Gap case," Mason (2000) finds that the price of the durable good is constant through time.[12]

Mason (2000) is the closest paper to ours. He describes monopoly pricing of a durable good with a continuum of consumers under complete information. Assuming stationary network effects in the primary market, he concludes that such effects may invalidate the strong form of the Coase conjecture: he shows that the firm may serve the customers gradually instead of instantaneously, in one go.[13] However, the weak form of Coasian result is preserved since socially optimal pricing (price equals marginal cost) prevails in the Markov Perfect Equilibrium.

Differently from Mason (2000), in the primary market we consider non-stationary network effects.[14] Moreover, as we allow the durable good monopolist to participate in the aftermarket as well, we introduce an additional source of (non-stationary) network effects, the AMNE. To our knowledge, ours is the first work to shed some light on how non-stationary primary and aftermarket network effects play entirely different roles in terms of their distinct influences on the dynamics of the problem faced by a monopolist producing a durable good. Among other results, we show that, under non-stationary network effects, the existence of PMNE alone does not invalidate the Coase conjecture. In our set-up, the existence of non-stationary

[12] The difference in results between Mason (2000) and Bensaid and Lesne (1996) is due both to the fact that they consider different cases (No Gap versus Gap) and that the discrete time formulation implies that there is a non-degenerate time interval of commitment.

[13] Mason (2000) does not consider the case in which the durable good producer is also involved in the provision of CGS. Accordingly, the network effects addressed in Mason (2000) correspond to direct PMNE (in our terminology).

[14] In our paper consumers also benefit from further expansion of of the network after they have bought the durable good, i.e., there are non-stationary network externalities, according to the definition by Mason (2000).

AMNE is a necessary condition for the failure of the Coase conjecture.

The rest of the paper is organized as follows. Section 2 describes the main ingredients of the model. Section 3 provides a detailed description of the primary market and the aftermarket. Section 4 characterizes the Markov perfect equilibrium for different magnitudes of the AMNE, Section 5 discusses the implications for welfare and for regulation, and Section 6 concludes. Finally, the Appendix includes a detailed analysis of some aftermarkets in which network effects (direct or indirect) may arise, and provides detailed mathematical proofs.

2 The Basic Model

We consider a monopolist producing a perfectly durable good at a constant marginal cost c.[15] The firm also participates in an aftermarket where complementary goods and services (CGS) are provided to its base of consumers.

Consumers are infinitely lived and the surplus they derive from purchasing the durable good depends on (i) its intrinsic characteristics; (ii) the direct network benefits it may generate; (iii) the benefits yielded by future consumption of CGS; and (iv) the price of the durable good.

The price of the durable good at instant s is denoted by $p(s)$. Regarding (i), we assume that consumers have heterogeneous views on the intrinsic valuation of the durable good. More precisely, consumers, indexed by θ, are uniformly distributed in the interval $[0,1]$. Higher θ types value the durable good more highly. In what concerns (ii), the size of the monopolist's customer base at instant s is denoted by $D(s)$[16] and the (direct) primary market network externalities (PMNE) yielded at time s are assumed to be proportional to the customers base and are denoted by $\omega D(s)$, where the parameter $\omega \geq 0$ represents the intensity of the direct PMNE. The term $\theta + \omega D(s)$ represents the instantaneous primary utility[17] delivered by the durable good to a type θ-consumer, at instant s.

[15] In line with Mason (2000), we assume that (a) there is no depreciation and no capacity constraint; (b) the monopolist must sell, rather than rent the output; and (c) the time horizon is infinite.

[16] In other words, $D(s)$ is the fraction of consumers that have already bought the equipment at instant s.

[17] By primary utility, we mean the value of benefits from the durable good that does not depend on the

Regarding (iii), we denote by $Z^e(s)$ the expected CGS consumer surplus obtained from CGS consumption at instant s. Since all agents are forward looking and have rational expectations, firms and consumers' beliefs about the future are confirmed in equilibrium, implying that the expected CGS consumer surplus, $Z^e(s)$, is equal to the actual one, denoted by $Z(s)$. In particular this means that, when AMNE exist,[18] consumers are perfectly able to anticipate the evolution of the monopolist's network and the future magnitude of AMNE.

Denoting by $V(\theta, t)$ the discounted expected lifetime utility obtained by a consumer type $\theta \in [0, 1]$ who chooses to buy the durable good at instant t, we have, under non-stationary network effects,

$$V(\theta, t) = \int_t^{\infty} [\theta + \omega D(s)] \, e^{-r(s-t)} ds + \int_t^{\infty} Z^e(s) e^{-r(s-t)} ds - p(t), \tag{1}$$

where r stands for the discount rate and $s > t$.

3 The Aftermarket and the Primary market

In this section, we specify in more detail our assumptions about the primary market and the aftermarket. We start by studying the aftermarket because consumers' anticipated valuations about future CGS consumption play an important role on their decisions concerning the purchase (or not) of the durable good. By deriving consumers' optimal decision in the aftermarket, we will be able to compute their expected life-time CGS consumer surplus, conditional on having purchased the durable good at a given time t. This will allow us to determine how the optimal time of purchase of the durable good varies across consumer types.

---

consumption of CGS.

[18] The next section provides further details on the nature and the modelization of AMNE (which occur when the enjoyment of CGS is increasing in the size of the consumer base in the primary market).

3.1 The aftermarket

When computing $V(\theta, t)$ consumers need to anticipate $Z^e(s)$, $s > t$. Since our consumers are forward-looking and have rational expectations, it holds that $Z^e(s) = Z(s)$. The value of $Z(s)$ depends on (i) the intrinsic characteristics of CGS, (ii) the AMNE, and (iii) the price of CGS. The intrinsic characteristics of the CGS may be exogenous and time-invariant, or may evolve with the network size.[19] Thus, to compute $Z(s)$, consumers need to correctly anticipate the future evolution of $D(s)$ (which is the basis for future AMNE) as well as the future price of CGS. In our set-up, it is assumed that CGS are non-durable, and the CGS provider(s) is (are) unable to commit to future prices.[20] The no-commitment assumption, which implies static profit maximization in the aftermarket, consistently extends to the non-durables the usual assumption underlying the Coase Conjecture literature, according to which the durable good monopolist is unable to commit to future prices of the durable good. This rules out non-credible strategies in which the firm would promise at time t to lower the price in the aftermarket at some time $t' > t$, in order to make the primary good more attractive to forward-looking consumers.

In light of the above remarks, let $Z(D(s))$ denote the equilibrium CGS consumer surplus at instant s and suppose:

$$Z(D(s)) = \gamma_1 + \phi_1 D(s), \tag{2}$$

with $\gamma_1 \geq 0$;[21] and $\phi_1 \geq 0$, meaning that the equilibrium CGS surplus (weakly) increases with the size of the consumer base of the durable's good producer (due to AMNE). The specification (2) requires equilibrium CGS consumer surplus to be increasing linearly with $D(s)$.[22]

Analogously, let $\pi^A(D(s))$ denote the equilibrium instantaneous profit made by the monopolist producer of the durable good in the aftermarket at instant s. Then $\frac{\pi^A(D(s))}{D(s)}$ is its

[19] See examples 1 and 2 in the Appendix for the first case, and examples 3(a) and 3(b) in the Appendix for the second case.

[20] Kuhn and Padilla (1996) also make a similar no commitment assumption.

[21] γ_1 measures the equilibrium GGS surplus in the absence of network effects.

[22] The linear affine specification is made for the sake of tractability.

equilibrium instantaneous aftermarket profit per customer. Let us suppose that:

$$\frac{\pi^A(D\,(t))}{D\,(t)} = \gamma_2 + \phi_2 D(t), \tag{3}$$

with $\gamma_2 \geq 0$. For most of the analysis, we assume $\phi_2 \geq 0$, meaning that the monopolist's equilibrium aftermarket profit per consumer increases with its consumer base.[23]

In Appendix A, we describe in detail some illustrative examples in which equations (2) and (3) are derived from alternative specifications of competition in the CGS market. Example 1 deals with direct AMNE and CGS competition à la Cournot. This example is suitable to study the provision of homogeneous software programs or homogeneous applications for video calls. Example 2 considers direct AMNE and price competition with differentiated CGS. This example is suitable to study the provision of horizontally differentiated software or applications. Example 3 considers indirect AMNE and monopoly provision of CGS. In this example, we deal with markets in which the variety or quality of CGS (like software, applications, repairing services,....) grow endogenously as the network expands. Finally, example 4 addresses the case of a derivative aftermarket, in which the durable good producer is not directly involved in the provision of CGS but it owns a platform which independent CGS suppliers need to access in order to sell their goods/services to the consumer base of the durable good producer.[24] This example is suitable to analyze business strategies such as the ones adopted by Apple (who owns iTunes, the exclusive platform to provide apps for iPhones or iPads) or Amazon (who owns the Kindle store).

3.2 The primary market

Since the durable good monopolist benefits from CGS sales, from the perspective of the integrated firm, the per period marginal profitability (in the aftermarket) of supplying one additional unit of durable good is equal to $\gamma_2 + 2\phi_2 D(t)$.This leads us to define the *effective*

[23] In fact, the crucial assumption for the failure of the Coase Conjecture is that equilibrium instantaneous profits per consumer in the aftermarket are increasing in the consumer base of the durable good. Assuming an affine function is only for the sake of tractability.

[24] In example 4, the market structure resembles a two-sided monopoly market.

marginal cost of supplying one additional unit of the durable good as follows:

$$m(D(t)) \equiv rc - \gamma_2 - 2\phi_2 D(t), \tag{4}$$

where $m(D(t))$ is equal to the amortized manufacturing cost per-period[25] minus the marginal instantaneous profit (with respect to the customer base) derived from CGS sales.

On the consumers' side, when computing $V(\theta, t)$, forward-looking consumers anticipate that $Z^e(s) = Z(D(s))$. This allows us to compute, for each consumer, the value of purchasing the durable good at time t. Plugging equation (2) into (1), we obtain the expected life-time utility, discounted back to time zero, of a type-θ-consumer if she buys the durable good at time t:

$$e^{-rt}V(\theta, t) = e^{-rt}\left[\frac{\theta + \gamma_1}{r} + (\omega + \phi_1)\int_t^{\infty} D(s)e^{-r(s-t)}ds - p(t)\right]. \tag{5}$$

Each type-θ consumer must decide whether she will ever buy the equipment and, if so, what is the optimal time to buy it. These two decisions are, in principle, distinct. Nonetheless, they are often confused in the literature. A consumer of type θ will buy the durable good if and only if there exists some date t' at which it will provide her with a positive expected life-time utility, i.e. such that $V(\theta, t') \geq 0$. When this condition is met, the consumer needs to choose the optimal date of purchase $t(\theta)$ to maximize (5). Since consumers have rational expectations, the equilibrium evolution of the equipment price is determined by a simple arbitrage condition that follows from the first-order condition of this problem. The second-order condition implies that higher types of consumers buy the equipment not later than lower types. The following Lemma, proved in Appendix A, provides the non-arbitrage condition.

Lemma 1 *For a consumer of type θ, the optimal purchase date $t(\theta)$ must satisfy*

$$\frac{dp(t(\theta))}{dt} = rp(t(\theta)) - [\theta + \gamma_1 + (\omega + \phi_1) D(t(\theta))]. \tag{6}$$

[25]Producing one additional unit of the durable good costs c to the firm at moment t, when that unit is manufactured. Assuming a perfect capital market, this is equivalent to incurring a cost rc per unit of time from t until infinity. Kühn and Padilla (1996) also used this equivalence.

where $t(\theta)$ must be non-increasing in θ, i.e. $\frac{dt(\theta)}{d\theta} \leq 0$.[26]

Denote by $\theta(t)$ the lowest consumer type who has already bought the equipment at time t. From Lemma 1, $\theta(t)$ is non-increasing in t. The assumption of uniform distribution of types over $[0, 1]$ implies $\theta(t) = 1 - D(t)$. Accordingly, the non-arbitrage condition stated in Lemma 1 leads consumers to formulate their expectations about the evolution of the price of the durable good according to the following rule:

$$\frac{dp(t)}{dt} = rp(t) - b(D(t)), \tag{7}$$

where we have defined

$$b(D(t)) \equiv 1 - D(t) + \gamma_1 + (\omega + \phi_1) D(t). \tag{8}$$

We can interpret $b(D(t))$ as representing the *instantaneous full benefit* at time t that the equipment confers to the *marginal* customer $\theta(t)$.[27]

Since the customer $\theta(t)$ buys the equipment at instant t, it must be the case that an infinitesimal postponement of the purchase has a marginal cost (namely, forgone enjoyment) which is balanced by a marginal monetary benefit. The former is measured by $b(D(t))$, whereas the latter is equal to $rp(t) - \frac{dp(t)}{dt}$. Condition (7) states that, in equilibrium, these terms must cancel out for the marginal consumer $\theta(t)$; for otherwise, she could improve her lifetime utility by changing the timing of her purchase. Let us elaborate on this.

From (8), the marginal cost of postponing infinitesimally the purchase is the sum of three terms: *(i)* the intrinsic "stand-alone benefit" of the equipment to the marginal consumer, $\theta(t) = 1 - D(t)$; *(ii)* the direct PMNE, $\omega D(t)$; and *(iii)* the instantaneous net benefit obtained from the CGS, $\gamma_1 + \phi_1 D(t)$.The marginal benefit of postponing infinitesimally the purchase consists of the interest income $rp(t)$ and the cost savings due to the change in the equipment price $-\frac{dp(t)}{dt}$. (We will show that, in our model, the price cannot rise.)

[26]It can be easily shown that the SOC holds at equilibrium for all the cases studied in the paper. The proof is available from the authors upon request.

[27]Note that $\frac{1}{r}b(D(t))$ may be interpreted as the present value of life-time benefits that would accrue to the consumer $\theta(t)$, if there were no further network expansions. If there are expectations of further expansion of the network, the present value of life-time benefits is higher than $\frac{1}{r}b(D(t))$.

Integrating (7) yields the equilibrium price path

$$p(t) = \int_t^{\infty} b(D(s))e^{-r(s-t)}ds, \tag{9}$$

which is perfectly anticipated by forward-looking consumers.[28]

Remark 1 In the case of stationary network effects, equation (5) would write

$$e^{-rt}V(\theta, t) = e^{-rt}\left[\frac{\theta+\gamma_1}{r} + (\omega+\phi_1)\frac{D(t)}{r} - p(t)\right]. \tag{10}$$

Accordingly, using the same argument as in the proof of Lemma 1, we would obtain the arbitrage equation

$$\frac{dp(t(\theta))}{dt} = rp(t(\theta)) - [\theta + \gamma_1 + (\omega+\phi_1) D(t(\theta))] + (\omega+\phi_1)\frac{1}{r}\frac{dD(t(\theta))}{dt}, \tag{11}$$

which differs from (6) by the last term on the right-hand side. This would lead consumers to form their expectations about the evolution of the price of the durable good according to the following rule:

$$\frac{dp(t)}{dt} = rp(t) - b(D(t)) + (\omega+\phi_1)\frac{1}{r}\frac{dD(t)}{dt}, \tag{12}$$

instead of (7). Condition (12) exactly corresponds, mutatis mutandis, to Mason's equation 4.[29] Upon integration, we obtain:

$$p(t) = \int_t^{\infty} (1 - D(s) + \gamma_1 + (\omega+\phi_1) D(t))e^{-r(s-t)}ds \tag{13}$$

or equivalently,[30]

$$p(t) = \int_t^{\infty} (b(D(s))e^{-r(s-t)}ds + (\omega+\phi_1)\int_t^{\infty} [D(t) - D(s)]\, e^{-r(s-t)}ds. \tag{14}$$

[28] We have imposed the condition $\lim_{t\to\infty} e^{-rt}p(t) = 0$, reflecting the fact that the price of a durable good must be bounded, given that utility is bounded above.

[29] We are grateful to a referee for drawing attention to the difference between the two differential equations, which reflects the difference between Mason's assumption (stationary network effects) and ours (non-stationary ones).

[30] From (13) it is obvious that the durable price is bounded below by the value of the network effect (see Bensaid and Lesnes,1996, Proposition 4, for a similar result). This is larger than the marginal cost only in the Gap Case which Mason did not consider.

For any given time path $D(s)$, the price described by equation (9) would exceed the price described by equation (14) by the term

$$(\omega + \phi_1) \int_t^{\infty} [D(s) - D(t)] \, e^{-r(s-t)} ds,$$

which is positive as long as the network size is expected to increase. This difference reflects the fact that, under stationary effects, the customer does not benefit from the future expansion of the network.

Let us show that under non-stationary network effects, the consumer who purchases the durable good at time t expects a positive life-time surplus that depends only on the equilibrium time path of the durable good producer's network. From (5),

$$V(\theta(t), t) = \frac{\theta(t) + \gamma_1}{r} + (\omega + \phi_1) \int_t^{\infty} D(s) e^{-r(s-t)} ds - p(t).$$

Substituting in the above equation for $p(t)$ using (9), we finally obtain

$$V(\theta(t), t) = \int_t^{\infty} [D(s) - D(t)] \, e^{-r(s-t)} ds$$

where we have made use of the fact that $\theta(t) = 1 - D(t)$.(This shows that the surplus obtained by the last consumer is zero.)

Since in equilibrium the customer base is non-decreasing, $D(s) \geq D(t)$ for all $s \geq t$, we conclude that $V(\theta(t), t) \geq 0$. Consumers who find it optimal to purchase the equipment at t, rather than at some some later time $t' > t$, derive from that purchase a positive expected lifetime utility. If the network size is strictly increasing over time, this utility is strictly positive. This means that some consumer types with valuations that are marginally below $\theta(t)$ do not buy the equipment at t but they would strictly prefer buying the equipment to never buying it. They simply delay their purchase in order to benefit from future lower prices.

The monopolist's instantaneous profits in the primary market are equal to

$$\pi^{PM}(t) = q(t) \, [p(t) - c],$$

where $q(t)$ represents the equipment quantity sold by the monopolist at instant t.

In the following section, we derive the Markov Perfect Equilibrium (MPE) of this game. In a MPE, the players' strategies (i.e. the monopolist's output strategy in the primary market and the consumers' price expectations) depend only on the state variable, which is the size of the monopolist's installed base of consumers, $D(t)$.

The nature of the equilibrium turns out to depend crucially on the relative position of the curve representing the *effective marginal cost* of supplying one additional unit of the durable good, $m(D)$, as defined by equation (4), and the curve representing the *instantaneous full benefit* of the equipment, $b(D)$, to the marginal customer. In what follows, in order to sharpen our analysis, we assume that $b(0) > m(0)$, i.e. the instantaneous full benefit of the equipment to the highest-valuation consumer, if she were the only consumer, exceeds the effective marginal cost of supplying the durable good to this consumer. This assumption is sufficient to ensure that it is worthwhile for the monopolist to produce a positive output. We state this as Assumption A1.

Assumption A1: $b(0) > m(0)$, i.e., $1 + \gamma_1 > rc - \gamma_2$.

Remark 2: If Assumption A1 were violated, the strategy of never producing the durable good, coupled with consumer expectations of a constant equipment price forever at $\frac{b(0)}{r}$ would be an equilibrium for some set of parameter values.[31] This equilibrium would coexist, for some subset of parameter values,[32] with an equilibrium in which the market is covered in finite time. This corresponds to situations when the production of durable goods may become profitable only thanks to the network effects. Finally, there would also be room for situations where no MPE would exist.[33]

Note that, for $\phi_2 > 0$, the curve $m(D)$ is downward sloping. Then the per period marginal profitability (in the aftermarket) of supplying one additional unit of the durable good is increasing with the size of the monopolist's network. Therefore, the effective marginal

[31] Provided that $b(0) \leq m(\frac{1}{2})$. The proof is available upon request.

[32] When $m(1) < b(1)$ in addition to $b(0) < m(1/2)$.

[33] When both $b(0) > m(\frac{1}{2})$ and $b(1) < m\left(\frac{1}{2}\right)$.

cost of supplying one additional unit of the durable good decreases with D, for $\phi_2 > 0$. In contrast, the slope of the curve $b(D)$ can be negative or positive. In fact, when the network of the monopolist firm expands, two effects arise. First, consumers benefit from greater network effects ($\omega + \phi_1$ measures the marginal network benefit for consumers). This increases the instantaneous full benefit that the equipment confers to the marginal customer $\theta(t)$. However, this expansion implies that the marginal customer itself is a lower type. This second effect, when combined with the first, determines the net change in the instantaneous benefit that the equipment confers to the (lower type) marginal customer.

Notice that $b(1) = \omega + (\gamma_1 + \phi_1)$ while $m(1) = rc - \gamma_2 - 2\phi_2$. Recalling that D can only take on values in the interval $[0, 1]$, and that $b(0) > m(0)$ (Assumption A1), we distinguish three mutually exclusive and exhaustive cases: Case 1 (the No Gap case), Case 2 (the Small Gap case) and Case 3 (the Large Gap case), depending on the intensity of AMNE.[34] Figure 1 below illustrates where each regime holds in the space (ω, ϕ_2). The figure is drawn for $r = 0.07$, $c = 10$, $\gamma_1 = 0.05$, $\gamma_2 = 0.1$, $\phi_1 = 0.05$. It shows that, provided $\phi_2 > 0$, the No Gap case (NG) arises for weak AMNE, the Small Gap case (SG) arises for intermediate AMNE, whereas the Large Gap Case arises for strong AMNE, where the precise definitions of "weak AMNE", "intermediate AMNE", and "strong AMNE" are given below.

INSERT FIGURE 1 HERE

The figure also shows that, *provided that* $\phi_2 > 0$, i.e. there are strictly positive AMNE and the durable good producer is able to extract part of the surplus generated by such AMNE, the roles of PMNE and AMNE in the configuration of the possible MPE regimes are similar. In fact, as long as $\phi_2 > 0$, the type of regime depends on the intensity of overall network effects (i.e. it depends on the sum of PMNE and AMNE).

Case 1 (The No Gap Case) The No Gap Case arises when there exists a customer

[34] Although all the conditions are written in terms of AMNE (strong, weak, intermediate aftermarket network effects), *as long as AMNE are strictly positive*, the critical factor is the intensity of overall network effects, understood as the sum of AMNE and PMNE.

base $\underline{D} \in (0,1]$ such that, at $\underline{D}$, the marginal customer's instantaneous full benefit, $b(\underline{D})$, is equal to the monopolist's effective marginal cost, $m(\underline{D})$. In other words, there is no gap between the valuation of the person at the margin of the customer base, $\underline{D}$, and the monopolist's marginal cost of servicing her. We will show that, in this case, the durable good monopoly, starting with any initial customer base $D_0 < \underline{D}$, will restrict the size of its customer base to $\underline{D}$, i.e., it will serve only the relatively high valuation consumers, those with index $\theta \in [1-\underline{D}, 1]$. Since $b(D)$ and $m(D)$ are linear and since $b(0) > m(0)$ (by Assumption A1), the No Gap Case arises if and only if $b(1) \leq m(1)$, i.e., iff

$$\phi_1 + 2\phi_2 \leq rc - (\gamma_1 + \gamma_2) - \omega. \tag{15}$$

Since ω and (ϕ_1, ϕ_2) are, respectively, the primary and aftermarket network parameters, the No Gap Case arises if and only if the aftermarket network effects are *weak*, in the sense of inequality (15).[35]

The Gap Cases

If condition (15) is not satisfied, then $b(1) > m(1)$, i.e., there is a positive gap between the benefit to the lowest valuation consumer type, and the firm's effective marginal cost if it covers the whole market. In the literature on durable good monopoly without network effects, this situation is known as the Gap Case, with the well-known results that the monopolist will immediately cover the whole market and make a strictly positive profit (Bulow, 1982). Interestingly, in our model with network effects, these results do not carry over without substantial qualifications. In fact, instead of a Gap Case, we have two different Gap Cases, with completely different equilibrium characteristics: Case 2 (The Large Gap Case) and Case 3 (The Small Gap Case.)

[35] Condition (15) can be re-written as $\omega + \phi_1 + 2\phi_2 \leq rc - (\gamma_1 + \gamma_2)$, showing that the condition for the No Gap case really depends on the overall network effects (i.e. the sum of PMNE and AMNE). Yet, the roles of PMNE and AMNE are substantially different because the existence of AMNE is a necessary condition for the Coase conjecture to fail, whereas the same is not true for PMNE. In subsection 4.2.2, we indeed show that non-stationary PMNE alone do not invalidate the Coase Conjecture. It is also worth noting that the roles played by ϕ_1 and ϕ_2 are not symmetric. In fact, to have a violation of the Coase conjecture, we need $\phi_2 > 0$, which requires the existence of strictly positive AMNE and, in addition, part of the surplus they create must be absorbed by the producer of the durable good. Differently, the role of ϕ_1 is somewhat similar to the role played by ω.

Given Assumption A1, a Gap Case (i.e., $b(1) > m(1)$) implies that at all possible installed customer bases (all possible values of D in the interval $[0,1]$), the benefit to the marginal consumer, $b(D)$, exceeds the effective marginal cost of serving that customer, $m(D)$. It might be tempting to conjecture that, in such a case, the monopolist will want to serve the whole potential market, i.e., to achieve $D = 1$, either immediately, or asymptotically. However, upon reflection, a key consideration is whether the total cost of supplying the whole market instantaneously is lower than the total revenue that consumers are willing to pay if all consumers were served immediately at the first instant.

Now, the total effective cost of supplying the whole market instantaneously is the capitalized value of the area under the per-period effective marginal cost schedule, the area A defined by

$$A \equiv \int_0^1 m(D)dD.$$

Since $m(D)$ is linear, the area A is equal to $m(1/2)$, and its capitalized value is $\frac{1}{r}m(1/2)$. This is to be compared with the price that the lowest-valuation consumer is willing to pay, $\frac{1}{r}b(1)$.(The latter is equal to the total revenue obtained in the primary market if the whole market is covered in one go). If $b(1) \geq m(1/2)$, we say that we are in the Large Gap Case. If $b(1) < m(1/2)$, we say that we are in the Small Gap Case.

Case 2 (The Large Gap Case)

The Large Gap case is defined by the inequality $b(1) \geq m(1/2)$. This condition is equivalent to the following condition

$$\phi_1 + \phi_2 \geq rc - (\gamma_1 + \gamma_2) - \omega. \tag{16}$$

When this condition holds, we say that the AMNE are strong.

Clearly, in the special case where $\phi_2 = 0$,then whenever a Gap Case arises, i.e., $\phi_1 + 2\phi_2 > rc - (\gamma_1 + \gamma_2) - \omega$, it is the Large Gap Case.

Case 3 (The Small Gap Case)

The Small Gap case is defined by $m(1/2) > b(1) > m(1)$.The first strict inequality is equivalent to

$$rc - (\gamma_1 + \gamma_2) - \omega > \phi_1 + \phi_2 \tag{17}$$

while the second strict inequality, $b(1) > m(1)$, is simply the negation of the No Gap Condition (15). The Small Gap Case arises if and only if the AMNE are "intermediate" in the sense that $\phi_1 + 2\phi_2 > rc - (\gamma_1 + \gamma_2) - \omega > \phi_1 + \phi_2$. Obviously, for a Small Gap Case to exist, one requires that $\phi_2 > 0$. The small gap case cannot arise when only PMNE exist.

The properties of the Markov perfect equilibrium differ fundamentally across the three cases. This will be demonstrated in the next section.

Remark 3

While we focus on positive network effects, it is simple to extend the analysis to the case of negative network effects (i.e. congestion effects). If all network effects were negative, there would exist only two possible cases: the No Gap case and the Large Gap case. Differently from the case of positive AMNE, with congestion effects in the aftermarket, the Coase conjecture remains valid.[36]

Figure 2 below illustrates the configuration of MPE when we allow for congestion effects. In the Large Gap case, the Coase conjecture of immediate sale to all customers holds both in the case of positive and negative network effects. The Small Gap case only exists for positive AMNE (requiring $\phi_2 > 0$). Finally, the No Gap case arises both under positive or negative network effects, but its properties are considerably different. When $\phi_2 > 0$, the Coase conjecture of immediate sale does not hold: sales will instead take place gradually (independently of the positive or negative network effects arising in the primary market). On the contrary, when $\phi_2 \leq 0$, the Coase conjecture is valid.

INSERT FIGURE 2 HERE

[36] The proof of the results in Remark 3 is available from the authors, upon request.

4 Markov Perfect Equilibrium

In a MPE, the monopolist uses a Markovian output strategy (denoted by G) and the consumers' expectations about the evolution of prices can be represented by a Markovian price function (denoted by ζ), such that (i) given consumers' price expectations, the monopolist's Markovian output strategy maximizes its profits for all possible (date, state) pairs and (ii) given the monopolist's output strategy, the Markovian price function underlying consumers' expectations about the evolution of the equipment prices is consistent with rational expectations.

4.1 Some preliminary considerations

Consider first the Marvokian price function representing consumers' expectations about the evolution of the equipment price. All consumers have a common Markovian price expectation function $\zeta(D)$, where ζ is a function of the state variable D. Using condition (9), the price expectations function is

$$\zeta(D(t)) = p(t) = \int_t^{\infty} b(D^*(s))e^{-r(s-t)}ds, \tag{18}$$

or, written in full,

$$\zeta(D(t)) = \int_t^{\infty} \left[1 - D^*(s) + \gamma_1 + (\omega + \phi_1)\, D^*(s)\right] e^{-r(s-t)}ds, \tag{19}$$

where $\{D^*(.)\}_t^{\infty}$ is the time path of the state variable D induced by the strategic behavior of the monopolist from time t on, given that the state variable at time t takes the value $D(t)$.

Consider now the monopolist's Markovian output strategy. Such a strategy specifies how the monopolist intends to sell the durable good. For example, in some intervals of the state space, the monopolist may choose to sell the durable good gradually whereas in some other intervals of the state space, she may prefer to sell a lumpy amount. The output strategy is said to be Markovian if it is a rule G which tells the firm the amount of the durable good to sell at time t, based only on the knowledge of its current customer base, $D(t)$. A strategy G

is a best reply to the consumers' expectations rule ζ if it yields a time path of sales $q^*(t)$ that maximizes total discounted profits of the firm for all starting (date, state) pairs $(t, D(t))$:

$$\Pi(t) = \int_t^{\infty} \left[\pi^A(s) + \pi^{PM}(s)\right] e^{-r(s-t)} ds. \tag{20}$$

In other words, we look for a solution to the problem

$$\max_{q(s)} \Pi(t)$$

$$\text{subject to} \quad \frac{dD(s)}{ds} = q(s), \ D(t) \text{ given,}$$

that generates a price path that is consistent with (19).

Recall that a strategy G defines how the monopolist intends to sell the durable good, namely the circumstances in which she sells lumpy amounts of the durable good, as well as those in which she sells the good gradually. Formally, a Markovian strategy G is the specification of (i) a collection of disjoint intervals of the state space, $I_1, I_2, ..., I_m$, in which the monopolist plans to sell lumpy amounts of the durable good, where $I_i \equiv [a_i, b_i] \subset [0, 1]$, (ii) a lumpy sale function $L_i(.)$ corresponding to each interval I_i, such that $L_i(.) \geq 0$ specifies an upward jump in the state variable, so as to increase the customer base from D to $D + L_i(D)$, with $0 \leq D + L_i(D) \leq 1$ where $D \in I_i$, and (iii) a gradual sale function $g(.)$ defined for all $D \notin I_i$, such that

$$q(t) = \frac{dD(t)}{dt} = g(D(t)) \text{ for } D \notin I_i.$$

Definition 1 *A Markov-perfect equilibrium is a pair (G, ζ) such that, (i) given the price function ζ, the strategy G maximizes the integrated firm's payoff, starting at any (date,state) pair $(t, D(t))$ and (ii) given G and $(t, D(t))$, the price function ζ satisfies the rational expectation property (19).*

In what follows, we turn to a complete characterization of the Markov Perfect Equilibrium. We will show that in the No Gap case, the monopolist will serve only a fraction of the market, and she will do so either (a) gradually, with the price of the durable good falling along the way, or (b) instantaneously, with a constant price (i.e., no intertemporal price discrimination). Policy (a) is optimal only if AMNE are present, with $\phi_2 > 0$. If there are no AMNE, policy (b) is optimal. These results will be established in sub-section 4.3.2.

It might be tempting to conjecture that if $b(1) > m(1)$, eventually the market will be fully covered. We prove below that this conjecture does not always hold true. When AMNE take place and the monopolist is able to capture some of the surplus they generate (which is represented by the parameter $\phi_2 > 0$), there are two different sets of circumstances:

(i) If the AMNE are strong, i.e., in the "Large Gap" case, where $b(1) \geq \int_0^1 m(D)dD$, the monopolist always covers all the market in one go. (See sub-section 4.3.1.)

(ii) If the AMNE are intermediate, i.e., in the "Small Gap" case, where $\int_0^1 m(D)dD > b(1) > m(1)$, there is a strictly positive probability that the market is not fully covered. (See sub-section 4.3.3.) Note that the Small Gap case cannot arise if $\phi_2 = 0$.

4.2 Benchmarks

Before stating our main results, it is useful to consider two benchmark scenarios. In the first benchmark scenario, there are no network effects. In the second benchmark scenario, there are only primary market network effects (no AMNE).

4.2.1 First benchmark scenario: no network effects

In the absence of any type of network effects, we have $\omega = 0$ and $\phi_1 = \phi_2 = 0$. In this case, only the No Gap and the Large Gap cases may arise, depending on the sign of the expression $rc - (\gamma_1 + \gamma_2)$.

In what follows, we show that, in the "No Gap" case, the strong form of the Coase conjecture always holds in the absence of network effects, despite the existence of positive aftermarket benefits.[37]

Lemma 2 (The No Gap Case in the absence of network effects) *Assume* $(\gamma_1 +$

[37] In the absence of AMNE, the aftermarket benefits are measured by the parameters γ_1 and γ_2.

$\gamma_2) \leq rc$, *i.e., the "No Gap" case in the absence of any type of network effects. The Markov Perfect Equilibrium has the following Coasian properties:*

(i) The equilibrium price function is a constant: $\zeta(D) = c - \frac{\gamma_2}{r} = \frac{1}{r} b\left(\underline{D}\right) = \frac{1}{r} m(\underline{D})$.

(ii) Starting at any $D < \underline{D}$, *the monopolist' s equilibrium strategy is the lumpy sale strategy* $L(D) = \underline{D} - D$, *where* $\underline{D} = 1 - rc + (\gamma_1 + \gamma_2)$.

(iii) The firm's value is $J(D) = \frac{\gamma_2 D}{r}$ *for all* $D \in [0, \underline{D}]$ *with, in particular,* $J(0) = 0$.[38]

Proof: Apply the proof of Proposition 1 in Appendix, and take the limit $\phi_2 \to 0$.■

When $(\gamma_1 + \gamma_2) > rc$, the Large Gap case arises. In that case, as usual in the literature without network effects, the price is constant, equal to the present value of the stream of net benefits to the lowest valuation consumer and the monopolist's equilibrium strategy is to supply the whole market immediately.[39]The value of the firm is strictly positive.

This subsection makes the point that adding an aftermarket (without network effects) to an otherwise traditional durable goods model does not change the standard results in the durable good literature.

We now turn to the second benchmark scenario, to show that adding only primary market network effects (i.e. $\omega > 0$ but $\phi_1 = \phi_2 = 0$) also does not bring any new result.

4.2.2 Second benchmark scenario: primary market network effects only

When there are only PMNE, we have $\omega > 0$ and $\phi_1 = \phi_2 = 0$. Again, only the No Gap and the Large Gap cases may arise. For $\omega > 0$ and $\phi_2 = 0$, it can be easily shown that the Coasian dynamics arising in Lemma 2 remain valid for $(\omega + \gamma_1 + \gamma_2) \leq rc$ (the No Gap case), and the addition of PMNE (alone) affects only the size of the monopolist's steady

[38] Thus, if the firm starts with $D = 0$, the present value of its stream of net revenue is zero: the total production cost of the durable goods, $c\underline{D}$, minus the present value of the stream of CGS profits, $\gamma_2 \underline{D}$, is just equal to the revenue from the sales of the durable goods, $\underline{D} b(\underline{D})/r$. If a firm inherits a market with $D_0 > 0$, it will need to produce only $\underline{D} - D_0$ units, earning a total revenue of $[\underline{D} - D_0]\, b(\underline{D})/r$ from sales of durable goods, which, together with CGS profit $\gamma_2 \underline{D}/r$, exceeds production cost $c\,[\underline{D} - D_0]$. This difference equals the value $J(D_0) = \gamma_2 D_0 / r$.

[39] The proof of this result is formally a special case of the proof concerning MPE with strong network effects, by setting the relevant network effect parameters to zero.

state network, with $\underline{D} = \frac{1-rc+\gamma_1+\gamma_2}{1-\omega}$. In the Large Gap case, again the traditional results hold.

This shows that the Coase conjecture is consistent with the existence of non-stationary PMNE. This result is in sharp contrast to Mason (2000). Analyzing the No Gap case, and assuming stationary PMNE, Mason (2000) shows that (i) the monopolist sets price at the marginal cost and earns zero profits[40], and (ii) yet, the strong form of the Coase Conjecture fails since there is no instantaneous adjustment of the durable good stock to its steady state level. Differently from Mason (2000), in our model, when only non-stationary PMNE arise, we get a lumpy adjustment of D to its steady state level. This makes the point that the different ways in which the network effects are modeled (non-stationary in our model, versus stationary in Mason's) lead to substantially different results.

4.3 MPE with aftermarket network effects

We now consider the case where aftermarket network effects exist, i.e. when at least one of the two parameters ϕ_1 and ϕ_2 are positive. Our main findings are as follows. In the Large Gap case, the standard result applies: immediate coverage of the entire market, and positive profit for the monopolist. In the No Gap case, but with AMNE reflected in $\phi_2 > 0$, we obtain the results that (a) the monopolist sells the durable good gradually, (b) the speed of convergence is a decreasing function of ϕ_2, and (c) the monopolist's profit is zero. A new type of MPE emerges in the Small Gap Case (which arises only if $\phi_2 > 0$): the monopolist's equilibrium strategy displays drastically different modes of behavior, depending on the current size of the customer base. This is reported in Proposition 2 below.

Notice that the standard Coasian results apply if we suppose that $\omega > 0$ and $\phi_1 > 0$ but $\phi_2 = 0$, a hypothetical situation in which all the benefits from aftermarket network effects would accrue to consumers. In fact, the source of the violation of the Coase Conjecture in our model is the fact that AMNE, when partly appropriated by the durable good producer,

[40] We could easily reproduce his results in our framework, using the arbitrage equation (11) which we derived for the case of stationary network effects.

introduces a convexity of instantaneous equilibrium profits in the non-durable good market with respect to the stock of the durable good.[41]

In what follows, we derive equilibrium outcomes when there exist network effects in the aftermarket, of which some benefits go the durable good producer ($\phi_2 > 0$). We will show that in that under these conditions, we can get a failure of the Coase conjecture where we did not get one before. This finding is independent of the existence (or not) of PMNE.

4.3.1 MPE with strong AMNE (the Large Gap case)

In this sub-section, we tackle the simplest case. We show that if there are strong AMNE, the firm's equilibrium strategy is to serve *all* customers and cover the whole market *immediately*.

We have defined the Large Gap case by the condition $b(1) \geq m(1/2)$. Since $m(D)$ is linear, this condition is equivalent to

$$\frac{b(1)}{r} \geq \frac{1}{r} \int_0^1 m(D) dD. \tag{21}$$

This inequality indicates that supplying the whole spectrum of consumer types in one go is profitable. This condition is equivalent to $\phi_1 + \phi_2 \geq rc - \omega - \gamma_1 - \gamma_2$, meaning that the AMNE effects are sufficiently strong.

It is easy to verify that the Markov-perfect equilibrium in the Large Gap Case has the following properties. (See Appendix B for the proof.) First, consumers expect that the price of the durable good is constant and it is equal to the present value of the stream of net benefits to the lowest valuation consumer buying the durable good. This means that the equilibrium price function is

$$\zeta(D) = \frac{1}{r} (\gamma_1 + \phi_1 + \omega) = \frac{b(1)}{r} = p^*. \tag{22}$$

[41] In Kuhn and Padilla (1996) the violation of the Coase conjecture was also due to the convexity of instantaneous profits in the non-durable good market with respect to the stock of the durable good. However, they do not consider any kind of network effects.

Second, the monopolist's equilibrium strategy is to supply the whole market immediately, i.e., $D(t) = 1$ for $t > 0$. In other words, starting from any $D \in [0, 1)$, the firm uses the lumpy sale strategy $L(D) = 1 - D$. As a result, the firm's value in the Large Gap case is

$$J(D) = (1 - D)(p^* - c) + \frac{1}{r}[\gamma_2 + \phi_2] \geq 0 \tag{23}$$

where the term inside the square brackets is the capitalized value of the stream of profit in the aftermarket when all customers are served. Note that $J(0) = \frac{1}{r}(b(1) - m(1/2)) \geq 0$. The possibility that $b(1) < rc$ exists, and in that special case the equilibrium price p^* is smaller than the production cost c. Since $J(D) \geq 0$, any loss in the primary market is more than offset by the positive aftermarket profits.

The firm's initial value, $J(D_0)$, is *strictly* positive.[42] This resembles the standard Gap Case in the literature on durable good monopoly without network effects, in which the lowest-valuation consumer's intrinsic utility from the durable good is larger than the constant marginal production cost. The bold line in Figure 3 below depicts the equilibrium Markovian price function (multiplied by r) under strong network effects.[43]

INSERT FIGURE 3 HERE

4.3.2 MPE with weak AMNE (the No Gap case)

In this sub-section, we consider the case where AMNE are sufficiently weak, so that there exists $\underline{D} \in (0, 1]$ for which $b(\underline{D}) = m(\underline{D})$. The condition for the No Gap case, inequality (15), is equivalent to $m(1) \geq b(1)$, which means that the effective marginal cost of servicing the lowest-valuation consumer type $\theta = 0$ is larger than the marginal benefit yielded to her, thereby excluding the possibility of full market coverage.

In the No Gap case, we will show that, provided that there are strictly positive AMNE, $\phi_2 > 0$, the equilibrium strategy of the monopolist consists in selling the equipment *gradu-*

[42] Except in the rasor edge case where the following conditions hold simultaneously, $b(1) = m(1/2)$ and $D_0 = 0$.

[43] It is drawn for $r = 0.05, c = 5, \omega = 0.1, \gamma_1 = 0.05, \phi_1 = 0.1$, $\gamma_2 = 0.1$ and $\phi_2 = 0.2$.

ally until the (generically) interior steady state $\underline{D}$ is reached, consequently the lowest type consumers never buy the equipment, i.e., the market is never completely covered (see Proposition 1). An additional result is that the equipment price is always lower than the marginal production cost, and it decreases gradually.[44]

The gradual evolution of the network size means that the strong version of the Coase conjecture fails. Nevertheless, the weak version of the Coase conjecture holds: at the time the monopolist starts production, the value of the discounted stream of future aggregate profits is zero. The bold line in Figure 4 depicts the equilibrium Markovian price function (multiplied by r) under weak network effects.[45]The full characterization of the MPE under weak AMNE is provided in Proposition 1.

INSERT FIGURE 4 HERE

Proposition 1 (**The No Gap Case, with AMNE**) *Assume that the aftermarket network effects are positive, with* $\phi_2 > 0$*, but weak enough so that* $\phi_1 + 2\phi_2 \leq rc - (\gamma_1 + \gamma_2 + \omega)$*. Then there exists a unique* $\underline{D} \in (0, 1]$*, such that* $m(\underline{D}) = b(\underline{D})$*. The steady state network size is* $\underline{D} = \frac{1 - rc + \gamma_1 + \gamma_2}{1 - (\phi_1 + 2\phi_2 + \omega)}$.

(i) The Markovian equilibrium price function, for all $D \in [0, \underline{D}]$*, is*

$$\zeta(D) = c - \frac{1}{r}(\gamma_2 + 2\phi_2 D) = \frac{m(D)}{r}, \tag{24}$$

(ii) The equilibrium value of the firm is strictly convex and increasing in D

$$J(D) = \frac{\pi^A(D)}{r} = \frac{\gamma_2 D + \phi_2 D^2}{r}, \tag{25}$$

(iii) The monopolist's equilibrium strategy is to sell gradually, and the rate of sale at time t *is*

$$q(t) = \frac{r}{2\phi_2}\left[b(D(t)) - m(D(t))\right]. \tag{26}$$

[44]See equation (A.6) in the Appendix, where $p = c - J' < c$.

[45]It is drawn for $r = 0.05, \omega = 0.1, \gamma_1 = 0.05$, $\phi_1 = 0.1$, $\gamma_2 = 0.1$, $\phi_2 = 0.2$ and $c = 18$.

This implies that $q(t)$ asymptotically approaches zero as the network size approaches $\underline{D}$. Starting at $D(0) = 0$, the size of the network increases over time, such that

$$D(t) = (1 - e^{-\psi t})\underline{D}. \tag{27}$$

The speed of convergence is $\psi = \frac{1-(\phi_1+2\phi_2+\omega)}{2\phi_2} > 0$, it is decreasing in ϕ_2.

Proof: See Appendix B.

From Proposition 1, the equilibrium price is $p(t) = \frac{m(D(t))}{r}$. This implies that the price is below the marginal production cost, c. Moreover, the equilibrium equipment price is decreasing through time, until $\underline{D}$ is reached. As $D(t)$ expands, in the aftermarket, the marginal profitability of serving an additional customer increases (due to AMNE), therefore the monopolist's effective marginal cost of serving an additional consumer becomes lower (see equation (4)), leading to a reduction in the equilibrium price of the durable good. As result, the price paid by early consumers is higher than the one paid by later consumers, i.e., the monopolist is practicing intertemporal price discrimination. However, her scope for intertemporal price discrimination is limited because rational consumers expect the price function $p(t) = \frac{m(D(t))}{r}$ and the monopolist's pricing must satisfy the no-arbitrage condition.

Regarding the evolution of the installed base, Proposition 1 implies a gradual adjustment until the steady state level $\underline{D}$ is reached. The steady state $\underline{D}$ is increasing with ϕ_2. Intuitively, a greater ϕ_2 means a higher aftermarket profitability per customer and therefore the monopolist is interested in selling his durable good to a larger set of consumers.

In addition, the greater is ϕ_2, the slower is the speed of convergence to the steady state. This means that in equilibrium the monopolist prefers to slow down the adoption of the durable good (i.e. to opt for a slower rate of price decrease) when the AMNE are stronger (provided they remain sufficiently weak to be consistent with the No Gap case). The reason is as follows. The monopolist faces the consumers Markovian price expectation rule $\zeta(D) = \frac{m(D)}{r}$. The greater is ϕ_2, the steeper is the schedule $m(D)/r$, implying that, for any given increase in D, the price falls by more. The monopolist, not wanting the price to fall too fast, has the incentive to slow down the rate of adoption. In contrast, if the schedule $m(D)/r$ were horizontal (no AMNE) or even increasing (negative AMNE), the monopolist would be induced to reach the equilibrium steady-state instantaneously.

Proposition 1 also implies that the value of the firm evaluated at any $D \in [0, \underline{D}]$ is just equal to the discounted stream of net returns in the CGS market that would be obtained if that D were kept constant for ever. The firm initially incurs losses in the primary market but is is able to recoup them through aftermarket profits, as $D(t)$ increases over time: all customers (including those who have bought the durable good earlier on) buy more and more CGS as the network expands. From (25), we have $J(0) = 0$, i.e., starting at $D = 0$, the monopolist expects to gain nothing by selling gradually as compared with choosing a zero output forever. Nonetheless, starting at $D = 0$, refraining from production and sale is not a Markov perfect equilibrium. The rationale behind this result is the following: if the monopolist were to choose a zero output, from (9) it would follow that the expected equipment price would be constant for ever at $\frac{b(0)}{r}$, which would of course induce her to sell, given Assumption A1. As a result, the monopolist prefers to sell the durable good and therefore, the firm's value, which is initially zero at $D(0) = 0$, increases through time as the network of the durable good producer expands.

For the sake of completeness, let us considers the hypothetical case where, perhaps because of some errors in the past, a fraction $D^{\#} > \underline{D}$ of consumers has bought the durable good. Corollary 1 states what the firm would do in this (off-the equilibrium) subgame.

Corollary 1 *In the hypothetical case under which the monopolist faces a network size $D^{\#}$ greater that the desired steady-state $\underline{D}$, as the monopolist cannot buy back what she has sold, she will stay put at $D^{\#}$, and the value of the firm, starting from $D^{\#}$, is equal to:*

$$J(D^{\#}) = \frac{\gamma_2 D^{\#} + \phi_2 (D^{\#})^2}{r} \text{ for } D^{\#} \in (\underline{D}, 1].$$

Proof: See Appendix B.

The Corollary shows that as long as the monopolist is unable to buy back what she has sold, the (continuing) equilibrium strategy starting from this point $D^{\#}$(which is off the equilibrium path) consists in staying put at $D^{\#}$, and the value of the firm, starting from $D^{\#}$, is simply the capitalized value of the stream of the aftermarket profits. In this case, those consumers with low valuations such that $\theta < 1 - D^{\#}$ can only be in equilibrium if they expect to gain nothing in buying the durable good, which is true if they expect the price $\zeta(D) = \frac{b(D)}{r}$ for all $D > D^{\#}$. This price is lower than $m(D^{\#})/r$, and the monopolist has no incentive to expand the market.

4.3.3 MPE with intermediate AMNE (the Small Gap case)

We say that the network effects are intermediate if $m(1/2) > b(1) > m(1)$, i.e..

$$\omega + \phi_1 + 2\phi_2 > rc - (\gamma_1 + \gamma_2) > \omega + \phi_1 + \phi_2. \tag{28}$$

For this condition to hold, it is necessary that $\phi_2 > 0$. The condition $b(1) > m(1)$ is simply the negation of the No Gap case. Condition $m(1/2) > b(1)$ means that the revenue from selling the durable good to all types of consumers in one go, $b(1)/r$, is less than the cost of durable good production to supply the whole potential market minus the capitalized value of the constant stream of profit in the aftermarket. Therefore it is not profitable to cover the market instantaneously: the price would be to low to compensate for the effective cost of producing for the whole market. On the other hand, to cover the whole market gradually with price falling steadily along the curve $m(D)/r$ (as in the No Gap case) until $D = 1$would not be a credible strategy, because the inequality $m(1)/r < b(1)/r$ indicates that the monopolist would have an incentive to charge the last customer the price $b(1)/r$, but such a jump in price would not be consistent with the non-arbitrage condition. It turns out that the optimal strategy of the monopolist is quite interesting in the Small Gap case.

We will show that there exists a critical value of customer base $\widetilde{D} \in (0, 1)$ such that: (i) if for some reasons the monopolist starts with some D *above* this critical value, the optimal strategy is to make a lumpy sale and cover the whole market immediately;[46] (ii) if she starts with some D *below* this critical value, the optimal strategy is to expand her customer base gradually, until $\widetilde{D}$ is reached; (iii) at $\widetilde{D}$, she plays a mixed strategy: (a) with probability λ, she makes a lumpy sale $1-\widetilde{D}$ so that the whole market is covered immediately, while (b) with probability $1-\lambda$, she stops selling the durable good. As is always the case with equilibrium mixed strategy, the two courses of actions (a) and (b) yield the same payoff to the firm. The critical installed base $\widetilde{D}$ is such that, starting at $\widetilde{D}$, the monopolist's future profits from the "stop-production" policy equals the value of profit gained from a lumpy sale covering the market. The probability λ which characterizes the mixed strategy is chosen so as to eliminate any arbitrage opportunities on the consumers' side.

Let us first establish the existence of a unique critical value $\widetilde{D}\in(0,1)$ which satisfies the following condition

$$\frac{\gamma_2\widetilde{D}+\phi_2\left(\widetilde{D}\right)^2}{r}=(1-\widetilde{D})\left(\frac{b(1)}{r}-c\right)+\frac{\gamma_2+\phi_2}{r} \tag{29}$$

The left-hand side is the capitalized value of the perpetual constant flow of profits in the aftermarket, $\gamma_2\widetilde{D}+\phi_2\left(\widetilde{D}\right)^2$, if, starting at $\widetilde{D}$, the durable good monopolist refrains from adding to its customer base. The right-hand side is the alternative payoff if, given that its existing customer base is $\widetilde{D}$, the firm decides to make a lumpy sale of additional output to cover the whole market instantaneously, by selling $(1-\widetilde{D})$ in one go, at the price $p=b(1)/r$. Under this alternative policy, the firm earns a higher perpetual constant flow of profits in the aftermarket, $\frac{\gamma_2+\phi_2}{r}>\frac{\gamma_2\widetilde{D}+\phi_2\left(\widetilde{D}\right)^2}{r}$, but at the cost of selling the durable goods at a loss, because $\frac{b(1)}{r}-c$ is negative in the Small Gap case.

It can be shown[47] that in the Small Gap case there exists a unique $\widetilde{D}\in(0,1)$ that satisfies

[46] Because, given that $D>\widetilde{D}$ has been served, the relevant production cost is only $(1-D)c$.

[47] To demonstrate the existence of a unique $\widetilde{D}\in(0,1)$, define the function $K(D)$ given by

$$K(D)=\gamma_2+\phi_2+(1-D)\left(\gamma_1+\phi_1+\omega-rc\right)-\left(\gamma_2 D+\phi_2 D^2\right).$$

This function is concave and quadratic, with $K(1)=0$. Note that, under condition (28), $K(0)<0$ and $K'(1)<0$. From this and the concavity of $K(D)$, it follows that there exists a unique $\widetilde{D}\in(0,1)$ such that $K\left(\widetilde{D}\right)=0$.

eq. (29), with

$$\widetilde{D} = \frac{1}{\phi_2}\left(rc - (\gamma_1 + \gamma_2 + \phi_1 + \phi_2 + \omega)\right). \tag{30}$$

An increase in the intensity of AMNE, i.e., in ϕ_1 or ϕ_2, will make $\widetilde{D}$ smaller. When ϕ_1 increases, $b(1)$ increases because the instantaneous full benefit for the lowest valuation consumer increases. Analogously, when ϕ_2 increases, the effective marginal cost of serving an additional consumer goes down, meaning that $m(1/2)$ goes down. Accordingly, if we increase ϕ_1 or ϕ_2 sufficiently, $b(1)$ will approach $m(1/2)$,and $\widetilde{D}$ will approach zero, meaning that, in the limit, with $b(1) = m(1/2)$, the monopolist sells everything in one go (i.e., Large Gap case obtains, when AMNE are sufficiently strong). In contrast, if we decrease ϕ_1 or ϕ_2 sufficiently, $b(1)$ goes down and $m(1)$ goes up, implying that $\widetilde{D}$ will approach 1 and therefore, in the limit, the lumpy adjustment in D will never take place. In this limiting case, the behavior of the monopolist is similar to the No Gap case (in which AMNE must indeed be sufficiently weak).

We see that the Small Gap case is intermediate between the Large Gap case and the No Gap case. Therefore we expect that the equilibrium of this case has a combination of features of the equilibrium of the other two cases: a range $\left[0, \widetilde{D}\right)$of values of D in which it is optimal to sell gradually, and a range $\left(\widetilde{D}, 1\right]$ of values of D in which the firm finds it optimal to cover the whole market in one go. In fact, as will be shown below, the value function $J(D)$ in the Small Gap case has two segments. Over the interval $\left[0, \widetilde{D}\right)$, $J(D)$ is strictly convex, and is equal to $\pi^A(D)/r$ just as in equation (25) of the No Gap case, and the corresponding equilibrium price must be $\zeta(D) = \frac{m(D)}{r}$. Over the interval $\left(\widetilde{D}, 1\right]$, $J(D)$ is linear, and is equal to $(1-D)(\frac{b(1)}{r} - c) + \frac{\gamma_2+\phi_2}{r}$, just as in equation (23) of the Large Gap case.[48]

Note that $\widetilde{D}$ is not a steady state. In fact, starting from any $D_0 \in \left[0, \widetilde{D}\right)$, $D(t)$ will reach $\widetilde{D}$ at some finite time T, and as soon as $\widetilde{D}$ is reached, the firm plays a mixed strategy:

[48] At $\widetilde{D}$, there is the kink in the value function. The left-hand derivative minus the right-hand derivative equals $\frac{1}{r}\left[(rc - (\gamma_1 + \gamma_2)) - (\omega + (\phi_1 + 2\phi_2))\right]$, a negative number in view of equation (28). Therefore the value function is globally convex. We do not have smooth-pasting here, because the Markovian price function is discontinuous.

(a) with probability λ, it makes a lumpy sale $1-\widetilde{D}$ so that the whole market is covered immediately, and (b) with probability $1-\lambda$, it stops selling the durable good. Any consumer who has bought the durable good prior to T can expect that, if she were to re-sell it at time T, she would obtain the low price $b(1)/r < m(\widetilde{D})/r$ with probability λ and, with probability $1-\lambda$, at the high price, $b(\widetilde{D})/r$, which is what the marginal consumer $\widetilde{\theta} = 1-\widetilde{D}$, would be willing to pay (if the monopolist stops producing at time T). Thus the expected price of the durable good at time T is

$$Ep(T) = \lambda \frac{b(1)}{r} + (1-\lambda)\frac{b(\widetilde{D})}{r}.$$

To eliminate any gains from arbitrage, the monopolist's choice of λ must be such that no speculator can gain by buying just before T and re-selling at T; therefore, in equilibrium,

$$\lim_{t \uparrow T} p(t) = \lambda \frac{b(1)}{r} + (1-\lambda)\frac{b(\widetilde{D})}{r} \tag{31}$$

Since $\zeta(D) = m(D)/r$ for $D \in [0, \widetilde{D})$, condition (31) is equivalent to

$$\frac{m(\widetilde{D})}{r} = \lambda \frac{b(1)}{r} + (1-\lambda)\frac{b(\widetilde{D})}{r}. \tag{32}$$

To make precise the above intuitive discussion, we state the following proposition:

Proposition 2 *When primary and aftermarket network effects are such that* $\phi_2 > 0$ and $\omega + \phi_1 + 2\phi_2 > rc - (\gamma_1 + \gamma_2) > \omega + \phi_1 + \phi_2$, *or, equivalently,* $m(\frac{1}{2}) > b(1) > m(1)$ *(Small Gap),*

(i) The consumers' equilibrium expected price function is

$$\begin{cases} \zeta(D) = c - \frac{1}{r}(\gamma_2 + 2\phi_2 D) \equiv \frac{m(D)}{r} & \text{if } D \in [0, \widetilde{D}) \\ E\zeta(D) = c - \frac{1}{r}(\gamma_2 + 2\phi_2 \widetilde{D}) \equiv \frac{m(\widetilde{D})}{r} & \text{if } D = \widetilde{D} \\ \zeta(D) = \frac{b(1)}{r} = \frac{1}{r}(\gamma_1 + \phi_1 + \omega) < \frac{m(\widetilde{D})}{r} & \text{if } D \in (\widetilde{D}, 1] \end{cases} \tag{33}$$

Thus the expected price function is piece-wise continuous, and has a jump discontinuity immediately to the right of $\widetilde{D}$.

(ii) If $D \in (\widetilde{D}, 1]$, *the monopolist's equilibrium strategy is the lumpy sale strategy* $L(D) = 1-D$. *If* $D \in [0, \widetilde{D})$, *the monopolist's equilibrium strategy is to sell gradually the durable good*

at a rate given by equation (A.10). At $D = \widetilde{D}$, the monopolist uses a mixed strategy: (a) with probability λ, cover the market by selling in one go the quantity $1-\widetilde{D}$ at the price $b(1)/r$; and (b) with probability $1-\lambda$, stop selling the durable good; where $\lambda b(1) + (1-\lambda)b(\widetilde{D}) = m(\widetilde{D})$:

$$\lambda \frac{1}{r}(\gamma_1 + \phi_1 + \omega) + (1-\lambda)\frac{1}{r}\left(\gamma_1 + \phi_1\widetilde{D} + \omega\widetilde{D} + 1 - \widetilde{D}\right) = c - \frac{1}{r}(\gamma_2 + 2\phi_2\widetilde{D}). \tag{34}$$

(iii) the firm's value is

$$\begin{cases} J(D) = \frac{\gamma_2 D + \phi_2 D^2}{r} & \text{for all } D \in \left[0, \widetilde{D}\right] \\ J(D) = \frac{(1-D)(\gamma_1+\phi_1+\omega-rc)+\gamma_2+\phi_2}{r} & \text{for all } D \in \left[\widetilde{D}, 1\right] \end{cases} \tag{35}$$

Proof: See the Appendix.

Figure 5 below depicts the Markovian equilibrium price function (in bold), in the Small Gap case.[49]

INSERT FIGURE 5 HERE

5 Implications for Welfare and Regulations

To discuss implications for welfare and regulations, we consider of example 1 in the Appendix, and focus on the case of the durable-good monopolist also has monopoly power in the CGS market, i.e. $N = 1$. Then $\gamma_1 = \gamma/8$, $\phi_1 = \phi/8$, $\gamma_2 = \gamma/4$ and $\phi_2 = \phi/4$.

At each instant t,when the customer base reaches $D(t)$, the instantaneous social surplus is the sum of the monopolist's profits in the aftermarket and in the primary market, $\pi^A(t) + \pi^{PM}(t)$, and of the aggregate consumers surplus $U(t)$ obtained by those who have bought the good, which equals

$$\begin{aligned} U(t) &= \int_{1-D(t)}^{1} \left[\theta + (\omega + \frac{\phi}{8})D(t) + \frac{\gamma}{8}\right] d\theta - p(t)q(t) \\ &= \left(D(t) - \frac{D(t)^2}{2}\right) + D(t)\left(\frac{\gamma + \phi D(t)}{8} + \omega D(t)\right) - p(t)q(t). \end{aligned}$$

[49] It is drawn for $r = 0.05, \omega = 0.1, \gamma_1 = 0.05$, $\phi_1 = 0.1$, $\gamma_2 = 0.1$, $\phi_2 = 0.2$ and $c = 13$.

The monopolist's profit at t is

$$\pi^A(t) + \pi^{PM}(t) = \frac{\gamma D(t) + \phi D(t)^2}{4} + [p(t) - c]\, q(t).$$

The function $\pi^A(t)$ reflects the assumption of monopolistic pricing of CGS (for otherwise, with perfectly competitive behavior, CGS price would be zero, and aftermarket profit would be zero).

To determine whether the equilibrium time path of $D(t)$ under the Markov-perfect equilibrium is socially efficient or not, we ask what the outcome would be if a social planner can choose $D(t)$. We consider two different scenarios under the social planner. In the first scenario, the social planner cannot regulate monopoly in the CGS market. In the second scenario, the social planner can force competitive pricing of CGS goods: CGS price must be equal to CGS marginal cost.

Welfare Comparison Under Scenario 1

Under Scenario 1, we assume that the social planner can dictate the output rate $q(t)$ for the durable good, but cannot prevent monopoly power in the CGS market.

Then, at time $t = 0$, with $D(0) = 0$, the present value of the stream of future aggregate discounted social surplus is

$$\begin{aligned} W &= \int_0^\infty (U(t) + \pi^A(t) + \pi^{PM}(t))e^{-rt}dt \\ &= \int_0^\infty \left(D(t) - \frac{D(t)^2}{2} + D(t)\left[\frac{3}{8}(\gamma + \phi D(t)) + \omega D(t)\right] - cq(t) \right) e^{-rt}dt, \end{aligned}$$

Recall that $q(t) = \frac{dD(t)}{dt}$. Integrating $\int_0^\infty e^{-rt}c\frac{dD(t)}{dt}dt$ by parts, noting that $D(t)$ is bounded and that $D(0) = 0$, yields $\int_0^\infty e^{-rt}rcD(t)dt$. Then

$$W = \int_0^\infty \left[\frac{D(t)^2}{2}(2\omega + \frac{3}{4}\phi - 1) + D(t)(1 - rc + \frac{3\gamma}{8})\right] e^{-rt}dt. \tag{36}$$

Accordingly, conditional on monopolistic pricing in the CGS market, maximizing social welfare over time requires to set immediately and forever the value of the stock of the durable good at the value D^{opt} which maximises the bracketed term, which is a second-order polynomial in $D(t)$,

$$R(D) \equiv \frac{D^2}{2}(2\omega + \frac{3}{4}\phi - 1) + D(1 - rc + \frac{3\gamma}{8}) \tag{37}$$

We continue to maintain Assumption A1. Then we can state

Proposition 3

Assume that the monopoly pricing in the CGS market cannot be regulated.

(i) If $\frac{3}{4}\phi + 2\omega \geq rc - \frac{3}{8}\gamma$ *then the welfare maximizing policy is to set* $D(t) = D^{opt} = 1$, $\forall t \geq 0$, *i.e. to cover the whole market in one go.*

(ii) if $\frac{3}{4}\phi + 2\omega < rc - \frac{3}{8}\gamma$ *(very weak network effects), the welfare maximizing policy is to set* $D(t) = D^{opt} = \frac{1-rc-\frac{3}{8}\gamma}{1-\frac{3}{4}\phi-2\omega} \in (0,1)$, $\forall t \geq 0$, *i.e. to supply the durable goods immediately in one go to consumers whose intrinsic utility for the durable good exceeds* $\theta^{opt} \equiv 1 - D^{opt}$.

Proof

(i) The function $R(D)$ has the following properties: $R(0) = 0$ and $R'(0) > 0$. Consider two mutually exclusive cases, (a) $\frac{3}{4}\phi + 2\omega - 1 \geq 0$, and (b) $\frac{3}{4}\phi + 2\omega - 1 < 0$. Recall that Assumption A1 implies $1 - rc + \frac{3}{8}\gamma > 0$. In case (a), $R(D)$ is strictly increasing in D, therefore $R(1) > R(D)$ for all $D \in [0, 1)$.

In case (b), $R(D)$ is strictly concave. If $\frac{3}{4}\phi + 2\omega \geq rc - \frac{3}{8}\gamma$ then $R'(1) \geq 0$, therefore, recalling $R(0) = 0$ and $R'(0) > 0$, we conclude that $R(1) > R(D)$ for all $D \in [0, 1)$.

(ii) If $rc - \frac{3}{8}\gamma > \frac{3}{4}\phi + 2\omega$, then, because of Assumption A1, $1 > \frac{3}{4}\phi + 2\omega$ and hence $R(K)$ is concave. In this case, $R(0) = 0, R'(0) > 0$ and $R'(1) < 0$. Therefore the function $R(D)$ attains a maximum in the interior of $(0, 1)$. The maximum occurs at $\frac{1-rc+\frac{3}{8}\gamma}{1-\frac{3}{4}\phi-2\omega}$. It is easy to check, given Assumption 1, that this value is strictly positive and < 1, for this case.

Notice that $rc - \frac{3}{8}\gamma > \frac{3}{4}\phi + 2\omega$ implies that $rc - \frac{3}{8}\gamma > \frac{5}{8}\phi + \omega$, and the latter inequality defines the No Gap Case of the monopoly model.■

Comparing the social optimum with the durable good monopoly outcome, we can identify two sources of inefficiency of the Markov Perfect Equilibrium. Firstly, the monopolist may sell a smaller aggregate amount of durable good than the socially optimal one. Secondly, whereas the social planner always finds it optimal to sell in one go, the monopolist may sell gradually. In the latter case, market outcomes lead to phenomena of excessive inertia in the network expansion (compared with the welfare maximizing solution).

In the Large Gap case, i.e. when $rc \leq \left(\frac{3}{8}(\gamma+\phi)+\omega\right)$, the MPE coincides with the social optimum. Indeed, $rc \leq \left(\frac{3}{8}(\gamma+\phi)+\omega\right)$ implies $rc \leq \left(\frac{3}{8}\gamma+\frac{3}{4}\phi)+\omega\right)$ and then, from Proposition 3(i), $D^{opt}=1$. The equilibrium and the socially optimal strategies coincide: the firm in both cases sells in one go one unit of equipment to all consumers (i.e. the market is fully covered).

In the Small Gap case, i.e. when $\frac{5}{8}\phi+\omega > rc-\frac{3}{8}\gamma > \frac{3}{8}\phi+\omega$, the market is partially covered with probability $1-\lambda$, and is eventually fully covered at equilibrium with probability λ, but the welfare maximizing solution would require that it is fully covered[50]. Furthermore, the monopoly's equilibrium strategy is first to sell gradually whereas it would be socially optimal to cover the market in one go. So both sources of inefficiency are potentially present.

In the No Gap case, i.e. when $rc-\frac{3}{8}\gamma \geq \frac{5}{8}\phi+\omega$, the two sources of inefficiency are in play. First, the monopolist sells a lower aggregate amount of durable good than would be socially optimal, namely $\underline{D} = \frac{1-rc-\frac{3}{8}\gamma}{1-\frac{5}{8}\phi-\omega} < D^{opt} = \min\{\frac{1-rc-\frac{3}{8}\gamma}{1-\frac{3}{4}\phi-2\omega}, 1\}$. Second, sales take place gradually whereas it would be optimal to sell in one go. The reason why aggregate output of the durable good may fall short of its optimal value has little to do with the usual effect of monopoly power. Indeed, the equilibrium value $\underline{D}$ of the steady-state output equals its *Walrasian* value, it is determined by the equality between the marginal utility of the durable good (given that CGS output k is smaller than the social optimal output of CGS, which is $k^{opt}=1$) to the marginal customer and its effective marginal cost to the firm. It is suboptimal mainly because this does not account for *consumption externalities*: when deciding whether or not to buy one unit of durable, the marginal customer does not consider that, due to network effects, her decision will affect the utility of inframarginal consumers.

[50] Indeed $rc \leq \left(\frac{3}{8}\gamma+\frac{5}{8}\phi)+\omega\right) \Rightarrow rc \leq \left(\frac{3}{8}\gamma+\frac{3}{4}\phi)+\omega\right) \Rightarrow D^{opt}=1$.

Welfare Comparison Under Scenario 2

Finally, let us consider what would be the socially optimal output of the durable good if the social planner can regulate the aftermarket. Since the cost of production of CGS is zero, the social planner would require that CGS price be $\rho(t) = 0$. Then each consumer would purchase $k = 1/2$ and her CGS utility would be

$$[\gamma + \phi D(t)] [k(t) - \frac{1}{2}k^2(t)] = \frac{\gamma + \phi D(t)}{2}$$

At each instant t,when the customer base is $D(t)$, the instantaneous social surplus is the sum of the monopolist's profits $\pi^{PM}(t)+\pi^A(t) = \pi^{PM}(t)+0$ (because $\rho = 0$) and of the aggregate consumers surplus $U(t)$ obtained by those who have bought the good, which equals

$$\begin{aligned} U(t) &= \int_{1-D(t)}^{1} \left[\theta + (\omega + \frac{\phi}{2})D(t) + \frac{\gamma}{2}\right] d\theta - p(t)q(t) \\ &= \left(D(t) - \frac{D(t)^2}{2}\right) + D(t)\left(\frac{\gamma + \phi D(t)}{2} + \omega D(t)\right) - p(t)q(t). \end{aligned}$$

It follows that, at time $t = 0$, with $D(0) = 0$, the present value of the stream of future aggregate discounted social surplus is

$$\begin{aligned} W &= \int_0^\infty (U(t) + 0 + \pi^{PM}(t))e^{-rt}dt \\ &= \int_0^\infty \left(D(t) - \frac{D(t)^2}{2} + D(t)\left[\frac{4}{8}(\gamma + \phi D(t)) + \omega D(t)\right] - cq(t)\right) e^{-rt}dt, \end{aligned}$$

Then, after integration by parts, we obtain

$$W = \int_0^\infty \left[\frac{D(t)^2}{2}(2\omega + \frac{4}{4}\phi - 1) + D(t)(1 - rc + \frac{4\gamma}{8})\right] e^{-rt}dt.$$

Accordingly, conditional on marginal cost pricing ($\rho = 0$) in the CGS market, maximizing social welfare over time requires to set immediately and forever the value of the stock of the durable good at the value D^{opt} which maximises the bracketed term, which is a second-order polynomial in $D(t)$

$$\overline{R}(D) \equiv \frac{D^2}{2}(2\omega + \phi - 1) + D(1 - rc + \frac{4\gamma}{8})$$

Then we can state our final result:

Proposition 4

Assume marginal cost pricing ($\rho = 0$) *in the aftermarket.*

(i) If $\phi + 2\omega \geq rc - \frac{4}{8}\gamma$ *then the welfare maximizing policy is to set* $D(t) = D^{opt} = 1$, $\forall t \geq 0$, *i.e. to cover the whole market in one go.*

(ii) If $\phi + 2\omega < rc - \frac{4}{8}\gamma$ *(very weak network effects), the welfare maximizing policy is to set* $D(t) = D^{opt} = \frac{1-rc-\frac{4}{8}\gamma}{1-\phi-2\omega} \in (0,1)$, $\forall t \geq 0$, *i.e. to supply the durable goods immediately in one go to consumers whose intrinsic utility for the durable good exceeds* $\theta^{opt} \equiv 1 - D^{opt}$.

6 Conclusion

This paper analyses the dynamic problem faced by a monopolist firm that produces a durable good (in the primary market) and also participates in the corresponding aftermarket, where complementary goods and services are provided. The consumption of the durable good is subject to both primary and aftermarket network effects, yielding strategic complementarities that are new to the literature on dynamic pricing of a durable good with an aftermarket.

We characterize the evolution of the monopolist's equilibrium network and the equilibrium price trajectories, when both *non-stationary* primary and aftermarket network effects take place. Considering first the case of PMNE alone, we find that the different way in which the network effects are modeled (stationary versus non-stationary effects) is crucial. We show that the Coase Conjecture is consistent with the existence of non-stationary PMNE: in equilibrium there is a lumpy adjustment of the durable good stock to its steady state level. This result is in sharp contrast with the previous literature dealing with stationary PMNE (see, e.g. Mason, 2000).

When positive AMNE arise, results are fundamentally distinct. In the No Gap case where AMNE are weak, the monopolist prefers to expand its network gradually and she never covers the entire market (the monopolist would be interested in covering the entire market in one go only if aftermarket network effects were null). The price of the durable good is below its marginal production cost and the monopolist practices intertemporal price discrimination.

For sufficiently strong AMNE (the Large Gap case), the monopolist always covers the entire market in one go. The monopolist fixes a durable good price which may be above or below its marginal production cost, and, taking into account its profit in the aftermarket, the firm's value is positive.

For intermediate network effects (the Small Gap case), our results depend on whether the current consumer base of the monopolist producer is above or below a critical network size. In the former case, the dynamics are similar to the Large Gap case, whereas in the latter case the dynamics correspond to the ones observed in the No Gap case. When the monopolist's consumer base is equal to the critical network size, the monopolist stops producing with a certain probability, and, with the complementary probability, she covers the whole market immediately.

Our model can be enriched by extension in several dimensions. In our future research, we intend to study the monopolist's dynamic behavior under the threat of entry of an imperfectly substitute durable good.

Acknowledgements: Joana Resende acknowledges financial support from Cef.up and FCT through research grant PTDC/EGE-ECO/115625/2009. The authors are grateful to the editor and the referees for their very helpful comments and suggestions.

APPENDIX

Appendix A - Aftermarket network effects: Examples

Example 1: Direct AMNE and competition *à la* Cournot

In this example, we first consider the case of direct PMNE and direct AMNE. This means that consumers benefit directly when others buy the durable good: they gain both because the value of the durable good itself is higher (due to direct PMNE), and because the value of CGS also increases (due to direct AMNE).

This example is suitable to address some features of the Operating Systems (OS) industry. First, an OS can be reasonably treated as a durable good, with the software programmes being the corresponding CGS (see Economides, 2000). Second, OS often entail direct PMNE.[51] Finally, OS are frequently associated with AMNE. The latter can be indirect (when the variety/quality of the software available for a certain OS is increasing in the number of its users) and/or direct (since the utility of a given software often depends on the potential number of individuals with whom the consumers may exchange files).[52] There are also other real-world examples of a similar nature. Consider for instance the case of tablets (e.g. iPad) and video calls apps (e.g. Facetime). In this example, the iPad can be seen as the durable good, whereas Facetime would be a CGS. Several types of direct network effects may arise in this context. First, there may exist direct PMNE, namely, when consumers buy an iPad because, among other reasons, the device itself is considered fashionable on the eyes of other consumers, yielding a conspicuous consumption effect[53] akin to positive network external-

[51] For example, Cabral (2011) argues that *"The most obvious source of network effects is direct network effects. Take the example of operating systems. If I use Windows OS then, when I travel, it is more likely I will find a computer that I can use (both in terms of knowing how to use it and in terms of being able to run files and programs I carry with me)."*

[52] Regarding the possibility of DNE in the software industry, Page and Lopatka (2000) argue that *"Similarly, the value to an individual of a particular word processing program, say WordPerfect, likely will depend in part on the number of others who select WordPerfect and with whom the individual expects to exchange files. This effect is diminished to the extent that conversion between programs is possible, but, so long as conversion is imperfect or costly, the effect persists."*

[53] Regarding this type of effect, Dritsa and Zacharias (2013) point that "*Although there is a wide number of firms that produce such devices of similar quality, there are many consumers who prefer to buy an iPhone or a Blackberry as they are convinced that the specific gadgets will confer status to them. The Apple paradigm is*

ities (as studied by Grilo et al., 2001). Second, there are also direct AMNE: for example, we expect the utility of Facetime for a user to be increasing with the number of other iPad users with whom they can communicate.

In light of the features of the OS/software markets, in Example 1, we take as given the variety/quality of the available software (ignoring indirect AMNE, which are addressed in Example 3) and we concentrate on direct network effects arising for a certain type of software.

Concerning the structure of the aftermarket, in this example we suppose that a monopolist producer of the durable good is also a CGS producer who competes à la Cournot with other independent suppliers of CGS. Coming back to the OS examples, our modelling choice is appropriate to study situations in which the producer of a certain OS (e.g. Windows OS) is also involved in the provision of a certain type of software (e.g. a word processing software like Microsoft Word) but it faces the competition of independent software suppliers providing very close substitutes (e.g. Word Process, Word Perfect, Google Docs, ...). Example 2 relaxes the assumption of perfect substitutability of CGS.

For now, we assume that in the aftermarket the monopolist producer of the durable good and $N-1$ independent firms provide homogenous CGS at a constant marginal cost, which, w.l.o.g., is normalized to zero.

Consumers who already own an equipment derive utility $Z(t)$ from the consumption of $k(t)$ units of CGS at time t, with:

$$Z(t) = \gamma[k(t) - \frac{1}{2}k^2(t)] + \phi D(t)[k(t) - \frac{1}{2}k^2(t)] - \rho(t)k(t).$$

The term $\gamma > 0$ measures the magnitude of the "stand alone value of CGS"[54], for a given consumption level $k(t)$. The term $\phi D(t)[k(t) - \frac{1}{2}k^2(t)]$ corresponds to the (direct) network benefit associated with the consumption of CGS, with $\phi > 0$ measuring the intensity of

also applied in the market for tablet PCs where iPad, amongst all other brand names, is acknowledged as the product that confers status to its purchasers." Regarding this aspect, note that this conspicuous consumption effect may not take place but this is not crucial to our model, since only aftermarket network effects may induce a failure in the Coase conjecture.

[54] In the literature about network effects, the "stand alone value" refers to the value of a good that is independent of its network size (Katz and Shapiro,1985).

network effects. $\rho(t)$ stands for the unit price of CGS at instant t. Assuming that consumers who already own a durable good maximize their instantaneous utility, we obtain the total demand for CGS at instant t, given by $D(t)\left[1-\frac{\rho(t)}{\gamma+\phi D(t)}\right]$. Considering the outcome of the Cournot game played by the CGS suppliers, we obtain the instantaneous, equilibrium price of the CGS, equal to $\frac{\gamma+\phi D(t)}{N+1}$. Accordingly, the model yields (i) the equilibrium *instantaneous CGS utility* specification (2), with $\gamma_1 = \frac{\gamma}{2}\left(\frac{N}{N+1}\right)^2$ and $\phi_1 = \frac{\phi}{2}\left(\frac{N}{N+1}\right)^2$; and (ii) the monopolist's equilibrium profit per customer in (3), with $\gamma_2 = \frac{\gamma}{(N+1)^2}$ and $\phi_2 = \frac{\phi}{(N+1)^2}$.

Example 2: Direct AMNE and price competition with differentiated CGS

In Example 1, it was assumed that CGS providers offer a homogeneous good. However, there are many situations in which the goods and services sold in the aftermarket are horizontally differentiated (for example, the software and/or applications available to a certain OS are often horizontally differentiated). The model described above can be easily adapted to deal with this possibility. In what follows, we show that the properties of the equilibrium utility and profit specifications (in equations (2) and (3), respectively) remain valid in the context of direct AMNE and price competition with differentiated products.

We assume the existence of $N > 3$ varieties of CGS and only one of these varieties is supplied by the durable good producer. Following Ottaviano, Tabuchi and Thisse (2002), we assume that consumers, facing CGS prices $\rho_i(t)$, get the net utility level $Z(t)$ from the consumption of $k_i(t)$ units of each CGS i at time t,

$$Z(t) = [\gamma + \phi D(t)]\,[\sum_{i=1}^{N} k_i(t) - \frac{1-\eta}{2}\sum_{i=1}^{N} k_i^2(t) - \eta(\sum_{i=1}^{N} k_i(t))^2] - \sum_{i=1}^{N} \rho_i(t)k_i(t),$$

where γ now measures the stand-alone value for any CGS consumption bundle $\mathbf{k}(t) = (k_1(t), k_2(t), ..., k_N(t))$, ϕ measures the intensity of DNE arising in the aftermarket, η measures the degree of complementarity/substitutability among the CGS.[55]

[55] We suppose $\eta \in (\frac{1}{3-N}, 1)$, the two extreme values corresponding to the cases of perfect complements and substitutes, respectively.

Considering the Bertrand equilibrium, in which firms set $\rho_i(t)$ non-cooperatively, we obtain the equilibrium *instantaneous CGS utility* specification (2), where $\gamma_1 = \gamma\vartheta_1$ and $\phi_1 = \phi\vartheta_1$, with $\vartheta_1 = \frac{N(1+(N-2)\eta)^2(1+(4N-1)\eta)}{2(2+(N-3)\eta)^2(1+(N-1)\eta)^2}$. Analogously, each Bertrand competitor's equilibrium profit per customer is given by (3), where $\gamma_2 = \gamma\vartheta_2$ and $\phi_2 = \phi\vartheta_2$, with $\vartheta_2 = \frac{(1-\eta)(1+\eta(N-2))}{(2+(N-3)\eta)^2(1+(N-1)\eta)}$.

Example 3: Indirect AMNE and monopoly provision of CGS

While the previous examples address aftermarkets with direct network effects (DNE), in this section we explicitly deal with indirect network effects (INE). Network externalities are indirect when the variety or quality of CGS is increasing with the size of the network. Real life reveals a plethora of situations of aftermarkets in which INE may arise. For example, in the case of software, we often observe that the variety/quality of software or applications available for a certain OS is increasing with the number of users of this OS. Similarly, the number/quality of videogames available to a certain console is an increasing function of number of consumers using a similar model (see, e.g., Clements and Ohashi, 2005 or Gretz, 2010).

In order to show that the properties of the specifications (2) and (3) are compatible with INE, we develop two models. In the first model, we introduce endogenous quality of CGS, whereas in the second model we consider an endogenous number of varieties of CGS. To keep the analysis simple, in this case we consider monopoly provision of CGS. Regarding this last assumption, we are aware that in many aftermarkets, there is a considerable degree of competition (e.g., videogame industry) and it would actually be possible to develop a more complicated model in which two or more firms compete in an aftermarket with INE (with the properties of the specifications (2) and (3) remaining valid). For the sake of simplicity, Example 3 deals with monopoly provision of CGS. Some real world examples of monopolization of aftermarkets include some types of software in which OS producers embrace tie-in strategies.[56] Carlton and Waldman (2009) argue that the typical example of

[56] In this respect, Choi and Stefanadis (2001) argue that *"In recent years, tying in the high-technology sector*

aftermarket monopolization would be the cases of Kodak, Data General, Xerox, or, more generally, any situation in which a durable good producer also has the monopoly of repairs for its own product. In the last examples, there are usually no direct network effects (since the number of users does not affect the value of the durable good or the repairing services), but indirect AMNE may arise (as the quality and the number of available repairing points may be increasing with the customer base of the durable good producer). These real-world examples can be easily analyzed in the context of our model by *(i)* assuming indirect AMNE; and *(ii)* considering $\omega = 0$ (which does not affect the nature of results since only AMNE are critical to the failure of the Coase conjecture).

Example 3(a) Endogenous quality of CGS Assume that the producer of the durable good is the monopolist supplier of CGS. We have a model where the quality of CGS changes over time. The firm chooses at instant t the quality $\mu(t)$ of the CGS it sells to its $D(t)$ locked-in consumers. The net utility derived from the consumption $k(t)$ units of CGS with quality level $\mu(t)$ is

$$Z(t) = \mu(t)(k(t) - \frac{1}{2}k^2(t)) - \rho\,(t)\,k(t),$$

where $\rho\,(t)$ denotes the unit price.

Maximization of $Z(t)$ with respect to $k(t)$ yields the individual demand function $k(\rho\,(t)) = 1 - \frac{\rho(t)}{\mu(t)}$. Assuming that the marginal production cost is zero and that the cost of supplying quality $\mu(t)$ above a minimum level $\underline{\mu}$ is a quadratic function $\frac{b}{2}(\mu(t) - \underline{\mu})^2$, the instantaneous profit made in the aftermarket is

$$\pi^A(\rho(t), D(t), \mu(t)) = \left(\rho\,(t) - \frac{\rho^2\,(t)}{\mu\,(t)}\right) D(t) - \frac{b}{2}(\mu(t) - \underline{\mu})^2, \tag{A.1}$$

where $\underline{\mu}$ is a minimum quality level.

has been one of the hottest issues in antitrust economics. When, for example, practitioners try to explain the extraordinary success of Microsoft, they often stress the use of tie-in sales as a defense against entrants. By tying complementary products to its PC operating system, Microsoft allegedly creates an "applications barrier to entry," fending off potential competitors".

At instant t, the monopolist provider of CGS chooses both the quality $\mu(t)$ and the unit price $\rho(t)$ that maximize its instantaneous aftermarket profit (A.1). It is easy to show that the monopolist will provide higher quality CGS as the network D expands. After some manipulation, we can obtain the equilibrium *instantaneous CGS utility* specification (2), with $\gamma_1 = \frac{\mu}{8}$ and $\phi_1 = \frac{1}{32b}$. Analogously, the monopolist's equilibrium profit per customer is given by (3), with $\gamma_2 = \frac{\mu}{8}$ and $\phi_2 = \frac{1}{32b}$.

Example 3(b) Endogenous number of varieties of CGS In this example, we analyze the situations in which INE arise because the number of varieties available in the aftermarket grows with the network size, D. Suppose the monopolist provides a continuum of varieties. We will show that this continuum grows as the network expands. At instant s, the CGS utility obtained by a consumer who owns the durable good and consumes $k_i(s)$ units of variant type i produced by the monopolist at instant s is equal to:

$$Z(s) = \alpha \int_0^{N(s)} k_i(s)di - \frac{\beta - \delta}{2} \int_0^{N(s)} k_i^2(s)di - \frac{\delta}{2}(\int_0^{N(s)} k_i(s)di)^2 - \int_0^{N(s)} \rho_i(s)k_i(s)di,$$

where $N(s)$ is the number of varieties the monopolist makes available at time s, $\beta > 0$ and $\alpha > 0$.[57] δ is a parameter that measures the degree of substitutability between the available varieties. They are perfect substitutes when $\delta = 1$, imperfect substitutes when $\delta \in (0,1)$, complements when $\delta < 0$ and independent when $\delta = 0$. The variable $\rho_i(s)$ denotes the price charged by the firm for CGS type i at instant s.

The consumer's FOC with respect to the consumption $k_i(s)$ of variety i is obtained as

$$\alpha - (\beta - \delta)k_i(s) - \delta \int_0^{N(s)} k_i(s)di - \rho_i(s) = 0.$$

At a given point of time, assuming a zero cost of production and quadratic costs of having a larger variety range than some minimum level $\underline{N}$), we obtain that, *at any instant* s, the

[57] The interpretation of these parameters is the standard one proposed in the literature dealing with endogenous number of varieties.

profit maximizing production of CGS of type i is $k_i^*(s) = \frac{\alpha}{2\beta+(N(s)-1)\delta}$, $\forall i \in [0, N(s)]$. The firm's aftermarket gross profits are then equal to $\frac{D(s)N(s)\beta\alpha^2}{(2\beta+(N(s)-1)\delta)^2}$. In the particular case of independent varieties, i.e. $\delta = 0$, they are a linearly increasing function of the variety range and one obtains the equilibrium number of varieties as $N^*(s) = \underline{N} + \frac{\alpha^2 D(s)}{4b\beta}$. In that case,[58] we obtain the equilibrium *instantaneous CGS utility* specification (2), with $\gamma_1 = \underline{N}\frac{\alpha^2}{8\beta}$ and $\phi_1 = \frac{\alpha^4}{32b\beta}$. Analogously, the monopolist's equilibrium profit per customer is given by (3), with $\gamma_2 = \frac{\alpha^2}{4\beta}$ and $\phi_2 = \frac{\alpha^4}{32b\beta}$.

Example 4: Two-sided market

In this example, we consider a situation in which the durable good producer is not itself directly involved in the provision of CGS but it owns a platform, which constitutes the only vehicle through which the CGS can be provided. For example, Apple sells the durable good iPad and it is also the owner of the iTunes platform, which is used by the app providers to sell their applications to the iPad users.[59] Similarly, Amazon sells Kindles (a durable electronic book reader) and it owns the Kindle Store where independent suppliers can sell their e-books to the kindle owners. In this case, the durable good producer is supplying a two-sided platform in which CGS consumers and suppliers interact with each other. As pointed out by McMurrer (2011), this type of business model can be thought of as a new "derivative aftermarket", meaning that it constitutes an aftermarket "created not through a contract, or a necessarily contingent relationship to the primary market, but by *programming what would otherwise be compatible and open technologies to be a closed system."*

In order to apply our model to the situation described above, we assume N independent firms providing independent CGS. We consider a linear quadratic utility specification, so

[58] Note that $\lambda = 0$ is necessary only for getting a linear-quadratic profit function. We conjecture that the equilibrium profit remains convex in D in the cases of complementary $(\lambda < 0)$ and substitute goods.

[59] The same business model is used by Apple in the case of iPhones and applications. Regarding this market, McMurrer (2011) points out that *"When Apple launched its App Store as a part of the iTunes Music Store, it implemented the process for creation and download of third-party software in such a way as to preserve its total control over content. Though Internet access is not limited, the iPhone's firmware blocks installation of any software not downloaded from Apple's App Store. Apple maintains complete control over the App Store catalog, requiring developers to register with Apple, pay a fee, and submit apps to Apple for approval before an app will be made available for download."*

that consumers who already own an equipment, get utility $Z(t)$ from the consumption of $k_i(t)$ units of each CGS i at time t, with:

$$Z(t) = \sum_{i=1}^{N} k_i(t) - \frac{1}{2}\sum_{i=1}^{N} k_i^2(t) - \sum_{i=1}^{N} \rho_i(t) k_i(t),$$

where $\rho_i(t)$ denotes the price of CGS_i at instant t.

Assuming that CGS suppliers have a zero marginal production cost, in equilibrium, the individual revenue (per customer) of the supplier of CGS_i is $k_i^* \rho_i^* = \frac{1}{4}$. To keep the analysis simple, we suppose that at each instant t, the CGS suppliers have to pay an access fee $\tau(t)$ to the platform and they bear an additional cost equal to cN, with $c > 0$.[60] Then, the individual profit of a CGS supplier at instant t is

$$\pi^{CGS}(t) = \frac{D(t)}{4} - cN - \tau.$$

Under free entry, the aftermarket profit obtained by the durable good producer is equal to $\pi^A(D, \tau) = \left(\frac{\frac{D(t)}{4} - \tau(t)}{c}\right)\tau(t)$, yielding an optimal access fee equal to $\tau(t)^* = \frac{D(t)}{8}$. Then, the equilibrium number of CGS suppliers equal to $N^*(t) = \frac{D(t)}{8c}$. Thus, the equilibrium *instantaneous CGS utility* is given by the specification (2), with $\gamma_1 = 0$ and $\phi_1 = \frac{1}{64c}$; whereas durable good's equilibrium profit per customer is given by (3), with $\gamma_2 = 0$ and $\phi_2 = \frac{1}{64c}$.

Appendix B - Proofs

Proof of Lemma 1

(i) Differentiating (5) with respect to t yields the first order condition

$$e^{-rt}\left[rp(t) - \theta - \gamma_1 - (\omega + \phi_1) D(t) - \frac{dp(t)}{dt}\right] = 0. \quad \text{(A.2)}$$

Multiplying both sides by $e^{rt} > 0$ and rearranging yields equation (6).

[60] This additional cost can be interpreted as an advertising cost, for example. This specification is consistent with the fact that the cost of advertising effectively a new CGS (e.g. a new application) is increasing in the number of existing applications (since it becomes harder to highlight an application vis à vis the competing apps).

(ii) From (A.2), we obtain the implicit function

$$\psi(t,\theta) \equiv rp(t) - \theta - \gamma_1 - (\omega + \phi_1) D(t) - \frac{dp(t)}{dt} = 0$$

This gives us

$$\frac{dt}{d\theta} = -\frac{\partial\psi/\partial\theta}{\partial\psi/\partial t}$$

where $\partial\psi/\partial t < 0$ by the second order condition, and $\partial\psi/\partial\theta = -1$.Thus $\frac{dt(\theta)}{d\theta} \leq 0$. ■

Proof of the MPE with strong AMNE (the Large Gap case)

(i) Given the lumpy sale strategy $L(D) = 1 - D$, rational expectations require that consumers hold the following price function $\zeta(D) = \frac{b(1)}{r}$. This function is constant.

(ii) Given the consumers' price expectation function $\zeta(D) = \frac{b(1)}{r}$, the firm's problem is, given any $D_0 \in [0,1]$, where D_0 stands for $D(0)$, to choose the time path of sale to maximize the integrated firm's value:

$$J(D_0) = \max \int_0^\infty e^{-rt}\left[\gamma_2 D(t) + \phi_2 D^2(t) + \frac{dD(t)}{dt}\frac{1}{r}(b(1) - rc)\right] dt$$

Integration by parts, noting that $D(t)$ is bounded, yields

$$J(D_0) = \max_{1 \geq D(t) \geq D_0} \int_0^\infty e^{-rt}[\gamma_2 D(t) + \phi_2 D^2(t) + (D(t) - D_0)(b(1) - rc)]dt. \tag{A.3}$$

Let us denote the expression inside [...] by $H(D(t), D_0)$:

$$H(D(t), D_0) \equiv \gamma_2 D(t) + \phi_2 D(t)^2 + (D(t) - D_0)(b(1) - rc)$$

For $\phi_2 > 0$, $H(D(t), D_0)$ is strictly convex in $D(t)$. Therefore the maximum of this expression with respect to $D(t)$ occurs either at $D = D_0$ or at $D = 1$. It can be easily seen that the maximum occurs at $D = 1$ for all possible $D_0 \in [0,1]$ if and only if we are in the Large Gap case[61], so that $\phi_1 + \phi_2 \geq rc - \gamma_1 - \gamma_2 - \omega$.

[61] This proof is available from the authors upon request.

Then the value of the firm, i.e., the maximized value of integral (A.3), which is obtained by setting $D(t) = 1$ for all $t > 0$, is

$$J(D_0) = \frac{H(1, D_0)}{r} = \frac{\gamma_2 + \phi_2}{r} + (1 - D_0)\left[\frac{b(1)}{r} - c\right]$$

$$J(D_0) = \frac{H(1, D_0)}{r} \geq \frac{H(D_0, D_0)}{r} = \gamma_2 D_0 + \phi_2 D_0^2. \tag{A.4}$$

In particular $J(0) = \frac{1}{r}(b(1) - m(1/2)) \geq 0$. Notice that the value function $J(D)$ is linear in D and it is higher than the capitalized value of the instantaneous aftermarket profit, $(\gamma_2 D + \phi_2 D^2)$, for all $D \in [0, 1]$, with equality only at $D = 1$. ■

Proof of Proposition 1 The Hamilton-Jacobi-Bellman equation for the monopolist is

$$rJ(D) = \max_q \{\gamma_2 D + \phi_2 D^2 + [\zeta(D) - c]\, q + J'(D)q\}, \tag{A.5}$$

where the time index has been omitted. Since this equation is linear in q, the optimal q is finite if and only if

$$\zeta(D) - c + J'(D) = 0, \text{ for all } D \in (0, 1). \tag{A.6}$$

Substituting this into the HJB equation (A.5), we obtain

$$J(D) = \frac{\gamma_2 D + \phi_2 D^2}{r}, \text{ for all } D \in (0, 1). \tag{A.7}$$

In light of (A.6), the Markovian equilibrium price function is:

$$\zeta(D) = c - J'(D) = c - \frac{\gamma_2 + 2\phi_2 D}{r} = \frac{m(D)}{r}. \tag{A.8}$$

Thus

$$p(t) = \zeta(D(t)) = \frac{m(D(t))}{r}. \tag{A.9}$$

Differentiating with respect to time we obtain

$$\dot{p}(t) = \frac{1}{r} m'(D)\dot{D}(t) = -\frac{2\phi_2}{r} q(t)$$

Taking into account equations $\dot{p} = rp - b(D)$, and (A.9), the equilibrium output rate is

$$\begin{aligned} q(t) &= \frac{r}{2\phi_2}\left[b(D(t)) - m(D(t))\right] \\ &= \frac{r}{2\phi_2}\left[(1 - rc + \gamma_1 + \gamma_2) - D(t)\left(1 - \phi_1 - 2\phi_2 - \omega\right)\right] \end{aligned} \tag{A.10}$$

Note that q becomes zero when D reaches the value $\underline{D}$ defined by $b\left(\underline{D}\right) = m\left(\underline{D}\right)$:

$$\underline{D} = \frac{1 - rc + \gamma_1 + \gamma_2}{1 - \phi_1 - 2\phi_2 - \omega} \leq 1 \tag{A.11}$$

Assumption A1 implies that the numerator is positive. The denominator is greater than the numerator because we are dealing with the No Gap Case, yielding $0 \leq \underline{D} \leq 1$. With $q\left(t\right) = \frac{r}{2\phi_2}\left[b(D(t)) - m(D(t))\right]$, we see that, given $\phi_2 > 0$, the output rate q is positive if and only if $D < \underline{D}$. The stability of the steady state is ensured, and consequently sales are gradual. Replacing the optimal value of $q\left(t\right)$ in $D\left(t\right) = \int_0^t q\left(s\right) ds$, given $D\left(0\right) = 0$, we obtain $D\left(t\right) = (1 - e^{-\psi t})\underline{D}$, where $\psi = \frac{r}{2\phi_2}\left(1 - \phi_1 - 2\phi_2 - \omega\right) > 0$ is the rate of convergence. To complete the proof, note that all the necessary conditions for an equilibrium are satisfied under condition (A.8). It remains to verify that the monopolist, starting with any $D < \underline{D}$, is never interested in selling a discrete amount $\widehat{D} - D$, such that $D < \widehat{D} \leq \underline{D}$, prior to embarking on gradual sales according to (A.10). To demonstrate this, note that the value of selling a positive discrete amount $\widehat{D} - D$ at time $t = 0$ is given by $J(\widehat{D}) - J'(\widehat{D})(\widehat{D} - D)$ which is strictly lower than $J(D)$ since, from (A.7),

$$J(\widehat{D}) - J'(\widehat{D})(\widehat{D} - D) - J(D) = -\frac{\phi}{4r}(\widehat{D} - D)^2 < 0.$$

This shows that selling gradually is always better than selling discrete amounts.[62]

Proof of Corollary 1

Given the conditions in Corollary 1, consumers with low valuations such that $\theta < 1 - D^{\#}$ are only in equilibrium if they do not expect to gain anything in buying the durable good.

[62]Obviously, this argument also rules out a jump to $\widehat{D} = \underline{D}$. Also, note that serving the entire market in one go would not be part of a Markov perfect equilibrium. The necessary condition for selling in one go cannot be met when network effects are weak.

This suggests the following price expectation function

$$\begin{cases} \zeta(D) = \frac{m(D)}{r} & \text{for } D < D^{\#} \\ \zeta(D) = \frac{b(D)}{r} & \text{for } D \in \left[D^{\#}, 1\right] \end{cases}$$

To prove Corollary 1, we must verify that given the above price expectation function, the firm finds it optimal to choose the corner solution $q(D) = 0$ whenever $D \in \left[D^{\#}, 1\right]$. With the value function $J(D) = (\gamma_2 D + \phi_2 D^2)/r$, it is indeed true that the corner solution $q(D) = 0$ whenever $D \in \left[D^{\#}, 1\right]$ does indeed satisfy the HJB equation:

$$\max_{q \geq 0}\{\gamma_2 D + \phi_2 D^2 + \left[\zeta\left(D\right) - c\right] q + J'(D)q\} = rJ(D)$$

where the FOC is satisfied:

$$\frac{b(D)}{r} - c + \frac{\gamma_2 + 2\phi_2 D}{r} \leq 0 \text{ for } D \in \left[D^{\#}, 1\right]. \blacksquare$$

Proof of Proposition 2

(i) Given that the firm's lumpy sale strategy $L(D) = 1 - D$ for all $D \in (\widetilde{D}, 1]$, rational expectations on the part of consumers imply that $\zeta(D) = \frac{1}{r}\left(\gamma_1 + \phi_1 + \omega\right) = \frac{b(1)}{r}$ for all $D \in (\widetilde{D}, 1]$. And given the consumers' function $\zeta(D) = \frac{1}{r}\left(\gamma_1 + \phi_1 + \omega\right) = \frac{b(1)}{r}$ for all $D \in (\widetilde{D}, 1]$, the firm's maximization problem, for any given $D_0 > \widetilde{D}$, is the same as in equation (A.3), hence its optimal strategy is $L(D) = 1 - D$ for all $D \in (\widetilde{D}, 1]$. The value function is therefore

$$J(D) = \frac{(1 - D)(\gamma_1 + \phi_1 + \omega - rc) + \gamma_2 + \phi_2}{r} \text{ for } D \in \left(\widetilde{D}, 1\right]$$

Turning to the interval $\left[0, \widetilde{D}\right)$, given to the output strategy $g(D)$ defined by equation (A.10), for all $D \in [0, \widetilde{D})$, the same argument as that used in the proof of in Proposition 1 applies to show that the price function $\zeta(D) = c - \frac{\gamma_2 + 2\phi_2 D}{r} = \frac{m(D)}{r}$ satisfies the rational expectations requirement for all $D \in [0, \widetilde{D})$. And given the price function $\zeta(D) = \frac{m(D)}{r}$ for $D \in [0, \widetilde{D})$, the firm's optimal response is to use the output strategy $g(D)$ defined by equation (A.10).

Now, since sales are gradual for all $D \in [0, \widetilde{D})$, customers will purchase if $D < \widetilde{D}$, which occurs if and only if[63]

$$\lim_{D \uparrow \widetilde{D}} \zeta(D) = \zeta(\widetilde{D}),$$

i.e.

$$\zeta(\widetilde{D}) = c - \frac{1}{r}(\gamma_2 + 2\phi_2 \widetilde{D}), \tag{A.12}$$

where $\zeta(\widetilde{D})$ denotes the expected equipment price at $D = \widetilde{D}$. Condition (A.12) is necessary to support equilibrium, because if $\zeta(\widetilde{D}) < c - \frac{1}{r}(\gamma_2 + 2\phi_2 \widetilde{D})$ then, for some small $\varepsilon > 0$, consumers whose type is in interval $\left(\widetilde{\theta}, \widetilde{\theta} + \varepsilon\right)$ would not want to buy the equipment when $D \in \left(\widetilde{D} - \varepsilon, \widetilde{D}\right)$, where $\widetilde{\theta} \equiv 1 - \widetilde{D}$; and if $\zeta(\widetilde{D}) > c - \frac{1}{r}(\gamma_2 + 2\phi_2 \widetilde{D})$ agents would make speculative gains by purchasing the durable good immediately before D reaches $\widetilde{D}$, and resell it an instant later.

(ii) Given (33), any deviation by the monopolist implying a discontinuous variation in D in the interval $[0, \widetilde{D})$ can be ruled out as was shown in the proof of Proposition 2. Given the definition of $\widetilde{D}$, a deviation implying an upward jump from some $D \in [0, \widetilde{D})$ to 1 is ruled out[64] since it would yield a payoff $\frac{(1-D)(\gamma_1+\phi_1+\omega-rc)+\gamma_2+\phi_2}{r} < \frac{\gamma_2 D + \phi_2 D^2}{r}$. Given that $\zeta(D)$ is constant in the interval $(\widetilde{D}, 1]$, an argument similar to that used in the proof of the MPE with strong AMNE shows that it is always better to jump from any D in this interval to 1 than to stop at D or to jump to any other value in $(D, 1)$. Finally, when $D = \widetilde{D}$, the monopolist is indifferent between two actions: (a) selling $1 - D$ immediately or (b) stopping to sell the equipment. The two actions yield by construction the same payoff to the firm since

$$\frac{\gamma_2 D + \phi_2 D^2}{r} = \frac{(1 - \widetilde{D})(\gamma_1 + \phi_1 + \omega - rc) + \gamma_2 + \phi_2}{r}.$$

Being indifferent between actions (a) and (b), the firm can choose action (a) with probability λ and action (b) with probability $1 - \lambda$. The value of λ must be determined such that

[63] This condition requires that there is no discontinuity of the price expectation function at $D = \widetilde{D}$. Such a discontinuity would be eliminated by arbitrage.

[64] A deviation to some value in $\left(\widetilde{D}, 1\right)$ is even less profitable.

consumers cannot gain by arbitrage. The consumers will rationally expect an equipment price equal to

$$\lambda \frac{b(1)}{r} + (1-\lambda) \frac{b\left(\widetilde{D}\right)}{r}.$$

Using condition (A.12), we then obtain equation (34) which determines the equilibrium value of λ.■

References

[1] Amir, R. , 2005, "Supermodularity and complementarity in economics: an elementary survey" Southern Economic Journal 71, 636-660.

[2] Amir, R. and N. Lazzati, 2011, "Network effects, market structure and industry performance", Journal of Economic Theory 146, 2389-2419.

[3] Ausubel, L. and R. Deneckere, 1989, "Reputation and Bargaining in Durable Goods Monopoly," Econometrica, 57(3), 511–532.

[4] Bensaid B and J.-P. Lesne, 1996, "Dynamic monopoly pricing with network externalities, International Journal of Industrial Organization, 14(6), 837-855

[5] Bond, E. and L. Samuelson, 1984, "Durable Goods Monopoly with Rational Expectations and Replacement Sales", Rand Journal of Economics, 15, 336–345.

[6] Borenstein,S., Mackie-Mason, J.K. and Netz, J.S., 2000. "Exercising Market Power in Proprietary Aftermarkets," Journal of Economics & Management Strategy, 9(2), 157-188.

[7] Bulow, J. I., 1982, "Durable Goods Monopolists", Journal of Political Economy, 90(2), 314-32.

[8] Cabral, L, D. Salant, and G. Woroch, 1999, "Monopoly pricing with network externalities", International Journal of Industrial Organization, 17(2), 199-214

[9] Cabral, L., 2011, “Dynamic price competition with network effects” Review of Economic Studies, 78, 83-111.

[10] Carlton, D. W, and M. Waldman, 2010. "Competition, Monopoly, and Aftermarkets." Journal of Law, Economics, and Organization, 26(1), 54-91.

[11] Chen J., Harrington J. and U. Doraszelski, 2009, Avoiding Market Dominance: Product Compatibility in Markets with Network Effects, Rand Journal of Economics, 49(3), 455-485

[12] Chen, Z., W. Thomas and W. Stanbury, 1998,. "Refusals to Deal and Aftermarkets," Review of Industrial Organization 13, (1-2), 131-151.

[13] Choi, J. P. and C. Stefanadis, Christodoulos, 2001. "Tying, Investment, and the Dynamic Leverage Theory," RAND Journal of Economics, 32(1), 52-71.

[14] Clements, M. T.and H. Ohashi, 2005. "Indirect Network Effects and the Product Cycle: Videogames in the US", Journal of Industrial Economics, Wiley Blackwell, vol. 53(4), 515-542.

[15] Coase, R., 1972, “Durability and Monopoly, Journal of Law and Economics, 15, 143–149.

[16] Dockner, E., 1992,“A Dynamic Theory of Conjectural Variations,” Journal of Industrial Economics 40: 377-395.

[17] Dockner, E. and G. Sorger,1996, “Existence and Properties of Equilibria for a Dynamic Game on Productive Assets,” Journal of Economic Theory 71: 209-227.

[18] Doganoglu, T., 2003, "Dynamic Price Competition with Consumption Externalities", Netnomics, 5(1), 43-69

[19] Driskill, R., 2007, "Monopoly and Oligopoly Supply of a Good with Dynamic Network Externalities", mimeo.

[20] Driskill, R., 1997, “Durable-Goods Monopoly, Increasing Marginal Cost, and Depreciation,” Economica 64(253), 137-54.

[21] Driskill, R. and S. McCafferty, 2001, “Monopoly and Oligopoly Provision of Addictive Goods,” International Economic Review 42(1): 43-72.

[22] Dritsa, E. and E. Zacharias, 2012. "Price Competition in a Duopoly Characterized by Positional Effects," mimeo, Working Papers 12-21, NET Institute.

[23] Economides, N. (2000). "Durable Goods Monopoly with Network Externalities with Application to the PC Operating Systems Market" Quarterly Journal of Electronic Commerce 1.3,193-201.

[24] Farrell, J. and P. Klemperer, 2007, Coordination and Lock-In: Competition with Switching Costs and Network Effects, in Handbook of Industrial Organization, 3, M. Armstrong and R. Porter (eds.), North-Holland

[25] Fudenberg, D., and J. Tirole, 2000, Pricing a Network Good to Deter Entry, Journal of Industrial Economics, 48, 373-390

[26] Gabszewicz, J. J. and F. Garcia, 2008, "A note on Expanding Networks and Monopoly Pricing", Economics Letters, 98(1), 9-15

[27] Gabszewicz, J. J. and F. Garcia, 2007, "Optimal Monopoly Price Paths with Expanding Networks", Review of Network Economics, 6(1), 42-49

[28] Gretz, R. T., 2010. "Hardware quality vs. network size in the home video game industry," Journal of Economic Behavior & Organization, 76(2), 168-183.

[29] Grilo, I., O. Shy, and J. Thisse, 2001, "Price Competition When Consumer Behavior is Characterized by Conformity or Vanity", Journal of Public Economics, 80: 385–408

[30] Griva, K. and N. Vettas, 2011, “Price competition in a differentiated products duopoly under network effects” Information Economics and Policy 23, 85-97.

[31] Gul, F., H. Sonnenschein, and R. Wilson, 1986, “Foundations of Dynamic Monopoly and the Coase Conjecture,” Journal of Economic Theory, 39, 155–190.

[32] Hilli, A., D. Laussel and N.V. Long, 2013, "Large shareholders, monitoring, and ownership dynamics: Toward pure managerial firms?", Journal of Economic Dynamics and Control, 37, 666–679

[33] Kahn, C., 1987, “The Durable Goods Monopolist and Consistency with Increasing Costs,” Econometrica, 54, 274–294.

[34] Karp, L., 1996a, “Monopoly Power can be Disadvantageous in the Extraction of a Durable Nonrenewable Resource,” International Economic Review, 37(4), 825–849.

[35] Karp, L. 1996b, “Depreciation Erodes the Coase Conjecture,” European Economic Review, 40, 473–490.

[36] Katz, M. and C. Shapiro, 1985, Network Externalities, Competition and Compatibility, American Economic Review, 75 (3), 424-440.

[37] Katz, M. and C. Shapiro, 1994. Systems Competition and Network Effects. Journal of Economic Perspectives, Vol. 8(2): 93-115.

[38] Kühn, K-U. and A. J. Padilla, 1996. "Product Line Decisions and the Coase Conjecture," RAND Journal of Economics, 27(2), 391-414.

[39] Laussel, D., Montmarin, M. and N. V. Long, 2004, Dynamic Duopoly with Congestion Effects, International Journal of Industrial Organization, 22(5): 655-677

[40] Laussel, D. and J. Resende, 2013, "Dynamic competition in aftermarkets with network effects", forthcoming Journal of Mathematica Economics

[41] Laussel, D. and N.V. Long, 2012, "Vertical Disintegration: A Dynamic Markovian Approach", Journal of Economics & Management Strategy, 21, 745-771

[42] Markovich, S.: Snowball: a Dynamic Oligopoly Model with Indirect Network Effects, Journal of Economic Dynamics & Control, 32, 909-938 (2008)

[43] Markovich, S. and J. Moenius (2009), "Winning While Losing: Competition Dynamics in the Presence of Indirect Network Effects," International Journal of Industrial Organization, 27: 346-357

[44] Mason, R. (2000), "Network externalities and the Coase conjecture", European Economic Review 44: 1981-1992

[45] Mitchell, M. and A. Skrzypacz: Network Externalities and Long-run Market Shares, Economic Theory, 29 (3), 621-648 (2006)

[46] Morita, H. and Waldman, M., 2004, "Durable Goods, Monopoly Maintenance, and Time Inconsistency," Journal of Economics and Management Strategy, 13 (2), 273-302.

[47] McMurrer, M. (2011), "Exclusive Gadget: Apple & AT&T Antitrust Litigation and the iPhone Aftermarkets", Journal of Corporation Law; Vol. 36(2), 495-512.

[48] Ottaviano, G, T. Tabuchi and J.F. Thisse (2002), Agglomeration and Trade Revisted, International Economic Review 43 (2), 409-435.

[49] Page, W. H. and Jopatka, J.E., 2000, "Network Externalities" in Encyclopedia of Law and Economics, Volume I, The History and Methodology of Law and Economics , B. Bouckaert and G. De Geest (eds.), Edward Elgar, 2000, 952-980.

[50] Schwartz, M. and Werden, G. J., 1996, "A Quality-Signaling Rationale for Aftermarket Tying", Antitrust Law Journal, 64, 387-404.

[51] Shapiro, C., 1995, "Aftermarkets and Consumer Welfare: Making Sense of Kodak", Antitrust Law Journal, 63, 483-511.

[52] Stokey, N., 1981, “Rational Expectations and Durable Goods Pricing,” Bell Journal of Economics, 12, 112–128.

[53] Xie, J. and Sirbu, M., 1995, "Price competition and compatibility in the presence of positive demand externalities," Management Science 41.5: 909-926.

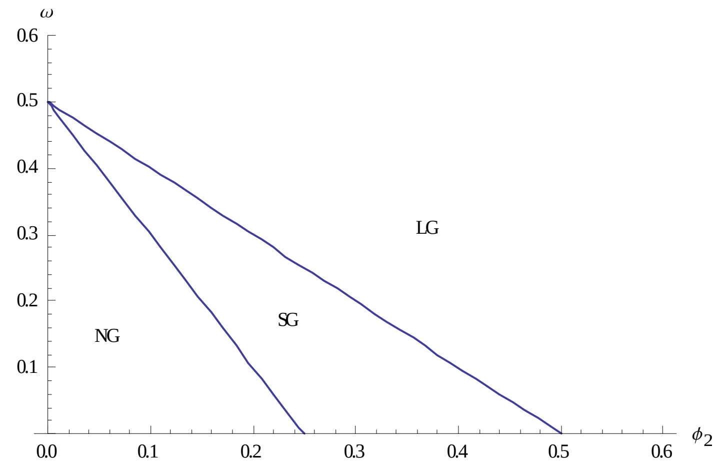

Figure 1: Network Effects and MPE Regimes

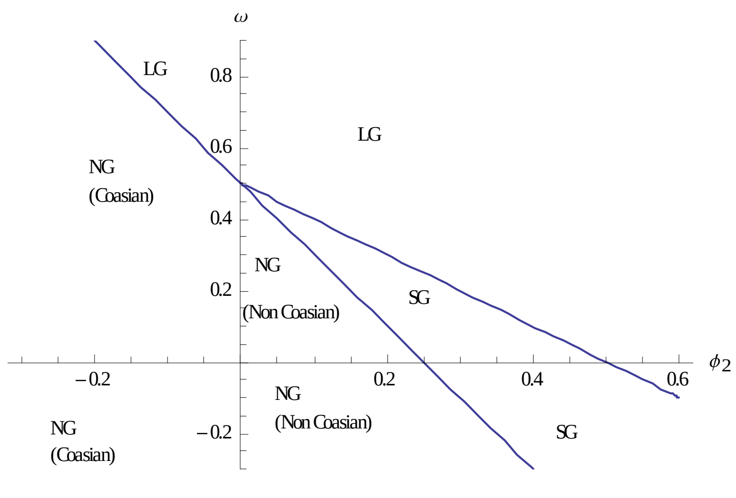

Figure 2: Network effects (positive/ negative) and MPE regimes

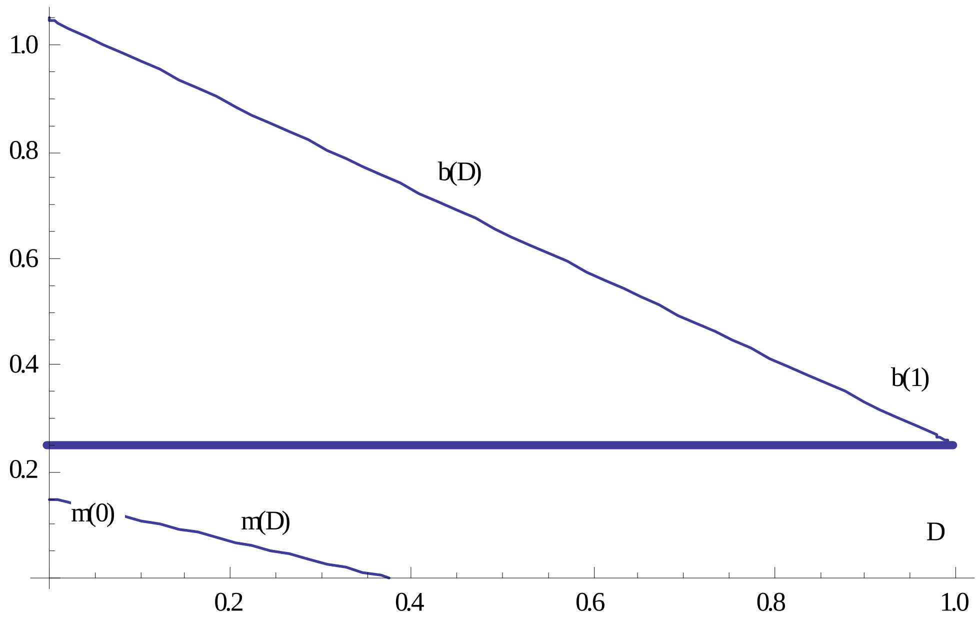

Figure 3: Equilibrium price $r\xi(D)$ in the Large Gap Case

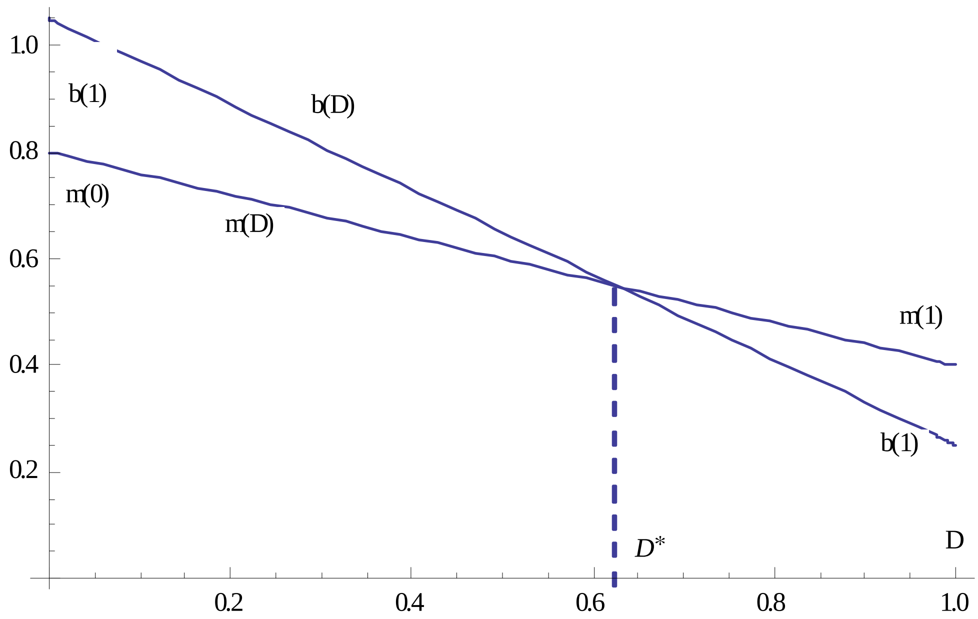

Figure 4: Equilibrium price $r\xi(D)$ in the No Gap Case

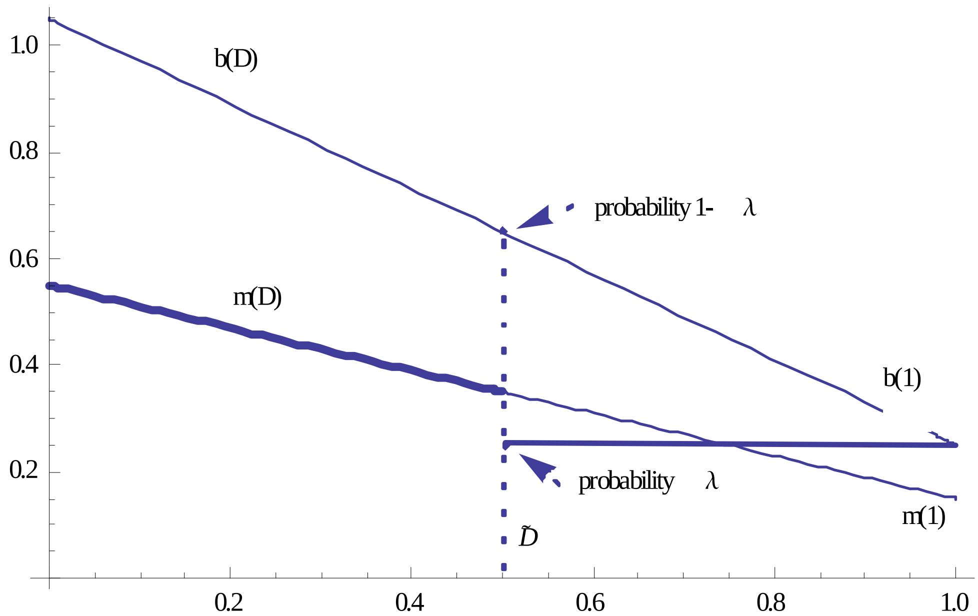

Figure 5: Equilibrium price $r\xi(D)$ in the Small Gap Case

Récents cahiers de recherche du CIREQ
Recent Working Papers of CIREQ

Si vous désirez obtenir des exemplaires des cahiers, vous pouvez les télécharger à partir de notre site Web http://www.cireqmontreal.com/cahiers-de-recherche

If you wish to obtain copies of the working papers, you can download them directly from our website, http://www.cireqmontreal.com/cahiers-de-recherche

07-2013 McCausland, W.J., A.A.J. Marley, "Bayesian Inference and Model Comparison for Random Choice Structures", juillet 2013, 26 pages

08-2013 Cardia, E., P. Gomme, "The Household Revolution : Childcare, Housework, and Female Labor Force Participation", juillet 2013, 44 pages

09-2013 Dutta, R., S. Horan, "Inferring Rationales from Choice : Identification for Rational Shortlist Methods ", mars 2013, 51 pages

10-2013 Dutta, R., R. Ishii, "Coordinating by Not Committing : Efficiency as the Unique Outcome", août 2013, 38 pages

11-2013 Amarante, M., "A Representation of Risk Measures", octobre 2013, 8 pages

12-2013 Amarante, M., "A Characterization of Exact Non-atomic Market Games", octobre 2013, 8 pages

13-2013 Dufour, J.-M., L. Khalaf, M. Voia, "Finite-Sample Resampling-Based Combined Hypothesis Tests, with Applications to Serial Correlation and Predictability", octobre 2013, 23 pages

14-2013 Zhang, H.J., J.-M. Dufour, J.W. Galbraith, "Exchange Rates and Commodity Prices : Measuring Causality at Multiple Horizons", septembre 2013, 41 pages

15-2013 Gaudet, G., P. Lasserre, "The Taxation of Nonrenewable Natural Resources", octobre 2013, 41 pages

16-2013 Benchekroun, H., G. Gaudet, "On the Effects of Mergers on Equilibrium Outcomes in a Common Property Renewable Asset Oligopoly", octobre 2013, 33 pages

17-2013 Andersson, T., L. Ehlers, "An Algorithm for Identifying Agent-k-Linked Allocations in Economies with Indivisibilities", novembre 2013, 7 pages

01-2014 Alvarez-Cuadrado,F., N.V. Long et M. Poschke, "Capital-Labor Substitution, Structural Change and the Labor Income Share", janvier 2014, 55 pages

02-2014 Long, N.V., "The Green Paradox under Imperfect Substitutability between Clean and Dirty Fuels", février 2014, 23 pages

03-2014 Doko Tchatoka, F., J.-M. Dufour, "Identification-Robust Inference for Endogeneity Parameters in Linear Structural Models", janvier 2014, 29 pages

04-2014 Amarante, M., "What Is Ambiguity?", mars 2014, 51 pages

05-2014 Ehlers, L., "Strategy-Proofness and Essentially Single-Valued Cores Revisited", avril 2014, 22 pages