Centre interuniversitaire de recherche
en économie quantitative

CIREQ

Cahier 09-2008

MERGER PERFORMANCE UNDER UNCERTAIN EFFICIENCY GAINS

Rabah AMIR,
Effrosyni DIAMANTOUDI and
Licun XUE

Cahier 09-2008

MERGER PERFORMANCE UNDER UNCERTAIN EFFICIENCY GAINS

Rabah Amir, Effrosyni Diamantoudi and Licun Xue

CIREQ, Université de Montréal
C.P. 6128, succursale Centre-ville
Montréal (Québec) H3C 3J7
Canada

téléphone : (514) 343-6557
télécopieur : (514) 343-5831
cireq@umontreal.ca
http://www.cireq.umontreal.ca

Dépôt légal - Bibliothèque nationale du Canada, 2008, ISSN 0821-4441
Dépôt légal - Bibliothèque et Archives nationales du Québec, 2008
ISBN-13 : 978-2-89382-564-9

Merger Performance under Uncertain Efficiency Gains*

Rabah Amir
Department of Economics, University of Arizona, USA & CIREQ

Effrosyni Diamantoudi
Department of Economics, Concordia University, Canada & CIREQ

Licun Xue
Department of Economics, McGill University, Canada & CIREQ

June 2008

Abstract

In view of the uncertainty over the ability of merging firms to achieve efficiency gains, we model the post-merger situation as a Cournot oligopoly wherein the outsiders face uncertainty about the merged entity's final cost. At the Bayesian equilibrium, a bilateral merger is profitable provided the non-merged firms sufficiently believe that the merger will generate large enough efficiency gains, *even if ex post none actually materialize*. The effects of the merger on market performance are shown to follow similar threshold rules. The findings are broadly consistent with stylized facts. An extensive welfare analysis is conducted, bringing out the key role of efficiency gains and the different implications of consumer and social welfare standards.

JEL Classification Codes: D43, L11, L22.

Key words and phrases: Horizontal merger, Bayesian Cournot equilibrium, Efficiency gains, Market performance.

---

*This paper has benefitted from the comments of Francis Bloch, Claude d'Aspremont, Dennis Mueller, Christian Schultz and Christine Zulehner. Diamantoudi and Xue gratefully acknowledge the hospitality of CORE, where this research was initiated. *Corresponding author:* Rabah Amir, Department of Economics, University of Arizona, Tucson, AZ 85721, USA. *E-mail:* ramir@eller.arizona.edu.

1 Introduction

Mergers and acquisitions constitute a major feature of the economic landscape of most industrialized countries. To provide an idea of the resources involved, over the period 1981-1998, there were nearly 70,000 merger announcements worldwide, each worth at least 1 million U.S. dollars, of which nearly 45,000 were actually implemented. The average deal was valued at 220 million (base year 1995) U.S. dollars (Gugler, Mueller, Yurtoglu and Zulehner, 2003; henceforth GMYZ). Mergers have been an important source of increase in market concentration, particularly outside the U.S. (Schmalensee, 1989). An extensive empirical and theoretical literature has explored the motives behind mergers and their impact on business activity. While both approaches have yielded useful insights, allowing industrial economists to reach a consensus on some aspects of merger performance, important discrepancies exist between key theoretical findings and stylized facts from empirical and event studies.

By their very nature, mergers pose a complex conceptual challenge, wherein structure and conduct are inextricably intertwined. The pioneering work of Salant, Switzer and Reynolds (1983), henceforth SSR, showed that in the context of a symmetric Cournot oligopoly with linear demand and costs, for a merger to be profitable, it should comprise a pre-merger market share of at least 80%. This result forms the so-called "merger paradox". Allowing the merging firms to exploit production synergies in some way, thereby lowering their post-merger costs, leads to a wider scope for profitable mergers (Perry and Porter, 1985, Farrell and Shapiro, 1990, and McAfee and Williams, 1992). A similar result holds under sufficiently concave demand (Fauli-Oller, 1996). By contrast, postulating Bertrand competition with differentiated products, Deneckere and Davidson (1985) establish that every merger would be profitable.[1]

While some degree of controversy, mostly of a quantitative sort, persists, the empirical literature has delineated some important stylized facts. On the key issue of profitability, in the largest cross-national study to date, GMYZ reports that nearly 60% of all horizontal mergers were profitable, with this proportion being higher in services than in manufacturing. As for sales (or revenues), it is essentially the other way around, with nearly 60% of merged firms experiencing a drop in sales. A similar negative effect is also reported for the post-merger market shares of the merged firms (Mueller, 1985). On the other hand, two other broad-based studies concluded that the profitability of acquired firms declined after the merger for U.K. firms (Meeks, 1977)

[1] The Bertrand paradigm has been widely used in numerical simulation of the effects of mergers: See e.g. Werden and Froeb (1994).

and for U.S. firms (Ravenscraft and Scherer, 1987).[2] The overall conclusion one can draw from this rather mixed picture is that while horizontal mergers have a limited negative impact on sales and market share, they do not appear to have, on average, a clear-cut effect on profitability.

It is widely held that mergers typically lead to price increases.[3] Kim and Singal (1993) find a 10% increase for airline mergers)[4]. Regarding the effects on share prices, initially the target firm's shareholders earn a substantial premium of about 30% on the merger while those of the acquirer tend to have more variable fortunes, with an average on the low side (Mueller, 1985). For the merged firm, the overall initial effect is a substantial rise in share value, which however turns into a subsequent fall in value a few years after the merger. For firms outside a merger, the evidence does not seem conclusive for recent times, but Banerjee and Eckard (1998) report significant losses of about 10% for the merger wave at the turn of the 19th century in the U.S.

As to the crucial issue of whether mergers generate efficiency gains, the evidence is not direct as such gains are difficult to estimate, but rather deductive[5]. While many studies, including Ravenscraft and Scherer (1987), report little support for a positive relationship, GMYZ concludes that 29% of all mergers engendered efficiency gains, as suggested by observed increases in both profits and sales. Naturally, it is very difficult to disentangle the efficiency gain and the market power effects due to a merger. On the other hand, there appears to be a consensus reached on the basis of case studies and casual observation that while some mergers were successful in securing substantial efficiency gains[6], there is great variability on this issue.

In view of the lack of congruence between theoretical and empirical findings[7], the primary challenge of theoretical work on mergers is to come up with alternative models of merger behavior that would close this gap. This paper constitutes an attempt in this direction within the framework of static analysis. The novel ingredient is that all

[2] There are many other studies on this central point, and the results are quite mixed. In particular, specific studies involving OECD countries have produced divergent results (Mueller, 1980).

[3] Interestingly, while the Cournot and Bertrand models yield strongly divergent conclusions on the profitability of mergers, they nonetheless do agree in their prediction that mergers increase prices.

[4] There are surprisingly few studies to this effect, particularly in view of the prominence of consumer surplus as a key criterion in the antitrust review process for mergers in the U.S. and elsewhere.

[5] That mergers often result in savings on fixed costs, via inefficient plant shut-downs, personnel consolidation and R&D expenses, is a well-accepted proposition. Mergers also arguably require substantial one-time transaction costs to be implemented. We follow the literature in ignoring these effects.

[6] For instance, a case study in Scherer et. al. (1975) reports a 40% increase in output per worker. Other success stories may be found in Fisher and Lande (1983). Using a structural model, Pesendorfer (2003) finds positive evidence for mergers in the paper industry.

[7] Observe that with the above stylized fact on profitability, the conclusions reached under the Cournot and the Bertrand approaches to mergers are equally far off the mark, in opposite directions.

the firms in the industry face uncertainty as to the efficiency gains, in terms of variable costs, that the merged firm could achieve. The efficiency gains may correspond, for example, to the claim made by the merging firms to the antitrust agency, possibly appropriately discounted by the rival firms, or to a past average achieved by comparable mergers in related industries. Pre-merger competition is modelled as a standard Cournot oligopoly with identical firms while short-run post-merger competition involves a Bayesian Cournot equilibrium, with the merged firm alone being informed about its true cost. As simplifying assumptions, we take demand and costs to be linear, and the uncertainty to be binomial.

This simple formulation seems appropriate in view of the stylized facts on mergers. Indeed, for the merger to obtain antitrust approval in most countries, the candidate firms have to convincingly document scope for significant efficiency gains, via the exploitation of organizational and production synergies. In most cases, the approval of a merger presumes that the antitrust authority has been swayed by the firms' claims of lurking efficiency gains. Likewise, the initial surge in share prices provides some support for the presumption that the merger is likely to lead to strong efficiency gains, as an increase in market power alone would be unlikely to yield the concomitant increase in expected profits. Another point is that the firms in the industry frequently react with apprehension to a merger announcement by two of their rivals. These typical facts lend credence to the postulate that all concerned parties generally hold beliefs about the prospect of efficiency gains that are naturally captured by a Bayesian model. Indeed, the revised Section 4 of the Horizontal Merger Guidelines issued by the U.S. Department of Justice and the Federal Trade Commission in 1997 states that "efficiencies are difficult to verify and quantify, in part because much of the information relating to efficiencies is uniquely in the possession of the merging firms. Moreover, efficiencies projected reasonably and in good faith by the merging firms may not be realized".[8] Further discussion in support of our Bayesian setting is given in Section 5.2.

Recent studies have also proposed settings where uncertainty plays a key role. Chone and Linnemer (2006) analyze a very general model with multi-product firms and uncertainty over efficiency gains of the merged firms. In contrast to the present paper, this uncertainty fully resolves before post-merger market competition takes place, but

[8] Fisher and Lande (1983) assert that "efficiencies still are enormously difficult to predict on a case-by-case basis...". Likewise, according to FTC chairman Robert Pitofsky, the efficiencies defense is "easy to assert and sometimes difficult to disprove" (Quoted in J. Kattan (1994), Efficiencies and merger analysis, *Antitrust Law Journal*, 62, 513). One is tempted to add that if all the federal agencies empowered to ascertain the prospects of efficiency gains admit to the complexity of the task, the rival firms and outside analysts of the industry will typically find it beyond hope. As a consequence, these outsiders have no option after the merger other than to engage in Bayesian behavior.

nonetheless affects the ex ante social welfare of the merger for antitrust authorities, and hence their approval decision. Depending on the nature of competition and on the demand specification, social welfare may be convex (e.g. for linear demand) or concave in these gains, so that uncertainty may enhance or lower the approval chances. In a setting with uncertainty over demand or costs, Banal-Estanol (2007) shows that there are added incentives for mergers arising from merged firms sharing their private signals.[9]

One of our main results states that if the non-merged firms believe with a sufficiently high probability that the merged firm will experience a high enough efficiency gain, the merger will be profitable, even if one takes the worst-case scenario for the merged firm, wherein it ends up not experiencing any efficiency gain at all. Similar threshold rules are shown to govern the effects of a merger on the merged firm's and outsiders' outputs as well as on industry price, using worst case, best case and expected term scenarios.

In all theoretical models with complete information and no efficiency gains, whether based on Cournot or on Bertrand competition, mergers always exert a positive externality on non-merged firms. In a Bayesian formulation, the nature of this externality also follows a threshold rule depending on the same pair of parameter values, so that it may well be negative. Similar remarks may be made about market shares and sales. The set of possible outcomes following a merger is substantially expanded, with one or both the merged firm and the outsiders, or neither of them, being possible beneficiaries.

For both consumer surplus and social welfare, the worst-case benchmark yields a negative effect of mergers while the latter two lead to thresholds depending again on the belief and the efficiency gain levels. The threshold rule associated with the ex ante and worst case benchmarks confirms the central role played by expected efficiency gains in gauging the welfare effects of mergers, as in common antitrust practice in many countries. Another main conclusion of the paper is that an ex-ante profitable merger is necessarily social-welfare, but not always consumer-welfare, improving. This result provides support for a laisser-faire policy if the decisive criterion rests on social welfare, but not if it rests on consumer welfare. This underscores the importance of the selection of a decision criterion for antitrust approval of a merger.

The present set-up also demonstrates that the merging firms have a strong incentive to overstate the extent of their potential efficiency gains ex ante, not only to secure approval of the merger by antitrust authorities, but also to twist the terms of Bayesian Cournot competition in their favor, in a short-run perspective.

All in all, our results form a major departure from the complete information equi-

[9] Other recent studies include Banal-Estanol et. al. (2007) and Mialon (2007), who investigate the effects of internal reorganization by a merged entity and Spector (2003), who consider the effects of mergers when allowing for entry into the industry.

librium analysis of the literature starting with SSR. Particularly noteworthy is the fact that the novel features of the present paper hold even in the worst-case ex post outcome of no efficiency gains. In such a case, the *only difference* between the post-merger markets in this paper and in SSR is the *first-to-know advantage* of the merged firm inherent in the Bayesian Nash concept. An important consequence of this difference is that, unlike most previous theoretical results, our conclusions are quite consistent with many empirical findings and stylized facts on the effects of mergers on profitability, sales and market shares, both for the merged firm and for the outsiders to the merger.

This *first-to-know advantage* thus emerges as a natural candidate for the fundamental asymmetry that mergers seem to trigger in favor of the merged firm. By its very nature, this new type of asymmetry is transitory, as are most investigated effects of mergers. In this sense, the present theory constitutes a short-run analysis, but the short run is where most of the interest in mergers actually lies[10]. In addition, we can add a plausible dynamic extrapolation of our model to capture the resolution of uncertainty over the merged firm's cost. Our results are consistent with GMYZ's finding that over their five-year data window, from one year to the next, realized profits increased for profitable mergers but decreased for unprofitable mergers (see Section 5 where a dynamic extension of the model is discussed). A similar mechanism may be invoked to account for the initial substantial rise in share values that typically accompanies merger announcements, which often ends up spiraling downwards after one to three years.

This paper is organized as follows. After a model description in Section 2, the effects of mergers on market performance are presented in Section 3, followed by a detailed welfare analysis in Section 4. Section 5 is devoted to some dynamic extensions. All computations, proofs and quantitative illustrations are gathered in an Appendix.

2 The Model

In the pre-merger situation, consider an industry composed of $n+1$ identical firms, each with constant unit cost c, choosing quantity levels of a homogenous product in a market with inverse demand $P = a - bQ$, with $a > c > 0$ and $b > 0$. Each firm's pre-merger Cournot equilibrium output and profits are then

$$q = \frac{a-c}{b\,(2+n)} \qquad \text{and} \qquad \pi = \frac{(a-c)^2}{b\,(2+n)^2}.$$

[10]Indeed, a long run analysis would have to disentangle other potential contributing factors, including industry-specific or economy-wide shocks, entry and exit, other mergers within the same industry, etc. Recall also that most empirical studies consider horizons extending only three to five years.

We consider a bilateral (two-firm) merger only. In the post-merger situation, we postulate that ex ante the firms in the industry are uncertain over the final unit cost of the merged firm, and thus engage in a Bayesian Cournot game. Assuming a binomial outcome, the firms believe that with probability p the merged firm will end up with marginal cost $c_l < c$, thus having experienced efficiency gains $\Delta c \equiv c - c_l$, while with probability $(1-p)$ its cost will remain at the pre-merger level c. Here, c_l may for instance be an average value attained by comparable mergers, or the value reflected in post-merger simulations accepted by the merger authorities, or the actual value claimed by the merging firms. In conformity with the Bayesian Nash concept, the value of p is common knowledge (and the same) amongst all firms, and reflects the perception that all agents (firms, antitrust authority and financial markets) have formed about the merged firm's ability to achieve the posited efficiency gain, given the information available to them about the case and possibly all previously treated similar mergers.

While the model is clearly very stylized and makes use of several strong (though common) simplifying assumptions, the principles behind the conclusions derived in the paper will be seen to be robust to more general specifications, but only at an intuitive level since a general analysis would be intractable.[11]

Ex post, only the merged firm gets to observe its cost (or type) and thus condition its output decision on it. Each outsider firm chooses only one output level to maximize its ex ante p-expected profits. Let q_m^h and q_m^l be the merged firm's quantities conditional on being the low-cost type (i.e. with unit cost c_l) and high-cost type (i.e. with unit cost c) respectively, and Eq_m be its expected quantity. Each *outsider's* quantity is denoted by q_o. The Bayesian Nash equilibrium quantities are as follows[12] (recall that *after* the merger, the industry has n firms, and $\Delta c = c - c_l > 0$ is the efficiency gain):

$$\begin{aligned} q_o &= \frac{a-c-p\Delta c}{b\,(n+1)}, \qquad q_m^l = \frac{2(a-c)+\Delta c(1+n+p(n-1))}{2b\,(n+1)} \\ q_m^h &= \frac{2(a-c)+p\Delta c(n-1)}{2b\,(n+1)} \qquad \text{and} \qquad Eq_m = \frac{a-c+np\Delta c}{b\,(n+1)}. \end{aligned}$$

All these quantities are strictly positive if one assumes[13] $p < \frac{a-c}{\Delta c}$. The corresponding

[11]Nonetheless, two specific assumptions deserve further discussion. The first is that while one might argue that the different agents involved might hold different beliefs about the value of p, we have no theory at present for a Bayes-Nash concept with multiple priors. The second point is that it would be realistic to allow for the possibility that a merger might increase the final cost of the merged firm. Ignoring this, as we do, might be justified by the fact that the merging firms have the option of keeping the pre-merger production set-up unchanged in both their plants, thus guaranteeing themselves cost c.

[12]The computational details and cumbersome expressions are in Appendix 8.1.

[13]Observe that certainty-equivalence holds, due to the linear specification, in that the merged firm's expected output is the p-weighted average of its outputs in the corresponding full informa-

market prices are

$$P^l = \frac{2\left(a + nc_l\right) + \Delta c(n-1)(p+1)}{2\left(n+1\right)}$$

$$P^h = \frac{2\left(a + nc\right) + p\Delta c\left(n-1\right)}{2\left(n+1\right)} \quad \text{and} \quad EP = \frac{a + cn - p\Delta c}{n+1}.$$

The expected equilibrium profits of each outsider firm and the profits of *the merged* firm, conditional on its cost type, c or c_l, are respectively

$$E\pi_o = \frac{\left(a - c - p\Delta c\right)^2}{b\left(n+1\right)^2}, \qquad \pi_m^h = \frac{\left(2\left(a-c\right) + p\Delta c\left(n-1\right)\right)^2}{4b\left(n+1\right)^2}$$

$$\text{and } \pi_m^l = \frac{\left(2\left(a-c\right) + \Delta c\left(1 + n + p\left(n-1\right)\right)\right)^2}{4b\left(n+1\right)^2}$$

Note that it is always the case that $E\pi_o \leq \pi_m^h \leq \pi_m^l$, with equality throughout if and only if $p = 0$. In other words, the informational asymmetry created by the merger always works in favor of the merged firm, which now outperforms its rivals even in the worst case situation wherein their costs are all equal. Whether this informational rent is sufficient to compensate for the fact that the merged firm must now divide its profit between its two pre-merger partners is investigated in the following section.

3 Effects on Market Performance

This section provides a detailed account of the consequences of the merger on profits and outputs for both the merged firm and the outsider firms, as well as on industry price. In dealing with these effects, several options are possible. One is obviously to use expected profits and outputs at the Bayesian Cournot equilibrium. This profit measure is arguably the relevant indicator that determines the movement and magnitude of the merged firm's share price. We also study the worst-case scenario, wherein the merged firm fails to achieve any ex-post efficiency gains at all, so that its post-merger realized profits are given by π_m^h, as well as the best-case scenario with realized profits π_m^l.

In the worst case scenario, the merger is profitable if $\pi_m^h > 2\pi$, the solution of which leads to one of our main results (note that as π_m^h is clearly the lowest possible realized profit, the threshold values below are the most conservative ones).

Proposition 1 *If the non-merged firms believe sufficiently, i.e. with*

$$p > p^{h*} = \frac{2\left(a-c\right)\left(\left(\sqrt{2}-2\right) + \left(\sqrt{2}-1\right)n\right)}{\Delta c\left(n+2\right)\left(n-1\right)}$$

tion oligopolies. This property suggests that our conclusions extend to more general formulations, instead of our stylized binomial version, of the Bayesian feature of this model.

that the merged firm will experience large enough efficiency gains

$$\Delta c > \frac{2\,(a-c)\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{(n+2)\,(n-1)}$$

then the merger will be profitable, even in the worst case scenario. These gains can occur only if the original cost is high enough.

In the way of comparative statics, it can be shown that the profitability of a two-firm merger is enhanced by having a lower level of demand (a), a higher initial unit cost (c), or a higher initial number of firms (n). This last point constitutes a departure from the complete information case analyzed by SSR.

In expected rather than worst-case terms, mergers are more likely to be profitable for the merging firms. It is easy to see that $E\pi_m = (1-p)\pi_m^h + p\pi_m^l \geq \pi_m^h$, so that the threshold for ex ante merger profitability, p^* (see the Appendix) is lower, thus providing a wider scope for profitable mergers.

In the best case scenario, when efficiency gains do realize, the resulting profit π_m^l clearly satisfies $\pi_m^l \geq E\pi_m \geq \pi_m^h$, so the probability threshold, p^{l*}, is even lower than p^*, where p^{l*} is given by[14]

$$p^{l*} = \frac{2(a-c)\left[n(\sqrt{2}-1)-(2-\sqrt{2})\right]-(n+1)(n+2)\Delta c}{(n-1)(n+2)\Delta c}.$$

We now investigate the effects of the merger on outputs and outsiders' profits.

Proposition 2 *(a) The merged firm expands output in the worst case scenario (resp., in the best case scenario) if and only if*

$$p \geq p_m^h = \frac{2n(a-c)}{\Delta c(n-1)(n+2)} \quad \left(\textit{resp.},\ p \geq p_m^l = \frac{2n(a-c)-(n+1)(n+2)\Delta c}{(n+1)(n+2)\Delta c}\right).$$

(b) In the worst case scenario, a merger always raises industry price, and in the best case scenario, industry price rises if and only if

$$p > p^l = \frac{(n+1)(n+2)\Delta c - 2(a-c)}{\Delta c(n+1)(n-1)}.$$

(c) In expected terms, the merger increases an outsider firm's profit and output, the merged firm contracts output and industry price rises if and only if

$$p \leq p_o = \frac{a-c}{\Delta c(n+2)}\ .$$

[14] As in Proposition 1 we need conditions on both Δc and c.

Upon observing that the Bayesian game at hand satisfies *certainty equivalence*,[15] one can provide a clear-cut intuition for the expected terms case using standard Cournot thinking.[16] To this end, let $Q_0(Eq_m)$ denote the (ex ante) aggregate reaction curve of the outsiders and $Eq_m(Q_0, p)$ the merged firm's ex ante reaction curve. Note that the former is independent of p while the latter is not. Figure 1 depicts $Q_0(Eq_m)$ and $Eq_m(Q_0, p)$ for three different values of p: $p_o, p' < p_o$ and $p'' > p_o$. When $p = p_o$, the certainty-equivalent efficiency gain reflected in the expected cost $p_o c_l + (1 - p_o)c$ is just sufficient in the world of Farrell and Shapiro (1990) to make the outputs of the merged firm and (thus) of the outsiders, as well as industry price, remain at their pre-merger levels. It follows that the outsiders' profit remains at its pre-merger level while that of the merged firm increases (due to its lower cost, i.e. $p_o c_l + (1 - p_o)c < c$).

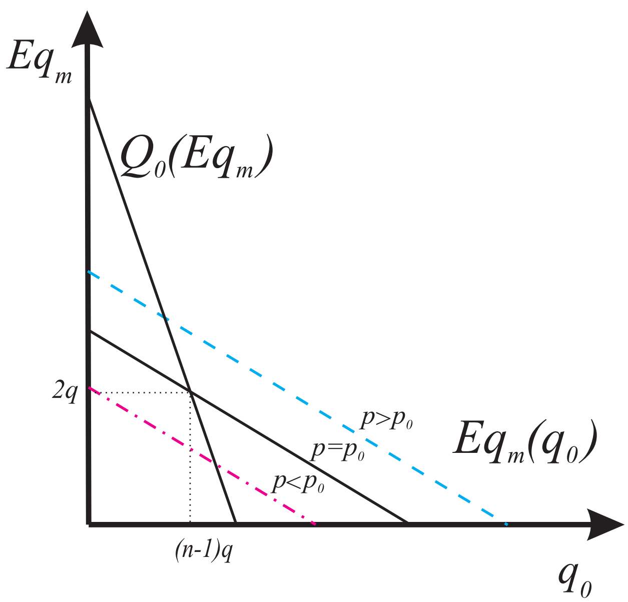


Figure 1.

An increase in p, say from p_o to p', shifts $Eq_m(Q_0, p)$ upwards, thus raising the output of the merged firm and lowering the output of the outsiders by a lower amount, due to the slopes of reaction curves lying in $(0, -1)$, see e.g. Amir (1996). It follows that industry price also rises. The intuition is that as p rises, the outsiders contract their outputs as they expect to compete against a more efficient merged firm that will expand output. The opposite statement holds for a decrease from p_o to p''.

[15]This refers to the property that the Bayesian Nash solution in expected terms coincides with the standard Cournot equilibrium where the merged firm has its expectd cost, $pc_l + (1 - p)c$, with certainty.

[16]We are grateful to a referee of this Journal for suggesting the following intuitive interpretation.

Figure 2 below summarizes all the possibilities in expected terms. It also includes the threshold belief, p^*, above which the merged firm will be profitable in expected terms. In Appendix 8.2 (Figure 2 explanation), it is shown that $p^* < p_o$. A more detailed exposition of all the possible ex-post realizations (high and low costs) is given in Figure 4 in Appendix 8.2. Depending on the belief and efficiency gain levels, a much richer picture emerges, relative to the full information model of SSR. All possible combinations on the changes of output by the merged firm and the outsiders can emerge. Due to the informational asymmetry, the usual business stealing argument does not quite hold in the worse case scenario, where both the merged firm and the outsiders actually contract output when $p_o \leq p \leq p_m^h$. Likewise, the worst-case profits of the merged firm and the expected profits of the outsiders can both increase (for $p^* \leq p \leq p_o$). More interestingly, one novel feature (for static models) that emerges here is that outsider firms may actually be harmed by a merger (also see Davidson and Mukherjee, 2007). This provides theoretical support for the conclusion by Banerjee and Eckard (1998) that outsider firms experienced profit losses during the 19th century merger wave in the U.S. It also lends credence to a prevalent belief that firms are often apprehensive of the prospect of a merger between two of their rivals.

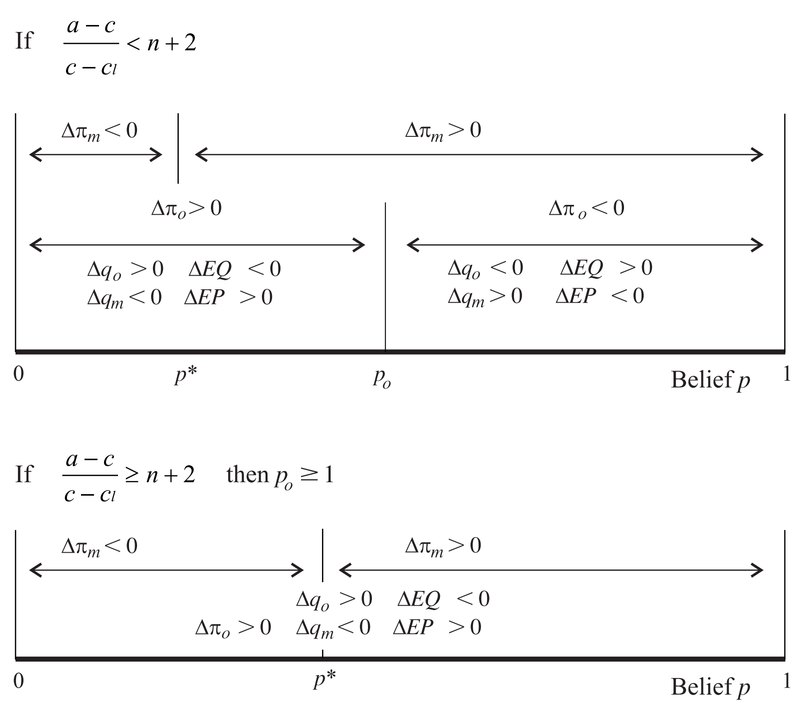


Figure 2. Market performace in expected terms.

All in all, the emerging picture squares well with the stylized facts, even from a (suggestive) quantitative standpoint. The theory at hand predicts a narrower scope for output expansion ($p \geq p_m^h$) than for profitability of the merger ($p \geq p^{h*}$), since $p^{h*} < p_o$ as shown in the explanation of Figure 2 in Appendix 8.2. The corresponding empirical averages reported by GMYZ are about 40% and 60%, respectively[17]! Thus a merger may be profitable in our setting in case the merged firm contracts output, even if this means a loss in market share (recall that output and market share reductions for the merged firms take place in a majority of cases, Mueller, 1985).

These results suggest that the informational advantage underlying the Bayesian Cournot concept endows the merged firm with a new form of market power relative to the outsider firms. Whether this advantage is sufficient to overturn the usual mechanism that makes mergers more favorable to outsiders than to insiders depends on the levels of the potential efficiency gains and of the associated belief.

Recall that in the full information model of SSR, each partner of a merged firm wishes to reduce output in the post-merger situation as it now takes into account the negative externality it inflicts on its merging partner. Non-merging firms react to this contraction by expanding output, due to strategic substitutability. The resulting price increase is then not sufficient to imply higher profits for the merged firm.[18]

By contrast, in the present Bayesian setting, the merged firm exploits its informational advantage that lies in the inability of the outsiders to adapt their outputs to its true, but unknown unit cost. Depending on the belief held by the outsiders, this new advantage may well lead to the merged firm producing more than before the merger, despite the fact that the aforementioned externality effect is still present here. While a tendency for the outsiders' output to move in the opposite direction is still there, there is a range of values of p (between p_0 and p_m^h) for which all firms decrease their output after the merger, even in the worst case scenario. Similarly, there is a range of p (between p^{h*} and p_o) such that all firms' profits increase. This new diversity of strategic behavioral patterns is due to the interaction between the informational market power of the merged firm with the well-known effects of mergers in the standard SSR model (also see Gaudet and Salant, 1991). As a result, real-life merger behavior emerges as being compatible with static equilibrium theory, at least in the short run.

[17] If one were to include savings on fixed costs in efficiency gains, the scope for profitability would widen while the merged firm's output would remain unaltered. This would further reinforce our results.

[18] By contrast, in the Bertrand case with substitute products, the merging partners raise their prices while outsider firms react by raising prices as well, due to strategic complementarity. So prices rise even more, and the merger ends up being profitable to all firms.

4 Welfare Analysis

Producer surplus, PS, consumer surplus, CS, and total (social) welfare, TW, before the merger are easily found to be:

$$PS = \frac{(n+1)(a-c)^2}{b(2+n)^2}, \quad CS = \frac{(c-a)^2(n+1)^2}{2(n+2)^2 b} \text{ and } TW = \frac{(c-a)^2(n+1)(n+3)}{2(n+2)^2 b}.$$

In evaluating the welfare effects of a merger, we consider all the three possible evaluation benchmarks: worst-case scenario, best-case scenario, and expected terms.

We start our analysis with producer surplus. Ex-post producer surplus, conditional on the realized cost being high and low are $PS^h = \pi_m^h + (n-1)\pi_o^h$ and $PS^l = \pi_m^l + (n-1)\pi_o^l$ respectively. Expected producer surplus is $EPS = pPS^l + (1-p)PS^h$.

Often, antitrust authorities take consumer welfare as the key indicator in merger cases. So a separate analysis of consumer surplus is highly desirable. Ex-post consumer surpluses, conditional on the actual realized cost, are $CS^h = \frac{1}{2}(a - P^h)Q^h$ and $CS^l = \frac{1}{2}(a - P^l)Q^l$. Expected consumer surplus is clearly the weighted average of the two conditional consumer surpluses: $ECS = pCS^l + (1-p)CS^h$. All these expressions are in Appendix 8.3 (with the proofs in Appendix 8.5).

Proposition 3 *(a) In the worst-case scenario, a merger increases producer surplus, and lowers consumer surplus and social welfare.*

(b) In expected terms, a merger increases producer surplus, and may increase or decrease consumer surplus and social welfare.

(c) In the best-case scenario, a merger may increase or decrease producer surplus and consumer surplus.

In addition to part (b), we have fully characterized the expected welfare change with a threshold belief (see ADX, 2008), along with an illustration indicating that a merger will be beneficial to society as a whole, unless p and Δc are both rather small.[19] Interestingly, this may be viewed as a Bayesian-Cournot analog of Williamson's (1968) classical efficiency defense, arguing that a small efficiency gain is sufficient to make a merger welfare-improving in a competitive economy.

Part (a) follows in part from Proposition 2 since in case of no efficiency gains ex post, price always increases, and so consumer surplus decreases.

In the worst-case scenario as well as in expected terms, a merger is beneficial to the industry as a whole as in the standard model of SSR. How these gains are divided between the insiders and the outsiders to the merger in our Bayesian setting depends

[19] This numerical observation was confirmed to be robust via other simulations.

of course on the usual pair of key parameters $(p, \Delta c)$. Interestingly, it is only when efficiency gains are realized ex-post that the industry as a whole may be adversely affected by a merger. In this case, the conjunction of the first-to-know advantage and the efficiency superiority of the merged firm may lead to a reduction in outsiders' profits exceeding the merger's profit expansion. This suggests one plausible explanation for the frequently observed apprehensive reaction of outsider firms to a merger.

According to Proposition 3, not surprisingly, if antitrust authorities adopted an absolutely conservative standard requiring that consumer surplus or social welfare increase in the worst case scenario, then no merger would ever be allowed. On the other hand, going by the ex-ante or best-case standards, the conclusion depends on the levels of p and Δc, in conformity with existing practice.

In view of our Bayesian setting and of the prospect for an efficiency gain, the relationship between consumer and producer surpluses is more nuanced than in the standard SSR model. It makes sense here to ask whether an ex-ante profitable merger will necessarily improve consumer welfare. Example 7 in the appendix settles this question in the negative. Hence, antitrust authorities acting on the basis of consumer surplus should not adopt an a priori laisser-faire merger policy.

We now consider the same question with social welfare as the criterion. While Proposition 3 reveals similar effects of mergers on consumer and social welfare from a qualitative standpoint, an important divergence for merger policy is brought out in the next result, relating the private and social incentives for a merger.

Proposition 4 *Whenever a merger is profitable in expected terms for the merged firm, it will increase social welfare in the best-case scenario as well as in expected terms, i.e.,*

$$\Delta E\pi_m > 0 \Longrightarrow \Delta ETW = ETW - TW > 0 \text{ and } \Delta TW^l = TW^l - TW > 0.$$

This is consistent with Farrell and Shapiro's (1990, Proposition 5) finding that, under some conditions on demand and costs that are satisfied by our linear setting, if a merger with sure efficiency gains (i.e. with $p = 1$ here) is profitable to the merging firms, it will also be welfare improving.

The implications of this result are significant in that they suggest that if two firms' wish to merge were fully based on an expected profit calculation, then, on the basis of a social welfare criterion, antitrust authorities should adopt a laissez-faire policy. However, if efficiency gains do not materialize ex post, then the merger will always be detrimental to society, irrespective of the ex-ante private incentives (Proposition 3). To the extent that it is widely believed that in some cases, mergers are, at least partly, motivated by such reasons as managerial self-promotion, hubris or empire-building, the implications of Proposition 4 should be viewed with due care.

A continuing controversy in merger policy is whether the key decisive criterion for approving mergers should be consumer or social welfare. Our results neatly bring out the commonalities and the divergences in appropriate public policy responses depending on which standard is adopted. With the somewhat intermediate standard given by the sum of expected consumer surplus and outsiders' profits, proposed by Farrell and Shapiro (1990), the present model would not prescribe a laissez-faire policy.

5 Further Results and Observations

This section presents two extensions of interest, not considered so far. The first deals with how our Bayesian setting would affect m-firm mergers, and the second with how uncertainty over efficiency gains might resolve over time. We then discuss a possible dynamic extension of the present analysis.

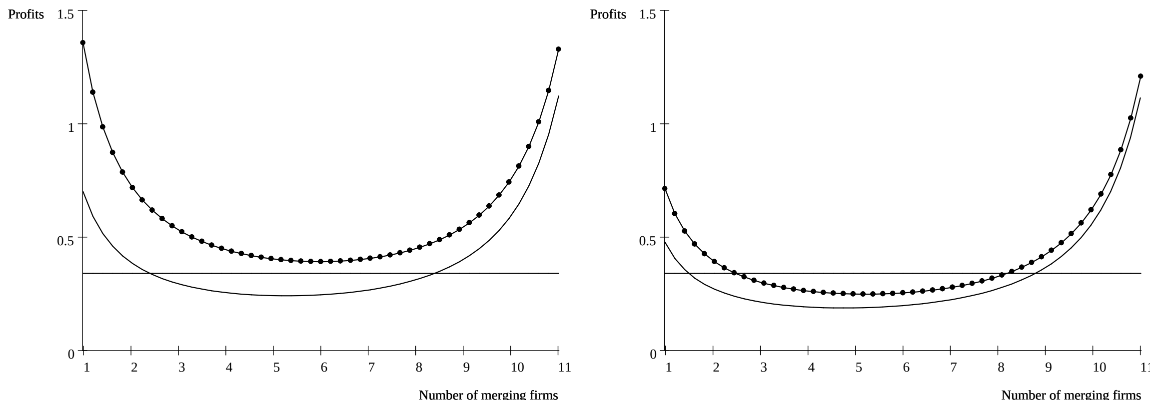
5.1 The Profitability of Multilateral Mergers

Extending consideration to multilateral mergers here, we provide plausible illustrations showing that (i) larger mergers do not necessarily fare better than bilateral mergers and (ii) larger mergers may not even be profitable when bilateral mergers are.

With s merging firms, each firm's profit in the worst-case and ex-ante (expected) benchmarks are respectively

$$\frac{\pi_m^h}{s} = \frac{\left(2a - 2c + p\left(c - c_l\right)\left(n - s + 1\right)\right)^2}{4sb\left(n - s + 3\right)^2} \text{ and } \frac{E\pi_m}{s} = p\frac{\pi_m^l}{s} + (1-p)\frac{\pi_m^h}{s}.$$

Consider $n = 10$, $a = 10$, $c = 3$, $c_l = 2$, $b = 1$ and either $p = 0.25$ or $p = 0.6$. Each of the following two graphs is a plot of π_m^h/s (solid lines) and $E\pi_m/s$ (dotted lines) as functions of s in the two cases.



Profits when $p = 0.6$ Profits when $p = 0.25$

Figure 3. Per partner profits.

In the case where $p = 0.6$, in expected terms, all mergers are profitable, and bilateral mergers are preferred by the merged firms to multilateral ones, at least up to 9 partners. On the other hand, in the worst case scenario the only profitable mergers are those involving $2, 10$ or all 11 firms. By contrast, recall that for this example, only mergers with $9, 10$ or 11 firms are profitable in the complete information Cournot model of SSR. Analogously, when $p = 0.25$, in the worst case scenario, no merger involving less than 9 firms is profitable, while in expected terms a bilateral merger is profitable!

These findings are obviously consistent with the observed reality that virtually all mergers involve two firms, another noteworthy divergence from previous models.

5.2 On the "Dynamics" of Profitability

This subsection discusses some possible dynamic extensions of the static Bayesian approach to mergers, paying close attention again to the stylized facts. A finer empirical finding of GMYZ is that, over their five-year data window, from one year to the next, realized profits increased for profitable mergers but decreased for unprofitable mergers. This seemingly strange finding turns out to be quite consistent with our results if one adds a plausible dynamic extrapolation capturing the resolution of uncertainty over time in our model. Assuming that profitable mergers tend to be those that indeed generate efficiency gains, the outsiders progressively learn about these gains, realize with more and more certainty that they face a lower-cost merged rival and react accordingly. Likewise, if unprofitable mergers are identified with those that failed to generate efficiency gains, rivals will progressively find out over time that they face a high-cost merged rival, and the latter's profits will move accordingly lower. In both cases, the process eventually settles at the full information Cournot equilibrium that reflects the true efficiency gains actually achieved by the merger. This argument conveys clearly the sense in which the present analysis is of a short-run nature.

In this scenario, it is important to note that the learning process envisioned is not related to signaling; it is rather nonstrategic and is based on information about p gathered via firm reports, leaks in the investigative press, etc. Alternatively, it is reasonable to postulate that due to exogenous noise, for instance in demand or in macroeconomic variables, the outside firms cannot fully learn the true efficiency gains of the merged firm in one, or even a few, periods.[20]

In the way of illustrative insight, consider the following example where the merger decision is based on expected merger profits.

Example 5 *Let* $a = 10, b = 1, n = 10, c = 7, p = 0.5,$ *and* $c_l = 6.8 \in (6.6049, 6.8280)$. *That is,* c_l *is between the 2 critical values of* c_l *in the tables following Proposition 1. As a result,* $\pi_m^h = 0.0984 < 2\pi = 0.125$ *and* $\pi_m^l = 0.1711 > 2\pi = 0.125$. *If the merger decision is based on ex ante expected profits, then, ex post, with probability* p *the cost is* c_l *and the profits are* $\pi_m^l = 0.1711 > 2\pi = 0.125$ *while with probability* $(1-p)$ *the cost is* c *and profits are* $\pi_m^h = 0.0984 < 2\pi$. *The subsequent (complete information) Cournot game is either one with the merged firm having cost* c_l *or one with the merged firm having cost* c. *In the former case, the merged firm's profits are* $0.2066 > \pi_m^l = 0.1711$ *and in the latter case, its profits are* $0.074\,38 < \pi_m^h = 0.0984$.

An other plausible dynamic extension would be to a multi-period framework allowing for signaling by the merged firm and learning on the part of outsider firms. Such an extension would produce an even more diverse set of possible equilibrium outcomes, including separating and pooling (perfect Bayesian) equilibria. In the following example, the separating equilibrium yields an outcome that is fully consistent with the data on profit dynamics described above (though many other outcomes can emerge in other plausible examples).

Consider the industry in Example 5 operating over two periods with discount factor $\delta = 0.7$. It can be verified[21] that the following strategies and system of beliefs constitute

[20] There is quite a bit of anectodal evidence in support of this slow learning of the extent of efficiency gains in mergers. For instance, in a report to the Federal Trade Commission advocating a two-stage process to review efficiency claims, one ex ante and one ex post, Brodley (1996) argues that "the ex post proceeding should normally be held between three and five years after the ex ante determination. Efficiency realization generally will require a longer time period than that used in competitive analysis of mergers".

[21] The associated computations are tedious, and are available from the authors upon request.

a separating (perfect Bayesian) equilibrium:

$$\begin{aligned}
\text{First period choices:} \quad & q_m^l = 0.5398, q_m^h = 0.3503, q_o = 0.2550 \\
\text{Second period choices:} \quad & q_m^l = 0.4546, q_m^h = 0.2727, q_o^h = 0.2727, q_o^l = 0.2546 \\
\text{Beliefs:} \quad & \Pr(\text{low cost} \mid q < 0.5398) = 0, \Pr(\text{low cost} \mid q \geq 0.5398) = 1
\end{aligned}$$

In this equilibrium, the low-cost merged firm's profits would be 0.1947 in the first period and rise to 0.2066 in the second, while a high-cost merged firm's profits would be 0.1227 in the first period and go down to 0.0744 in the second. Prior to the merger, the total profits of the two firms are 0.1250 per period. Thus, comparing period by period, the merger is profitable for the low-cost merged firm but unprofitable for the high-cost one. Furthermore, in ex-ante terms, using discounted expected profits, the merger is profitable (0.2125 and 0.2992 before and after the merger, respectively). Note that in a separating equilibrium, all the information is fully revealed after the first period.

This two-period example also admits a pooling equilibrium, with beliefs that do not survive the "intuitive criterion": Let the merged firm produce 0.3636 in the first period regardless of costs and each outsider produce 0.26364; however, to sustain such behavior in equilibrium, extreme beliefs such as $\Pr(\text{high cost}|\ q \neq 0.3636) = 1$ are necessary.

While such strategic dynamic models are appealing, going beyond two periods in this context is unfortunately intractable, due in part to equilibrium multiplicity.

6 Conclusion

This paper argues that many of the circumstances surrounding mergers call for a theoretical model wherein the firms outside the merger face a new type of rival, characterized by unknown unit costs, reflecting their natural initial uncertainty about the ability of the merged firm to achieve any (of the claimed) efficiency gains. This pervasive uncertainty also affects the approval decision of antitrust authorities, and triggers the favorable response by financial markets. Within the obvious confines of a static model, the proposed Bayesian Cournot equilibrium leads to an outcome that is broadly consistent with much of the empirical evidence on the industry effects of mergers, on profits, price and market shares for the merged firm as well as for outsiders, at least in the short run. All in all, the model at hand reflects a simple and natural modification of the standard Cournot approach, based on an inherent informational advantage of the merged firm over outsiders, bringing about a surprising level of congruence with stylized facts.

In terms of welfare, mergers lower consumer and social welfare for sure only in the worst-case scenario. In the other two scenarios, welfare depends on the levels of belief and the efficiency gain. An ex-ante profitable merger is necessarily beneficial for expected social welfare but not necessarily for expected consumer welfare. Overall, these results vindicate the central role assigned to efficiency gains in merger policy.

7 References

1. Amir, R. (1996): Cournot oligopoly and the theory of supermodular games, *Games and Economic Behavior*, **15**, 132-148.

2. Banal-Estanol, A. (2007): Information-sharing implications of horizontal mergers, *International Journal of Industrial Organization,* **25***,* 31-49*.*

3. Banal-Estanol, A., I. Macho-Stadler and J. Seldeslachts (2006): "Endogenous mergers and endogenous efficiency gains: The efficiency defense revisited, *International Journal of Industrial Organization,* to appear*.*

4. Banerjee, A. and W. Eckard (1998): Are mega-mergers anti-competitive? Evidence from the first great merger wave, *Rand Journal of Economics,* **29**(4), 803-827.

5. Besanko, D. and D. Spulber (1993): Contested Mergers and Equilibrium Antitrust Policy, *Journal of Law, Economics and Organization*, **9**, 1-29.

6. Borenstein, S. (1990): Airline Mergers, Airport Dominance, and Market Power, *American Economic Review,* **80**, 400-404.

7. Brodley, J. F. (1996), Proof of efficiencies in mergers and joint ventures, *Antitrust Law Journal*, 64, 575.

8. Chone, P. and L. Linnemer (2008): Assessing horizontal mergers under uncertain efficiency gains, *International Journal of Industrial Organization,* to appear.

9. Davidson, C. and A. Mukherjee (2007), Horizontal mergers with free entry, *International Journal of Industrial Organization*, **25**, 157-172.

10. Deneckere, R. and C. Davidson (1985): Incentives to form coalitions with Bertrand competition, *Rand Journal of Economics,* **16**, 473-486.

11. Farrell, J. and C. Shapiro (1990): Horizontal mergers: An equilibrium analysis, *American economic Review,* **8**0, 107-126.

12. Fauli-Oller, R. (1997): On Merger Profitability in a Cournot Setting, *Economics Letters,* **54**(1), 75-79.

13. Fisher, A. and R. H. Lande (1983): Efficiency Considerations in Merger Enforcement, *California Law Review,* **71**.

14. Gaudet, G. and S. Salant (1991): Increasing the profits of a subset of firms in oligopoly models with strategic substitutes, *American Economic Review*, **81**, 658-665.

15. Gugler, K., Mueller, D. C., Yurtoglu, B.B. and C. Zulehner (2003): The effects of mergers: An international comparison, *International Journal of Industrial Organization*, **21**, 625-654.

16. Kim, E. H. and V. Singal (1993): Mergers and market power: Evidence from the airline industry, *American economic Review,* **83**, 549-569.

17. McAfee, R. P. and M. A. Williams (1992): Horizontal mergers and antitrust policy, *Journal of Industrial Economics,* **40**, 181-187.

18. Meeks, G. (1977): Disappointing marriage: A study of the gains from merger, *Cambridge University Press.*

19. Mialon, S. (2008), Efficient horizontal mergers: The effects of internal capital reallocation and organizational form, *International Journal of Industrial Organization*, to appear.

20. Mueller, D. C. (1985): Mergers and market share, *Review of Economics and Statistics,* **67**, 259-267.

21. Mueller, D. C. ed. (1980): The determinants and effects of mergers: An international comparison, Oelschlaeger: Cambridge, MA.

22. Perry, M. K. and R. H. Porter (1985): Oligopoly and the incentive for horizontal merger, *American Economic Review,* **75**(1), 219-227.

23. Pesendorfer (2003), Horizontal mergers in the paper industry, *RAND Journal of Economics*, **34**, 495-515.

24. Ravenscraft, D. J. and F. M. Scherer (1987): Mergers, sell-offs, and economics efficiency, Brookings Institution, Washington, D.C.

25. Salant, S. W., Switzer, S. & R. J. Reynolds (1983): Losses from horizontal merger: The effect of an exogenous change in industry structure on Cournot-Nash equilibrium, *Quarterly Journal of Economics,* **98**(2), 185-199.

26. Scherer, F. M., Backstein, A., Kaufer, E. and R. D. Murphy (1975): The economics of multiplant operation: An international comparison study, *Harvard University Press*, Cambridge.

27. Schmalensee, R. (1989): Inter-industry studies of structure and performance, *Handbook of Industrial Organization*, *Vol II*, 951-1010.

28. Spector, D. (2003), Horizontal mergers, entry and the efficiencies defense, *International Journal of Industrial Organization,* **21**, 1591-1600.

29. Williamson, O. (1968): Economies as an antitrust defense: The welfare trade-offs, *American Economic Review,* **58**, 18-36.

30. Werden, G. J. and L. M. Froeb (1994), The effects of mergers in differentiated products industries: Logit demand and merger policy, *Journal of Law, Economics and Organization*, **10**, 407-426.

8 Appendix

This Appendix gathers the computational details, some extra figures, the quantitative illustrations that complement the results, and the proofs of the results in the text.

8.1 Finding the Bayesian Cournot Equilibrium

This part provides the computational details of Section 2. Each (outsider) firm's expected payoffs are:

$$E\pi_i = p\pi_i^h + (1-p)\pi_i^l = aq_i - bEQ_{-i}q_i - bq_i^2 - cq_i.$$

The 1$^{\text{st}}$ order condition yields the best response function: $q_i = \frac{a-bEQ_{-i}-c}{2b}$. As everybody knows the cost of $n-1$ firms but not the cost of the n^{th} firm, the best response function of the *each outsider* becomes $q_o(Eq_m) = \frac{a-c-bEq_m}{bn}$.

The *merged* firm's best reaction curves are:

$$q_m^h(q_o) = \frac{a-c-b(n-1)q_o}{2b},\ q_m^l(q_o) = \frac{a-c_l-b(n-1)q_o}{2b} \text{ and } Eq_m(q_o) = \frac{a-(p\Delta c+c_l)-b(n-1)q_o}{2b}.$$

Each *outsider's* and the merged firm's outputs are presented in the text.

The three versions of total output are:

$$Q^l = \frac{2n\left(a-c\right)+\Delta c\left(n+1-p\left(n-1\right)\right)}{2b\left(n+1\right)},\ Q^h = \frac{2n\left(a-c\right)-p\Delta c\left(n-1\right)}{2b\left(n+1\right)}$$

$$\text{and } EQ = \frac{n\left(a-c\right)+p\Delta c}{b(n+1)}.$$

The corresponding prices are presented in the text.

The conditional expected profits of each outsider firm are:

$$\pi_o^h = E\pi_o + \frac{p\Delta c\left[(a-c)-p\Delta c\right]}{2b(n+1)} \text{ and } \pi_o^l = E\pi_o + \frac{\Delta c(1-p)\left[p\Delta c-(a-c)\right]}{2b(n+1)}$$

The expected profits of *each outsider* firm and conditional profits of the merged firm are again presented in Section 2. The expected profits of the merged firm is $E\pi_m = p\pi_m^l + (1-p)\pi_m^h$, or

$$E\pi_m = \frac{p^2\Delta c^2\left(3n+1\right)\left(n-1\right)+p\Delta c\left(8n\left(a-c\right)+\Delta c\left(n+1\right)^2\right)+4\left(c-a\right)^2}{4b\left(n+1\right)^2}$$

The change in expected profits is $\Delta E\pi_m = \pi_m - 2\pi$, or

$$\frac{\left(p^2\left(3n+1\right)\left(n-1\right)\left(n+2\right)^2\Delta c^2+p\left(n+2\right)^2\Delta c\left(8n\left(a-c\right)+\Delta c\left(n+1\right)^2\right)-4\left(c-a\right)^2\left(n^2-2\right)\right)}{4b\left(n+1\right)^2\left(2+n\right)^2}$$

Threshold belief above which expected profits are higher than pre-merger profits:

$$p^* = \frac{\left(\begin{array}{c}-(n+2)\left[8n(a-c)+\Delta c\left(n+1\right)^2\right]\\ +\sqrt{(n+2)^2\left[8n(a-c)+\Delta c\left(n+1\right)^2\right]+16(3n+1)(n-1)\left(a-c\right)^2\left(n^2-2\right)}\end{array}\right)}{2(3n+1)(n-1)(n+2)\Delta c}$$

8.2 Extra Figures and Explanations

This part contains supplementary figures and their explanations.

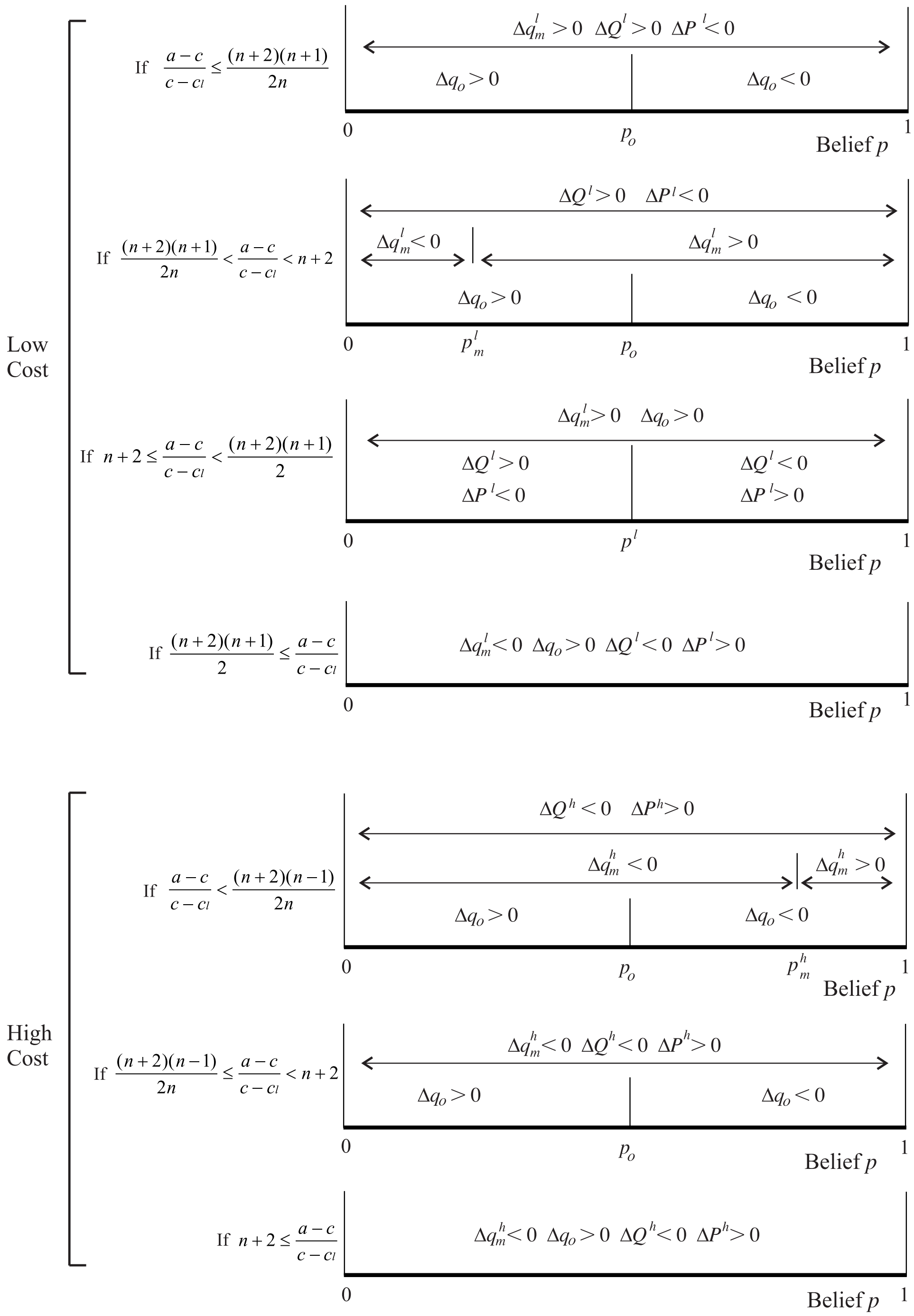


Figure 4. Market performance in low and high cost realizations.

Figure 2 explanation. Note that if $\frac{a-c}{c-c_l} \geq n+2$ then $p_o \geq 1$ hence, $E\pi_o \geq \pi$ and $q_o \geq q$ for all $p \in [0,1]$. If the merger is profitable in the worst case scenario, then $p > p^{h*}$ and it easy to show that $p_o > p^{h*}$ for all $(a-c) > 0$ and for all $\Delta c > 0$ since the inequality reduces to $n > -1$ which is always true. If the merger is profitable in the worst case scenario then it is profitable in expected terms as well, i.e., $p^* < p^{h*}$, hence, $p^* < p_o$. It follows immediately from the expressions presented in the previous section that $EQ > Q$ if and only if $p > p_o$. ■

Figure 4 explanation. For information regarding p_o see explanation of Figure 2.

Low Cost:

Further note that $p_m^l \geq 0$ if and only if $\frac{a-c}{c-c_l} \geq \frac{(n+2)(n+1)}{2n}$ and $p_m^l \leq 1$ if and only if $\frac{a-c}{c-c_l} \leq n+2$. It is easy to see that $\frac{(n+2)(n+1)}{2n} < n+2$.Observe that $p_m^l < p_o$ if and only if $\frac{a-c}{c-c_l} < n+2$ but otherwise both p_o and p_m^l are greater than 1 hence their relationship is insignificant. Similarly, notice that $p^l \leq 1$ if and only if $\frac{a-c}{c-c_l} \geq n+2$ and $p^l \geq 0$ if and only if $\frac{a-c}{c-c_l} \leq \frac{(n+2)(n+1)}{2}$. Again it is easy to see that $n+2 \leq \frac{(n+2)(n+1)}{2}$ is always true. Finally, the inequality $Q^l > Q$ reduces to $p < p^l$. Observe that if p^l is effective (i.e., $p^l < 1$) p_o and p_m^l are ineffective (i.e., greater than 1).

High Cost:

Note that it is always the case that $p_m^h \geq 0$ and $p_m^h \leq 1$ if and only if $\frac{a-c}{c-c_l} \leq \frac{(n+2)(n-1)}{2n}$. Similarly, it is easy to show that $p_m^h > p_o$ for all $(a-c) > 0$ and for all $(c-c_l) > 0$ since the inequality reduces to $n > -1$, which is always true. Finally the inequality $Q^h > Q$ reduces to $2(a-c) + p(c-c_l)(n-1)(n+2) > 0$, which is always true. ■

8.3 Welfare analysis details

This part provides the computational details of Section 4. We obtain:

$$PS^h = \frac{(2(a-c) + p\Delta c(n-1))^2}{4b(n+1)^2} + (n-1)\left(\frac{a-c-p\Delta c}{b(n+1)}\right)\left(\frac{2(a-c) + p\Delta c(n-1)}{2(n+1)}\right)$$

$$\begin{aligned} PS^l \;=\; & \frac{(2(a-c) + \Delta c(1+n+p(n-1)))^2}{4b(n+1)^2} + \\ & (n-1)\left(\frac{a-c-p\Delta c}{b(n+1)}\right)\left(\frac{2(a+nc_l) + \Delta c(n-1)(p+1)}{2(n+1)} - c\right) \end{aligned}$$

Expected producer surplus, EPS, is:

$$EPS = p\frac{(2(a-c) + \Delta c(1+n+p(n-1)))^2}{4b(n+1)^2} + (1-p)\frac{(2(a-c) + p\Delta c(n-1))^2}{4b(n+1)^2} + (n-1)\frac{(a-c-p\Delta c)^2}{b(n+1)^2}$$

The change in expected producer surplus,$\Delta EPS = EPS - PS$, is:

$$\begin{aligned}\Delta EPS &= p\frac{\left(2\left(a-c\right)+\Delta c\left(1+n+p\left(n-1\right)\right)\right)^2}{4b\left(n+1\right)^2}+\left(1-p\right)\frac{\left(2\left(a-c\right)+p\Delta c\left(n-1\right)\right)^2}{4b\left(n+1\right)^2}\\ &\quad +(n-1)\frac{\left(a-c-p\Delta c\right)^2}{b\left(n+1\right)^2}-\left(n+1\right)\frac{\left(a-c\right)^2}{b\left(2+n\right)^2}.\end{aligned}$$

The consumer surpluses are:

$$\begin{aligned}ECS &= p\left(\frac{2n\left(a-c_l\right)-\Delta c(n-1)(p+1)}{8b\left(n+1\right)^2}\right)\left(2n(a-c)+\Delta c(1+n-p(n-1)\right)\\ &\quad +(1-p)\frac{1}{2b}\left(\frac{2n\left(a-c\right)-p\Delta c\left(n-1\right)}{2\left(n+1\right)}\right)^2\end{aligned}$$

$$CS^h = \frac{1}{2b}\left(\frac{2n\left(a-c\right)-p\Delta c\left(n-1\right)}{2\left(n+1\right)}\right)^2$$

$$CS^l = \left(\frac{2n\left(a-c_l\right)-\Delta c(n-1)(p+1)}{8b\left(n+1\right)^2}\right)\left(2n(a-c)-\Delta c(n-1)(p+1)\right)$$

The expected total welfare is given by:

$$\begin{aligned}ETW &= p^2\Delta c^2\left(5n+7\right)\left(n-1\right)+p\Delta c\left[8\left(a-c\right)\left(n+2\right)+3\Delta c\left(n^2+2n+1\right)\right]\\ &\quad +4n\left(c-a\right)^2\left(n+2\right)\end{aligned}$$

The change in expected total welfare is then:

$$\Delta ETW = \frac{\left(\begin{array}{c} p^2\left(5n+7\right)\left(n-1\right)\left(n+2\right)^2\Delta c^2+p\left(n+2\right)^2\Delta c\left(8\left(a-c\right)\left(2+n\right)+3\Delta c\left(n+1\right)^2\right)\\ -4\left(c-a\right)^2\left(2n+3\right)\end{array}\right)}{8\left(n+1\right)^2 b\left(n+2\right)^2}$$

The threshold belief above which post-merger expected total welfare is higher is

$$p' = \frac{\left(\begin{array}{c} -(n+2)\left[8(a-c)(n+2)+3\Delta c\left(n+1\right)^2\right]+\\ \sqrt{(n+2)^2\left[8(a-c)(n+2)+3\Delta c\left(n+1\right)^2\right]+16(n-1)(5n+7)\left(a-c\right)^2\left(2n+3\right)}\end{array}\right)}{2(5n+7)(n-1)(n+2)\Delta c}$$

8.4 Illustrations

This part of the appendix provides some insight of a quantitative nature into some of the results of Section 4.

(a) *The values of* $(p, \Delta c)$ *and expected social welfare.*

Consider the parameter values $a = 10$, $b = 1$, $n = 10$, and $c = 3$.

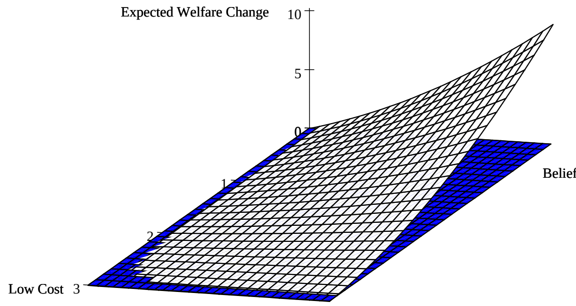


Figure 5.

It is clear that unless the belief p and the efficiency gain are extremely low, a merger will be advantageous to society, i.e., expected total welfare will increase.

Example 6 *Consider the parameter values $a = 10$, $b = 1$, $n = 10$ and $c = 7$.*

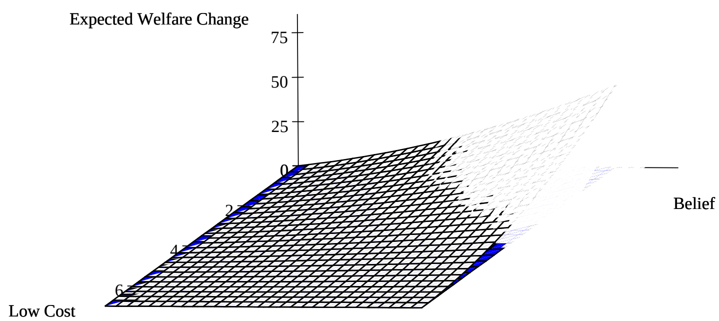


Figure 6.

The observations made in the previous example concerning the impact of a merger to society are reinforced. In fact, as can be shown to hold for consumer surplus as well, the opportunities for a socially beneficial merger are increased when the starting cost is higher. Naturally, higher starting costs provide more opportunities for efficiency gains that are advantageous to consumers as well.

8.5 Proofs

Proof of Proposition 1. The merged firm is profitable if $\pi_m^h > 2\pi$ which reduces to

$$p > \frac{2\,(a-c)\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{(c-c_l)\,(n+2)\,(n-1)} = p^{h*}$$

It is easy to see that $p^{h*} > 0$ if $n \geq 2$. Moreover, $p^{h*} < 1$ if the cost gains are sufficient:

$$\frac{2\left(a-c\right)\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{\left(n+2\right)\left(n-1\right)} < c - c_l.$$

For the lower bound on cost gains to be feasible ($c - c_l < c$) it must be that the pre-merger cost is at least:

$$\frac{2a\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{2\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)+\left(n+2\right)\left(n-1\right)} < c.$$

Note that the latter lower bound on c is always below a. ■

Proof of Proposition 2. (a)In the worst case scenario, the merged firm expands if and only if $q_m^h \geq 2q$, or $p \geq p_m^h = \frac{2n(a-c)}{\Delta c(n-1)(2+n)}$. In the best case scenario, it expands if and only if $q_m^l \geq 2q$, or $p \geq p_m^l = \frac{2n(a-c)-(c-c_l)(n+1)(n+2)}{\Delta c(2+n)(n-1)}$.(b)Setting $P^h > P$ reduces to $2(a-c) + p\Delta c(n-1)(n+2) > 0$, which is always true. Setting $P^l > P$ reduces to $p > p^l = \frac{(n+1)(n+2)\Delta c - 2(a-c)}{\Delta c(n+1)(n-1)}$. (c)Outsider firms benefit from the merger and expand their output if $E\pi_o \geq \pi$ and $q_o \geq q$. Both inequalities reduce to $p \leq p_o = \frac{a-c}{\Delta c(n+2)}$. Similarly, the merged firm contracts its output if $Eq_m \leq 2q$, which reduces to $p \leq p_o$. Setting $EP > P$ reduces to $p < \frac{a-c}{\Delta c(n+2)} = p_o$. ■

Proof of Proposition 3. (a)The change in Producer Surplus in the worst case scenario is given below:

$$\Delta PS^h = \frac{\left(2n\left(a-c\right)-p(n-1)\Delta c\right)\left(2(a-c)+p(n-1)\Delta c\right)}{4b\left(n+1\right)^2} - (n+1)\frac{\left(a-c\right)^2}{b\left(2+n\right)^2}.$$

Recall that we require that $p < \frac{a-c}{\Delta c}$ for an interior solution. Now observe that ΔPS^h is increasing in p since

$$\frac{d\Delta PS^h}{dp} = \frac{2(n-1)^2\Delta c^2\left(\frac{a-c}{\Delta c}-p\right)}{4b\left(n+1\right)^2} > 0.$$

Hence, it suffices to show that $\Delta PS^h|_{p=0} > 0$. Indeed

$$\Delta PS^h|_{p=0} = \frac{\left(a-c\right)^2}{b}\left(\frac{n}{\left(n+1\right)^2} - \frac{n+1}{\left(2+n\right)^2}\right) > 0.$$

The change in consumer surplus in the worst case scenario is given below:

$$\Delta CS^h = \frac{1}{2}\frac{\left(2n(a-c)-p\left(n-1\right)\Delta c\right)^2}{4b\left(n+1\right)^2} - \frac{1}{2}\frac{\left(a-c\right)^2\left(n+1\right)^2}{\left(n+2\right)^2 b} < 0$$

Observe that ΔCS^h is a decreasing function of p as $2n(a-c) - p\left(n-1\right)\Delta c > 0$. It is easy to see that at $p = 0$ where ΔCS^h takes its highest value it is already negative.

The change in Total Welfare in the worst case scenario is given by the formula below:

$$\begin{aligned}\Delta TW^{h} &= \frac{\left(2n\left(a-c\right)-p(n-1)\Delta c\right)\left(2(n+2)(a-c)+p(n-1)\Delta c\right)}{8b\left(n+1\right)^{2}} \\ &\quad -(n+1)\frac{\left(a-c\right)^{2}}{b\left(2+n\right)^{2}}-\frac{1}{2}\frac{\left(a-c\right)^{2}\left(n+1\right)^{2}}{\left(n+2\right)^{2}b}.\end{aligned}$$

It is easy to see that ΔTW^h is a decreasing function of p, thus, it suffices to show that $\Delta TW^h < 0$ at $p = 0$ where ΔTW^h takes its highest value. Indeed

$$\Delta TW^{h}|_{p=0}=\frac{(a-c)^{2}}{b}\left(\frac{-2n-3}{2\left(n+1\right)^{2}\left(2+n\right)^{2}}\right)<0.$$

(b)To show $\Delta EPS > 0$ we take its derivative w.r.t. p and see that it is positive:

$$\frac{d\Delta EPS}{dp}=\frac{2p\left(3n+5\right)\left(n-1\right)\left(n+2\right)^{2}\Delta c^{2}+\left(n+2\right)^{2}\Delta c\left(8\left(a-c\right)+\left(1+n\right)^{2}\Delta c\right)}{4b\left(n+1\right)^{2}\left(2+n\right)^{2}}>0$$

So $\Delta EPS > 0$ if $\Delta EPS|_{p=0} > 0$. Then observe that $\Delta EPS|_{p=0} = \frac{(a-c)^2\left(n^2+n-1\right)}{b(n+1)^2(n+2)^2} > 0$.

As Figures 5 and 6 illustrate the expected total welfare can increase or decrease depending on the parameters. The following example proves that the change in ECS can be positive or negative, depending on the parameters of the model.

Example 7 *Consider the parameter values $a = 10$, $b = 1$, $n = 10$ and $c = 3$. The dark shaded surface is the 0 plane whereas the light shaded area denotes the change in consumer surplus. Clearly, the higher the belief and the lower the cost, the more likely it is for consumer surplus to increase.*

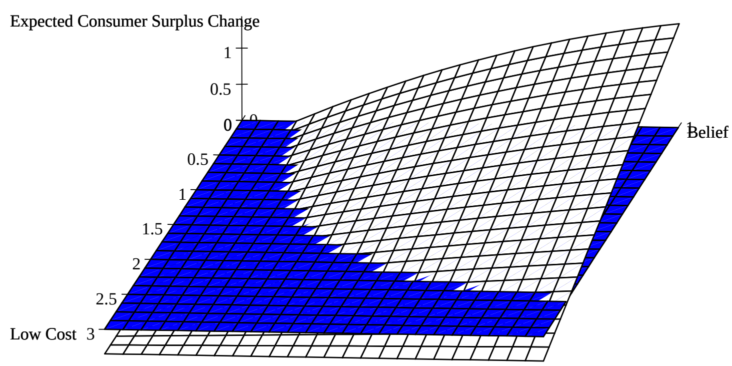


Figure 7.

(c)It is easy to see that when the parameters take the following values: $a = 10, b = 1$, $n = 10$, $c = 7$, $p = 0.5$, $c_l = 6.6200$ the change in PS^l becomes $\Delta PS^l = -2.650\,9 \times 10^{-2}$. Similarly, when the parameters are $a = 10, b = 1$, $n = 10$, $c = 7$, $p = 0.5$, $c_l = 5.6200$ the change in PS^l becomes $\Delta PS^l = 0.607\,38$.

Consumer surplus in the best-case scenario follows a similar pattern. For example with parameters $a = 10, b = 1$, $n = 10$, $c = 7$, $p = 0.5$, and $c_l = 6.95$, the change in consumer surplus is $\Delta CS^l = -0.02184\,3$. But when $c_l = 5.62$, the change becomes $\Delta CS^l = 1.\,132\,9$. ■

Proof of Proposition 4. Observe that for $\Delta E\pi_m > 0$ it suffices to show that

$$p^2 (3n+1)(n-1)(n+2)^2 \Delta c^2 + p(n+2)^2 \Delta c \left(8n(a-c) + \Delta c (n+1)^2\right) - 4(c-a)^2 \left(n^2-2\right) > 0.$$

Let the coefficient of p^2 be denoted by

$$\alpha_m = (3n+1)(n-1)(n+2)^2 \Delta c^2,$$

the coefficient of p be denoted by

$$\beta_m = (n+2)^2 \Delta c \left(8n(a-c) + \Delta c (n+1)^2\right)$$

and the constant be denoted by

$$\gamma_m = -4(a-c)^2 \left(n^2-2\right).$$

Note that $\alpha_m, \beta_m > 0$, hence $\Delta E\pi_m$ is increasing in p.

Now observe that for $\Delta ETW > 0$ it suffices to show that

$$p^2 (5n+7)(n-1)(n+2)^2 \Delta c^2 + p(n+2)^2 \Delta c \left(8(a-c)(2+n) + 3\Delta c (n+1)^2\right) - 4(c-a)^2 (2n+3) > 0.$$

Similarly, let the coefficient of p^2 be denoted by

$$\alpha_w = (5n+7)(n-1)(n+2)^2 \Delta c^2,$$

the coefficient of p be denoted by

$$\beta_w = (n+2)^2 \Delta c \left(8(a-c)(2+n) + 3\Delta c (n+1)^2\right)$$

and the constant term by $\gamma_w = -4(a-c)^2 (2n+3)$. Again, note that $\alpha_w, \beta_w > 0$, hence ΔETW is increasing in p.

Next note that $\alpha_w > \alpha_m$, $\beta_w > \beta_m$ and $\gamma_w > \gamma_m$. When $\Delta E\pi_m(p^*) = 0$ we have $p^{*2}\alpha_m + p^{*2}\beta_m = -\gamma_m$. since $\alpha_w > \alpha_m$ and $\beta_w > \beta_m$ we have $p^{*2}\alpha_w + p^{*2}\beta_w >$

$p^{*2}\alpha_m + p^{*2}\beta_m$ hence $p^{*2}\alpha_w + p^{*2}\beta_w > -\gamma_m$ but $\gamma_m < \gamma_w \Leftrightarrow -\gamma_m > -\gamma_w$. Thus, $p^{*2}\alpha_w + p^{*2}\beta_w > -\gamma_w$ which implies that $p^{*2}\alpha_w + p^{*2}\beta_w + \gamma_w > 0 \Leftrightarrow \Delta ETW(p^*) > 0$.

To conclude, when $p = p^*$ we have $\Delta E\pi_m(p^*) = 0$ and $\Delta ETW(p^*) > 0$ and for all $p > p^*$ we have $\Delta E\pi_m(p^*) > 0$ and $\Delta ETW(p^*) > 0$ since both $\Delta E\pi_m(p)$ and $\Delta ETW(p)$ are increasing functions of p as argued earlier.

From $\Delta TW^h < 0$ and $\Delta ETW > 0$ it follows that $\Delta TW^l > 0$. ■