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Joint Search with No Information: An Inefficient Immediate Agreement Theorem *

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Abstract

The no-information case of a finite horizon joint search problem between two players with conflicting preferences is studied. It is shown that if the players have convex preferences and are patient enough, then they abandon their search by accepting the first period alternative.

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1 Introduction

Important economic decision making units, from households to monetary policy committees, typically consist of multiple individuals. Recent research (see Wilson (2001), Compte and Jehiel (2010), Albrecht et al. (2010) and Moldovanu and Shi (2013)) has carefully analyzed the search behaviour of such units and how it may be different from that of a single agent. A rich set of results has emerged linking the predictability of the chosen alternative and the expected search length to patience, preference heterogeneity and information asymmetry of group members.

In this short paper I identify a set of circumstances in which a pair of individuals choose to give up on the benefit of searching by accepting the first alternative that comes along.[1] The result obtains when: the search horizon is finite (N objects), players only observe how the current object compares to those that came before, players care only about how their chosen object ranks relative to all N, the two players have opposite preferences, preferences are more discerning among more preferred alternatives and the players are sufficiently patient.

The finite horizon search structure is appropriate, for instance, when groups must conclude their search by a deadline and on any given day they can inspect at most one alternative. In most economic search models agents make draws from a known distribution. Not only do they observe the exact payoff their current sample brings, they know precisely how it compares to the distribution of objects they face. Our setting corresponds to the opposite benchmark of "no-information" and is a multi-agent version of the so called *secretary problem*.[2] It is appropriate when the agents have little information about the market they face. Caring about the relative rank of the chosen alternative is typical if the search is for a positional good or for instance, if the search is for an alternative among rival objects such as competing startups in the same industry, where the eventual payoff depends on how the chosen alternative eventually compares to all others. The assumption of opposite preferences makes our environment similar to a bargaining setup, a single-dimensional one in particular. The final assumption on preferences imposes convexity. It is a natural assumption if the stakes are very high at the top and not so much at the bottom. For instance, the

[1] The result easily extends to larger groups that nevertheless require the consent of two such individuals.

[2] DeGroot (2005) has an excellent treatment of the basic *secretary problem* and a lot of the notation in the current paper is borrowed from there.

difference in returns between the best and the second best ranked alternative may be significant while among poorly ranked alternatives the returns may be so small as to make any difference between them even smaller.

All the papers mentioned at the start typically make some sort of concavity assumption, either on preferences directly or on the known distribution of rewards. Since they study an infinite horizon setting, such assumptions are necessary to guarantee the existence of stationary equilibria, which they mostly focus on. Due to the existence of a deadline, the current model is itself non-stationary. The resulting non-stationary structure of equilibria typically makes it very difficult to analyze. Kamada and Muto (2015) is the only paper in the economics literature (to the best of my knowledge) that analyzes such a setting and in considerable detail.[3] They assume, however, that the group samples from a known distribution. Interestingly they find that search duration may not shrink to zero in the limit as search friction vanishes. This contrasts with the earlier papers where the time to acceptance shrinks to zero as the group becomes more patient. To be clear, these asymptotic zero expected search duration results are of an entirely different nature to the immediate agreement result of this paper, in which search literally ends at the first round.

2 Model and Result

Id and Superego can jointly choose one out of N objects. They observe the objects sequentially. Upon observing an object, Id and Superego decide sequentially whether to accept it or reject and continue the search.[4] If both choose to accept then they obtain said object and the search ends. The N objects are ranked from 1 to N. The order in which they are observed is random with all orderings equally likely. The only information the players obtain by observing a given object is its relative rank among those that have already been inspected. An r'th period observation is denoted by the pair (a, r) for $r \in \{1, 2, \ldots, N\}$ and $a \in \{1, \ldots, r\}$, where a is the rank of the current

[3]There is an operations research literature, surveyed in Abdelaziz and Krichen (2007) that studies multi-agent versions of the *secretary problem*. The paper in that literature closest to this setting is Sakaguchi and Mazalov (2004). They assume, however, that when the two players disagree on an object, the outcome corresponds to each player's choice with some exogenously given probability. Alpern and Gal (2009) and Alpern et al. (2010) too study a two player version of the secretary problem. In their model search ends when at least one player chooses to accept unless the other player vetoes, and each player can veto only a finite number of times.

[4]The sequential structure is meant to avoid equilibria like those in the simultaneous voting version in which both players always reject.

object relative to the r objects inspected so far.

The two players care about the relative rank of their chosen alternative with respect to all N objects. In particular, the object ranked n generates a strictly positive utility, $U(n) > 0$, for Id.[5] Furthermore, Id strictly prefers lower ranked objects, $U(n) > U(n+1)$ for all $n < N$. The object ranked n brings Superego a utility of $V(n) > 0$. Superego's preference is the opposite of Id's (ordinally), in that it satisfies

$$U(p) > U(q) \Leftrightarrow V(p) < V(q)$$

for any $p, q \in \{1, 2, \ldots, N\}$.[6]

Preferences are said to be *convex* if the difference in utility from adjacently ranked objects is higher the more the player prefers these objects. So, for Id this would require that $U(n) - U(n+1)$ is decreasing in n while for Superego it requires that $V(n+1) - V(n)$ is increasing in n. Both Id and Superego discount the future at a rate $\delta \in (0, 1)$. Finally, if all objects are rejected then both get a utility of 0.

Theorem. *If the players have convex preferences and are sufficiently patient then in any subgame perfect equilibrium of the game, search ends with the first object being accepted.*

Proof. Suppose the choice of when to stop searching rested solely on Id. Id's expected payoff from stopping after observation (a, r) is denoted by $U_0(a, r)$. Of course,

$$U_0(a, N) = U(a)$$

Note that the probability with which an object ranked a at the r'th observation becomes ranked b at the n'th observation with $n > r$ and $b \geq a$ is given by

$$\frac{\binom{b-1}{a-1}\binom{n-b}{r-a}}{\binom{n}{r}}$$

[5]An object's rank without any further qualifiers refers to its relative rank among all N objects.

[6]Note that it is not required that Superego's utility from the various ranked objects is *cardinally* the opposite of Id's.

The expected payoff from stopping at (a, r) is therefore

$$U_0(a,r) = \sum_{b=a}^{N-r+a} \frac{\binom{b-1}{a-1}\binom{N-b}{r-a}}{\binom{N}{r}} U(b)$$

Observation 1: In equilibrium Id's continuation payoff in the N'th period before observing the object is

$$\frac{1}{N}\sum_{b=1}^{N} U(b).$$

This is because Id and Superego would both accept the object in the N'th period irrespective of its rank since it, at least, brings a positive payoff. The alternative would be to reject and get 0. The expected payoff formulation then follows from the fact that all ranks for the N'th period object are equally likely.

Observation 2:

$$\frac{1}{r}\sum_{b=1}^{r} U_0(b,r) = \frac{1}{N}\sum_{b=1}^{N} U(b) \qquad \forall r.$$

The equation above says that if in period r before observing the rank of the object, Id and Superego committed to accepting it, then Id's expected payoff that period would be the same as his expected payoff in period N from the same commitment of accepting whatever object they receive, before observing its rank. The reason is simple. Since all orders in which the objects are observed are equally likely, the probability that the next object observed is of a given rank is the same irrespective of the rank. [7]

We now focus on values of r strictly above 1.

Let

$$a^*(r) = \begin{cases} \frac{r}{2}+1 & \textit{if } r \textit{ even} \\ \frac{r+1}{2} & \textit{if } r \textit{ odd} \end{cases}$$

Observation 3: Accepting $a^*(r)$ on the r'th observation gives Id no higher an expected payoff than a gamble that gives expected payoffs $U_0(a^*(r), r+1)$ and $U_0(a^*(r)+1, r+1)$ with equal probability.

Indeed, if r is odd, the two give the same expected payoff. Notice that an object

[7] Note that this is very different from the probability that an object *already observed* to have a particular relative rank out of r will eventually have some rank (out of N).

ranked $a^*(r)$ out of r when r is odd, has a half half chance of ending up ranked $a^*(r)$ or $a^*(r+1)$ out of $r+1$ objects. Therefore accepting $a^*(r)$ out of r would give a payoff of $U_0(a^*(r), r+1)$ and $U_0(a^*(r)+1, r+1)$ with equal probability. If r is even then an object ranked $a^*(r)$ out of r has a $\frac{1}{2} - \frac{1}{2r+2}$ probability of staying at rank $a^*(r)$ out of $r+1$ and $\frac{1}{2} + \frac{1}{2r+2}$ probability of ranking $a^*(r)+1$ out of $r+1$. The expected payoff from this is less than the equal probability $U_0(a^*(r), r+1)$ and $U_0(a^*(r)+1, r+1)$ gamble since $U_0(a^*(r), r+1) > U_0(a^*(r)+1, r+1)$.

Observation 4: Id strictly prefers the gamble $\left(\frac{1}{r+1}, U_o(b, r+1)\right)_{b=1,\dots,r+1}$ to the gamble $\left(\frac{1}{2}, U_o(a^*(r), r+1)\right)\left(\frac{1}{2}, U_o(a^*(r)+1, r+1)\right)$.[8]

If r is odd, then observe that the gamble $\left(\frac{1}{2}, U_o(a^*(r), r+1)\right)\left(\frac{1}{2}, U_o(a^*(r)+1, r+1)\right)$ second order stochastically dominates $\left(\frac{1}{r+1}, U_o(b, r+1)\right)_{b=1,\dots,r+1}$. Since Id's preference is convex, the observation follows.

If r is even, then observe that the gamble $\left(\frac{1}{2}, U_o(a^*(r)-1, r+1)\right)\left(\frac{1}{2}, U_o(a^*(r), r+1)\right)$ second order stochastically dominates $\left(\frac{1}{r+1}, U_o(b, r+1)\right)_{b=1,\dots,r+1}$. Again since Id's preference is convex he prefers the latter to the former gamble. Further, the gamble $\left(\frac{1}{2}, U_o(a^*(r)-1, r+1)\right)\left(\frac{1}{2}, U_o(a^*(r), r+1)\right)$ first order stochastically dominates $\left(\frac{1}{2}, U_o(a^*(r), r+1)\right)\left(\frac{1}{2}, U_o(a^*(r)+1, r+1)\right)$. As a result Id strictly prefers $\left(\frac{1}{r+1}, U_o(b, r+1)\right)_{b=1,\dots,r+1}$ to the gamble $\left(\frac{1}{2}, U_o(a^*(r), r+1)\right)\left(\frac{1}{2}, U_o(a^*(r)+1, r+1)\right)$.

Observation 5: If patient enough, Id would strictly prefer to wait till period N than accept $a^*(r)$ in period r.

Observations 3 and 4 show that Id strictly prefers $\left(\frac{1}{r+1}, U_o(b, r+1)\right)_{b=1,\dots,r+1}$ to accepting $a^*(r)$ in period r, which brings $U_0(a^*(r), r)$. We know from observations 1 and 2 that the expected payoff from the gamble $\left(\frac{1}{r+1}, U_o(b, r+1)\right)_{b=1,\dots,r+1}$ is the same as Id's expected payoff in period N from not having accepted any of the earlier objects. Id's expected payoff in period r from waiting till period N would be $\delta^{N-r}\frac{1}{N}\sum_{b=1}^{N} U(b)$. If Id is patient enough this would be arbitrarily close to $\frac{1}{N}\sum_{b=1}^{N} U(b)$. Since by the above arguments

$$\frac{1}{N}\sum_{b=1}^{N} U(b) > U_0(a^*(r), r)$$

the observation follows.

Observation 6: If patient enough, Id would strictly prefer to wait till period N than accept a in period r, with $a \geq a^*(r)$, for all $r \geq 2$.

[8]Gambles are often denoted as $(p_i, x_i)_{i=1}^{k}$, meaning that the outcome is x_i with probability p_i for $i \in \{1, \dots, k\}$.

Note first that $U_0(a,r) < U_0(a^*(r),r)$ for all $a > a^*(r)$. The observation then simply follows from observation 5. Importantly, this means that no such object would be accepted in period r, since Id would reject it.

Observation 7: If patient enough, Superego would strictly prefer to wait till period N than accept a in period r, with $a \leq a^*(r)$ for all $r \geq 2$.

This observation follows from the same set of arguments used for Id, accounting for Superego's preference being the opposite (ordinally) of Id's.

Observations 6 and 7 show that any object observed in a period $1 < r < N$ would necessarily be rejected by some player. Finally, observe that accepting the first period object brings Id an expected payoff of $\frac{1}{N}\sum_{b=1}^{N} U(b)$ which is the same as the expected in period N. This is true for Superego too, with an expected payoff of $\frac{1}{N}\sum_{b=1}^{N} V(b)$. As long as $\delta < 1$, accepting the first period object must then bring a strictly higher payoff than waiting till period N. In equilibrium, therefore, Id and Superego must end the search in the first period itself. This concludes the proof.

□

3 Discussion

Notice that the rationale for immediate agreement is not driven by extreme impatience or agents' indifference towards the alternatives, as is usually the case. The no-information environment, too, typically favours longer searches compared to the known distribution case. This is because the learning incentive to search in the former is missing in the latter, since the known distribution is usually assumed to be independently distributed across time. Nevertheless, in the current setup, both players correctly anticipate that any learning will only lead to a divergence of views in terms of acceptability of the object. Since learning is only valuable to the extent that it will help in choosing a more appropriate alternative, the perceived deadlock leads the two to optimally abandon search in the first period.[9]

Most economic search environments (say a first job search) lie between the two benchmarks of known distribution and no-information. A tractable way to incorporate elements of both settings in a model would be to think of each object as having two distinct attributes, both of which an agent cares about (for instance) in an addi-

[9]This is closer, in spirit, to the result of Yildiz(2003), where correctly anticipated future disagreements precipitates an immediate agreement.

tive way.[10] For one of those attributes (say salary) the agent could, upon sampling, directly infer the resulting payoff and compare it to how objects in the market are distributed along that attribute. For the other (say work environment, cohesiveness of the workgroup, current projects) the agent could have no information and could therefore only compare a given draw to what she has observed before. Notice that the question of which information setting will have greater influence does not simply depend on how much the agents care about salary versus work environment. Indeed even if salary mattered a whole lot more than work environment quality, minimal variance in the former could make the latter be the deciding factor, making the equilibrium search behaviour more in line with the no-information environment.

Inefficiency: The search behaviour captured by the theorem above is inefficient in the following sense. Suppose Id and Superego were represented by a single individual, Ego, instead, whose payoff was a weighted average of the two $W = \alpha U + (1 - \alpha)V$ for some $\alpha \in [0, 1]$. If Ego were to decide when to stop searching instead, we would have the standard *secretary problem*. We could then compare W under Ego's optimal strategy and the utilitarian welfare with weights $(\alpha, 1 - \alpha)$ generated by the equilibrium behaviour of Id and Superego. It is obvious that Ego can do no worse, since she can choose to stop at period 1 if that is what maximizes W given optimal continuation play. Typically, however, Ego would do better and this would involve searching beyond the first alternative. This is particularly true when the discount factor is close to 1.

Search under known distribution: Fixing the rest of the model and assuming instead that Id and Superego have a very clear understanding of the market they are searching would typically generate the opposite prediction. With a very high probability (often 1), search would continue till the last period. For instance, this would be the prediction of a model in which Id and Superego make N *i.i.d.* draws of X from a uniform distribution with support $[0, 1]$, with Id's utility, say U, strictly increasing and Superego's, say V, strictly decreasing in the realized value x, both strictly convex. Since in equilibrium they would accept the N'th draw for sure, to find any earlier draw acceptable it would have to give Id and Superego a payoff at least as high as $\delta^{N-1}E(U(X))$ and $\delta^{N-1}E(V(X))$, respectively. For high values of δ and due to the

[10] This is the approach taken by Molodvanu and Shi(2013) where an individual's utility from an object is the weighted average of private returns to that individual and the returns to everyone else. Their goal is to model interdependent preferences.

convexity of U and V, such draws of X, if they were to exist, would occur with very small probability.

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